Dynamic Pricing and Inventory Management: Theory and Applications

Renyu (Philip) Zhang

Olin Business School
Washington University in St. Louis

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Dissertation Defense

Committee: Nan Yang (Co-chair), Fuqiang Zhang (Co-chair), Amr Farahat, Jake Feldman, John Nachbar
Motivation

Key operations decisions of a firm to deliver (physical) products:
- Price;
- Inventory.
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- Dynamic, uncertain, and (possibly) competitive market environment.
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  - Price;
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- Dynamic, uncertain, and (possibly) competitive market environment.

- Emerging trends in
  - Technology and marketplace (e.g., social networks, customer behaviors);
  - Society (e.g., sustainability concerns).
Running Questions of Interest

- How would these emerging market trends influence the operations decisions and profitability of a firm?
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- How would these emerging market trends influence the operations decisions and profitability of a firm?

- What pricing and inventory strategies could a firm use to leverage these trends?
Outline

- Network externalities (monopoly setting) (Yang and Z, 2015a)
- Network externalities (oligopoly setting) (Yang and Z, 2015b)
- Trade-in remanufacturing (Zhang and Z, 2015)
- Scarcity effect of inventory (Yang and Z, 2014)
- Comparative statics method (Yang and Z, 2016)
- Conclusion
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- Network externalities (monopoly setting) (Yang and Z, 2015a)

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- Conclusion
Network Externalities

- Dynamic Pricing and Inventory Management under Network Externalities. (Yang and Z, 2015a)
Xbox and Xbox Live

Xbox game console and Xbox live online gaming network.
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- Customers are more willing to purchase an Xbox if there are more players on Xbox Live.
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- Xbox game console and Xbox live online gaming network.
- Customers are more willing to purchase an Xbox if there are more players on Xbox Live.
- Microsoft's strategies:
  - 50$ discount for Xbox buyers who guarantee to join Xbox live for 1 year (Tech. Times 2015).
  - 33% discount for Xbox live gold membership in Feb. 2015 (Geek Wire 2015).
Network Externalities

- Network externalities (NE):
  - The value of a product to customers increases with the number of other customers who purchase the same product (Economides 1996).
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  - The value of a product to customers increases with the number of other customers who purchase the same product (Economides 1996).

- Social networks make network externalities everywhere.
Questions of Interest

- What is the impact of network externalities upon a firm’s price and inventory policy?

- What strategies can a firm use to leverage network externalities?
Literature Review

- Network economics:

  - Compatibility and technology adoption (Katz and Shapiro 1985, 1986); financial market (Diamond 1982); pricing (Dhebar and Oren 1986); network structure (Ballester et al. 2006, Chen and Zhou 2013, 2015).

- Joint pricing and inventory management:

  - Single-period (Petruzzi and Dada 1999); multi-period (Federgruen and Heching 1999); fixed ordering cost (Chen and Simchi-Levi 2004a, 2004b, 2006); random yield risk (Li and Zheng 2006); lost-sales (Huh and Janakrman 2008).

- Inventory management with intertemporal demand correlations:

  - Myopic policy (Johnson and Thompson 1975); non-stationary demand (Graves 1999); joint forecasting and replenishment (Aviv 2002).
Model Setup

- $T$—period stochastic inventory system, labeled backwards $\{T, T - 1, \ldots, 1\}$, full backlog.

- Objective: maximize the total expected discounted profit.

- Dynamic price and inventory adjustments.

- Purchasing cost $c$, holding cost $h$, backlogging cost $b$, and discount factor $\alpha$. 
Model: Network Externalities

- There exists a service/communication network associated with the product.
  - $N_t$: network size in period $t$.
  - $r_n(\cdot)$: per-period profit from the network, concavely increasing in the network size.
Model: Network Externalities

