Common Problems

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Abstract

How do shared problems affect politics and policy in a divided society? We examine some common intuitions about preference polarization and policy-making in light of such problems and show that the relationships they imply are fundamentally contingent. When actors’ individual costs from a policy addressing a commonly shared problem differ, their preferences over the appropriate policy respond asymmetrically to increases in the magnitude of the problem. In a broad range of circumstances such increases can give rise to increased polarization, but may also simultaneously yield policy adjustments rather than entrenchment of gridlock. The association of polarization and gridlock is contingent on two underlying factors: how the problem responds to the policy solution, and the location of the status quo policy when the extent of the problem changes. We illustrate the model’s logic by comparing U.S. national policy making in the Progressive Era and the present.
Introduction

A nearly ubiquitous narrative of contemporary politics is one of despair about polarization. A polarized political system is mired in policy gridlock, with every issue transformed into a partisan battle. The partisan rancor might be set aside if only we were to focus on a pressing common problem: catastrophic climate change, a dangerous real estate bubble, or an out-of-control national debt, for example. While this narrative seems so intuitive as to be nearly a platitude, recent experience clearly belies that hope. In this note, we provide a simple formal account of collective decision-making that suggests that there may be a good reason to doubt both the rationale for the hope and the one for despair.

In our model, actors in a polity experience a common harm. A policy can mitigate the harm, but actors bear its costs asymmetrically. A helpful example is air pollution: all citizens may be harmed by sulfur-dioxide emissions, but citizens in coal-producing regions would disproportionately bear the economic consequences of emissions controls. Consequently, citizens differ in their preferred levels of regulation. An increase in the magnitude of the harm will increase the preferred scope of remedial policy of all citizens. However, rates of increase will generally differ across citizens. In a broad and behaviorally plausible array of circumstances relating to how citizens experience the mitigating effects of the policy, the rate of increase in the preferred scope of remediation will be higher for actors who, because they face relatively low remediation costs, prefer higher levels of the policy.

The conjunction of these effects has several implications for politics and policy. First, under the circumstances referred to above, a uniform increase in the harm – a common problem – will produce greater polarization with respect to policy solutions, rather than greater consensus. Second, in supermajoritarian settings, that increase in polarization may yet be accompanied by policy innovation, rather than entrenched gridlock. Third, in such settings, the net effect of an increase in the collective harm may be a welfare improvement for citizens in the polity. The occurrence of some or all of these effects depends on three factors highlighted below: the relationship between the policy solution and the experienced
harm, the location of the status quo policy at the time of the shock to the magnitude of the harm, and the institutional structure in which policy making takes place.

The possibility of simultaneous yet non-coincidental occurrence of polarization and policy innovation suggests that the commonly posited relationship between the two is contingent, and that more polarization does not, in and of itself, imply more gridlock.\footnote{The simultaneous occurrence of polarization and policy innovation is consistent with canonical gridlock models (e.g., Krehbiel, 1998; Brady and Volden, 1998), in which electoral shocks that both widen and shift the gridlock interval are not ruled out a priori. Whereas those accounts treat both changes as exogenous, in our analysis they are endogenous to the same underlying common source – the change in the magnitude of the common problem.} The contingency of this relationship on the location of the policy status quo points to the importance of placing the latter at the center of the accounts of the dynamics of policy-making failure. We illustrate the implications of this idea with a comparison of U.S. national policy making in the Progressive Era and the present.

**The Model**

Suppose a continuum of actors, indexed by $i$, with measure one.$^2$ Depending on the context, one may think of the actors as individuals (e.g., in a centralized system), or states (e.g., in a federal one). Each actor suffers a common problem, e.g., pollution. Let $g(\omega, r)$ represent the harm to an actor associated with a level of the (unremediated) common problem $\omega$ and a regulatory policy imposed centrally, $r$. We assume that harm is increasing in the magnitude of the problem itself at an increasing rate and decreasing in regulatory solution at a decreasing rate.$^3$

There is an actor-specific cost of regulation, $\alpha_i r$, with the cdf of $\alpha_i \in \mathbb{R}^+$ given by $P(\alpha_i)$.

