

Household Choices and Child Development*

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Abstract

The growth in labor market participation among women with young children has raised concerns about its implications for child cognitive development. We estimate a model of the cognitive development process of children nested within an otherwise standard model of household behavior. The household makes labor supply decisions and provides time and money inputs into the child quality production process during the development period. Our empirical results indicate that both parents' time inputs are important for the cognitive development of their children, particularly when the child is young. Money expenditures are less productive in terms of producing child quality. Comparative statics exercises demonstrate that cash transfers to households with children have small impacts on child quality due to the relatively low impact of money investments on child outcomes and the fact that a significant fraction of the transfer is spent on other household consumption and the leisure of the parents.

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1 Introduction

Economic theory does not provide unambiguous predictions regarding the impact of parental employment on the welfare of children. Family income is necessary to provide for the consumption of family members and for market-purchased investments in children, but there are opportunity costs associated with the time parents supply to the labor market beyond foregone leisure. From a societal perspective, perhaps the most important output of the household is the number and the “quality” of children it produces. It is helpful to think of there being a production technology for child quality, in which some initial endowment of child quality at birth is augmented during the development process by inputs of time contributed by parents, siblings, other relatives, paid child-care workers, teachers, etc., and by various types of goods purchased in the market, such as formal schooling, toys, books, and sporting goods. Given the household’s objectives, its mode of decision-making, and the constraints it faces over time, it makes a sequence of time allocation, consumption, and investment decisions at each stage of the child development process. Ignoring the possibility of borrowing and saving for the moment, the more income the household has at a point in time, the more that can be spent on child investment goods, among other things. By the same token, decreasing time with the child, holding other inputs fixed, leads to worse child quality outcomes. How does the household properly balance these trade-offs?

Our focus is on the estimation of a child outcome technology, or production function, that includes as arguments a limited number of (potentially observable) factors of production, as well as functions characterizing the dynamic evolution of the budget constraint of the household. Our model utilizes a reasonably standard life cycle framework, in which parents face constraint sets that evolve over time. The Child Development Supplements of the Panel Study of Income Dynamics, hereafter referred to as the CDS, gives us a substantial amount of useful information, but only at a few points in time. The keys to being able to use such limited (in a panel data sense) information to estimate a model of child development are (1) assuming age-invariance or parametric age-dependence of the functions and processes describing the households’ objectives and constraints and (2) the use of simulation-based estimation techniques that allow us to “fill in” the large numbers of gaps we face in our data on the development process at the household level.

Even under the restrictive assumptions made for purposes of computational tractability and for the identification of model parameters, we find that we are able to fit reasonably well the observed patterns of household income, labor supply, child investment, and child outcomes. Using the parameter estimates, we analyze the impact of changes in the time inputs of mothers and fathers on the child development process. Of course, both of these processes are endogenous within the model, so that any changes in the relationship between them must be generated by changes in the parental wage and/or nonlabor income processes and the prices of consumption and investment goods. The model is able to generate some complex behavioral links between the child quality and employment processes in the household, which we believe shed light on dynamic interrelationships observed in the

raw data and the possible impacts of parental labor market shocks on the welfare of their children. Our results indicate that the time inputs of both parents are extremely important in the cognitive development process, particularly for young children, and we conclude that the importance of the time fathers spend with children for cognitive development has not been emphasized enough in most previous research on this subject.

We are also able to examine the potential impacts of monetary transfers to the household on child outcomes, and we explore how the timing of transfers differentially affects child cognitive ability at the end of the development process. Our results here are somewhat surprising. First, given that time inputs are generally more productive than money expenditures, the impact of monetary transfers is small. This would be the case even if all of the transfer was spent on the child, which it is not due to the fungibility of dollars and the fact that the household values leisure and consumption in addition to child quality. Regarding timing, we find that the largest impacts come when the transfer is received towards the end of the development process. This is due primarily to monetary expenditures on investment in the child having relatively larger effects later in the development process. In the conclusion of the paper, we relate our findings to those of Cunha and Heckman (2008) and Cunha et al. (2010), who present empirical evidence supporting the claim that early childhood interventions are the most efficacious.

Although our model is only defined for intact families with a given number of children, we do formulate and estimate the model for the cases of one- and two-child households. Our two-child model allows us to address two key issues concerning the allocation of resources within families with multiple children. First is the question of economies of scale in child quality production. What is the productivity of resources provided by parents to siblings jointly, through shared time with parents or shared (public) material resources, compared to the productivity of resources provided privately to each child, through separate “alone” time with parents or child-specific investments of market goods? The second question concerns parental preferences across children. To what extent do parents prefer to equalize their children’s outcomes relative to maximizing the child quality of the “best” child? Both issues are connected, and as we emphasize above, it is necessary to understand both the technology and preferences to form a complete picture of the child development process and accurately forecast the effects of policy interventions.

There is an extensive literature in economics on parental and public investment in children and child outcomes. Recent surveys have persuasively argued that children’s cognitive and non-cognitive outcomes are largely determined early in life (e.g., Carneiro and Heckman (2003); Ermisch and Francesconi (2005)). Inputs supplied by families and others outside the household during early childhood play a very significant role in later cognitive, social, and behavioral outcomes. Todd and Wolpin (2003, 2007) estimate a dynamic child quality production function that views child development as a cumulative process, with the final child quality level being determined by heritable endowments and the sequence of family and school inputs supplied during the developmental period. Their estimating framework allows for unobserved endowment effects, potentially endogenous input choices,

and for cumulative effects of child investments at early stages of the development process. Their results indicate that both contemporaneous and lagged inputs matter in the production of current achievement, and that it is important to allow for unobserved child-specific endowment effects and the endogeneity of inputs. Cunha and Heckman (2008) and Cunha et al. (2010) estimate a dynamic factor model of child cognitive and non-cognitive outcomes in analyzing the process of skill formation, taking into account the problems of the endogeneity of inputs and the unobserved nature of both the inputs and outputs. They find that early environments play a large role in shaping later outcomes, and conclude that children’s cognitive and non-cognitive outcomes are largely determined early in life.

Our research builds on these previous studies by estimating the production technology of child cognitive ability within an explicit model of household choices. This strategy accomplishes the goal of “correcting” for the endogeneity of inputs in the estimation of the production technology, as in the work by Todd and Wolpin and Cunha et al., but also allows us to estimate the household preferences that lead to these input decisions, albeit with explicit assumptions about the form of household utility. This enables us to conduct more realistic policy experiments by manipulating the time and budget constraints that the household faces (e.g. through income transfers) in order to understand how households adjust their input choices to changes in the policy environment and how this ultimately impacts the child development process. A limitation of our approach relative to Cunha et al. is that we only consider household investments in cognitive development (which is also the case in Todd and Wolpin). However, in contrast to the work of Cunha et al., which considers only a single child investment good, households in our model make a number of specific input choices, ranging from various time inputs to child good expenditures, each with a child age-specific productivity. Our model allows us to incorporate a rich variety of household level data, including parental labor supply, wages, and non-labor income, and to relate these data to the child development process.

Perhaps the closest antecedent to this paper is Bernal (2008). She estimates a dynamic model of mothers’ choices in an effort to eliminate the potential biases that may arise as a result of the fact that women who work and use child care may be systematically different from women who do not, allowing for feedback from the child’s cognitive development level to the mother’s work decision. She allows for the mother’s wage process to be endogenous, in that wage offers are a function of the mother’s work history. We instead assume that the wage process for both parents is exogenous. She also explicitly considers the child care decisions of mothers, which is not a feature of our model. However, she assumes that fathers have no active role in the child development process and that all time that the mother does not spend in the labor market is spent investing in the child. In contrast to Bernal (2008), Todd and Wolpin (2003, 2007), and Liu et al (2010), all of whom use proxy measures for mothers’ time investments, such as mothers’ employment, we use detailed information on the time children spend in different activities with both parents.

We find that mother’s time is a crucial input in the production process of child outcomes, and that the father’s time is almost equally productive, especially in some stages

of development. Using detailed time budget information, we observe that mothers and fathers spend a considerable amount of time away from their jobs and with their children. The time parents spend actively or passively engaged with their children has an effect on cognitive development that decreases with the child's age, particularly in the case of mothers. Our estimates indicate that money expenditures on the child have an impact on cognitive development that increases with the child's age, though their impact at any age is modest at best.

An important contribution of this research is the ability to trace the connections between the level of household income and child development (Blau (1999), Locken et al. (2012)). A higher level of family income does not necessarily indicate a higher level of family resources being devoted to children. This is due to the fact that, for most households, household income is primarily generated by labor market earnings, and these require substantial time commitments from parents. To the extent that parental time investments are important factors in producing good cognitive outcomes in their children, this tends to decrease the resources devoted to the children. This channel may dampen or even reverse the assumed positive relationship between income and child development. Even when households are provided higher levels of nonlabor income, we find the impact on child outcomes is small, due to the limited value of investment goods purchased in the market for increasing cognitive ability and to the fact that households use a substantial proportion of such income gains to obtain additional parental leisure and household consumption goods. In a companion paper (Del Boca et al. (2012)), we have utilized our model estimates to more thoroughly investigate how conditional cash transfers can be designed to increase child quality in the most cost-effective manner.