- There exists a service/communication network associated with the product.
  - $N_t$: network size in period $t$.
  - $r_n(\cdot)$: per-period profit from the network, concavely increasing in the network size.
- $V + \gamma(N_t)$: willingness-to-pay of a customer (Katz and Shapiro 1985).
  - $V$: customer type, uniformly distributed on $(-\infty, \tilde{V}_t]$.
  - $\gamma(\cdot)$: concavely increasing in $N_t$.
  - Homogeneous centrality of each customer.
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- $p_t \in [\underline{p}, \bar{p}]$: sales price of the product.
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Demand in period $t$

$$D_t(p_t, N_t) = \bar{V}_t + \gamma(N_t) - p_t + \xi_t.$$ 

- $\{\xi_t\}$: i.i.d. continuously distributed demand perturbations with $\mathbb{E}[\xi_t] = 0$.
- $D_t(p_t, N_t) \geq 0$ for all $N_t$ and $p_t$. 
Model: Network Size Dynamics

\[ N_{t-1} = \eta N_t + \theta D_t(p_t, N_t) + \epsilon_t. \]
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- \( \eta \in [0, 1] \): carry-through rate of network size.
Model: Network Size Dynamics

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- \(\eta \in [0, 1]\): carry-through rate of network size.
- Two customer segments: social and individual customers.
  - \(\theta \in (0, 1]\): fraction of social customers.
  - \(1 - \theta\): fraction of individual customers.
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- \( \{\epsilon_t\} \): i.i.d. continuous market size dynamics perturbations, \( \mathbb{E}[\epsilon_t] = 0 \).
Sequence of Events

- The firm reviews inventory $I_t$ and network size $N_t$. 
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- Inventory carried over to the next period; network size updated.
Dynamic Program Formulation

\[ v_t(l_t, N_t) = \text{the maximal expected discounted profit in periods } t, t - 1, \cdots, 1, \]
\[ \text{with starting inventory level } l_t \text{ and network size } N_t \text{ in period } t. \]

Terminal condition: \[ v_0(l_0, N_0) = cl_0. \]
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Bellman Equation

\[ v_t(I_t, N_t) = cI_t + \max_{x_t \geq I_t, p_t \in [\underline{p}, \overline{p}]} J_t(x_t, p_t, N_t), \text{ where} \]

\[ J_t(x_t, p_t, N_t) = cI_t + E \left[ \min \{ p_t, \bar{p} \} \right] D_t(p_t, N_t) + \left[ r_n(N_t - 1) \right] \]

\[ (\text{Revenue}) - (\text{Procurement cost}) + \left( h(x_t D_t(p_t, N_t)) + b(x_t D_t(p_t, N_t)) \right) \]

\[ (\text{Holding and backlogging cost}) + v_{t+1}(x_t D_t(p_t, N_t), N_{t+1}) \]
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\[ J_t(x_t, p_t, N_t) = -cl_t + \mathbb{E}\{p_tD_t(p_t, N_t) + r_n(N_{t-1})\} \]
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v_t(l_t, N_t) = cI_t + \max_{x_t \geq l_t, p_t \in [\underline{p}, \bar{p}]} J_t(x_t, p_t, N_t), \text{ where } \\
J_t(x_t, p_t, N_t) = -cI_t + \mathbb{E}\left\{ p_t D_t(p_t, N_t) + r_n(N_{t-1}) \right\} - c(x_t - l_t)
\]

\[
\text{Revenue} \quad \text{Procurement cost}
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Revenue

\[ -h(x_t - D_t(p_t, N_t))^+ - b(x_t - D_t(p_t, N_t))^+ \]

Procurement cost

Holding and backlogging cost
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- Revenue
- Procurement cost

$$-h(x_t - D_t(p_t, N_t))^+ - b(x_t - D_t(p_t, N_t))^-$$

- Holding and backlogging cost

$$+\alpha v_{t-1}(x_t - D_t(p_t, N_t), N_{t-1}) | N_t \}.$$  

- Future profit
Optimal Policy

\( (x_t^*(l_t, N_t), p_t^*(l_t, N_t)) \): the optimal decisions in period \( t \).

The network-size-dependent base-stock/list-price policy is optimal:

- If \( l_t \leq x_t(N_t) \), order up to \( x_t(N_t) \) and charge a list price \( p_t(N_t) \).

- If \( l_t > x_t(N_t) \), order nothing and charge an inventory-dependent price.

\( x_t(N_t) > 0 \).
State Space Dimension Reduction

We can reduce the state space dimension from two to one.
State Space Dimension Reduction

We can reduce the state space dimension from two to one.

