First-Year Summer Project:

Pricing and Inventory Management under Fluctuating Costs

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1 Industry Practice

In the recent past, commodity prices have experienced not only dramatic rises but also sharp falls. Such unpredictable price fluctuations prevail in today’s unstable global economic climate, presenting great challenges for the firm’s pricing and inventory (procurement and processing) decisions. In the manufacturing industry where the profit of a firm highly depends on the prices of raw materials or items containing large amounts of such materials, companies are forced to take the commodity risk management, which deals with the price fluctuations of raw materials, seriously. According to a survey by the Efficio Consulting, more than 55 percent of procurement executives said commodity price instability was their single biggest challenge (Jenkinson, 2011). It is also estimated that an approximate loss of half a billion Euros in 2010 due to higher raw material costs (Sebastian and Maessen, 2010). In the global market, it is completely normal for firms to have overseas sourcing opportunities. The uncertainty in exchange rate, labor cost and tax rate will all contribute to a firm’s fluctuating input costs if he lies in the global supply chain. For example, long-term growth in worldwide demand, coupled with a slowdown in agricultural production growth, reduced global stockpiles of basic commodities like corn, soybeans, wheat, and rice. Lower stocks, in turn, made it more likely that new sources of demand, or disruptions to supply, precipitated sharply changing prices. See Schwartz (2009) for reference in the fluctuating food commodity prices and Crenshaw et al. (2010) for the random and rapidly increasing oil prices.

1.1 Factors that Drive Price Fluctuation

The up-and-downs of raw material prices are primarily driven by supply and demand (Sebastian and Maessen, 2010). The Japan earthquake in 2010, for example, inevitably raised quite a large spectrum of industry products. A change in government, legislative bodies, other foreign policy makers, or military control can constrain oil production, especially in countries like Iran, Iraq and Nigeria. The supply disruption will naturally give rise to acute price increase. Other traditional factors like reserve level, free capacity and current inventory all contribute significantly to the fluctuation of several storable commodities like oil and natural gas.

Most economic analysts agree that the simple laws of supply and demand are working but speculation in the market has caused an increase in demand, and is the major debate among economists and financial analysts (Crenshaw et al. 2010). Speculative and investment-grade money entering the commodities market has attributed to the recent
price increase. People want to own contracts, and this has created additional demand. In an effort to spread out the risk and take advantage of opportunities, investment funds, retirement funds and insurances are investing in the commodity market, according to Sebastian and Maessen, 2010. It is also emphasized that speculations only strength the up and down trend. They do not set specific trend, but go wherever looks best.

The uncertain exchange and interest rates may also be a contributing factor of some commodity price fluctuations. For example, since oils are priced using dollars, the varying status of the US dollar is another cause of the fluctuating oil price. In 2007, low rates and decreasing dollar value together lead to high oil prices (Crenshaw etal. 2010).

Last but not least, the trading market condition is also another complicated factor that affect commodity prices (See Nationwide Insurance Company website http://www.nationwide.com/rss/gas-prices-fluctuation.jsp, for example.). There are usually three different markets that play a role in the price of commodities like gasoline.

- **Contract market**

Contracts among oil companies, dealers, refineries and independent dealers crucially predetermine the price of a lot of commodities, say oil, gas, copper and several food commodities.

- **Spot market**

The spot market can be used to fill the gaps in the contracts market by matching companies with surplus commodities to those that need more. Of the three markets, the spot market is the only one where actual commodities, say barrels of oil, are traded. Best deals are also found there because buyers and sellers are not bound by contracts.

- **Futures market**

Crude oil, for example, is traded on the New York Mercantile Exchange, although contracts are rarely fulfilled. More than 5 billion barrels of oil were reportedly traded on the futures market during a seven-year period, however, only 31,000 were actually delivered. The information on the status of the commodity market motivates the fluctuation. In general, consumer gasoline prices will follow the trends of the crude oil futures market.
1.2 Managing Price Fluctuation

To manage the risk of commodity price fluctuation, the firm makes several attempts, say inventory management, contract optionality and passing cost increase to customers to dampen the influence of such volatility. The fundamental idea is that the should assess risks and opportunities from price volatility accurately and adapt its inventory and pricing decisions accordingly.

Intuitively speaking, the firm should pass the cost uncertainty to customers. This suggestion is correct but somewhat trivial. However, this is really an uneasy task. Customers are hardly willing to bear the cost risk alone if they do not have any chance to pass on the costs to his buyers. For some products, the value added to it is very high and the firm will earn a high gross margin. For example, branded pharmaceuticals and specialized electronics (e.g. IPAD) all fall into this category. The firm not only has high freedom to price the product but also enjoys low dependence on the raw materials. Hence, the firm’s pricing mainly depends on the demand (customer) side instead of the supply side. Therefore, raw material price fluctuations will not have very big impact on the firm’s pricing and inventory decisions, neither will it greatly affect its profit. For other products, however, the margin is very low and the raw material costs contribute to a large proportion of the the product. Products of this category include, for example, modeled plastic and petrochemicals. In this case, the high competitive intensity is the reason why costs increase cannot be converted into price change. The firm will suffer greatly from raw material price fluctuations because the pricing freedom it has is very limited.

In the face of rapid price fluctuation, the first thing a firm needs to do is to move away from rigid pricing and procurement systems (Sebastian and Maessen, 2010). Annual contracts, for example, may not be a suitable choice in the highly volatile current raw material market. In this dynamic market and cost environment, shorter terms (quarterly or monthly contracts) will enable the firms to better responsively and accurately assess the price opportunities. The gap between contact prices and spot market prices also decrease with quarterly or monthly contracts. Shortening the negotiation cycle can also create more opportunities for the firm to better deal with raw material cost fluctuations. It is also a good idea to add escalation clauses so that the firm can call for an adjustment in the event of an increase or decrease in certain costs. See Kinlan and Roukema (2011), for example. The escalation clause also helps firms to cover the costs resulting from fluctuations.