In Section 2 we present the model. Section 3 contains a discussion of estimation issues, the data utilized in the empirical work, and some descriptive empirical results. In Section 4 we present the model estimates, and comparative statics exercises and a few policy experiments are presented in Section 5. Section 6 concludes.

2 Model

This section develops the model that serves as the basis of our empirical analysis. As noted in the Introduction, the model is based on a set of assumptions that allows us to derive closed-form solutions to the household's dynamic optimization problem; it is the simple form of the life-cycle demand functions that allows us to include a large number of inputs and household labor supply decisions in a tractable way. The special characteristics of the decision rules also allow us to sort out identification issues when we discuss estimation of the model in the following section.

We solve and estimate the model for the cases of one- and two-child families. The generality of the specification of household preferences and child quality production technologies

makes it difficult, if not impossible, to endogenize fertility decisions within our framework.¹ For ease of exposition and notational simplicity, we devote most of this section to the analysis of the one-child household case. At the end of the section we discuss the two-child family version of the model. This specification involves a much larger set of time inputs in the cognitive ability production processes of the two children, a modification of household preferences, and, unfortunately, a corresponding increase in notational complexity. Basic issues regarding identification and estimation are similar in the two cases.

2.1 Timing and Preferences (One Child Case)

The model begins with the birth of a child. The household makes decisions in each period of the child’s developmental phase, where the child’s age (or stage in the development process) is indexed by t . Parents make investments in child quality from the first period, $t = 1$, through the last developmental period, M . In period $M + 1$ the child begins the next stage of development, and outcomes in that stage are assumed to depend (in part) on the level of child quality attained at the onset of period $M + 1$.

In each period, the household makes seven choices: hours of work for each parent: h_{1t} (mother) and h_{2t} (father); time spent in “active” child care for each parent: $\tau_{1t}(a)$ (mother) and $\tau_{2t}(a)$ (father); time spent in “passive” child care by each parent: $\tau_{1t}(p)$ and $\tau_{2t}(p)$; and expenditures on “child” goods, e_t . Household utility in period t is a function of each parent’s hours of leisure, l_{1t} for the mother and l_{2t} for the father, the level of a consumption good purchased by the household, c_t , and the level of their child’s quality, k_t .² We assume a Cobb-Douglas household utility function and restrict the preference parameters to be stable over time:

$$u(l_{1t}, l_{2t}, c_t, k_t) = \alpha_1 \ln l_{1t} + \alpha_2 \ln l_{2t} + \alpha_3 \ln c_t + \alpha_4 \ln k_t, \quad (1)$$

where $\sum_j \alpha_j = 1$. In the empirical implementation of the model, we will allow heterogeneity in the parameter vector α across households.

Before we proceed to the description of the production technology, note that time with children is considered to be purely an investment in child quality. There is no direct utility from time with children, i.e. “enjoyment” of time with children or some effort cost

¹The usual route taken to endogenize quality and quantity of children is to define parental preferences over n and q , where n is the number of children and q is their average quality. Such modeling frameworks typically abstract from the growth process over the development period and are not designed to explain differences in the cognitive ability growth process between children in the same family. These phenomena are the focus of our research.

²Our “leisure” terms implicitly include any housework or home production either parent is engaged in. In a previous version of the model, we formally included housework as a separate time choice and allowed household production in consumption as well as in child quality. In the CDS data we found very little change in time devoted to housework over the child development period and concluded that we could simplify the model by neglecting housework, given that our goal was to characterize the dynamics of the investment process.

of this time. A model with these elements would be one where time investments had multiple outputs (both utility and child quality). In our model, the value of the child to the household is captured through the enjoyment of child quality, which depends on all time investments from both parents and the household's money investments in the child.³

2.2 Child Quality Production

Age $t + 1$ child quality is produced by the current level of child quality, k_t , parental time investments in the child of the active and passive kind, and expenditures on the child, all of which are made when the child is age t . We assume a Cobb-Douglas form for the child quality technology:

$$\begin{aligned} k_{t+1} &= f_t(k_t, \tau_{1t}(a), \tau_{2t}(a), \tau_{1t}(p), \tau_{2t}(p), e_t) \\ &= R_t \tau_{1,t}(a)^{\delta_{1,t}(a)} \tau_{2,t}(a)^{\delta_{2,t}(a)} \tau_{1,t}(p)^{\delta_{1,t}(p)} \tau_{2,t}(p)^{\delta_{2,t}(p)} e_t^{\delta_{3,t}} k_t^{\delta_{4,t}}, \end{aligned} \quad (2)$$

where $R_t > 0$ is the scaling factor known as total factor productivity, or TFP.

While the Cobb-Douglas form restricts the substitution possibilities at any point in time, we allow the productivities of the various inputs to vary over the age of the child. This allows us to capture the important insights in the economics and child development literatures that the marginal productivity of inputs varies over the stages of child development (for a useful survey, see Heckman and Masterov (2007)). As written in (2), the production technology is deterministic assuming knowledge of the $\{R_t\}_{t=1}^M$ and $\{\delta_t\}_{t=1}^M$ sequences. While it is moderately difficult to generalize our model solution to the case of a stochastic $\{\delta_t\}_{t=1}^M$ sequence, allowing for i.i.d. shocks in the $\{R_t\}_{t=1}^M$ sequence is not.

2.3 Dynamic Problem

Given wage offers and the current level of child quality, parents optimally choose their labor supply and child inputs to maximize the expected discounted sum of household utility over the development stage. The value function for the household in development period t is then

³In terms of time contributions of other family members in child investments, we found that about one-fourth of households in our sample use relatives to care for the child. The use of family members' care appears not to be a function of the level of education and/or incomes of the parents. We also found that family member care frequency (number of days per week) is relatively invariant with respect to the educational and income levels of the parents. In terms of incorporating other family members' time contributions, we face two practical problems as well. First, we do not have information on other family members' characteristics, nor do we know the reason that the relative is spending time with the child. Second, to incorporate other agents in our model would require that we consider the relatives' objective functions, alternative uses of time, etc. This is well beyond the scope of the current analysis.

$$V_t(S_t) = \max_{l_{1t}, l_{2t}, \tau_{1t}(a), \tau_{2t}(a), \tau_{1t}(p), \tau_{2t}(p), e_t} u(l_{1t}, l_{2t}, c_t, k_t) + \beta E_t V_{t+1}(S_{t+1}), \quad (3)$$

$$\text{s.t. } T = l_{jt} + h_{jt} + \tau_{jt}(a) + \tau_{jt}(p), \quad j = 1, 2 \quad (4)$$

$$c_t + e_t = w_{1t}h_{1t} + w_{2t}h_{2t} + I_t \quad (5)$$

where the vector of state variables S_t consists of the current level of child quality, the wage offers to the parents, and nonlabor income,

$$S_t = (k_t \ w_{1t} \ w_{2t} \ I_t),$$

β ($\in [0, 1)$) is the discount factor, and E_t denotes the conditional expectation operator with respect to the period t information set. The conditional expectation is taken with respect to the random variables appearing in the household's period $t+1$ problem, which include wages for both parents, household nonlabor income, and possibly R_{t+1} . The state variable vector at the birth of the child are the initial conditions of the problem, $S_1 = (k_1 \ w_{11} \ w_{21} \ I_1)$.

The constraint set faced by the household in period t consists of time and market good expenditures restrictions. We assume that each parent has a time endowment of T hours, and that this time is allocated between leisure, market labor supply, active time spent with the child, and passive time spent with the child. The last constraint is the expenditure constraint, and its form follows from our assumption that there is no saving and borrowing and that the prices of c_t and e_t are 1 in every period.

2.4 Terminal Value

We think about the child development process as lasting for M periods, and resulting in a “final” child quality level of k_{M+1} . Parental investments in child quality are limited to the first M period's of the child's life during the development period we study. We think of the child quality level k_{M+1} as an initial condition into another stage of the child development process, one that may (and almost surely does) include investment by the child in their own cognitive development, savings by parents and the child (possibly) for college costs, etc. Since the only truly dynamic process in our model is that of the child's cognitive development, the only “carry over” from the development stage we model is the child quality level at the beginning of the new development stage, k_{M+1} . We assume that the household's valuation of k_{M+1} at the beginning of the next development stage (i.e., period $M + 1$) is given by $\psi\alpha_4 \ln k_{M+1}$, where ψ is a free parameter to be estimated. We can write the period M optimization problem as

$$\begin{aligned} V_M(w_{1M}, w_{2M}, I_M, k_M) = & \\ & \max_{l_{1,M}, l_{2,M}, \tau_{1,M}(a), \tau_{1,M}(p), \tau_{2,M}(a), \tau_{2,M}(p), e_M} \alpha_1 \ln l_{1M} + \alpha_2 \ln l_{2M} + \alpha_3 \ln c_M + \alpha_4 \ln k_M \\ & + \beta\psi\{\delta_{1,M}(a) \ln \tau_{1,M}(a) + \delta_{2,M}(a) \ln \tau_{2,M}(a) + \delta_{1,M}(p) \ln \tau_{1,M}(p) \\ & + \delta_{2,M}(p) \ln \tau_{2,M}(p) + \delta_{3,M} \ln e_M + \delta_{4,M} \ln k_M\} \end{aligned}$$

2.5 Model Solution

We devote some time to describing the solution to the model, which will be important in evaluating the ability of the model to fit the data and, more formally, in assessing the ability of our proposed estimator to recover the primitive parameters that characterize the model.

