Dynamic Pricing and Inventory Management under Inventory-Dependent-Demand

Nan Yang and Philip (Renyu) Zhang

Olin School of Business
Washington University in St. Louis

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Main Results and Managerial Implications

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Motivation
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- Apple iPad, limits customers to two iPads per order in online sales (CBS News, 2010).
Impact of Inventory on Demand

Operations objective: avoid excessive inventory.

Drawbacks of high inventory levels:
- Poor turnover (sales = inventory).
- Significant purchasing holding and managing costs.
- Investment opportunity lost.
- Potential demand depressed.

Excessive inventory is an implicit negative indicator of the product's quality, popularity and freshness.

Future demand is negatively correlated with excessive inventory levels.
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Motivating Research Questions

I What is the structure of the optimal price and inventory policy under inventory-dependent-demand?

I How will inventory-dependent-demand influence the optimal policy?

I How will the flexibility in pricing and inventory disposal impact the optimal policy and the performance of the system?
Motivating Research Questions

- What is the structure of the optimal price and inventory policy under inventory-dependent-demand?

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Benefit of Dynamic Pricing

I Dynamic pricing is most effective for perishable products and products facing high demand and/or supply variability.

I Inventory-Dependent-Demand amplifies the demand volatility.

I Effective price adjustments can help stabilize the inventory levels, thus reducing the demand variability.
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Benefit of Inventory Disposal

Inventory disposal is an efficient way to liquidate surplus assets and reduce inventory holding and managing costs. Potential demand loss caused by high excessive inventory levels is also saved. Inventory disposal controls both the supply (service level) and demand (inventory-dependent-demand) sides of the story.
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Literature Review

Inventory dependent demand:
- Gerchak and Wang (1994),
- Urban (2005),
- Sapra et al. (2010).

Dynamic pricing under stochastic demand:
- Federgruen and Heching (1999),
- Chen and Simchi-Levi (2004 a,b, 2006).

Joint price & inventory control under inventory-dependent demand:
- Dana and Petruzzi (2001),
- Balakrishnan et al. (2008).

Our paper: Dynamic pricing and inventory control & stochastic demand negatively correlated with inventory.
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Models

We develop the following dynamic programming models to investigate the inventory-dependent-demand.

Model (1): perishable products.

Model (2): nonperishable products, without inventory disposal.

Model (3): nonperishable products, with inventory disposal.
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Notations and Assumptions

I periods in total, labeled backwards, full backorder.

It = the price set in period t.

pt is the optimal price.

c = the unit procurement cost.

It = the inventory level at the beginning of period t before replenishment.

xt = the inventory level in period t after replenishment before demand realization.

xt is the optimal service level.

D = \delta (pt, It, ϵ_t) = the random demand in period t, strictly decreasing in pt, decreasing in It.

ϵ_t is a random vector.

α = discount factor.
Notations and Assumptions

- $T$ periods in total, labeled backwards, full backorder.

- $p_t =$ the price set in period $t$. $p_t^*$ is the optimal price. $c =$ the unit procurement cost.

- $I_t =$ the inventory level at the beginning of period $t$ before replenishment.

- $x_t =$ the inventory level in period $t$ after replenishment before demand realization. $x_t^*$ is the optimal service level. Zero leadtime.

- $D_t = \delta(p_t, I_t, \epsilon_t) =$ the random demand in period $t$, strictly decreasing in $p_t$, decreasing in $I_t$. $\epsilon_t$ is a random vector.

- $\alpha =$ discount factor.
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- $\alpha =$ discount factor.
Model (1): Sequence of Events

In model (1), \( I_t \) is the inventory level at the end of period \( t+1 \), which is not usable but has impact on \( D_t \).

The sequence of events in period \( t \):

- Observes \( I_t \)
- Decides \((x_t, p_t)\)
- \( D_t \) realized

Period \( t \) starts
Period \( t-1 \) starts

Excessive demand costs \( \beta > c \) per unit.

\( I_t = x_t D_t \) perishes but has impact on \( D_t \).