Effectively managing the inventory may also help mitigate the risk of fluctuating input
costs. A better forecast of price and cost is thus essential. If the input cost is predicted as up-sloping, the firm should adjust the inventory up. This is called investment buying in the literature which is also widely used in pharmaceutical industry (Schwarz and Zhao 2011). Reversely speaking, if the cost is forecasted downward sloping, the firm should, instead, hold as little inventory as possible. The joint operational and financial hedging is also a widespread approach to address the highly fluctuating costs. The firms hardly put all their eggs in one basket so they usually outsource their inputs to multiple suppliers. The multiple sourcing strategy not only reduces the supply disruption risk but also enables the firm to dampen the procurement cost uncertainty. To better accomplish this goal, the firm’d better unbundle the costs of his suppliers and decide how much of the cost is driven by a particular commodity (Jenkinson, 2011 and Barosi and Busse, 2011). Financial hedging, on the other hand, also serve as a powerful weapon against input price fluctuations. Futures, swaps, options, and fixed-price agreements can all be employed for a cost to help avoid significant unexpected price increases (Barosi and Busse, 2011). Essentially, hedging raw materials or currencies on contract markets can be considered as paying a price for certainty and the firm should also bear the risk of price drop.

As an end note, we point out an innovative idea proposed by Monsanto’s former CEO R. J. Mahoney: when facing the rapid fluctuating prices, there are usually three choices: (1) risk it; (2) take contingent actions like hedging and (3) avoid it. Mr. Mahoney chose the third option. He was tired of having the wild swings in oil prices that decide their profitability because the company was upgrading oil to petrochemicals and their derivatives like plastics and fibers. As a consequence, he sold off some 8 billion dollars of chemical businesses and bought into biology, where their brains would determine results not the price of oil. Fortunately, it worked.

2 Literature Review

There is a large volume of literature that deals with the supply risk originating from the fluctuating procurement price. In theoretic literature, the underlying driven force of the price fluctuation is usually indebted to the dynamic imbalance between supply and demand and the behavior of the inventory holders that buy low and sell high (i.e. speculation). See Gustafson (1958), Deaton and Laroque (1992,1996), Chambers and Bailey (1996), Williams and Wright (1991) and Borenstein etal. (1997) for details. The rational expectation theory and how it influences the price movements dates back to Muth (1961) and is also applied in the study of production-inventory systems (2010) whereas
the stochastic speculative price theory is comprehensively treated in Kaldor (1939) and Samuelson (1970, 1973). The interaction of inventory and price dynamics is also carefully studied in the literature. See Pindyke (1994) or Geman (2005), for example. Based on the notion of convenience yield, Gibson and Schwartz (1990), Schwartz (1997), Schwartz and Smith (2000) and Casassus and Collin-Dufresne (2005) present some models that depict the stochastic evolution of commodity prices.

2.1 Production with Hedging and Speculation

In production economics literature, there are a stream of papers that address the issue of how a firm in the manufacturing industry should deal with price fluctuation, possibly with hedging and speculation. Sandmo (1971) and Batra and Ullah (1974) examines the behavior of a competitive firm being faced with input-hiring decisions under price uncertainty. Let $U(\pi)$ stands for the utility of the firm with profit $\pi$, where $U'(\pi) > 0$ and $U''(\pi) < 0$. Assume that the cost function $C(x)$ is strictly increasing. The firm’s total profit is

$$\pi = px - C(x),$$

where $p$ is the nonnegative random variable of the product price. The objective of the firm is to maximize the expected utility

$$\mathbb{E}\{U(px - C(x))\}.$$

Sandmo (1971) and Batra and Ullah (1974) show that the firm will underinvest and underproduce with price uncertainty giving rise to a declination in the firm’s output while the declination is increasing in the uncertainty of the price. This is intuitively correct because as the price uncertainty grows, the firm will bear more risk. For a risk-averse firm, it should produce less to reduce such additional risk.

Holthausen (1979) and Feder et al. (1980) demonstrate that using future contracts can help the firm to hedge or even speculate against price uncertainty. Suppose the firm sells the product either at stochastic price $p$ or as forward contract at the certain price $b$. The amount of output sold in the forward market is $h$. The objective function, defined as the expected utility of profit,

$$\mathbb{E}\{U(\pi)\} = \mathbb{E}\{U[p(x - h) + bh - c(x)]\}.$$  

The first order conditions with respect to $x$ and $h$ imply that the production quantity $x$ only depends on the price of the forward $b$, so the firm will not underproduce. Whether
the risk-averse firm should hedge its entire output \((h = x)\) or hedge less than then entire output \((0 < h < x)\) or speculate \((h > x\) and \(h < 0)\) depends on the comparison between the forward price \(b\) and the expected stochastic price \(\mathbb{E}\{p\}\). In short, if \(b = \mathbb{E}\{p\}\), the firm should hedge the entire production \((h = x)\); if \(b > \mathbb{E}\{p\}\), the firm should speculate by selling forward an amount greater than its output \((h > x)\), expecting to purchase the additional output in the future at a lower price in the open market; if \(b < \mathbb{E}\{p\}\), the firm should either hedge part of its production \((0 < h < x)\) or speculate by purchasing output in the forward market \((h < 0)\), expecting to sell the additional output at a greater price in the forward market. In a healthy economy, the firm should pay for certainty \((b < \mathbb{E}\{p\}\) but \(b\) is not too small compared with \(\mathbb{E}\{p\}\)) and the optimal policy is to hedge part of the production with forward contract \((0 < h < x)\). Otherwise, the firm may speculate by selling more than it produces \((h < 0\) or \(h > x)\).

Models that address the issue of optimal hedging with multiple futures and/or multiple periods are established in Anderson and Danthine (1981,1983), Meyer (1987) and Kamara (1993). Lapan et al. (1991), Lence (1995) and Moschini and Lapan (1992), among others, investigate the hedging behavior using options contracts.