**Theorem**

- If $I_T \leq x_T(N_T)$, $(x^*_t(I_t, N_t), p^*_t(I_t, N_t)) = (x_t(N_t), p_t(N_t))$ with probability 1.
- The optimal base-stock level and list price $(x_t(N_t), p_t(N_t))$ is the solution to the following dynamic program with a 1-dimensional state space:

\[
\pi_t(N_t) = \max_{x_t \geq 0, p_t \in [p, \bar{p}]} J_t(x_t, p_t, N_t), \text{ where }
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\[
J_t(x_t, p_t, N_t) = R_t(p_t, N_t) + \beta x_t + \Lambda(x_t - \bar{V}_t + p_t - \gamma(N_t))
\]

\[
+ G_t(\theta(\bar{V}_t - p_t + \gamma(N_t)) + \eta N_t),
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with $G_t(y) := \mathbb{E}\{r_n(y + \theta \xi_t + \epsilon_t) + \alpha \pi_{t-1}(y + \theta \xi_t + \epsilon_t)\}$, and $\pi_0(\cdot) \equiv 0$. 

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**Intuitions:**

- For all $N_t$ and $N_{t-1}$, $\mathbb{P}[x_t(N_t) - D_t(p_t(N_t), N_t) \leq x_{t-1}(N_{t-1})] = 1$.
- As long as $I_T \leq x_T(N_T)$, $I_t \leq x_t(N_t)$ for all $t$ with probability 1.
Managerial Implications of Network Externalities

Theorem

Compared with the benchmark case without NE,

**(a)** $x_t(N_t)$ is higher with the presence NE.
Managerial Implications of Network Externalities

Theorem

Compared with the benchmark case without NE,

(a) \( x_t(N_t) \) is higher with the presence NE.

- Network externalities lead to higher demand/base-stock level.

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Managerial Implications of Network Externalities

Theorem

Compared with the benchmark case without NE,

(a) \( x_t(N_t) \) is higher with the presence NE.

(b) There exists a threshold \( N_t \), such that

(i) \( p_t(N_t) \) is lower with the presence of NE if \( N_t < N_t \).

(ii) \( p_t(N_t) \) is higher with the presence of NE if \( N_t > N_t \).

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- Network externalities lead to higher demand/base-stock level.

- Impact of network externalities on the pricing policy:
  - A lower price to induce future demands with a small network size.
  - A higher price to exploit the better market condition with a large network size.
Intertemporal Tradeoff

- Dynamically balancing the tradeoff between generating profits and inducing demands.
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- Dynamically balancing the tradeoff between generating profits and inducing demands.

**Theorem**

If the market stationary ($\tilde{V}_T = \tilde{V}_{T-1} = \cdots = \tilde{V}_2 = \tilde{V}_1$),

(a) $x_T(\cdot) \geq x_{T-1}(\cdot) \geq \cdots \geq x_t(\cdot) \geq x_{t-1}(\cdot) \geq \cdots \geq x_2(\cdot) \geq x_1(\cdot)$.

(b) $p_T(\cdot) \leq p_{T-1}(\cdot) \leq \cdots \leq p_t(\cdot) \leq p_{t-1}(\cdot) \leq \cdots \leq p_2(\cdot) \leq p_1(\cdot)$. 

More weight on inducing future demands at the early stage of the sales season.

Numerical Results

Ignoring network externalities leads to a significant profit loss (30%+), especially with high network externalities intensity; high proportion of social customers; high network size carry-through rate.
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Ignoring network externalities leads to a significant profit loss (30%+), especially with

- high network externalities intensity;
- high proportion of social customers;
- high network size carry-through rate.
Effective Strategies to Exploit Network Externalities

Theorem
The firm's profit improves under Price discrimination; Network expanding promotion.

Employing an additional leverage (e.g., price or promotion) to (partially) separate generating current profits and inducing future demands.

Effective Heuristics
Dynamically maximize the total profit of a 5-period moving time window.
In period $t$, adopt the pricing and inventory policy that maximizes the profit in periods $t-1; t-2; t-3; t-4; t$.

Achieves an optimality loss of less than 2%.

It suffices to balance generating profits and inducing demands in the near future.
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Takeaways

- State space dimension reduction.

- Tradeoff: generating current profits and inducing future demands.

- Effective strategies to exploit network externalities.
Dynamic Competition under Market Size Dynamics: Balancing the Exploitation-Induction Tradeoff. (Yang and Z, 2015b)
Dynamic Competition under Network Externalities

- How about dynamic competition under network externalities?
  - Xbox (Microsoft) v.s. PlayStation (Sony).
How about dynamic competition under network externalities?