$^1$The simultaneous occurrence of polarization and policy innovation is consistent with canonical gridlock models (e.g., Krehbiel, 1998; Brady and Volden, 1998), in which electoral shocks that both widen and shift the gridlock interval are not ruled out a priori. Whereas those accounts treat both changes as exogenous, in our analysis they are endogenous to the same underlying common source – the change in the magnitude of the common problem.

$^2$The assumption of a continuum is a mathematical convenience and has no bearing on the substantive results that follow.

$^3$Formally, $\frac{\partial g}{\partial \omega} > 0$, $\frac{\partial g}{\partial r} < 0$, $\frac{\partial^2 g}{\partial \omega^2} \geq 0$, and $\frac{\partial^2 g}{\partial r^2} > 0$. 

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An actor’s utility is given by
\[ g(\omega, r) - \alpha_i r. \] (1)

To keep the focus on the political and policy consequences of the commonly experienced harm, we do not explicitly model the production choices of different actors and their consequences for the overall level of harm in a general equilibrium model of the economy. One can think of the cost heterogeneity as a reduced form for variation in the opportunity cost of foregone production stemming from a more stringent regulatory policy.

Let \( S \) represent the set of institutional structures that map the actors’ preference profile and the (exogenously given) status quo level of remediation, \( r^o \), into an equilibrium level of regulation \( r^* \). Let \( S \) denote a generic member of \( S \).\(^4\) We will assume that \( S \) consists of majority rule with an open agenda (under which, given single-crossing preferences, the median actor would be decisive\(^5\)), and the class of institutional structures that generate a gridlock interval in equilibrium – that is, a compact and convex interval of policies that cannot be beaten by another policy under the structure. Interpretively, policies falling within the gridlock interval may be expected to remain in effect, whereas policies falling outside of the interval may be expected to be amended under such an institutional structure to some policy within the interval. We will refer to the first set of policies simply as gridlocked.

Our definition of \( S \) admits such institutional structures as q-rules (Austen-Smith and Banks, 1999; Banks and Duggan, 2006) and legislative bargaining institutions with gatekeepers or veto players (Krehbiel, 1996, 1998; Brady and Volden, 1998; Chiou and Rothenberg, 2003; Cox and McCubbins, 2005). In much of what follows, we will focus on this set of gridlock interval-inducing institutions, and refer to them for simplicity as supermajoritarian (noting that one can conceive of institutions that would induce gridlock intervals even in the

\(^4\)Formally, let \( r^o \in \mathbb{R}_+ \) be the status quo federal policy; \( U(p(\cdot)) \) be the preference profile given the distribution of the \( \alpha_i \)'s, \( p(\cdot) \), and \( U \) be the set of all preference profiles. Then \( S := U \times \mathbb{R}_+ \rightarrow \mathbb{R}_+ \).

\(^5\)Below, we demonstrate single-peakedness, which is sufficient for single-crossing.
absence of supermajoritarian requirements for policy change).

The following lemma describes actors’ induced preferences over regulation:

Lemma 1  1. Each actor has single-peaked preferences with ideal regulation $\hat{r}_i$.

2. For all $i$, $\hat{r}_i$ is weakly increasing in $\omega$ and decreasing in $\alpha_i$ (and strictly if $\hat{r}_i > 0$).

Proof. All formal proofs in the Appendix. 

The lemma is intuitive: it states, simply, that each actor has a uniquely defined ideal level of regulation; unsurprisingly, this level is lower for actors that would incur a high cost of implementing the regulation, and higher when the common problem is more serious.

Our first main result concerns ideal point polarization: for any two actors $i$ and $j$, polarization is the absolute difference between their ideal levels of policy: $|\hat{r}_i - \hat{r}_j|$. A common metric in the empirical literature on U.S. politics used as a shorthand for political conflict, for example, is partisan polarization: the difference between the mean (or median) ideal points of Republican and Democratic members of Congress (e.g., McCarty, Poole, and Rosenthal, 2006).