### 2.2 Optimal Inventory Policy under Price Uncertainty

Quite a lot of papers make some attempt to understand the impact of stochastic spot prices on the optimal inventory control policy over the decades. For example, Zheng (1994) studies a single-item continuous-review inventory system with Poisson demand. In addition to the standard cost structure of a fixed setup cost and a quasiconvex expected inventory holding and shortage cost, special opportunities for placing orders at a discounted setup cost occur according to a Poisson process that is independent of the demand process. Let the single item inventory system faces a Poisson demand process of rate \(\lambda\). Unsatisfied demand is assumed to be fully backlogged. The continuously reviewed system has constant replenishment lead time. The ordinary setup cost is \(K\) per order whereas the discounted setup cost is \(k\) \((k < K)\). The discount opportunity epochs form a Poisson process, independent from the demand process, of rate \(\mu\). Therefore, the decision epochs form a Poisson process of rate \(\lambda + \mu\). The objective is to figure out the replenishment policy that minimizes the order (setup and unit) and operating (holding and backorder) costs. order policy The optimal policy is shown to be of \((s, c, S)\) type. i.e. an order is placed to raise the inventory position to \(S\) either when the inventory position drops to \(s\) or when the inventory position is below or at \(c\) \((c \geq s)\) when a discount opportunity occurs. The paper also develops an efficient algorithm for computing optimal
control parameters $\left(s^*, c^*, S^*\right)$. Similar models are also discussed in Friend (1960), Moinzadeh (1997) and Feng and Sun (2001) where discount opportunities occur randomly and Markovian.

Kalymon (1971) studies a multi-period stochastic inventory model in which the procurement price is driven by a Markovian process. In period $i$, $x_i$ refers to the inventory before ordering, $y_i$ is the inventory after ordering, $D_i$ is the demand that depends on the price of this period $p_i$. The price process $\{p_i\}$ is assumed to be a non-stationary Markov process. The period index $i$ is labeled backwardly.

$$C_i(y_i - x_i | p_i) \triangleq K_i \delta(y_i - x_i) + p_i(y_i - x_i)$$
is the ordering cost of period $i$, where $K_i \geq 0$ is the setup cost, $\delta(z) = 1$ for $z > 0$ and 0 for $z = 0$, and $y_i \geq x_i$.

$$L_i(y_i | p_i) \triangleq \mathbb{E}_{D_i}\{h(y_i - D_i) | p_i\}$$
denotes the expected holding and backorder costs where $h(z) \geq 0$ and is a convex function such that $h(0) = 0$, $\lim_{z \to \pm\infty} h(z) = \infty$. Let $f_i(x, p)$ be the minimum accumulative cost of the firm in period $i$ with inventory $x$ and facing price $p$. The Bellman equation is characterized as follows:

$$f_i(x, p) = \inf_{y \geq x} C_i(y - x | p) + L_i(y | p) + \alpha \mathbb{E}_{p_{i-1}, D_i}\{f_{i-1}(y - D_i, p_{i-1}) | p\},$$

where $\alpha$ is the discount factor and $f_0(x, p) \equiv 0$. Kalymon (1971) shows that the price-dependent $(s, S)$ is optimal, determines the bounds of the control bands $s(p)$ and $S(p)$ and discusses computational approaches exploiting structure.

Using single unit decomposition approach, Berling and Martínez-de-Albéniz (2011) makes some effort to characterize the optimal base-stock inventory policy when the purchase price either follows a geometric Brownian motion or an OU process. Assume that the firm needs a commodity as an input for production. The commodity bought in a spot market with quoted price $C(t) = e^{c(t)}$. The log-price $c(t)$ evolves in a one-factor model with drift $\mu(c)$ and variance $\sigma(c)^2$. i.e.

$$dc(t) = \mu(c) dt + \sigma(c) dW_t,$$

where $W_t$ is the standard (one dimensional) Brownian motion. For the geometric Brownian motion, $\mu(c) = \mu$ and $\sigma(c) = \sigma$ while for the Ornstein-Uhlenbeck process, $\mu(c) = -\kappa(c - \bar{c})$ and $\sigma(c) = \sigma$. The demand arrival process is a Poisson process of rate $\lambda$. The single-echelon continuous-time review system has constant lead time $L$. The holding cost
is $h$ per unit time whereas the backlog cost is $b$. The objective is, as usual, to minimize the long run average cost. Based on the continuous-time-review counterpart of the result in Kalymon (1970), the optimal inventory policy is an $(s(c), S(c))$ policy, where $s(c) = S(c) - 1$ because the setup cost is zero.

Denote $t_i$ as the ordering time of the $i$th unit of ordering and we assume that the placed order cannot be canceled. Let $T_i$ be the arrival time of the corresponding demand of the $i$th ordering and $r$ be the discount factor, so the objective is to minimize
\[
E \left\{ \int_{t=0}^{+\infty} \left[ h \sum_{i=1}^{+\infty} 1_{t_i+L \leq u \leq T_i} + b \sum_{i=1}^{+\infty} 1_{T_i \leq u \leq t_i+L} + C(t) \sum_{i=1}^{+\infty} \delta(t - t_i) e^{-ru} dt \right] \right\}
\]
\[
= \sum_{i=1}^{+\infty} E \left\{ \int_{t=0}^{+\infty} \left[ h 1_{t_i+L \leq u \leq T_i} + b 1_{T_i \leq u \leq t_i+L} + C(t) \delta(t - t_i) e^{-ru} dt \right] \right\}
\]
where $1$ is the indicator function and $\delta(\cdot)$ is the dirac function such that
\[
\int_{z=-\infty}^{+\infty} f(z) \delta(t-z) dz = f(t).
\]