\( V_P(t(I_t)) = \) the optimal expected profit to go with inventory level \( I_t \).
Model (1): Sequence of Events

- In model (1), \( l_t \) is the inventory level at the end of period \( t + 1 \), which is not usable but has impact on \( D_t \). \( x_t \geq 0 \).
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- The sequence of events in period $t$:
  
  - Period $t$ starts
  - Observes $I_t$
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  ![Sequence of Events Diagram]

  Period $t$ starts
  
  Observes $I_t$
  
  Decides $(x_t, p_t)$
  
  $D_t$ realized
  
  Period $t - 1$ starts

- Excessive demand costs $\beta(> c)$ per unit.

- $I_{t-1} = x_t - D_t$ perishes but has impact on $D_{t-1}$. 
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- \( V_t(I_t) = \text{the optimal expected profit to go with inventory level } l_t \).
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- Excessive demand costs $\beta(> c)$ per unit.

- $I_{t-1} = x_t - D_t$ perishes but has impact on $D_{t-1}$.

- $V_t^P (I_t)$ = the optimal expected profit to go with inventory level $I_t$. 
Model (1): Optimality Equation
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\[ V_t^P(l_t) = \max_{x_t, p_t} \mathbb{E}\{p_t D_t - c x_t - \beta (x_t - D_t)^- + \alpha V_{t-1}^P(x_t - D_t)\} . \]

\[ V_0^P(l_0) = 0. \]
Model (1): Optimality Equation

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- \( p_t D_t \) = revenue in period \( t \).
- \( c x_t \) = procurement cost in period \( t \).
- \( \beta(x_t - D_t)^- \) = penalty incurred by excessive demand in period \( t \).
- \( \alpha V_{t-1}^P(x_t - D_t) \) = the profit in later periods.
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Model (2): Sequence of Events

In model (2), \( I_t \) is the inventory level at the beginning of period \( t \), which is usable and has impact on \( D_t \). The sequence of events in period \( t \):

1. Observes \( I_t \)
2. Decides \((x_t, p_t)\)
3. \( D_t \) realized

Period \( t \) starts, Period \( t - 1 \) starts

- Procurement cost: \( c(x_t, I_t) \).

- \( I_t = x_t D_t \), which incurs cost: \( h I_t + t_1 + b I_t t_1 \).

\( VND_t(I_t) = \) the optimal expected profit to go with inventory level \( I_t \).
In model (2), $I_t$ is the inventory level at the beginning of period $t$, which is usable and has impact on $D_t$. $x_t \geq I_t$
Model (2): Sequence of Events

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- The sequence of events in period $t$:

  - Observes $I_t$
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- Procurement cost: $c(x_t - I_t)$. 
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  - \( D_t \) realized
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- Procurement cost: \( c(x_t - I_t) \).

- \( l_{t-1} = x_t - D_t \), which incurs cost: \( hI_{t-1}^+ + bI_{t-1}^- \).
**Model (2): Sequence of Events**

- In model (2), $l_t$ is the inventory level at the beginning of period $t$, which is usable and has impact on $D_t$. $x_t \geq l_t$

- The sequence of events in period $t$:
  - Observes $I_t$
  - Decides $(x_t, p_t)$
  - $D_t$ realized
  - Period $t$ starts
  - Period $t - 1$ starts

- Procurement cost: $c(x_t - l_t)$.

- $l_{t-1} = x_t - D_t$, which incurs cost: $hl_{t-1}^+ + bl_{t-1}^-$.

- $V_t^{ND}(l_t) =$ the optimal expected profit to go with inventory level $l_t$. 

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- $I_{t-1} = x_t - D_t$, which incurs cost: $hl_{t-1}^+ + bl_{t-1}^-$.

- $V_{t}^{ND}(I_t) =$ the optimal expected profit to go with inventory level $I_t$. 
Model (2): Optimality Equation

\[ V_t^{ND}(l_t) = \max_{x_t \geq l_t, p_t} \mathbb{E}\{p_tD_t - c(x_t - l_t) - h(x_t - D_t)^+ - b(x_t - D_t)^- \} + \alpha V_t^{ND}(x_t - D_t) \} \]

\[ V_0^{ND}(l_0) = -cl_0^- + sl_0^+ . \]
Model (2): Optimality Equation

\[ V_{t}^{ND}(l_{t}) = \max_{x_{t} \geq l_{t}, p_{t}} \mathbb{E}\{p_{t}D_{t} - c(x_{t} - l_{t}) - h(x_{t} - D_{t})^{+} - b(x_{t} - D_{t})^{-} \}
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\[ + \alpha V_{t-1}^{ND}(x_{t} - D_{t}) \}.
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- \( p_{t}D_{t} \) = revenue in period \( t \).
- \( c(x_{t} - l_{t}) \) = procurement cost in period \( t \).
- \( h(x_{t} - D_{t})^{+} + b(x_{t} - D_{t})^{-} \) = operational cost in period \( t \).
- \( \alpha V_{t-1}^{ND}(x_{t} - D_{t}) \) = the profit in later periods.
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- \( \alpha V_{t-1}^{ND}(x_t - D_t) \) = the profit in later periods.
Model (3): Sequence of Events