Let $Q \equiv \frac{g_{\omega r}(\omega, r)}{g_{rr}(\omega, r)}$ be the ratio of the cross-partial derivative of $g(\omega, r)$ with respect to $r$ and $\omega$ and the second derivative of $g(\omega, r)$ with respect to $r$. Then:

Proposition 1 Polarization between any states $i$ and $j$ with $\hat{r}_i < \hat{r}_j$ is decreasing in the magnitude of the common problem $\omega$ if and only if $Q(\hat{r}_i) < Q(\hat{r}_j)$ and increasing in the magnitude of the common problem $\omega$ if and only if $Q(\hat{r}_i) > Q(\hat{r}_j)$.

In order to interpret the result before proceeding to its implications, we make several observations. First, at the most basic level, the result says that an increase in the benefit of a collective policy can sometimes exacerbate, and at other times mitigate, political conflict. The key determinant is the shape of the harm function, as summarized by $Q$.

To interpret the condition on the shape of the harm function, consider the following example, which imposes additional structure on $Q$. Suppose that the harm from the common problem and benefit from remediation are multiplicatively separable, that is, $g(\omega, r) =$
Then a sufficient condition for polarization increasing in the magnitude of harm \( \omega \) is 
\[
\left( \frac{h''(r)}{h'(r)} \right) < 0.
\]
This is the definition of \textit{decreasing absolute risk aversion} (DARA) in the benefits to regulatory remediation (Arrow, 1965; Pratt, 1964). Informally, DARA implies that if one were able to monetize the benefits of remediation, actors experiencing a higher level of regulation (and so less harm from the common problem) would be less risk-averse with respect to the associated benefits. The condition is consistent with numerous functional forms (e.g., \( g(\omega, r) = \frac{\omega^k}{m+r} \) for \( k \geq 1 \) and \( m > 0 \)), as well as considerable observational and experimental evidence (e.g., Chavas and Holt, 1996; Saha, Shumway, and Talpaz, 1994).

If \( S \) is simple majority rule with an open agenda, then the equilibrium level of regulation corresponds to the ideal policy of the median actor, \( \hat{r}_m \). Suppose, however, that \( S \) is supermajoritarian and induces a gridlock interval as described above – a case particularly applicable in the U.S. national policy-making context. Let \( \hat{r}_l \) denote the ideal level of regulation of an actor at the extreme low end of the gridlock interval, and \( \hat{r}_h \) the ideal level for an actor at the extreme high end of the gridlock interval. Then assuming no cross-dimensional transfers, national policy may be gridlocked away from the median’s ideal point if it lies between \( \hat{r}_l \) and \( \hat{r}_h \).

Because Proposition 1 holds for any two states, it will hold for standard empirical measures of polarization: for example the distance between the median or mean ideal points of different “parties” (corresponding to a partition of \( \alpha \) into actors with high and low remediation costs). Given the foregoing, a variety of polarization of particular empirical interest is the \textit{width of the gridlock interval}, \( \hat{r}_h - \hat{r}_l \) (e.g., Krehbiel, 1998; Chiou and Rothenberg, 2003; but see Clinton, 2007 for a discussion of complications in estimation):\(^\text{6}\) naturally, the wider the gridlock interval, the more we might expect policy to be gridlocked, ceteris paribus. We will say that the gridlock interval \textit{expands} (contracts) in response to some exogenous stimu-

\(^{6}\text{Importantly, the width of the gridlock interval may or may not coincide empirically with partisan polarization, although the two measures are likely to be correlated. See, e.g., McCarty 2007, 235.}\)
lus if the width of the gridlock interval increases (decreases). Relatedly, we will say that the gridlock interval shifts rightward (leftward) in response to some exogenous stimulus if and only if both \( \hat{r}_h \) and \( \hat{r}_l \) increase (decrease). The following remark, which follows immediately from the conjunction of Lemma 1 and Proposition 1, describes changes to the position and width of the gridlock interval in response to a change in the level of pollution.

**Remark 1** Suppose \( S \) is supermajoritarian. As the magnitude of the common problem \( \omega \) increases, the gridlock interval shifts rightward and (a) expands if \( Q(\hat{r}_l) > Q(\hat{r}_h) \); (b) contracts if \( Q(\hat{r}_h) > Q(\hat{r}_l) \); or (c) maintains constant width otherwise.