Because the demand arrives as a Poisson process and the price process is also Markovian, whether $t_i = t$ (ordering now) only depends on the current inventory $k := i - D_{[0,t]}$ and log-price $c(t)$. The remaining arrival for customer $i$ only depends on $k$ and follows an Erlang distribution of rate $\lambda$ and index $k$. i.e. $T_i - t \sim E_k$. Therefore, the problem for the $i$th ordering can be reformulated as
\[
J(k, c(t)) = \min_{\tau \geq t} E \left\{ \int_{u=0}^{+\infty} \left[ h 1_{r- \tau + L \leq u \leq E_k} + b 1_{E_k \leq u \leq \tau - t + L} + e^{c(u)} \delta(u - \tau + t) e^{-ru} du \right] \right\},
\]
where $J(k, c(t))$ is the cost-to-go function at time $t$ with inventory $k$ and log-price $c(t)$. If the firm chooses to order one unit when the inventory is $k$, the cost should be $e^{c(t)} + \Pi(k)$, where $\Pi(k)$ is the expected net present value of future holding and backorder costs when the inventory is $k$. For a complete characterization of $\Pi(k)$, please refer to Berling and Martínez-de-Albéniz (2011). To get the Hamilton-Jacobi-Bellman equation that $J(k, c(t))$ satisfies, we observe that if the current customer has already arrived, i.e. $k \leq 0$, $J(k, c) = J(0, c)$ should satisfy the following HJB equation:
\[
b - r J(0, c) + \mu(c) \frac{\partial J}{\partial c}(0, c) + \frac{\sigma^2(c)}{2} \frac{\partial^2 J}{\partial c^2}(k, c) = 0,
\]
which follows directly from stochastic control theory. The boundary condition is also intuitive $J(0, c) \rightarrow b/r$ indicating that when the price is really high, it would be optimal to pay the backorder cost rather than ordering cost. If $k \geq 1$, the HJB equation can be
written as

\[-(r + \lambda)J(k, c) + \lambda J(k - 1, c) + \mu(c) \frac{\partial J}{\partial c}(k, c) + \frac{\sigma^2(c)}{2} \frac{\partial^2 J}{\partial c^2}(k, c) = 0.\]

The boundary condition can be summarized as

\[\lim_{c \to +\infty} J(k, c) = \frac{b}{r} \left( \frac{\lambda}{\lambda + r} \right)^k.\]

Under certain technical conditions, for every \(k\), there exists \(c^L_k\) and \(c^H_k\) such that, if \(c^L_k \leq c \leq c^H_k\), \(J(k, c) = e^c + \Pi(k)\) and \(J(k, c)\) solves the HJB equations otherwise. The computational approach is also proposed to the geometric Brownian motion model and the OU process model to determine \(c^L_k, c^H_k\) and equivalently \(S(c)\) in Berling and Martínez-de-Albéniz (2011).

There are also other papers that study the optimal inventory policy under price uncertainty and we only discuss a few here. Zipkin (1989) studies an inventory system with periodically varying demand and input price, which implies that the firm tends to order up to a higher inventory level when the price is low. Scheller-Wolf and Tayur (1998) considers a periodic-review inventory system with capacitated order quantity and fluctuating cost which depends on the exchange rate. Gavirneni (2004) shows that the order-up-to policy is optimal when the unit purchasing cost is fluctuating according to Markov chain and the order-up-to level is decreasing unit cost under certain conditions. Chen and Song (2001) derives structural policies for multi-echelon systems with Markov-modulated demand. In Golabi (1985), the demand is deterministic and must be completely satisfied. The paper determines how many periods of demand ahead the firm should satisfy when the ordering price is random.

In recent years, a new line of research that combines the classical inventory theory and commodity price theory mentioned above. See Haksoz and Seshadri (2007) for a review. The approximate approach is developed in Berling and Rosling (2005) and Berling (2006) whereas the periodic review models can be found in Goel and Gutierrez (2011a). Berling and Rosling (2005) discusses how the financial risks and option valuation will influence the \((R, Q)\) inventory policy under optimality while Berling (2006) studies the valuation of the inventory holding cost under the stochastic mean-reverting purchase price. In Goel and Gutierrez (2011a), a model for the procurement of traded commodities is developed and the optimal regulated band policy is proposed and algorithmically characterized. Yi and Scheller-Wolf (2003) investigates the optimal dual sourcing policy with a regular supplier and spot market.
2.3 Inventory Management with Financial Instruments

A great many of recent papers contribute to the application of financial instruments to hedge against fluctuating price risks in the operations management setting. Gaur and Seshadri (2005) contributes to the understanding of hedging inventory risk for a short life cycle or seasonal item when its demand is correlated with the price of a financial asset. Consider a single-period, single-item inventory model with stochastic demand, i.e., the newsvendor model. Assume that the demand forecast for the item is perfectly correlated with the price at time $T$ of an underlying asset that is actively traded in the financial markets.

Let $p$ denote the selling price of the item, $c$ the unit cost, $s$ the salvage value, $I$ the stocking quantity, and $D$ the demand. The firm purchases quantity $I$ at time 0 and demand occurs at a future time $T$. Demand in excess of $I$ is lost, and any excess inventory is liquidated at the salvage price of $s$. Without hedging, the cash flow at time 0 is therefore

$$\Pi^0_U(I) = -cI,$$

and at time $T$ is

$$\Pi^T_U = p \min\{D, I\} + s(I - D)^+.$$

Suppose the demand is perfectly correlated with the price of the financial asset, $S_T$: $D = a + bS_T$. Under, risk neutral measure, $S_0 = e^{-rT}\mathbb{E}_N\{S_T\}$. Under financial hedging with this asset (the details and proposed in Gaur and Seshadri, 2005), the profit of the firm is:

$$\Pi_H(I) = (p - s)bS_0 + e^{-rT}[(p - s)a + sI] - (p - s)be^{-rT}\mathbb{E}_N[S_T - (I - a)/b]^+ - cI.$$

We have that $E_N[\Pi_U(I)] = \Pi_H(I)$ and $Var(\Pi_H(I)) = 0$, where $\Pi_U(\cdot)$ stands for the total unhedged profit discounted to time 0. The paper also discusses the cases where the demand is not perfectly correlated with asset price and risk-averse firm. See Gaur and Seshadri (2005) for details. Caldentey and Haugh considers a similar problem in the continuous time setting. They consider the problem of dynamically hedging the profits of a firm when these profits are correlated with returns in the financial markets. In particular, The general problem of simultaneously optimizing over both the operating policy and the hedging strategy of the firm is analyzed in this paper. Akella etal. (2002) and Wu and Kleindorfer (2005) summarize the procurement and supplier risk management in B-to-B e-business markets. Dong and Liu (2007) derives the equilibrium forward contracts
on non-storable commodities between two firms that have mean-variance preference and negotiate their forward contracts through Nash bargaining.