- Xbox (Microsoft) v.s. PlayStation (Sony).

Generating current profits v.s. winning future market shares.

- Exploitation-induction tradeoff.
Main Findings

- Markov perfect equilibrium in a dynamic competition under network externalities.
- Inventory dynamics do not affect the equilibrium outcome.
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  - Inventory dynamics do not affect the equilibrium outcome.

- Exploitation-induction tradeoff:
  - Captured by a linear coefficient of market size.
  - When the coefficient is larger, price decreases and base-stock level increases.
Trade-in Remanufacturing

Apple’s Trade-in Program

In 2014, we collected 40,396 metric tons of e-waste through our take-back programs. That’s more than 75 percent of the total weight of the products we sold seven years earlier.

We recovered enough steel in 2014 that the equivalent could be used to build over 100 miles of railroad track.
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Get an Apple Store Gift Card for your old device.
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Strategic Customer Behavior

- How would strategic customers react to Apple’s trade-in recycling program?
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- Strategic customer behavior is prevalent in the electronics market due to frequent product introductions (Plambeck and Wang 2009).
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- Value of trade-in recycling/remanufacturing under different customer behaviors:
  - To the firm;
  - To the environment.
  - To the society.
Questions of Interest

- What is the value of trade-in remanufacturing to the firm and the environment under different customer behaviors?

- How should the government design the public policy that can induce the socially optimal outcome?
Literature

- Sustainable operations and remanufacturing:
  - Inventory control (Van der Laan et al. 1999); reverse channel structure (Savaskan et al. 2004); trade-in program (Ray et al. 2005); environmental impact (Agrawal et al. 2012).

- Strategic customer behavior:
  - Pricing (Bensako and Winston 1990); availability (Su and Zhang 2008); capacity rationing (Liu and Van Ryzin 2008); quick response (Cachon and Swinney 2009); product launches (Lobel et al. 2015).

- Trade-in rebates with forward-looking customers:
  - Price commitment (Van Ackere and Reyniers 1995); two product generations (Fudenberg and Tirole 1998); lemon problem (Rao et al. 2009).
**Model: First Period**

- $X$ customers arrive at the market; $X \sim F(\cdot)$. 
Model: First Period

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Trade-in Remanufacturing

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Model: Second Period

- New customers and repeat customers on the market.
Model: Second Period

- New customers and repeat customers on the market.
- The firm offers the second-generation product.
  - $(1 + \alpha)V = \text{second-generation product valuation (} \alpha \geq 0)$. 

The firm charges $p_n^2$ for new customers and $p_r^2$ for repeat customers. The trade-in rebate to collect the cores for remanufacturing is $0$. Customers make their second-period purchasing decisions. The firm remanufactures the used first-generation products.
Model: Second Period

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Solution Approach

- Customer behaviors: Strategic customers v.s. Myopic customers.

Solution Approach

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- Rational expectation (RE) equilibrium.
Solution Approach

- Customer behaviors: Strategic customers v.s. Myopic customers.


- Rational expectation (RE) equilibrium.

**Theorem**

An RE equilibrium exists.
Impact of Customer Behavior on Equilibrium Outcome

- Strategic customer behavior may benefit the firm under trade-in remanufacturing.
Impact of Customer Behavior on Equilibrium Outcome

- Strategic customer behavior may benefit the firm under trade-in remanufacturing.

**Theorem**

(a) With trade-in remanufacturing, equilibrium profit and price $\uparrow$ under strategic customer behavior if remanufacturing generates a high value to the firm.

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- Strategic customers anticipate the potential price discount.
- Remanufacturing ensures such discount is high enough.
- The firm may charge a higher price with strategic customers.
Value of Trade-in Remanufacturing to the Firm

- Revenue generating effect.
- Price discrimination effect.
- Early-purchase inducing effect (with *strategic customers* only).
Results

Value of Trade-in Remanufacturing to the Firm

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(a) Equilibrium profit ↑ if the firm adopts trade-in remanufacturing.
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**Numerical Results**

<table>
<thead>
<tr>
<th></th>
<th>Min</th>
<th>5th pctl</th>
<th>Median</th>
<th>95th pctl</th>
<th>Max</th>
<th>Mean</th>
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<tbody>
<tr>
<td>Strategic Customers</td>
<td>5.8</td>
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<td>2.5</td>
<td>8.1</td>
<td>11.7</td>
<td>3.1</td>
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</table>

**Table:** Profit Improvements of Trade-in Remanufacturing (%)
Environmental Value of Trade-in Remanufacturing

- Contrasting effects under different customer behaviors.
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- (MC) Price discrimination effect $\rightarrow$ higher profit from new customers $\rightarrow$ $Q_1$ $\downarrow$. 