Remark 1 elaborates on the result in Proposition 1 in the context of the gridlock interval: it shows that when an increase in pollution expands the gridlock interval, that increase happens not by a simultaneous leftward shift in \( \hat{r}_l \) and rightward shift in \( \hat{r}_h \); but rather by a rightward shift in \( \hat{r}_l \) and a larger rightward shift in \( \hat{r}_h \).

Insofar as it permits policies to remain in effect even when they deviate from the socially optimal one, gridlock is often understood as reflecting political “failure.” Hence, the width of the gridlock interval is taken as an indicator of the likelihood of such failure. However, that width is independent of the location of the status quo level of federal provision, \( r^0 \), and as such is not a sufficient statistic of gridlock itself. Our next result describes how the relationship between the width of the gridlock interval and gridlock respond to a change in pollution.

**Proposition 2** Suppose \( S \) is supermajoritarian and a change in the common problem \( \omega \) yields an expansion of the gridlock interval. The expansion will be accompanied by continued gridlock if and only if \( r^0 \) is sufficiently far from \( \hat{r}_l \) and \( \hat{r}_h \), and by policy change otherwise.

The implication of this result is that changes to the political environment that bring about greater polarization (wider gridlock interval) need not correspond to the persistence of gridlock. In fact, quite the contrary: such a change can facilitate a policy correction. To understand the intuition behind this proposition, suppose first that \( Q(\hat{r}_l) > Q(\hat{r}_h) \), so that an
increase in pollution yields an expansion and rightward shift in the gridlock interval. If the status quo policy is sufficiently close to \( \hat{r}_l \), an increase in the common problem will not only yield an increase in the width of the gridlock interval, but also increase \( \hat{r}_l \) to the point that the status quo policy now falls outside of the post-increase gridlock interval. The result, in equilibrium, will be policy change at the federal level in spite of the increase in polarization. Figure 1 displays this logic graphically. A similar intuition holds if \( Q(\hat{r}_h) > Q(\hat{r}_l) \), when a decrease in the common problem yields a leftward shift and increase in the width of the gridlock interval. In that case, a status quo policy sufficiently close to \( \hat{r}_h \) will fall outside of the post-decrease gridlock interval, yielding policy adjustment. By contrast, if a gridlocked status quo is sufficiently far from the edges of the gridlock interval, then policy will remain unchanged before and after the change in the magnitude of the problem.

We next explore the net welfare implications of the dynamics set into motion by an increase in the common problem, and in particular, the conditions under which a rise in polarization brought about by an increase in the common problem may be accompanied by an improvement in social welfare. To do so, we impose additional structure on the model in the functional form for an actor’s utility:

\[
 u_i(r) = -\frac{\omega}{1 + r} - \alpha_i r.
\]  

(2)

It is straightforward to demonstrate that \( \hat{r}_i = \max\{0, \sqrt{\frac{\omega}{\alpha_i}} - 1\} \), and that aggregate utility corresponds to the utility function of the actor with mean cost parameter, denoted \( \bar{\alpha} \). As noted above, this functional form satisfies DARA, and so polarization will be increasing in the magnitude of harm \( \omega \). Let \( \hat{r}_m \) be the legislative median’s ideal policy, and denote with \( \bar{r}_i(r) \) the policy that gives actor \( i \) the same utility as \( r \) (i.e., \( u_i(\bar{r}_m) = u_i(r) \)).

Apart from Majority Rule with Open Agenda, the following institutional structures have been focal in the analysis of policy-making in the U.S. in the recent literature:

**Pivotal Politics** *(Krehbiel, 1996, 1998)*. The median legislator sets the agenda.
Figure 1: Externalities, Polarization, and Policy Change

An increase in the magnitude of the common problem $\omega$ may widen the gridlock interval, but may also leave a status quo policy ($r^\circ$) out of equilibrium, yielding policy change.
by choosing a proposal \( r \) or accepting the exogenous status quo \( r^o \). The proposal is implemented if it is supported by a majority that includes the filibuster pivot and either the veto pivot or veto override pivot. Otherwise, the status quo policy remains in effect.

**Negative Agenda Control** (Cox and McCubbins, 2005). Any legislator can make a proposal, but a pivotal legislator \( l \) (e.g., the majority party caucus median) chooses whether to permit the proposal to the floor (open the gate). If the gate is opened, any proposal may be considered (the agenda is open) and the proposal that is not opposed by a majority of legislators is adopted. Otherwise, the status quo policy remains in effect.