The portfolio effects of option contracts and spot markets are analyzed in Fu et al. (2010) and Martínez-de-Albéniz and Simchi-Levi (2005) the former of which proposed an optimal procurement policy by the shortest monotone path algorithm and the latter establishes the optimality of a modified base-stock policy. Fu et al. (2010) develops a single period model that analyzes the value of portfolio procurement in a supply chain where a buyer can either procure parts for future demand from \( n \) sellers using fixed price contracts or from a spot market. The buyer of a single item faces random demand \( D \) and random spot market price \( P_s \) in a single period. For contract type \( i \), the reservation price is \( c_i \) whereas the execution price is \( h_i \). The buyer first purchases options from the \( n \) contracts to ensure \( Q_i \) units of supply before the realization of \( D \) and \( P_s \). After \( D \) and \( P_s \) are realized, the buyer decides how much, upto the quantity specified in the contract, to exercise (\( x_i \)) from each of the contracts signed with the suppliers and how much to procure from the spot market to satisfy the realized demand. Without loss of generality, assume that \( h_1 < h_2 < \cdots < h_n \) and \( c_1 > c_2 > \cdots > c_n \). The portfolio procurement problem can be formulated as

\[
\min_{Q_i \geq 0, i = 1, 2, \ldots, n} \left\{ \sum_{i=1}^{n} c_i Q_i + \mathbb{E}_{D, P_s} \left[ \min_{x_i, y} \left( \sum_{i=1}^{n} h_i x_i + P_s y \right) \right] \right\},
\]

subject to \( x_i \leq Q_i \) and \( x_i \geq 0 \) for \( i = 1, 2, \ldots, n \), \( y \geq 0 \) and \( \sum_i x_i + y \geq D \). A shortest-monotone path algorithm is provided in Fu et al. (2010) for the optimal portfolio procurement strategy of this problem. Again, this paper addresses the issue of the trade off between higher cost and uncertainty.

Wu and Chen (2010) further investigates the relation between commodity inventory and short-term price variation and characterizes the rational expectations equilibrium for an economy in which competitive production firms link a raw material market and a finished goods market, with uncertain and price-sensitive supply and demand. The individual firm controls production and two stages of inventory under uncertain input and output prices and operating costs. Let \( k_t = (k_{1t}, k_{2t}, \ldots, k_{nt})' \) be the \( n \) exogenous factors which follow the following stochastic differential equation

\[
dk_t = \mu_0(k_t)dt + \sigma_0(k_t)d\mathbf{w}_t,
\]

where \( \mu_0(\cdot) \) is an \( n \)-dimensional vector-valued function, \( \sigma_0(\cdot) \) is an \( n \times m \) matrix-valued function and \( \mathbf{w}_t \) is an \( m \)-dimensional standard Brownian motion. Let \( p_t = (p_{1t}, p_{2t})' \),
where \( p_{1t} \) is the raw material price and \( p_{2t} \) is the finished goods price. The price vector \( \mathbf{p}_t \) evolves according to the following stochastic differential equations

\[
d\mathbf{p}_t = \mu(\mathbf{p}_t, \mathbf{k}_t)dt + \sigma(\mathbf{p}_t, \mathbf{k}_t)d\mathbf{w}_t,
\]

where \( \mu(\cdot, \cdot) = (\mu_1(\cdot, \cdot), \mu_2(\cdot, \cdot)) \) and \( \sigma(\cdot, \cdot) \) is a \( 2 \times m \) matrix-valued function. At any time \( t \), the firm chooses \( \pi_t = (\lambda_t, q_t, s_t) \in \mathcal{U} = [\underline{\lambda}, \bar{\lambda}] \times [\underline{q}, \bar{q}] \times [\underline{s}, \bar{s}] \), where \( \lambda_t \) is the raw material procurement rate, \( q_t \) is the production rate, and \( s_t \) is the finished goods sales rate. \( \mathbf{x}_t = (x_{1t}, x_{2t})' \geq 0 \) is the firm’s inventory levels, where \( x_{1t} \) is the raw material inventory level and \( x_{2t} \) is the finished goods inventory level. Therefore,

\[
dx_{1t} = (\lambda_t - q_t)dt, \quad dx_{2t} = (q_t - s_t)dt.
\]

Because \( x_{1t} \geq 0 \) and \( x_{2t} \), \( q_t \leq \lambda_t \) if \( x_{1t} = 0 \) and \( s_t \leq q_t \) if \( x_{2t} = 0 \). \( h(\mathbf{x}, \mathbf{k}) \) is the physical holding cost and \( g(q, x, k) \) is all other operating cost. Let \( \rho(k_t) \) be the firm’s discount rate at time \( t \). So the cash flow at time \( t \) is discounted by \( e^{-R(t)} \), where

\[
R(t) = \int_0^t \rho(k_u)du.
\]

Let \( \mathcal{A} \) be the admissible control process. The firm’s objective is to choose \( \pi \in \mathcal{A} \) to maximize the expected long-run discounted profit

\[
V(\mathbf{x}, \mathbf{p}, \mathbf{k}) \sup_{\pi \in \mathcal{A}} \mathbb{E}\left\{ \int_{0}^{+\infty} e^{-R_t} (s_t p_{2t} - \lambda_t p_{1t} - g(q_t, x_t, k_t) - h(x_t, k_t)) dt \right\},
\]

where the expectation is taken under the policy \( \pi \).

The concavity of \( V(\cdot, \cdot, \cdot) \) is established in Wu and Chen (2010) and the optimal policy is characterized as several interrelated thresholds computed from the value function. The paper also considers the rational expectations equilibrium of the raw material market and finished goods market where the market prices are endogenously determined. The equilibrium can be computed without knowing the value function. Based on these models, it is found in Wu and Chen (2010) that inventory fluctuations lag behind price variations, and the length of the lags depend on how far the inventory is from the source of the supply or demand shocks. They also show that shocks are both dampened and delayed when propagating through the production stages, and that shocks have a prolonged effect on inventories and prices at both stages.