As is clear from the nature of the production technology, there are never any corner solutions to the household input choice problem during the investment period.⁴ However, we do allow for corner solutions in labor supply as labor supply for either or both parents may be 0 in any given period. We can write the conditional factor demands for child inputs, where we are conditioning on labor supply choices and nonlabor income, as

$$\tau_{1,t}^*(a) = (T - h_{1t}) \frac{\varphi_{1,t}(a)}{\alpha_1 + \varphi_{1,t}(a) + \varphi_{1,t}(p)} \quad (6)$$

$$\tau_{2,t}^*(a) = (T - h_{2t}) \frac{\varphi_{2,t}(a)}{\alpha_2 + \varphi_{2,t}(a) + \varphi_{2,t}(p)} \quad (7)$$

$$\tau_{1,t}^*(p) = (T - h_{1t}) \frac{\varphi_{1,t}(p)}{\alpha_1 + \varphi_{1,t}(a) + \varphi_{1,t}(p)} \quad (8)$$

$$\tau_{2,t}^*(p) = (T - h_{2t}) \frac{\varphi_{2,t}(p)}{\alpha_2 + \varphi_{2,t}(a) + \varphi_{2,t}(p)} \quad (9)$$

$$e_t^* = (w_{1t}h_{1t} + w_{2t}h_{2t} + I_t) \frac{\varphi_{3,t}}{\alpha_3 + \varphi_{3,t}} \quad (10)$$

where

$$\begin{aligned} \varphi_{l,t}(\xi) &= \beta \delta_{l,t}(\xi) \eta_{t+1}, \quad l = 1, 2; \quad \xi = a, p, \\ \varphi_{3,t} &= \beta \delta_{3,t} \eta_{t+1}. \end{aligned}$$

The sequence $\{\eta_t\}_{t=1}^{M+1}$ is defined (backwards-) recursively as

$$\begin{aligned} \eta_{M+1} &= \psi \alpha_4 \\ \eta_M &= \alpha_4 + \beta \delta_{4,M} \eta_{M+1} \\ &\vdots \\ \eta_t &= \alpha_4 + \beta \delta_{4,t} \eta_{t+1} \\ &\vdots \\ \eta_1 &= \alpha_4 + \beta \delta_{4,1} \eta_2. \end{aligned} \quad (11)$$

⁴If any factor is set at 0, then child quality will be 0 in all subsequent periods, and household utility diverges to $-\infty$ as $k \rightarrow 0$ whenever $\alpha_4 > 0$.

where η_t represents the period t marginal utility of (log) child quality to the household, i.e., $\eta_t = \partial V_t(S_t)/\partial \ln k_t$. The variable η_t reflects both the present period flow marginal utility of (log) child quality to the household, given by α_4 , and the discounted marginal value of child quality in terms of future utility. The latter value of current child quality depends on the discount factor and the technologically determined productivity of the current stock of child quality in producing future child quality, given by the time varying parameter $\delta_{4,t}$.

The solution to the spousal labor supplies problem in period t also has a simple form. Define two “latent” labor supply variables in period t by

$$\begin{aligned}\hat{h}_{1t} &= \frac{A_{1t} - A_{2t}B_{1t}}{1 - A_{2t}B_{2t}} \\ \hat{h}_{2t} &= \frac{B_{1t} - B_{2t}A_{1t}}{1 - A_{2t}B_{2t}},\end{aligned}\tag{12}$$

where

$$\begin{aligned}A_{1t} &= \frac{w_{1t}T(\alpha_3 + \varphi_{3,t}) - (\alpha_1 + \varphi_{1,t}(a) + \varphi_{1,t}(p))I_t}{w_{1t}(\alpha_1 + \alpha_3 + \varphi_{1,t}(a) + \varphi_{1,t}(p) + \varphi_{3,t})} \\ A_{2t} &= \frac{w_{2t}(\alpha_1 + \varphi_{1,t}(a) + \varphi_{1,t}(p))}{w_{1t}(\alpha_1 + \alpha_3 + \varphi_{1,t}(a) + \varphi_{1,t}(p) + \varphi_{3,t})} \\ B_{1t} &= \frac{w_{2t}T(\alpha_3 + \varphi_{3,t}) - (\alpha_2 + \varphi_{2,t}(a) + \varphi_{2,t}(p))I_t}{w_{2t}(\alpha_2 + \alpha_3 + \varphi_{2,t}(a) + \varphi_{2,t}(p) + \varphi_{3,t})} \\ B_{2t} &= \frac{w_{1t}(\alpha_2 + \varphi_{2,t}(a) + \varphi_{2,t}(p))}{w_{2t}(\alpha_2 + \alpha_3 + \varphi_{2,t}(a) + \varphi_{2,t}(p) + \varphi_{3,t})}.\end{aligned}$$

Given these latent labor supplies, we can define the actual optimal hour choices that satisfy the rationing constraint on the time allocations of the parents. If the latent labor supplies on the right hand sides are set to zero, it is apparent that the condition required for the conditional latent labor supplies to both be 0 is

$$(h_{1t}^* = 0, h_{2t}^* = 0) \Leftrightarrow A_{1t} \leq 0 \text{ and } B_{1t} \leq 0.$$

If both of these intercept terms are equal to or less than zero, then the household supplies no time to the market. For this to be the case, it is necessary that the household’s nonlabor income be strictly positive.

Going back to the “full” solutions to the model given in (12), if both of the solutions are positive, then both satisfy the time allocation constraints, and these are the solutions to the household optimization problem. If the latent labor supply of parent 1 is positive and that of parent 2 is negative, then $(h_{1t}^* = A_{1t}, h_{2t}^* = 0)$, while if the situation is reversed, the solution is $(h_{1t}^* = 0, h_{2t}^* = B_{1t})$. In summary, optimal labor supplies are

$$(h_{1t}^*, h_{2t}^*) = \begin{cases} (0, 0) & \text{if } A_{1t} \leq 0 \text{ and } B_{1t} \leq 0 \\ (A_{1t}, 0) & \text{if } A_{1t} - A_{2t}B_{1t} > 0 \text{ and } B_{1t} - B_{2t}A_{1t} < 0 \\ (0, B_{1t}) & \text{if } A_{1t} - A_{2t}B_{1t} < 0 \text{ and } B_{1t} - B_{2t}A_{1t} > 0 \\ (\hat{h}_{1t}, \hat{h}_{2t}) & \text{if } A_{1t} - A_{2t}B_{1t} \geq 0 \text{ and } B_{1t} - B_{2t}A_{1t} \geq 0 \end{cases}$$

Using these optimal labor supply choices, the investment decisions are determined using (6), (7), (8), (9), and (10) after substituting h_{1t}^* and h_{2t}^* into the functions.

The functional form assumptions we have made enable us to find analytic solutions to the household's dynamic investment problem. The cost of the assumptions is reflected in some of the properties of the solutions that we have just described. Most importantly, investment in any period is independent of the level of child quality entering the period. The assumptions of no borrowing and saving together with the Cobb-Douglas functional forms for household utility and cognitive development imply that household labor supply and investment decisions are independent of future actual or expected wages and nonlabor income levels when these processes are assumed to be exogenous.⁵

2.6 The Two-Child Household

The structure of our two child model is similar to the one-child case with a few notable and interesting exceptions. We first introduce the timing conventions utilized when the household is composed of two children. Let the period in which child j is born be denoted B_j , where without loss of generality we set $B_1 = 1$ and $B_2 > 1$ (that is, we do not consider the case of twins or children born in the same calendar year). In period $t = B_2$, the second child enters the household. The child investment period for the first child is from $t = 1$ to $t = M$ and for the second child from $t = B_2$ to $t = M + B_2 - 1$. For convenience define $M_1 = M$ as the terminal investment period for the first child and $M_2 = M + B_2 - 1$ as the terminal period for the second child.⁶

The household's preferences in the two child case are given by

$$u(l_{1t}, l_{2t}, c_t, k_t^1, k_t^2) = \alpha_1 \ln l_{1t} + \alpha_2 \ln l_{2t} + \alpha_3 \ln c_t + \alpha_4 \ln k_t^1 + \alpha_5 \ln k_t^2, \quad t = 1, \dots, M_2,$$

where k_t^j is the quality of child j in period t , and where $\alpha_k > 0$, $k = 1, \dots, 5$, and $\sum_{k=1}^5 \alpha_k = 1$. We see that this is a straightforward generalization of the utility function assumed in the one-child case.

An important question arises immediately, which is how to value k^1 and k^2 during periods in which one of the children is not present in this stage of the development process. Child 2 is only born in period B_2 and therefore his or her quality is not known in periods $t = 1, \dots, B_2 - 1$. In these periods, we assume that the household substitutes the expected value of that child's quality and derives utility from that. For every period before the second child is born, the argument k_t^2 is replaced with $E_0^2 k_t^2$, where E_0^2 denotes the expectation of

⁵The period t labor supply decision depends on the future only through the term η_{t+1} . As can be seen from (11), η_{t+1} is only a function of the discount factor, the preference weight on child quality, the terminal valuation parameter for child quality, and future production function parameters. Wages and nonlabor income values do not appear given our functional form assumptions and the lack of saving and borrowing opportunities.