We will consider the welfare consequences of shocks increasing common problems under these institutions and under the following assumptions that are standard in the corresponding models: \( \hat{r}_m \) lies between the ideal policies of two pivotal actors: the veto or veto override pivot and the filibuster pivot (Pivotal Politics model); and \( \hat{r}_l < \hat{r}_m \), where \( \hat{r}_l \) is the gatekeeper’s ideal policy (Negative Agenda Control model with a Low-Demand Gatekeeper).

**Proposition 3** Suppose \( \alpha_m = \alpha \), and each actor \( i \) has the utility function specified in (2). Then:

1. Under Majority Rule with Open Agenda and under Pivotal Politics, an increase in the common problem \( \omega \) never leads to an improvement in aggregate welfare.

2. Under Negative Agenda Control, there exists a nonempty set of shocks increasing the common problem \( \omega \) that lead to improvement in aggregate welfare if and only if \( r^o \) is sufficiently close to \( \hat{r}_l(\hat{r}_m) \) prior to the shock.

The intuition is as follows: an increase in the problem has both both direct and indirect effects on aggregate welfare. The direct effect is unambiguously negative: larger \( \omega \) is worse for everyone. The indirect effect is potentially positive: if the increase in \( \omega \) disequilibrates
the status quo, then the equilibrium adjustment may bring the policy closer to the social optimum than it was before the shock. If the shock is too small to push the status quo out of equilibrium, then the only effect is the direct one.

If, however, the shock is large enough, the indirect effect kicks in. Under majority rule with an open agenda, equilibrium adjustment to the social optimum will occur for any shock, but at a lower level of aggregate utility than before the shock. Under Pivotal Politics, shocks have to be quite large to yield the socially optimal policy as an equilibrium; under such circumstances, the direct effect predominates. Intermediate shocks, by contrast, induce partial adjustment, in effect truncating the size of the positive, indirect effect of policy change, and so again, the direct effect dominates.

By contrast, the Negative Agenda Control model leaves open the possibility of a relatively small shock yielding a large, discontinuous policy change to the social optimum. Under these circumstances, the indirect effect can predominate, yielding the overall improvement described in the Proposition.

**Polarization and Legislative Innovation in U.S. Politics**

A key implication of our analysis is that the conventional identification of increased polarization with policy gridlock (e.g., Mann and Ornstein 2012; Binder 2003; McCarty, Poole, and Rosenthal 2006, ch. 6) is incomplete. Yet, commentators are not wrong in identifying these features as definitive of contemporary politics. Work by McCarty, Poole, and Rosenthal (2006) has demonstrated that partisan polarization in Congress, defined as the ideological distance between the Republican and Democratic party caucuses, is at historically high levels unseen since the Progressive Era a century ago (1900-1916). And in the past two decades, major legislative accomplishments have been few and far between. Those that have passed (e.g., the Affordable Care Act, the Dodd-Frank Financial Reform Act, and the Stimulus Package) did so over the strenuous objections of a unified minority party. And McCarty (2007) documents an association between the increase in partisan polarization in the postwar period with a decline in legislative productivity.
And yet no observer would confuse our current politics with those of the Progressive Era. Despite similarly high levels of partisan polarization, that era is remembered for enormous legislative productivity and ferment, with landmark legislation that laid the groundwork for the 20th century national administrative state, passing with bipartisan majorities. Why is partisan polarization coincident with an expansion of national governance in one period and stalemate in another? Nominally, congressional institutions made obstructionist tactics easier then than today: before the adoption of Senate Rule 22 in 1917, debate in the senate could conclude only by unanimous consent. However, filibusters are far more common today (Wawro and Schickler, 2007; Binder and Smith, 1996).