In model (3), $I_t$ is the inventory level at the beginning of period $t$, which is usable and has impact on $D_t$. $x_t$ is the decision to purchase in period $t$.

The sequence of events in period $t$:

- Observes $I_t$
- Decides $(x_t, p_t)$
- $D_t$ realized

Period $t$ starts

$I_t$ = $x_t$ and $D_t$, which incurs cost: $hI_t + t_1 + bI_t$.

$V_t(I_t)$ = the optimal expected profit to go with inventory level $I_t$. 

Salvage value less procurement cost: $s(x_t, I_t) - c(x_t, I_t) + I_t$.
Model (3): Sequence of Events

- In model (3), $l_t$ is the inventory level at the beginning of period $t$, which is usable and has impact on $D_t$. $x_t \geq \min\{0, l_t\}$. 
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- The sequence of events in period $t$:

  1. Observes $I_t$
  2. Decides $(x_t, p_t)$
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  4. Period $t - 1$ starts
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- The sequence of events in period \( t \):
  - Period \( t \) starts
  - Observes \( I_t \)
  - Decides \((x_t, p_t)\)
  - \( D_t \) realized
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- Salvage value less procurement cost: \( s(x_t - I_t)^- - c(x_t - I_t)^+ \).
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- $V_t(I_t) =$the optimal expected profit to go with inventory level $I_t$. 

Model (3): Optimality Equation

\[ V_t(I_t) = \max_{x_t} \min_{\ell_t} f_0; I_t g; p_t E p_t D_t c(x_t I_t) + s(x_t I_t) + h(x_t D_t) + b(x_t D_t) + \alpha V ND_t. \]

\[ V_0(I_0) = cI_0 + sI_0. \]

\[ I_p t D_t = \text{revenue in period } t. \]

\[ I_c(x_t I_t) / s(x_t I_t) = \text{procurement cost/salvage value}. \]

\[ I_h(x_t D_t) + b(x_t D_t) = \text{operational cost in period } t. \]

\[ I_\alpha V t (x_t D_t) = \text{the profit in later periods}. \]
Model (3): Optimality Equation

\[ V_t(l_t) = \max_{x_t \geq \min\{0, l_t\}, p_t} \mathbb{E}\{ p_t D_t - c(x_t - l_t)^+ + s(x_t - l_t)^- - h(x_t - D_t)^+ - b(x_t - D_t)^- + \alpha V_{t-1}^{ND}(x_t - D_t) \} \]

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Main Results (Model 3)

I Inventory-dependent order-up-to/dispose-down-to list-price policy.

DisposeKeepOrder

It

Lt

It

Itp, xt, Lt and It are lower with inventory-dependent-demand.

Itp, xt, Lt and It are higher with inventory-disposal opportunity.

The benefits of dynamic pricing and inventory disposal (in profit) are as high as 1/3 (numerical).

When the wait-list effect is strong enough, Lt < It = 0.
Main Results (Model 3)

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\[ I_t^L \rightarrow I_t^H \rightarrow I_t \]

Order Keep Dispose

The benefits of dynamic pricing and inventory disposal (in profit) are as high as \( \frac{1}{3} \) (numerical).

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- $p_t^*, x_t^*, I_t^L$ and $I_t^H$ are lower with inventory-dependent-demand.
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\[ I_t^L \quad I_t^H \quad I_t \]

Order \quad Keep \quad Dispose

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Managerial Insights and Implications

- Inventory-dependent-demand strengthens overstocking risk by depressing potential demand.
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Suggestions:
- Never ignore inventory-dependent-demand.
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- Decrease sales price.
- Dispose unnecessary inventory (campus cafe).
- Take advantage of the wait-list effect (BMW and Apple).
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Thank you!

Questions?