⁶For example, with $M = 16$, the investment period of the first born would be $t = 1, 2, \dots, 16$, with $M_1 = 16$. If the second child is born in period $B_2 = 3$, the investment period for the second born would be periods $t = 3, \dots, 18$, with $M_2 = 16 + 3 - 1 = 18$.

the initial value of quality for child 2. Since this expectation is assumed to be constant and is not affected by any household decisions in the periods before birth, it has no impact on the decisions made with respect to investments in the first child or household labor supply.

At time $t = M_1 + 1$ and after, decisions in the household no longer affect the first child's quality level (at least in the stage of the development process we model). For these periods, we substitute the value of child quality at the end of that child's development process into the household utility function, so that $k_t^1 = k_{M_1+1}^1$, $t = M_1 + 2, \dots, M_2$. This is admittedly a thornier issue, since it is likely that this child is in another (unmodeled) stage of the development process to which the parents are contributing some household resources. Since we have no idea of how parental resources are allocated in other stages of the development process outside the age range covered by the CDS, we cannot incorporate these investments into our model. Indications from the data suggest that children spend increasing amounts of time in self-investment as they age, so that the time components of investment of the parents are not nearly as significant in later stages of development. However, money investments may be much greater during later stages of the development process (college tuition being an obvious example), which means that the amount of money available to be invested in the second (younger) child may be less than what is implied by our model structure. There is no question that this is a problem, but the impact of it on model estimates may not be severe if (1) there is not too large an age difference between the children and (2) if money investment effects on child development are not very large. We note that the average age difference between the first and second born child in our sample is 2.75 years, so that few younger children will have an older sibling in college during the early stages of the development process. In terms of the second condition, we find that time investments and inertia (i.e., the impact of the previous period's quality) are far more significant determinants of cognitive outcomes, especially in the early stages of development, than are money expenditures.

As in the one-child case, we specify a terminal value for child quality of each child, where the "terminal" period for both children is the period following the last developmental period for the second child, $M_2 + 1$. The terminal value at the end of this stage of the development process for child 1 is given by

$$\psi_1 \alpha_4 \ln k_{M_2+1}^1,$$

and for child 2 the terminal value at the end of this stage of the development process is

$$\psi_2 \alpha_5 \ln k_{M_2+1}^2.$$

As in the case of the one-child household, we treat the weights ψ_1 and ψ_2 as free parameters.

By only considering two-child households in which the children are of different ages, we know that there will exist periods in the interval $t = 1, \dots, M_2$ in which the household is making investments in only one child and periods in which investments are being made in both children. We assume that when only one child is in the active investment phase, the production technology for child quality is exactly as it was in the one-child household case.

Because the spacing between births is typically 5 years or less, most of the total development period $t = 1, \dots, M_2$ will be spent with the household making investments in both of the children simultaneously, and this is the case we will explicitly consider below.⁷ This period of “joint production” consists of periods B_2 through period M_1 . The child quality process in this case is similar to the one-child case except for the proliferation of inputs. As in the one-child case, we distinguish between active and passive time spent with a child. In the two-child case, however, an additional consideration is whether investment occurs with respect to one of the children present or both.⁸ In period t of child 1’s development stage their younger sibling is in stage $t' = t - B_2$. In period t then the production technology for child 1, aged t , is given by

$$\begin{aligned} k_{t+1}^1 &= R_t \tau_{1,t}(a, 0)^{\delta_{1,t}(a,0)} \tau_{1,t}(a, a)^{\delta_{1,t}(a,a)} \tau_{1,t}(a, p)^{\delta_{1,t}(a,p)} \\ &\quad \times \tau_{1,t}(p, 0)^{\delta_{1,t}(p,0)} \tau_{1,t}(p, p)^{\delta_{1,t}(p,p)} \tau_{1,t}(p, a)^{\delta_{1,t}(p,a)} \\ &\quad \times \tau_{2,t}(a, 0)^{\delta_{2,t}(a,0)} \tau_{2,t}(a, a)^{\delta_{2,t}(a,a)} \tau_{2,t}(a, p)^{\delta_{2,t}(a,p)} \\ &\quad \times \tau_{2,t}(p, 0)^{\delta_{2,t}(p,0)} \tau_{2,t}(p, p)^{\delta_{2,t}(p,p)} \tau_{2,t}(p, a)^{\delta_{2,t}(p,a)} \\ &\quad \times (e_t^1)^{\delta_{3,t}} (k_t^1)^{\delta_{4,t}}, \quad t = B_2, \dots, M_1, \end{aligned}$$

while for the younger child, aged t' , we have

$$\begin{aligned} k_{t'+1}^2 &= R_{t'} \tau_{1,t'}(0, a)^{\delta_{1,t'}(a,0)} \tau_{1,t'}(a, a)^{\delta_{1,t'}(a,a)} \tau_{1,t'}(a, p)^{\delta_{1,t'}(p,a)} \\ &\quad \times \tau_{1,t'}(0, p)^{\delta_{1,t'}(p,0)} \tau_{1,t'}(p, p)^{\delta_{1,t'}(p,p)} \tau_{1,t'}(p, a)^{\delta_{1,t'}(a,p)} \\ &\quad \times \tau_{2,t'}(0, a)^{\delta_{2,t'}(a,0)} \tau_{2,t'}(a, a)^{\delta_{2,t'}(a,a)} \tau_{2,t'}(a, p)^{\delta_{2,t'}(p,a)} \\ &\quad \times \tau_{2,t'}(0, p)^{\delta_{2,t'}(p,0)} \tau_{2,t'}(p, p)^{\delta_{2,t'}(p,p)} \tau_{2,t'}(p, a)^{\delta_{2,t'}(a,p)} \\ &\quad \times (e_t^2)^{\delta_{3,t'}} (k_t^2)^{\delta_{4,t'}}, \quad t = B_2, \dots, M_1; \quad t' = t - B_2, \end{aligned}$$

where $\tau_{j,t}(\zeta, \zeta')$ denotes the time spent by parent j in developmental period t in which they invest in the first child at level ζ and in the second child at level ζ' , where $\zeta, \zeta' \in \{a, p, 0\}$, with a indicating active investment in a child, p passive investment, and 0 indicating no investment (i.e., the child was not present while the parent was with the other child). The parameters $\delta_{j,t}(\zeta, \zeta')$ are interpreted in an exactly analogous manner. Note that the time inputs for the younger child are indexed according to the older child’s age t , while the productivity parameters for the younger child are indexed by the younger child’s age, reflecting the fact that the younger child is at a different point in the development process than their older sibling.

There are several things to note in this specification, mainly the restrictions that we have imposed across the production processes of the siblings. Most obvious is the restriction

⁷This is also the portion of the development period used in the estimation of model parameters using the two-child household sample.

⁸A distinct issue, which we do not consider, is whether both parents are present or not when there is time investment activity.

that all of the production parameters are specific to development period t , no matter what is the actual calendar time during which this developmental period occurs. Thus, for example, in any household investment period t when both children are present, the TFP for child 1 is R_t while the TFP for child 2 is R_{t-B_2} . For investment period t , we have assumed that

$$\delta_{j,t}(\zeta, \zeta') = \delta_{j,t}(\zeta', \zeta), \quad j = 1, 2$$

for all valid ζ and ζ' pairs (all pairs excluding $(0, 0)$). Furthermore, we assume that the productivity parameters of goods investments and previous child quality are equal for the two children when they are in the same developmental stage. We have also assumed that investment expenditures on the children are completely private.⁹ These restrictions aid in the solution and identification of the model and allow us to focus on the manner in which life cycle variation in resource constraints and household preferences contribution to intrafamily variability in children's cognitive outcomes.

During the household investment period in which both children are active in the investment process, the parental time constraints are given by

$$T = l_{j,t} + h_{j,t} + \tau_{j,t}(a, 0) + \tau_{j,t}(p, 0) + \tau_{j,t}(0, a) + \tau_{j,t}(0, p) \\ + \tau_{j,t}(a, a) + \tau_{j,t}(a, p) + \tau_{j,t}(p, a) + \tau_{j,t}(p, p), \quad j = 1, 2; \quad t = B_2, \dots, M_1.$$

The household budget constraint is

$$I_t + w_{1,t}h_{1,t} + w_{2,t}h_{2,t} = e_t^1 + e_t^2 + c_t, \quad t = B_2, \dots, M_1.$$

The production process for the two-child case is not well-defined for periods in which only one of the children is in the development process (neither for the first child before the second is born, nor for the second child after the first child has completed the development period after M_1). We think of the two-child household as facing the one-child household production process (2) in these periods. Since the two-child household case is only estimated over time periods in which both children are active in the development process, this question is not of direct concern to us here.¹⁰

The solutions to the two-child investment problem are essentially identical in form to those in the one-child investment case. They are presented in Appendix A.

⁹An alternative is to assume that the money expenditures on child investment goods are totally "public," that is, that the combined expenditures are spent on each child. An alternative in which some expenditures are public and some are private would lead to thorny identification problems when the model is taken to the data.