Recall from our formal analysis that actors’ preferred levels of policy are increasing in the extent of commonly experienced problems, and may increase faster for actors for whom the costs of regulation are lower. This yields the positive relationship between extent of the problem and ideal point polarization. The advent of the national market following industrialization in the late 19th Century (Bensel, 2000) was accompanied by both a rapid increase in externality-inducing production (e.g., in mining (Gordon and Hafer, 2013) and food and drug manufacturing (Young, 1989)) and by the increased public awareness of the presence of those externalities and their potentially detrimental effects. Contemporary political science scholarship gives us the other half of the relationship: a high degree of partisan polarization in Congress at the turn of the 20th century, which, per our model, is consistent with the disruptive changes brought about by industrialization. (To be sure, the formal analysis above holds constant the economic benefits associated with the increase in the common problem, and so accordingly, comparisons of net social welfare before and after industrialization are inadvisable.)

In contrast with today, however, the national government’s footprint at the outset of the Progressive Era was low. Viewed through the prism of our model, the increase in remediable harm from the excesses of industrialization pushed the low end of the gridlock interval upward, to the point where the previously gridlocked low level of status quo national policy was
now outside of the gridlock interval, enabling relatively consensual (in most cases) legislative changes to higher levels of regulation. The result was the policy innovation observed during the period. By contrast, in the contemporary period, the status quo federal footprint is relatively high, and increases in contemporary polarization correspond to conservative shifts among Republicans (McCarty et al., 2012), leaving the status quo firmly gridlocked.

**Conclusion**

In this brief note, we have considered the politics that may be expected to emerge when actors in a polity experience a common problem, but bear the costs of its solution asymmetrically. Such situations can give rise to patterns of political conflict and policy-making significantly at odds with common intuitions. In particular, we demonstrate that rather than creating consensus about appropriate policy responses, an increase in a common harm can generate increasing dissensus in the form of preference polarization. Under supermajoritarian systems where policy gridlock is possible, such increases in polarization may be accompanied by policy stagnation, but also by policy innovation. And under supermajoritarian institutions in which small changes in the status quo may give rise to large, disjunctive shifts in policy (e.g., negative agenda control), the increases in common problems that give rise to increases in polarization may, through such policy innovation, yield net social welfare improvements in equilibrium.

This sequence of insights calls into question the belief, common among commentators of U.S. politics, that political polarization is all deadweight loss. The *origins* of polarization are critical, as they may give rise to changes in policy that improve citizen welfare, depending on the status quo policy in effect and the collective decision-making structures in place. This, in turn, implies that preference polarization is not by itself a sufficient statistic for political dysfunction, and must be placed in its proper policy-making and institutional context.
References


Appendix

Proof of Lemma 1

i’s first order condition is given by

\[-g_r(\omega, r) - \alpha_i = 0.\]  \hspace{1cm} (A.1)

1. Single-peakedness of \(r_i^*\) is guaranteed by global concavity of \(-g(\omega, r)\) in \(r\).

2. Implicitly differentiating (A.1) with respect to \(\alpha_i\) yields

\[-g_{rr}(\omega, r) \frac{\partial r}{\partial \alpha_i} = 1,\]

or

\[\frac{\partial r}{\partial \alpha_i} = - \frac{1}{g_{rr}(\omega, r)} < 0.\]

Implicitly differentiating (A.1) with respect to \(\omega\) yields

\[-g_{r\omega}(\omega, r) - g_{rr}(\omega, r) \frac{\partial r}{\partial \omega} = 0,\]

or, simplifying,

\[\frac{\partial r}{\partial \omega} = - \frac{g_{r\omega}(\omega, r)}{g_{rr}(\omega, r)} > 0.\]  \hspace{1cm} (A.2)

■

Proof of Proposition 1

Let \(r_i^* < r_j^*\), so polarization is \(r_j^* - r_i^*\). Polarization is increasing in \(\omega\) if and only if

\[\frac{\partial r_j^*}{\partial \omega} - \frac{\partial r_i^*}{\partial \omega} > 0.\]  \hspace{1cm} (A.3)

Substituting for \(\frac{\partial r^*}{\partial \omega}\) from (A.2), (A.3) is equivalent to \(Q(r_i^*) > Q(r_j^*)\). ■
Proof of Proposition 2

There are two cases to consider:

1. $Q(r_i^*) > Q(r_j^*)$. Then an expansion of the gridlock interval implies an increase in $\omega$, which itself implies a rightward shift in $r_i^*$. By the definition of the gridlock interval, if and only if $r^o$ is sufficiently to the right of the post-increase $r_i^*$, gridlock will persist.