The value of a forward-looking dynamic programming policy is carried out in Devalka et al. (2011) which takes into consideration of the integrated optimization problem of procurement, processing and trade of commodities in a multi-period setting. Suppose the
time periods are indexed by \( n = 1, 2, \cdots, N \). The spot market price of the commodity in period \( n \) is \( S_n \). The deliver date of forward contract \( l \in \{1, 2, \cdots, L\} \) is \( N_l \) (\( N_1 < N_2 < \cdots, N_L \)). i.e. \( N_l - 1 \) is the last period that the firm can sell the output using contract \( l \). \( F^l_n \) denotes the period \( n \) forward price on contract \( l \) if \( n < N_l \). Let \( K \) be the per-period procurement capacity and \( C \) be the processing capacity. The cost of processing one unit of input commodity into output commodity is \( p \). The firm incurs per-period holding cost \( h_I \) (\( h_O \)) for input (output, respectively) commodity. Assume that \( h_O > h_I \). Suppose the risk free discount factor is \( \beta \). Under risk neutral measure, the output forward prices for each contract must satisfy

\[
\mathbb{E}_n\{F^{l+1}_m\} = F^l_n,
\]

for \( n < N_l, \forall l \), where \( \mathbb{E}_n\{\cdot\} \) is the conditional expectation operator on period \( n \). For each period \( n \leq N - 1 \), the firm should decide the quantity of the input commodity to be procured \( x_n \), the quantity to be processed \( m_n \) and the quantity of the output commodity to be committed for sale against forward contract \( l \) \( q^l_n \). Let \( Q_n(e_n) \) be the total output (input, respectively) inventory available at the beginning of period \( n \).

Let \( l^*(n) \) be the optimal forward contract that the firm commits against in period \( n \). Therefore,

\[
l^*(n) = \arg\max_{\beta^{N_l-n}F^l_n - h_O} \sum_{t=0}^{N_l-n-1} \beta^t : l \leq L, N_l > n \}.
\]

Let \( V_n(e_n, Q_n) \) be the optimal profit-to-go in period \( n \) and the stochastic dynamic program can be formulated in the following manner

\[
V_n(e_n, Q_n) = \max_{0 \leq x_n \leq K, 0 \leq m_n \leq \min\{C, e_n+x_n\}, 0 \leq q_{n}^{l^*(n)} \leq Q_n+m_n} \{[\beta^{N_{l^*(n)}-n}F^{l^*(n)}_n - h_O] \sum_{t=0}^{N_l-n-1} \beta^t q_n^{l^*(n)}] \}
\]

\[
- S_n x_n - pm_n - h_I (e_n + x_n - m_n) - h_O [Q_n + m_n - q_n^{l^*(n)}] + \beta \mathbb{E}_n[V_{n+1}(e_n + x_n - m_n, Q_n + m_n - q_n^{l^*(n)})] \}
\]

\[ (2.1) \]

where \( V_N(e_N, Q_N) = S_N e_N \) if \( Q_N \geq 0 \) and \(-\infty\) if otherwise. Devalka etal. (2011) shows that \( V_n(e_n, Q_n) \) is linear in \( Q_n \) and computes the marginal values of both input commodity inventory and output commodity inventory in closed forms. It is found in this paper that the optimal procurement and processing policies are governed by price-dependent inventory thresholds.

Unlike most papers that consider the single location supply chain, Goel and Gutierrez (2011b) analyzes a multiechelon distributional supply chain where the tradeoff between
spot price and forward price is resolved. The procurement and distribution model consists of a central warehouse and $N$ downstream retail locations. The model is cast within the framework of a periodic review inventory system where the objective is to minimize the present value of the expected costs over a planning horizon of $T$ periods. In period $t$, there are two modes of procurement, forward contracts and spot purchases. Commodity bought through a forward contract at period $t$ arrives at the beginning of period $t + 1$ while spot purchases arrive in the same period. The forward procurement transportation cost, denoted as $\alpha_t$, is smaller than the spot procurement transportation cost $\eta_t$. The unit cost of shipping the commodity from the terminal to retailer $i$ is $\gamma_i^t$.

The inventory position of the central warehouse is $x$ before procurement, and is $z$ after procurement ($z - x$) units from spot market; then the inventory position is raised to $y$ by forward procurement quantity $y - z$. The total inventory allocated to retailers must not exceed the upper echelon inventory level. $\sum_{i=1}^{N} v^i \leq z$. In period $t$, the spot price is $s_t$ and the forward price is $f_t$. The arbitrage free requirement implies that $f_t = \mathbb{E}_t^Q(s_\tau)$ for $\tau \geq t$, where the superscript $N$ stands for risk neutral measure.

At retailer $i$ in period $t$, the cost of each unit of unsatisfied demand is $p_i^t$ whereas the holding cost is $h_i^t$. So the expected holding and shortage cost for retailer $i$ in period $t$ is

$$L_i^t(v^i) = \mathbb{E}\{ h_i^t[v^i - \xi_i^t]^+ + p_i^t[v^i - \xi_i^t]^- \},$$

where $\xi_i^t$ is the stochastic demand faced by retailer $i$ in period $t$. The holding cost at the central warehouse is $h_t$. Let $v = (v^i)$, $w = (w^i)$, $\xi_t = (\xi_i^t)$ and $\zeta_t = \sum_{i=1}^{N} \xi_i^t$. The present value of the expected cost at time $t$ is represented by the following stochastic dynamic program

$$V_t(x, w) = \min_{z, y, v} \{(s_t + \eta_t)(z - x) + \beta(f_t + \alpha_t)(y - z) + \sum_{i=1}^{N} \gamma_i^t(v^i - w^i)$$

$$+ \sum_{i=1}^{N} L_i^t(v^i) + h_t(z - \sum_{i=1}^{N} v^i) + \beta\mathbb{E}_t^Q[V_{t+1}(y - \zeta_t, v - \xi_t))]\},$$

subject to $x \leq z \leq y$, $\sum_{i=1}^{N} v^i \leq z$ and $w^i \leq v^i$ for $i = 1, 2, \ldots, N$. The paper establishes the concavity of $V_t(\cdot, \cdot)$ and the first order conditions the optimal policy must satisfy and applies the Lagrangian relaxation approach to compute the near-optimal policy. The authors also show that the existence of commodity market and the information it conveys may lead to significant reductions in inventory-related costs.