¹⁰Conditional on the development level of either child at the beginning of the sample period (that is, conditional on their initial test scores), previous investment activities are immaterial to household decisions although they affect the evolution of realized child quality. Thus this restriction causes no problems in terms of estimating the two-child production process when both children are present. We would have to know the characteristics of the process when only one of the two children was present in order to do counterfactual simulations involving the entire development period. In this paper, we only conduct such exercises for the case of one-child households.

3 Econometric Issues

We begin by discussing some of our assumptions regarding the model specification. We focus attention on the one-child case. Issues are similar in the two-child case, and we discuss the modifications required to estimate that model specification in the following subsection. We estimate the two model specifications on separate samples of one- and two-child households.

3.1 The One Child Case

As noted above, we allow the production function parameters to vary with the age of the child, but do not allow any further heterogeneity in that function. That is, we assume that all families possess the same child production technology.¹¹ We economize on parameters by assuming that the input-specific productivity parameters are given by

$$\delta_{j,t}(\zeta) = \exp(\gamma_{j,0}(\zeta) + \gamma_{j,1}(\zeta)t), \quad j = 1, 2; \quad t = 1, \dots, M; \quad \zeta \in \{a, p\},$$

where $\gamma_{j,0}(\zeta)$ and $\gamma_{j,1}(\zeta)$ are parameters to be estimated. This specification constrains the time path of productivity parameters associated with any given time input to be monotonic in t . Similarly, the productivity parameter sequences associated with money investments in child cognitive ability and past child quality are given by

$$\delta_{l,t} = \exp(\gamma_{l,0} + \gamma_{l,1}t), \quad l = 3, 4; \quad t = 1, \dots, M.$$

The total factor productivity sequence is of the same form, although in this case we can also include a disturbance term

$$R_t = \exp(\gamma_{0,0} + \gamma_{0,1}t + \varpi_t),$$

where ϖ_t is a development-period specific shock to the household. This shock will have no impact on the decision rules of the household, therefore from this point of view it is

¹¹We have also estimated versions of the model in which the productivity of each parent's time in producing child quality is a function of that parent's level of education. We found no evidence that these parameters varied significantly by parental education and therefore have restricted attention to the homogeneous case.

It is possible to allow certain forms of "unobservable" heterogeneity in the production process parameters while still maintaining analytic solutions to the household's optimization problem (conditional on the unobserved heterogeneity draw). In particular, if the production function parameter associated with the factor ζ at child age t is written as

$$\delta_{i,j,t}(\zeta) = \exp(\phi_i + \gamma_{j,0}(\zeta) + \gamma_{j,1}(\zeta)t),$$

where ϕ_i is drawn from a distribution $F_\phi(X_i)$, with X_i denoting observable characteristics of household i , then the decision rules maintain a similar form to those we use (where $\phi_i = 0$ for all i). We have not implemented this generalization due to concerns about identification given the limited number of households in the one- and two-child samples.

not necessary to make any strong assumptions regarding the distribution of the sequence $\{\varpi_s\}_{s=1}^M$. However, for purposes of the identification discussion below, we will assume that $\varpi_t \stackrel{i.i.d.}{\sim} (0, \sigma_\varpi^2)$.

Household preferences are assumed to be fixed over time, however, we do allow heterogeneity in the household utility function. Each household's utility parameters are an i.i.d. draw from the distribution $G(\alpha; \theta)$, where G is a parametric distribution function characterized by the finite-dimensional parameter vector θ , and with the four dimensional vector $\alpha = (\alpha_1 \ \alpha_2 \ \alpha_3 \ \alpha_4)'$ defined such that $\sum_j \alpha_j = 1$, $\alpha_j > 0$, $j = 1, \dots, 4$. These restrictions are standard and ensure that utility is increasing in each argument and that the scale of the utility function is normalized since only relative utility matters in the household choice problem.

In order to impose the appropriate constraints on the preference parameters, the distribution G is constructed as follows. Let the 3×1 vector ν be normally distributed with

$$\nu \sim N(\mu_\alpha, \Sigma_\alpha),$$

where μ_α is a 3×1 vector and Σ_α is a 3×3 covariance matrix of full rank. Define $D = 1 + \sum_{j=1}^3 \exp(\nu_j)$. Then a draw ν from the trivariate normal is mapped into the preference parameters α as

$$\begin{aligned} \alpha_1 &= D^{-1} \exp(\nu_1) \\ \alpha_2 &= D^{-1} \exp(\nu_2) \\ \alpha_3 &= D^{-1} \exp(\nu_3) \\ \alpha_4 &= D^{-1} \end{aligned}$$

The c.d.f. of α is then given by

$$\begin{aligned} G(\alpha) &= \iiint \chi[D^{-1} \exp(\nu_1) \leq \alpha_1] \times \chi[D^{-1} \exp(\nu_2) \leq \alpha_2] \\ &\quad \times \chi[D^{-1} \exp(\nu_3) \leq \alpha_3] \times \chi[D^{-1} \leq \alpha_4] dF(\nu | \mu_\alpha, \Sigma_\alpha). \end{aligned}$$

The population distribution of α is characterized in terms of the parameter vectors μ_α and $vec(\Sigma_\alpha)$ (the vectorization of the nonredundant elements in Σ_α , of which there are six).

The wage offer processes are assumed to have the following structure:

$$\begin{bmatrix} \ln w_{1,t} \\ \ln w_{2,t} \end{bmatrix} = \begin{bmatrix} \mu_{1,t} \\ \mu_{2,t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix},$$

where

$$\begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} \stackrel{i.i.d.}{\sim} N \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} \right), \text{ for all } t$$

The terms $\mu_{1,t}$ and $\mu_{2,t}$ are the means of the log wage draws of the mother (1) and father (2) at time t . In our empirical work, we assume that

$$\ln \mu_{jt} = \mu_j^0 + \mu_j^1 s_j + \mu_j^2 age_{jt} + \mu_j^3 age_{jt}^2 + \mu_j^4 ybirth_{jt}, \quad j = 1, 2,$$

where s_j is the completed schooling level (which is time invariant) of parent j and $ybirth_{jt}$ is the year of parent j 's birth. Then μ_j^1 captures the labor market “return to schooling” for each parent, μ_j^2 and μ_j^3 capture age effects in the wage offer, and μ_j^4 captures any linear birth cohort effect. The disturbances in the parental wage equations are allowed to be correlated, which could arise through assortative mating on unobservable determinants of wages and sharing the same local labor market. We have estimated the model without allowing for temporal dependence in the disturbance process, though nothing in the structure of our model requires independence.¹²

In terms of the nonlabor income process, there are a large number of households with no nonlabor income in a given period, so we consider this process to be a truncated version of a latent variable process in levels (instead of logs). In particular, let

$$I_t^* = \mu_{3,t} + \varepsilon_{3,t}, \quad (13)$$

be the latent nonlabor income in period t , with a mean given by $\mu_{3,t}$ and where $\varepsilon_{3,t} \stackrel{i.i.d.}{\sim} N(0, \sigma_{33})$, for all t . The actual nonlabor income process is given by

$$I_t = \max(0, I_t^*), \quad \text{for all } t. \quad (14)$$

Since we found little relationship between the observed characteristics of parents and the nonlabor income process, $\mu_{3,t}$ is assumed to be constant across households and over time in the population.

3.2 The Two Child Case

The econometric specification for the two-child case is similar to that of the one-child case, except for the larger number of time inputs and the inclusion of one additional preference weight in the household utility function. In terms of the production technology, we now have

$$\delta_{j,t}(\zeta, \zeta') = \exp(\gamma_{j,0}(\zeta, \zeta') + \gamma_{j,1}(\zeta, \zeta')t), \quad t = 1, \dots, M, \quad j = 1, 2,$$

¹²Although allowing for dependence in these exogenous processes is straightforward and does not complicate the solution of the model, due to the nature of the data available to us we found it impossible to obtain credible estimates of the parameters characterizing dependence. Over the sample period, the PSID is administered every two years, while our model is based on annual time periods. Estimating autocorrelation parameters for a yearly process from biannual observations leads to a classic aliasing problem. When we allowed the disturbances in the wage equations to follow a first order autoregressive process, point estimates of the autocorrelation parameters were strongly negative, which we found not to be credible. In light of these negative outcomes, we have restricted the processes to be conditionally independent over time, with all dependence arising from the mean of the wage offer distribution being a function of time-dependent observable heterogeneity (the parent's age).

where $(\zeta, \zeta') \in \{(a, a), (a, p), (p, p), (a, 0), (p, 0)\}$. Other combinations, such as (p, a) , share the same parameters as those in this set due to our symmetry restrictions across the children (e.g., $\delta_{j,t}(p, a) = \delta_{j',t}(a, p)$, $j \neq j'$). The child investment expenditure productivities are given by

$$\delta_{3,t} = \exp(\gamma_{3,0} + \gamma_{3,1}t),$$

which are the same for each child, and the same is true of the parameter associated with last period's cognitive ability,

$$\delta_{4,t} = \exp(\gamma_{4,0} + \gamma_{4,1}t).$$

In terms of the household utility function, we denote the Cobb-Douglas utility parameter associated with the first (eldest) child by α_4 and with the younger child by α_5 . In estimating the model, we assume that

$$\begin{aligned} \alpha_4 &= \kappa \tilde{\alpha}_4 \\ \alpha_5 &= (1 - \kappa) \tilde{\alpha}_4, \end{aligned}$$

where the fixed scalar $\kappa \in (0, 1)$. The distributional assumptions on $\alpha = (\alpha_1 \alpha_2 \alpha_3 \tilde{\alpha}_4)'$ are the same as in the one-child case. We expect to find that the mean value of $\tilde{\alpha}_4 = \alpha_4 + \alpha_5$ will be larger in the two child case since the utility weight in the two child case is apportioned between two children instead of one.