2. $Q(r_i^*) < Q(r_j^*)$. Then an expansion of the gridlock interval implies a decrease in $\omega$, which itself implies a leftward shift in $r_h^*$. By the definition of the gridlock interval, if and only if $r^o$ is sufficiently to the left of the post-increase $r_h^*$, gridlock will persist.

■

Proof of Proposition 3

We will consider the consequences of a positive shock in harm sufficient to push the status quo, $\omega$, from $\omega_0$ to $\omega_1 = \omega_0 + \Delta$, with $\Delta > 0$. Let superscripts index elements before and after the shock where otherwise ambiguous, with 0 denoting before, and 1 after.

1. **Negative Agenda Control with an Open Agenda.**

   The equilibrium policy is given by

   $$r^* = \begin{cases} 
   r^o & \text{if } r^o \in \{\hat{r}_l(\hat{r}_m), \hat{r}_m\} \\
   \hat{r}_m & \text{otherwise.}
   \end{cases}$$

   \hspace{1cm} (A.4)

   Some algebra reveals

   $$\hat{r}_l(\hat{r}_m) = \frac{\sqrt{\alpha_m \omega}}{\alpha_l} - 1.$$  \hspace{1cm} (A.5)

   For a welfare enhancing shock to occur, three conditions must hold:

   *Condition 1. Ex Ante Gridlock.* $r^o > \hat{r}_l^0(\hat{r}_m^0)$. 

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**Condition 2.** Ex Post Ungridlocked. \( r^o < \hat{r}^1_l(\hat{r}^1_m) \). Substituting \( \omega = \omega_0 + \Delta \) into (A.5) and rearranging, this condition is equivalent to

\[
\Delta > \frac{\alpha_l^2 A - \alpha_m \omega_0}{\alpha_m} \equiv \Delta,
\]

where \( A = (1 + r_0)^2 \).

**Condition 3.** Welfare enhancement. If ex post ungridlocked, policy is amended to \( \hat{r}^1_l \). If \( r^* = \hat{r}^1_m \), aggregate ex post welfare evaluated at \( r^* \) simplifies to

\[
u^1_m(\hat{r}^1_m) = \alpha_m - 2\sqrt{\alpha_m(\omega_0 + \Delta)}.
\]

(A.6)

So condition 3 is equivalent to

\[-\frac{\omega_0}{1 + r_0} - \alpha_m r_0 < \alpha_m - 2\sqrt{\alpha_m(\omega_0 + \Delta)},\]

which simplifies to

\[\Delta < \frac{(\omega_0 - \alpha_m A)^2}{4\alpha_m A} \equiv \Delta.\]

The interval \((\Delta, \bar{\Delta})\) is nonempty if and only if \( \Delta < \bar{\Delta} \). Substituting from above, this condition is equivalent to

\[
\frac{\alpha_l^2 A - \alpha_m \omega_0}{\alpha_m} < \frac{(\omega_0 - \alpha_m A)^2}{4\alpha_m A}.
\]

This is a quadratic inequality in \( A \), satisfied for

\[-\frac{\omega_0}{2\alpha_l + \alpha_m} < A < \frac{\omega_0}{2\alpha_l - \alpha_m}.
\]

The first term in this triple inequality is negative, while the third is always positive. From condition 1, \( A > \frac{\alpha_m \omega_0}{\alpha_l^2} \). Comparing this lower bound on \( A \) with the upper bound
from the above, the latter exceeds the former if and only if $\alpha_l > \alpha_m$. By assumption, $\hat{r}_l < \hat{r}_m$, implying $\alpha_l > \alpha_m$. Turning again to the second part of the triple inequality, substituting for $A$, and solving yields

$$r^o < \sqrt{\frac{\omega_0}{2\alpha_l - \alpha_m} - 1},$$

which denotes the highest status quo for which a welfare-enhancing shock is feasible.