There are also many studies related to oil and natural gas. Yayur and Yang (2002) develops a model that characterizes the stable Markovian perfect equilibrium for the
storable commodity market with a small number of capacitated firms. In Secomandi (2010), a model is developed to understand the optimal commodity trading with a capacitated storage asset. When the commodity spot price evolves according to an exogenous Markov process, this work shows that the optimal inventory-trading policy of a risk neutral firm is characterized by two-stage and spot-price dependent basestock targets. The inventory trading decisions are made at given equally spaced points in time belonging to $\mathcal{T}$. $j \in \mathcal{J} = \{1, 2, \ldots, J\}$ denotes the $j$th decision made at time $\tau_j \in \mathcal{T}$. The length of review period is $T$. An action is denoted as $a$: A positive action corresponds to a purchase followed by an injection, a negative action to a withdraw followed by a sale, and zero is the do-nothing action. Assume that the receipt/delivery leadtime is zero.

The storage asset features minimum and maximum inventory features $\underline{x}$ and $\bar{x}$, so the feasible set is $\mathcal{X} = [\underline{x}, \bar{x}]$. $\bar{C} > 0$ ($C < 0$) is the injection (withdraw, respectively) capacity. The commodity spot price is assumed to be $s_j$ for $j \in \mathcal{J}$. $\{s_j\}_{j=1}^J$ is Markovian whereas $s_1$ is degenerate. The trading action $a$ depends on the realized spot price $s$. Let the payoff of this decision be $p_j(a, s)$. $\alpha^W \leq 1$ and $\alpha^L$ are the commodity price adjustment factors. Let $c^W$ and $c^L$ be the positive marginal withdrawal and injection costs. Therefore, $p_j(a, s) = -(\alpha^W s - c^W)a$ if $a < 0$, $0$ if $a = 0$ and $-(\alpha^L s + c^L)a$ if $a > 0$. A unit holding cost $h$ is also charged at each time $\tau_j$. The cash flows are discounted from time $\tau_j$ back to time $\tau_{j-1}$ using $\delta_j$. The Bellman equations that the optimal value function $V_j(x, s)$ should satisfy is as follows:

$$ V_j(x, s) = \max_a \left\{ p_j(a, s) - hx + \delta_j \mathbb{E}_j[V_{j+1}(x + a, s_{j+1})] \right\}, $$

where $V_j(x, s) = \max_a \left\{ -(\alpha^W s - c^W)a - hx \right\}$. The paper shows that with a capacitated commodity storage asset the qualification of low and high prices depends on the firm’s own inventory ability. The optimal policy structure is also fully characterized by two stage and spot price dependent base stock targets which can be easily computed.

Lai et al. (2011) develops a heuristic model for strategic valuation of the real option to store liquefied natural gas that integrates models of LNG shipping, natural gas price evolution, and inventory control and sale into the wholesale natural gas market.

### 2.4 Global Supply Chain Management

In the recent decade, global supply chain management emerges as a lively stream of research. Interested readers are referred to Kouvelis (1999), Cohen and Huchzermeier (1999) and Bouabatli and Toktay (2004) for literature reviews. Kogut and Kulatilaka (1994) considers a firm with operating flexibility to shift production between two man-
Manufacturing plants located in different countries. Consider a firm which is evaluating a project to invest in two manufacturing plants—one in the U.S. and the other in Germany. The plants are identical in their technological characteristics and differ only in the prices (evaluated in dollars) of the local inputs. The product of the firm is priced in a world market, say in U.S. dollars, and fluctuations of the Euro/$ exchange rate do not affect the dollar market price. The input cost rates in the two countries are equated by the real exchange rate

$$P_\$ = \theta P_G,$$

where $P_\$ is the input cost rate in the U.S., $P_G = P_ES$ is the dollar value of input cost rate in Germany, $P_E$ is the input cost rate valued in Euro, $S$ is the nominal exchange rate and $\theta$ is the effective real exchange rate. To elucidate the impact of exchange rate uncertainty, we model the evolution of $\theta$ as a discrete time stochastic process which tends to revert towards its equilibrium $\bar{\theta}$. The discrete-time mean reversion stochastic process for the real exchange rate can be written as

$$\Delta \theta_t = \lambda (\bar{\theta} - \theta_t) \Delta t + \sigma \theta_t \Delta Z_0,$$

where $\Delta Z_0$ follows a normal distribution of mean 0 and variance $\Delta t$. Suppose the minimum cost of producing one unit of output within time interval $\Delta t$ is $\psi^S = \psi(P_\$)$. Due to the identical technology, the unit dollar cost function of the plant in Germany must be $\psi^E = \psi(P_E \theta)$. By definition, $\psi(\cdot)$ is homogeneous of degree 1, so $\psi(P_E) = \psi(\theta P_\$) = \theta \psi(P_\$)$. When $\theta < 1$, the firm has more incentive to produce in Germany, whereas when $\theta < 1$, the firm has more incentive to produce in U.S. Without loss of generality, the costs of the U.S. plant is normalized to 1. So the dollar value of the German production is $\psi^G(\theta) = \psi(\theta P_\$)/\psi(P_\$) = \theta$.

Suppose the firm knows with certainty the realized value of $\theta_t$ at the beginning of each period. If switching plant location is costless, the optimal value in period $t$ $\mathcal{F}_t(\theta_t)$ satisfies the following Bellman equation

$$\mathcal{F}_t(\theta_t) = \min\{1, \psi^G(\theta_t)\} + \rho \mathbb{E}_t\{\mathcal{F}_{t+1}(\theta_{t+1})\},$$

where $\mathcal{F}_T(\theta_T) = \min\{1, \psi^G(\theta_T)\} = \min\{1, \theta_T\}$ and $\rho$ is the discount rate.