3.3 Measuring Child Quality

To derive the mapping between unobserved (latent) child quality, k_t , and measured child quality, k_t^* , we build on the approach utilized by psychometricians (see, e.g., chapter 17 in Lord and Novick (1968)). Consistent with prior research on this subject, we consider child quality to be inherently unobservable to the analyst, though we do assume that it is observable by household members, as it is a determinant of the household utility level and is a (potential) input into the decision-making process. In actuality, most cognitive test scores, such as the one used in our empirical work, are simple sums of the number of questions answered correctly by the test-taker. If a child of age t has a quality level of k_t , we consider the probability that they correctly answer a question of difficulty d to be

$$p(k_t, d).$$

It is natural to assume that p is nondecreasing in its first argument for all d and is nonincreasing in its second argument for all k_t . Taking the model to data, we assume that the Letter-Word test used in the empirical analysis consists of equally “difficult” questions, and we drop the argument d for simplicity.¹³

¹³This restriction is due to a data limitation: There is no information on the item response to each question in our data. We only know the total score for each child, not the individual answers to each question. If we had access to the item response data, we could infer the relative difficulty of a question by noting the proportion of children who answered it correctly.

Given a cognitive ability test consisting of NQ items of equal difficulty, the number of correct answers, k_t^* , is distributed as a Binomial random variable with parameters $(NQ, p(k_t))$. Note that the randomness inherent in the test-taking process ensures that the mapping between k and k^* is stochastic. The measurement process implies that a child of “quality” k_t has a positive probability of answering k^* questions correctly, $k^* = 1, \dots, NQ$. Our measurement model then achieves two goals: (i) we map a continuous latent child quality defined on $(0, \infty)$ into a discrete test score measure imposing the measurement floor at 0 and ceiling at $NQ < \infty$, and (ii) we allow for the possibility of measurement error so that a child’s score may not perfectly reflect her latent quality. Previous research has often used linear (or log linear) continuous measurement equations (e.g., Cunha and Heckman (2008) and Cunha et al. (2010)). Our approach differs from this in using a measurement process that explicitly recognizes the discrete and finite nature of the test score measure.

In order to estimate the model, we do have to take a position on the form of the function $p(k_t)$. In addition to it being nondecreasing in k_t , we would like to have it possess the properties: $\lim_{k_t \rightarrow 0} p(k_t) = 0$ and $\lim_{k_t \rightarrow \infty} p(k_t) = 1$. We choose the following function that satisfies these restrictions:

$$\begin{aligned} p(k; \lambda) &= \frac{\exp(\lambda_0 + \lambda_1 \ln k)}{1 + \exp(\lambda_0 + \lambda_1 \ln k)} \\ &= \frac{\exp(\lambda_0)k_t^{\lambda_1}}{1 + \exp(\lambda_0)k_t^{\lambda_1}}, \lambda_1 > 0. \end{aligned} \tag{15}$$

As in all factor models, we will have to restrict the values of λ_0 and λ_1 in order to identify other model parameters. We will set $\lambda_0 = 0$ and $\lambda_1 = 1$, so that the normalized function p is given by

$$\begin{aligned} p(k; \lambda_0 = 0, \lambda_1 = 1) &= \frac{\exp(\ln k)}{1 + \exp(\ln k)} \\ &= \frac{k}{1 + k}. \end{aligned}$$

For each child we observe measures of child quality at two different ages. We use the first measure of child quality as an initial condition. However, to solve the model, we require an initial level of latent child quality k_t , not the measure k_t^* . We map the initial measure into the initial latent child quality by assuming that, without previous observations on the process prior to our initial measure, we have “total ignorance” regarding a given individual’s value of p .¹⁴ We represent our initial prior as a Beta distribution with parameters $(1, 1)$,

¹⁴One referee has suggested that this “total ignorance” assumption could be relaxed if we incorporated other measured characteristics of the child and the household into the formation of the prior. This is an interesting suggestion, although it would require us to take a position on the manner in which such information was incorporated.

which is simply the uniform distribution on $[0, 1]$. We then observe the test score, allowing us to update our prior and produce a posterior distribution on p , which is also Beta (a conjugate distribution for the Binomial). The posterior distribution for p is then Beta with parameters $(1 + k_t^*, (NQ - k_t^*) + 1)$, where k_t^* is the number of correct answers out of the $NQ = 57$ items.

Knowledge of this posterior distribution is important for the implementation of the simulation-based estimator we define in detail below. To begin any simulation path for a household from the time of the first test score measurement for the child, we take pseudo-random draws of p . Let \tilde{p} be a given draw from the posterior distribution. We can invert (15) to obtain

$$k_t = \frac{\tilde{p}}{1 - \tilde{p}}.$$

From this initial value of k_t , we begin the construction of this particular sample path. When we get to the period of the second measurement, at which time the child is of age $t' > t$, the test score is viewed as a draw from a Binomial distribution with parameters $(NQ, p(k_{t'}))$ as described above. The measurement model for two-child families is the same for both children.

3.4 Identification

In this discussion we indicate the manner in which the behavioral parameters characterizing the model can be recovered in a reasonably straightforward manner.¹⁵ The estimator we actually implement has several advantages (both theoretical and practical), to be detailed below, over those discussed in this section. But in a reasonably complex dynamic model it is useful to develop some intuition as to the key sources of identifying information under our modeling assumptions.

Although the model contains a large number of endogenous variables, under our assumptions the data generating process (DGP) has a very simple dynamic structure. In estimating the child cognitive ability production technology, the observation period for sample household i begins when the child is of age t_i . The child's cognitive ability at age $t_i + 1$ is given by

$$\begin{aligned} \ln k_{i,t_i+1} &= \gamma_{0,0} + \gamma_{0,1}t_i + \exp(\gamma_{1,0}(a) + \gamma_{1,1}(a)t_i) \ln \tau_{i,1,t}(a) + \exp(\gamma_{2,0}(a) + \gamma_{2,1}(a)t_i) \ln \tau_{i,2,t_i}(a) \\ &\quad + \exp(\gamma_{1,0}(p) + \gamma_{1,1}(p)t_i) \ln \tau_{i,1,t_i}(p) + \exp(\gamma_{2,0}(p) + \gamma_{2,1}(p)t_i) \ln \tau_{i,2,t_i}(p) \\ &\quad + \exp(\gamma_{3,0} + \gamma_{3,1}t_i) \ln e_{i,t_i} + \exp(\gamma_{4,0} + \gamma_{4,1}t_i) \ln k_{i,t_i} + \eta_{i,t_i}, \\ &\equiv X(t_i, \tau_{i,1,t_i}(a), \tau_{i,2,t_i}(a), \tau_{i,1,t_i}(p), \tau_{i,2,t_i}(p), e_{i,t_i}, k_{i,t_i}; \gamma) + \omega_{i,t_i}; \quad i = 1, \dots, N, \end{aligned}$$

where N is the sample size. Under our assumptions on total factor productivity, the disturbances ω_{i,t_i} are independently distributed over time and across households. Then

¹⁵We ignore missing data issues for now, but we will provide a complete discussion of the structure of the data set below.

subject to the usual full rank condition on the matrix X , the nonlinear least squares estimator

$$\hat{\gamma}_{NLS} = \arg \min_{\gamma} \sum_{i=1}^N (\ln k_{i,t_i+1} - X(t_i, \tau_{i,1,t_i}(a), \tau_{i,2,t_i}(a), \tau_{i,1,t_i}(p), \tau_{i,2,t_i}(p), e_{i,t_i}, k_{i,t_i}; \gamma))^2 \quad (16)$$

is a consistent estimator of γ , that is, $\text{plim} \hat{\gamma}_{NLS} = \gamma$. The “full rank condition” in this case primarily means that not all households choose the same values of investments, which is trivially satisfied in the data, and that not all households have a child of the same age. Since the parameters characterizing the production function are a two-parameter, monotone function of age, it is enough that the sample contain children of two different ages for the full rank condition to be satisfied.^{16,17}

The argument given above requires that not only are child quality measures available for two successive years, it also assumes that child quality is observed. As discussed above, we only observe a test score measure k^* that allows us to generate a distribution of values of k given our prior distribution and knowledge of the parameters characterizing $p(k)$. If we had access to measures k_{i,t_i+1}^* and k_{i,t_i}^* , then the model and the measurement process imply a mapping between the two distributions that is a function of the observed inputs at time t_i , and the parameter vector γ . While estimates of γ could not be recovered simply using the NLS estimator described in (16), an alternative moment-based or maximum likelihood estimator could be defined to recover γ .

In estimating the nonlabor income process, which is assumed to be independent of the wage processes, there are no issues. The data generating process (DGP) for I_t is described in (13) and (14) under the i.i.d. assumption on the normally distributed disturbances. This model is simply a Tobit, and the mean and variance of the underlying normal distribution are estimable using, for example, a maximum likelihood estimator and only cross-sectional data.