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<td>49.2</td>
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<td>-10.2</td>
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<td>-5.5</td>
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Table: Environmental Impact Increases of Trade-in Remanufacturing (%)

Renyu (Philip) Zhang

Dissertation Defense

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- Contrasting effects under different customer behaviors.

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*Table:* Environmental Impact Increases of Trade-in Remanufacturing (%)
Interactions between Strategic Customer Behavior and Trade-in Remanufacturing

- Good news about strategic customer behavior.

- Tension between firm profitability and environmental sustainability.
Interactions between Strategic Customer Behavior and Trade-in Remanufacturing

- Good news about strategic customer behavior.

- Tension between firm profitability and environmental sustainability.
  - How should the government resolve this tension?
Government Intervention

- The objective of the government:
  - Maximize social welfare (firm profit + customer surplus - environmental impact).
**Government Intervention**

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- Most natural subsidization policy:
  - Subsidizes for remanufactured products only.
  - Leads to undesired outcomes.
Socially Optimal Government Policy

- Government subsidy/tax scheme $s_g = (s_1, s_2, s_r)$.
  - $s_1 =$ per unit subsidy/tax for first-generation products.
  - $s_2 =$ per unit subsidy/tax for second-generation products.
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**Theorem**

With strategic (myopic) customers, a linear subsidy/tax scheme \( s_g^* = (s_1^*, s_2^*, s_r^*) \) \((\tilde{s}_g^* = (\tilde{s}_1^*, \tilde{s}_2^*, \tilde{s}_r^*))\) can induce the social optimum.
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With strategic (myopic) customers, a linear subsidy/tax scheme $s_g^* = (s_1^*, s_2^*, s_r^*)$ ($\tilde{s}_g^* = (\tilde{s}_1^*, \tilde{s}_2^*, \tilde{s}_r^*)$) can induce the social optimum.

Implications:

- The government should subsidize/tax both product generations and remanufacturing.
- A linear subsidy/tax scheme can induce the social optimum.
Takeaways

- **Value of trade-in remanufacturing to the firm and the environment:**
  - Benefit of strategic customer behavior to the firm.
  - Tension between firm profitability and environmental sustainability.

- **Socially optimal government policy:**
  - Subsidies/taxes for both new and remanufactured products.
  - A simple linear subsidy/tax scheme to induce the social optimum.
Scarcity Effect of Inventory

- Dynamic Pricing and Inventory Management under Inventory-Dependent Demand. (Yang and Z, 2014, Operations Research)
Inventory level signals the popularity and quality of the product.
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Main Findings

- Optimal policy: a customer-accessible-inventory dependent order-up-to/display-up-to/dispose-down/list-price policy.
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- The scarcity effect of inventory strengthens overstocking risk.
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- The scarcity effect of inventory strengthens overstocking risk.

- Price and operational flexibilities help mitigate demand loss driven by high inventory levels.
Comparative Statics Analysis Method

- Comparative Statics Analysis Method of Inventory Management Models with Dynamic Pricing. (Yang and Z, 2016)
Comparative Statics Analysis

- Comparative statics analysis is an integral methodology in OM.
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- Develop a new comparative statics method for a general class of dynamic pricing and inventory management models.
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- Comparisons between decisions $\rightarrow$ comparisons between partial derivatives.

- Features of the new method:
  - Non-restrictive conditions;
  - Scalable;
  - Some optimal decisions can be non-monotone.
Conclusion

- How to optimize the price and inventory decisions?
  - Network externalities: Monopoly setting.
  - Network externalities: Oligopoly setting.
  - Trade-in remanufacturing.
  - Scarcity effect of inventory.
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Important and interesting implications of the emerging trends in technology, marketplace and society.
Thank you!

Questions?