If, otherwise, the switching is costly, and the switching cost from location $i$ to location $j$ is $\kappa_{ij}$ where 1 stands for U.S. and 2 stands for Germany. The Bellman equation then becomes

$$\mathcal{F}_t(\theta_t, 1) = \min\{[1 + \rho \mathbb{E}_t \mathcal{F}_{t+1}(\theta_{t+1}, 1)], [\kappa_{12} + \psi^G(\theta_t) + \rho \mathbb{E}_t\{\mathcal{F}_{t+1}(\theta_{t+1}, 2)\}]\}$$
and

\[ F_t(\theta_t, 2) = \min \{1 + \kappa_{21} + \rho \mathbb{E}_t F_{t+1}(\theta_{t+1}, 1), [\psi^G(\theta_t) + \rho \mathbb{E}_t \{F_{t+1}(\theta_{t+1}, 2)\}]\}, \]

where \( F_t(\theta_t, l) \) is the value of the project at time \( t \) and the location \( l \) is in operation in period \( t - 1 \). In general, there is a set \( \mathcal{L} = \{1, 2, \ldots, L\} \) of production locations with associated cost function \( \psi^l(\cdot) \), the Bellman equations can be expressed as follows

\[ F_t(\theta_t, l) = \min_{m \in \mathcal{L}} \{\kappa_{l,m} + \psi^m(\theta_t) + \rho \mathbb{E}_t \{F_{t+1}(\theta_{t+1}, m)\}\}, \]

where \( \kappa_{l,l} = 0 \), for \( l \in \mathcal{L} \). Using this dynamic programming framework, Kogut and Kulatilaka (1994) analyzes the benefits of multinational sourcing and operational flexibility under exchange rate uncertainty and shows that the management of across-border coordination has led to changes in the heuristic rules for performance evaluating and transfer pricing.

In a similar paper, Dasu and Li (1997) studies the structure of the optimal policies for a firm operating plants in different countries in the presence of exchange rate variability. It also shows that, under very general settings, the optimal policy is always a barrier policy and the optimal barriers can be computed by using linear programming techniques. Huchzermeier and Cohen (1996) investigates the valuation of the operational flexibility under exchange rate uncertainty and demonstrates how the global manufacturing strategy planning model framework can be utilized to analyze financial and operational hedging strategies.

In the global market, a firm may also apply financial hedging policies to mitigate the exchange rate risks. Ding etal. (2007) studies the integrated operational and financial hedging decisions faced by a global firm who sells to both home and foreign markets. The company has to invest in capacity before the selling season starts when the demand in both markets and the currency exchange rate are uncertain. The currency exchange rate risk can be hedged by delaying allocation of capacity and by buying financial option contracts. A two stage stochastic program model is developed to model the firm’s decisions. In the first stage, a production plan for the production facility is developed whereas approximated financial hedging contracts on foreign currency are purchased under the uncertainty in market demands and exchange rate. In the second stage, the demand and exchange rate realization is observed after which the firm makes production allocation (how many units to localize in each plant and how many to distribute in each market) decisions to maximize profits. Based on a risk-management formulation, Ding etal. (2007) derives the optimal joint capacity and financial option decision and shows that the firm’s financial hedging strategy ties closely and has impact on the firm’s operational strategy.
3 Future Research Directions

Based on our analysis of the industry practice and literature, there are various directions we think fruitful for future research.

We believe that the combined inventory and pricing management in the face of procurement price uncertainty is a promising direction of future direction. Intuitively speaking, pricing control adjusts the demand side of the system and has the following two effects: (1) it can serve as a lever to balance the demand to the most profitable level in accordance with the fluctuating procurement price; (2) the firm can apply price adjustment to pass the procurement cost risk to its customers. On the other hand, inventory management controls the supply side of the story. The firm should deal with both the trade off between understock and overstock as well as the risk of fluctuating prices. As is discussed in the first part of this paper, these two controls are jointly employed by firms in practice. However, very few papers in OM literature make some effort to analyze the optimal joint inventory and pricing policy while there are quite numerous papers that merely discuss the optimal inventory policy. Wu and Chen (2010) is maybe the only one that touches this issue, but this paper only establishes the rational expectation equilibrium but not the optimal inventory and pricing decision. To rigorously understand how the two sources of uncertainty (price and demand) correlate with the control methods of both supply (procurement) and demand (pricing) side is both interesting and challenging. There are some fundamental questions that need to be addressed, among which are, for example, the structure of the optimal policy and how the optimal policy should change with the procurement price. Additionally, the analysis of the firms with different degrees of pricing freedom is also interesting and promising. The optimal pricing and inventory policy of the firm with high pricing freedom and large gross margin must differ significantly from the one with limited market power and small margin. Very little literature tries to understand the value of pricing freedom and the optimal policy of low pricing freedom and low margin firm, which we believe is very important in practice.

Most papers in the literature that analyze the system facing price fluctuation only consists of a single echelon firm. It is a promising direction of future research to take into account the multi-echelon supply chain, where different stages of the system may compete or cooperate. Both the system-wise optimal policy for the multi-echelon supply chain and the equilibrium of a competitive setting are worth investigating. The information sharing and collaborative forecasting issue in the two-echelon supply chain is also very attractive. Generally speaking, the supplier will have a more precise forecast of price uncertainty whereas the retailer will be better aware of the trend of customer’s demand. As a result,
both players can potentially gain more profits through wise information sharing and collaborative forecasting strategies.

Some practitioners also suggest that the financial constraint a firm has also highly determines the optimal strategy of the firm. That is to say the amount of loan banks can lend will have a great impact on the optimal policy. This has yet been studied in the literature, either. It is an interesting, also practical, problem to analyze the optimal pricing and inventory policy when the inventory procurement is constrained by its financial condition. Joint operational and financial hedging policy under financial constraint is also not studied in the literature. However, it seems to us that this is also a common approach in practice so it deserves the attention of academia.

As is suggested by Mr. Mohoney (see the first section of this paper), the firm usually has three approaches in the face of rapidly increasing and fluctuating prices: (1) do nothing and take the risk straightforwardly; (2) take contingent actions to hedge the risk; and (3) avoid the risk by, for example, looking for an alternative supplier and even changing the business it runs. This idea also relates to the degree of freedom and flexibility the firm has. The more able the firm is to change its business, the higher degree of flexibility it enjoys. The basic question in this line of research is that will the benefit of avoiding fluctuating price risk over weighs the cost of changing a business. It is also interesting and relevant to point out an effective way to evaluate this change. All these are left to future research.

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References