The wage process, though exogenous, cannot be estimated in as simple a manner due to endogenous selection. While we have access to wage observations for multiple periods, wage observations are nonrandomly missing due to the significant number of corner solutions associated with labor supply choices. When one or both parents is not in the labor market, we do not observe the wage. Under our model specification, we can “correct” our estimator of model parameters for the nonrandomly missing data using the DGP structure from the model. In this case, both the wage processes and the parameters characterizing preferences and production technologies must be simultaneously estimated.

¹⁶If the production parameters were a quadratic function of child age, then the full rank condition would require that the sample contain children of at least three different ages, and so on.

¹⁷Note that we have only used data from periods t_i and $t_i + 1$ in defining the estimator. We could use data from periods in the development process after period $t_i + 1$ as well, which would, in general, improve efficiency. In this section, we are only concerned with consistency and we are presenting the minimal data requirements to achieve it.

The time-invariant one-child household utility function is characterized in terms of three free parameters, since $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 1$. Other parameters characterizing preferences that remain to be considered are the discount factor, β , and the parameter ψ that, in conjunction with α_4 , determines the terminal valuation of child quality. The β and ψ parameters are assumed to be homogeneous in the population.

If we condition on values of β and ψ , then the marginal distribution of α is nonparametrically identified given only one period of observed input demands, labor supplies, and total income per household. To see this, simply note that by the structure of the production and utility functions, input demands are positive in every period of the production process. The conditional (on labor supply and household income, β , and ψ) demand functions for inputs are functions of the γ vector and α . Given a consistent estimator for γ , then this conditional demand system can be inverted to yield unique values of α for each household in the sample. The empirical distribution of these values is the nonparametric maximum likelihood estimator of $G(\alpha)$, with the estimator conditional on estimates of the production function parameters and values of β and ψ .

What remains is the determination β and ψ . Under our assumption of homogeneity of these parameters, we could use any household in which both parents work, in conjunction with our estimates of α_1, α_2 , and α_3 for that household, to determine values of β and ψ . This is accomplished by using the labor supply decisions evaluated at the actual hours choices and wage offers to back these two values out.

The main objective of our discussion of identification has been to illustrate that there is a substantial amount of information regarding preferences and technology available in the data, even if we have ignored the crucial issue of missing data to this point. There are two problems with missing data in our sample. One is the gaps in the data that make it impossible to use successive observations on child quality along with input demands to estimate the production parameters directly, as in (16). We observe an imperfect measure of child quality in 1997, along with the factor demands in that year, but don't observe an indicator of the outcome of these choices until 2002, five years later. In between these dates, input decisions have been made and levels of child quality have been determined; these input decisions depend on wage and nonlabor income draws in the intervening years, possible shocks to productivity, etc. The only tractable way to fill in these gaps is to simulate the path of all of the state variables over this period using the DGP from the model.

The other type of missing data problem we face involves nonrandom missing data on wages. As mentioned above, wages have to be generated for the years between the observed child quality levels in any event.¹⁸ But in addition, when one or both parents supply no time to the market, the wage offer is not observed for the period. This type of selection

¹⁸While wage and nonlabor income is gathered at every interview date, PSID interviews are conducted every two years at this point in the survey, and our decision periods correspond to single years. For this reason, even the wage and nonlabor income processes have to be simulated between the times when the child quality measures are available.

is particularly troublesome when preferences are treated as random in the population. In this case, seeing a parent not supply time to the market is consistent with (1) that parent’s wage offer being low, (2) the household utility function weight on that parent’s leisure being high, (3) that parent’s time with the child being highly productive, or any combination of (1) - (3). In order to “extrapolate” preferences and wages when a large number of households have at least one parent out of the labor force requires parametric assumptions on both processes.

In the two child case the situation is essentially identical except for the larger number of inputs, all of which are measured in the data. There is also an additional parameter included in the preference distribution, since the value of “child services,” $\tilde{\alpha}_4$, is “split” between those emanating from the first and second child at a fixed level κ . We have also allowed for two separate parameters characterizing the value of the child’s cognitive level at the end of the development period, ψ_1 and ψ_2 .

To aid in identification in this model, additional sample characteristics are utilized in the method of simulated moments estimator that is described below. The addition of sample characteristics in the two child case and the fact that measures of all time investments are available results in no penalty in terms of the precision of the estimated production technology in two-child households. In both the one- and two-child case, it is difficult to obtain precise estimates of the rather flexible multivariate distribution of household preferences we have utilized, particularly the degree of dependence between the preference weights associated with the various goods. If we would have introduced an additional “unrestricted” preference parameter associated with the second child, this problem would have been exacerbated. By restricting the preference weights associated with the two children in the way that we have, through the addition of the share parameter κ , we have sought to minimize identification problems regarding the distribution of preferences in two-child households.

3.5 Data

We utilize data from the Panel Study of Income Dynamics (PSID) and the first two waves of the Child Development Supplements (CDS-I and CD-II). The PSID is a longitudinal study that began in 1968 with a nationally representative sample of about 5,000 American families, with an oversample of black and low-income families. In 1997, the PSID began collecting data on a random sample of the PSID families that had children under the age of 13 in the Child Development Supplement (CDS-I). Data were collected for up to two children in this age range per family. The CDS collects information on child development and family dynamics, including parent-child relationships, home environment, indicators of children’s health, cognitive achievements, social-emotional development and time use. The entire CDS sample size in 1997 is approximately 3,500 children residing in 2,400 households. A follow-up study with these children and families was conducted in 2002-03 (CDS-II). These children were between the ages of 8-18 in 2003. No new children were

added to the study (Hoffert et al. (1998)).

Starting in 1997, children’s time diaries were collected along with detailed assessments of children’s cognitive development. For two days per week (one weekday and either Saturday or Sunday), children (with the assistance of the primary caregiver when the children were very young) filled out a detailed 24 hour time diary in which they recorded all activities during the day and who else (if anyone) participated with the child in these activities. At any point in time, the children recorded the intensity of participation for parents: mothers and fathers could be actively participating or engaged with the child or simply around the child but not actively involved. We refer to the first category of time as “active” time and the second as “passive.” In the case of one-child households, we then utilize four categories of time inputs, active and passive time spent with each of the parents. We construct a weekly measure of each type of child investment time for the mother and father by multiplying the daily hours by 5 for the weekday and 2 for the weekend day (using a Saturday and Sunday report adjustment) and summing the total hours for each category of time.¹⁹

In the case of two-child households, the determination of time inputs is considerably more complex. In addition to the active-passive characterization, we further disaggregate the time the parents spend with their children into time with both children or time alone with only one of the children. We end up with a total of eight time investment “types” for each of the parents: active time with child 1 alone, active time with child 2 alone, active time with child 1 and active time with child 2, etc. (see Appendix A for more details).

Children’s cognitive skills are conceived broadly to include language skills, literacy, and problem-solving skills and are measured with the Woodcock Johnson Achievement Test-Revised (Woodcock and Johnson, 1989). In 1997, children aged 3-5 received the Letter-Word Identification and Applied Problem sub-tests. Children aged 6 and above received Letter-Word and Passage Comprehension sub-tests as well as Applied Problems and Calculation sub-tests. In the 2002-03 (second) wave, these tests were re-administered, with the exception of the Calculation sub-test. Given the wide range of ages to which the Letter-Word (LW) tests was administered, we use this test as our measure of child development. We use the raw scores on this exam rather than the age-standardized scores. The test contains 57 items (so that in terms of our discussion in Section 3.1, $NQ = 57$), and the range of possible raw scores is from 0 to 57.

For each household, we observe hours, hourly wages, and non-labor income during the 1997, 1999, 2001, and 2003 surveys reported for the previous year (1996, 1998, 2000, and 2002). The monetary values have been deflated and are all in 2001 dollars. All wage and income information is used in estimating the model, even though child investment and achievement information is only available in 1997 and 2002-03. A summary of the data

¹⁹To account for potential differences in time use across Saturday and Sunday time diary reports, we adjusted the time reports using the following: $\tilde{\tau}_i(\text{Adjusted Saturday report}) = \tau_i(\text{Raw Saturday Report}) * \frac{\bar{\tau}_{SUN}}{\bar{\tau}_{SAT}} * 2$ and $\tilde{\tau}_i(\text{Adjusted Sunday report}) = \tau_i(\text{Raw Sunday Report}) * \frac{\bar{\tau}_{SAT}}{\bar{\tau}_{SUN}} * 2$, where $\bar{\tau}_{SAT}$ and $\bar{\tau}_{SUN}$ are the average times spent in each category for those who make either a Saturday or Sunday report.

used in the estimation is given in the following table:

<i>Variable</i>	<i>Description</i>	<i>Survey Years</i>	<i>Model Years</i>	<i>Source</i>
$\{h_{1,t}, h_{2,t}\}$	Parental labor supply	1997,1999,2001	1996,1998,2000	PSID
$\{w_{1,t}, w_{2,t}, I_t\}$	Parental wages and nonlabor income	1997,1999,2001	1996,1998,2000	PSID
$\{k_t^*\}$	Letter word score	1997,2002	1997,2002	CDS
$\{\tau_{1,t}(a), \tau_{2,t}(a), \tau_{1,t}(p), \tau_{2,t}(p)\}$	Time spent with child by parent	1997,2002	1997,2002	CDS
X	Demographic characteristics	1997	1997-	PSID

We are interested in households in which both biological parents were present in both waves. Most of the variables we use in the model are collected from the primary caregiver of a child and for the head and wife of the household. Therefore, our initial sample selection results in households with children in the CDS who (1) have valid test scores in both waves of the CDS, and (2) are sons or daughters of the head of the household.