2. Pivotal Politics.

The equilibrium policy is given by

$$r^* = \begin{cases} 
\min \{\tilde{r}_l(r^o), \hat{r}_m\} & \text{if } r^o < \hat{r}_l \\
\max \{\tilde{r}_h(r^o), \hat{r}_m\} & \text{if } r^o > \hat{r}_h \\
\quad r^o & \text{otherwise.} 
\end{cases} \quad (A.7)$$

*Condition 1. Ex Ante gridlock: $\hat{r}_l^0 < r^o$. Substituting the expression for $i$’s ideal point and rearranging yields

$$\omega_0 < A\alpha_l, \quad (A.8)$$

where $A \equiv (1 + r^o)^2$.

*Condition 2. Ex post ungridlocked. $r^o < \hat{r}_l^1$. Substituting and rearranging yields

$$\Delta > A\alpha_l - \omega_0 \equiv \Delta' \quad (A.9)$$

There are two cases to consider. We proceed by demonstrating that a welfare enhancing shock is ruled out in both.

(a) $r^* = \hat{r}_m^1$. Some simple algebra reveals:

$$\tilde{r}_i(r^o) = \frac{\omega + \Delta}{\alpha_l(1 + r^o)} - 1.$$
\( r^* = \tilde{r}^1_m \) implies
\[
\sqrt{\frac{\omega_0 + \Delta}{\alpha_m}} - 1 < \frac{\omega_0 + \Delta}{\alpha_l(1 + r^o)} - 1,
\]
or, rearranging,
\[
\Delta > \frac{A\alpha_l^2}{\alpha_m} - \omega_0 \equiv \Delta''.
\] (A.10)

Comparing the expressions for \( \Delta' \) and \( \Delta'' \), the latter exceeds the former if and only if \( \alpha_l > \alpha_m \). By assumption, \( \tilde{r}_l < \tilde{r}_m \), implying \( \alpha_l > \alpha_m \). Therefore, \( \Delta'' \) is binding.

**Condition 3(a).** Substituting for aggregate ex post welfare evaluated at \( \tilde{r}_m \) from (A.6), ex post welfare exceeds ex ante welfare if and only if
\[
-\frac{\omega_0}{1 + r^o} - \alpha_m r^o < \alpha_m - 2\sqrt{\alpha_l(\omega_0 + \Delta)}
\]
or, substituting and rearranging
\[
\Delta < \frac{(A\alpha_m - \omega_0)^2}{4A\alpha_m} \equiv \overline{\Delta}'.
\] (A.11)

The set \((\Delta'', \overline{\Delta}')\) is nonempty if and only if \( \Delta'' < \overline{\Delta}' \). Substituting from (A.10) and (A.11) and rearranging yields
\[
\omega_0 > A(2\alpha_l - \alpha_m).
\]

Condition 2 requires \( \omega_0 < A\alpha_l \). There exists a value of \( \omega_0 \) that satisfies both conditions only if \( A(2\alpha_l - \alpha_m) < A\alpha_l \). This simplifies to \( \alpha_l < \alpha_m \), which is ruled out by assumption.

(b) \( r^* = \tilde{r}_l^1(r^o) \). From part 1, this implies
\[
\Delta < \frac{A\alpha_l^2}{\alpha_m} - \omega_0 \equiv \overline{\Delta}''.
\] (A.12)
If \( r^* = \tilde{r}_l(r^o) \), aggregate ex post welfare evaluated at \( r^* \) simplifies to

\[
\alpha_m - \alpha_l (1 + r^o) - \frac{\alpha_m (\omega_0 + \Delta)}{\alpha_l (1 + r^o)}.
\]

**Condition 3(b).** Comparing this to ex ante aggregate welfare (evaluated at \( \omega = \omega_0 \) and \( r = r^o \)), the shock is welfare enhancing if and only if

\[
\Delta < \frac{\alpha_l (\alpha_m - \alpha_l) (A - \frac{\omega_0}{\alpha_l})}{\alpha_m}.
\]

The right side of this inequality is strictly negative because \( \alpha_m < \alpha_l \) by assumption. Therefore, there exists no \( \Delta > 0 \) that enhances welfare.

3. **Majority Rule with an Open Agenda.** For any \( r^o, r^* = \tilde{r}_m^1 \). Given \( \bar{\alpha} = \alpha_m \), this is the social welfare optimizing policy. The result for this institutional structure then follows immediately from the envelope theorem.

\[\blacksquare\]