We estimate the model separately for one- and two-child families. In the case of two-child households, we drop those households that have more than two children during the sample period. Because our model requires defining joint time allocation with all children in multi-child households, we exclude households that report more than one child, but (1) only one child was randomly chosen to be interviewed in the CDS, (2) the second-born child had not been born yet by the time of the first interview in 1997 with the first born child, or (3) the children are twins (same reported age). In addition we drop observations with missing information on mother’s or father’s time with the child or missing age or education of either parent. We do not use wage observations if the reported (real) hourly wage is more than \$150 per hour, and do not use an income observation if the reported weekly nonlabor income is greater than \$1,000. Our total sample consists of 237 intact households, 105 one-child households and 132 two-child households.

3.6 Descriptive Statistics

Table 1 reports descriptive statistics for the sample we use in the estimation of the model. At the initial 1997 wave of the sample, the parent’s average age is 37.3 for fathers and 34.8 for mothers in one-child households and 36.1 and 33.8 for fathers and mothers in two-child households. Average years of schooling is similar across parents and household types at approximately 13.5 years. In 1997, the (first born) children are aged 3-12, with an average child age of 6.3 years for one-child households and 7.8 years for the older (first born) child in the two-child households. The average LW score for the children in one-child households is 23.9, and 32.6 (20.5) for the older (younger) child in the two-child households. Given the different ages of the children, it is not surprising that there are differences in average scores across the children. Figure 1 presents the average scores conditional on age and

there are no large (or statistically significant) differences across children in the two types of households.

Table 1 also provides descriptive statistics on labor supply, wages, and non-labor income. Average hours worked by fathers is about one-third larger than for mothers in one-child families, but about two-thirds larger than mothers in two-child families. Much of this difference in relative labor supply is due to the behavior of mothers: mothers have higher average labor market hours in one-child families than in two-child families (32.2 vs. 26.9 hours) and fathers work only slightly less (43.8 vs. 45.4) in one-child families. Average wages for fathers are about 27 percent larger than average mother's wages in one-child families, but more than 50 percent larger in two-child families. Fathers have lower average wages in one-child families than in two-child families (\$19.52 vs. \$23.07 per hour), but the difference is less than one dollar in the case of mothers. Average non-labor income is about \$105 per week in one-child families and \$142 in two-child families. In statistics not reported here, we found that two-thirds of both one- and two-child households had less than \$100 per week in nonlabor income.

Table 2 breaks down parental labor supply by the age of the child. Mother's labor supply, both at the extensive and intensive margins, is related to the age of children but the father's labor supply is largely constant throughout the development period. For one-child families, the fraction of mother's working at all increases from 75 percent when the child is age 3 to 82-88 percent for older children. At the intensive margin (i.e., for those supplying time to the market), the average hours of mother's work increases from 26 hours when the child is age 3 to nearly 40 hours when children are aged 12-15. For two child families, the gradient of the labor supply response for mothers is even sharper as mother's participation in the labor market increases from 65 percent when the younger child is aged 3 to 89 percent when the younger child is aged 12-15. Average hours of work for the mother also increase as the child ages, but is lower at each child age for mothers with two children than for mothers with a single child.

Table 3 provides evidence on the allocation of parental time as the child ages.²⁰ For one-child families, mothers spend almost twice as much active time with the children as fathers when the child is aged 3-5. This gap in active time closes for older children. When the children are young, both mothers and fathers spend much more of their total child investment time actively interacting with the child rather than in passive engagement. For older children, the parents are spending closer to equal amounts of time in passive and active engagement as the amount of active time declines for both mothers and fathers. Because of the sharp reduction in active time with the child, the mother's average total time with the child declines substantially as the child ages. However, the total time the father spends with the child increases slightly, which is due entirely to an increase in the father's passive time engagement.

²⁰We do not consider other family members' care besides that of the parents. About one-fourth of households use relatives' care and its usage is relatively invariant across levels of education and income.

For two-child families, Table 3 presents descriptive statistics for total active and passive time, combining all time spent with the child whether or not the sibling was present and also receiving parental time. We see a similar age profile in time allocation for active time as in one-child families: both children receive substantially more active time with the mother and father when young. However, the amount of active time with the mother and father is lower on average for the younger child in a two-child family than for the only child in a one-child family. Given the sample restriction that both children be included in the CDS survey for two-child households, we do not have older (second born) children less than 4 years of age in the sample. Examining the patterns in passive time, we see that while average hours in active time with the younger child at age 3 is less for mothers with two children, the average amount of passive time is higher. The total time with the younger child at age 3 (active and passive) is about the same for one-child and two-child families. For older children (aged 12-15), it is clear that children in two-child families receive less active and passive contact time from both parents than do children in one-child families.

Table 4 disaggregates parental time allocation for two-child families into various joint time categories. The top row displays the time spent by mothers and fathers in active time with the younger child alone (without the other sibling present). Mothers spend on average 4.5 hours actively engaging with the younger child alone, and fathers spend on average 2.4 hours in active engagement alone with the child. By far the largest time investments are made when the children both have active contact with a parent or when both children report passive contact. On average, active time for both siblings simultaneously accounts for 11.5 hours for the mother, and 7.1 hours for the father. Passive time for both siblings simultaneously accounts for 10.7 hours for the mother and 4.7 hours for the father.

3.7 Estimator

As in previous sections, the discussion is presented in some detail for the one-child case, and the section concludes with the amendments we have made when estimating parameters for two-child households.

The family data that are available consist of a sample of households with observed characteristics X , which includes time-invariant and time-varying characteristics, as well as information pertaining to children interviewed at various ages (where child age is indexed by t). The observed household characteristics include parental variables, such as the education and the ages of the parents when the child was born. For each mother and father in the household we observe: hours worked, hours spent with children (both active and passive), and repeated measures of child quality. In addition, we observe: (accepted) wages for both parents and total household income, as described below. Although data on some child specific expenditures are available, we do not utilize them in forming the estimator.²¹

²¹We made this decision because the distribution of reports of child-specific expenditures had what we considered to be too much mass in the left tail of the distribution. Our interpretation is that respondents were not properly attributing some of the household expenditures on public goods to children. In any case,

The estimator utilizes simulation methods. We first define a set of sample characteristics, which summarize the relationships in the sample at each survey date and across survey dates. Let the vector of sample characteristics in our sample of size N (households) be denoted by M_N . For each household i , we generate a set of S sample paths over the development period in the following manner. The empirical process begins in 1996 when the child is $t_i^0 - 1$ years of age in household i . Given the parent’s characteristics at the sample date, X_i^0 , we draw from the distribution of shocks to wages and nonlabor incomes, and in conjunction with the “mean shifters” in X_i^0 , we determine the initial wage and income draws. We also draw from the distribution of household preferences, $G(\alpha)$, and this draw stays with the household over the entire sample path. We solve the household’s decision problem in 1996 yielding labor supply and investment decisions. Then for the next period 1997, when the child is t_i^0 , we begin the solution for the full model since this is the first date we have access to child quality test scores. Using the 1997 Letter Word test score, k^* , we draw from the latent child quality distribution for k , as described in Section 3.3. The particular draw of k is an initial condition for this particular sample path. Since the production technology is assumed to be deterministic, we can then solve the household’s decision problem in 1997, yielding values of labor supply and investment decisions.²²

In 1998 we have no observations on child quality or investments, yet using the DGP we can simulate these values. Since the wage and nonlabor income processes are assumed to be conditionally (on observable characteristics) independently distributed over time, we draw new wage offers and nonlabor income for 1998. Child quality in 1998 is determined from child quality in 1997 and the inputs used in 1997. Household decisions for 1998 are then determined, and we repeat this process through the year 2002. In this manner we generate a sequence of wage and nonlabor income draws from 1996 through 2002, (latent) child quality realizations k_t in all years, and sequences of all of the other dependent variables in the model. “Filling in” the portion of the empirical sample path between actual data points that is required in this model with intertemporal dependencies is analogous to performing Monte Carlo integration so as to “integrate out” the missing data.

For the same household i , this process is repeated S times, so that in the end we have $S \times N$ sample paths. Using the simulated data set, we then compute the analogous simulated sample characteristics to those determined from the actual data sample. The characteristics of any simulated sample are determined by Ω , the vector of all primitive parameters that characterize the model, and the actual vector of pseudorandom number draws made in generating the sample paths. Denote the simulated sample characteristics

we think that it is difficult for any person, even an economist, to properly impute these values, and hence did not utilize them when forming the estimator. The implied estimates of money expenditures on children are larger than those reported, but we think that they are more representative of total expenditures on children when public good expenditures are “properly” assigned.

²²As we have noted above, as long as the shocks to $\ln(k_t)$ are mean-zero and additive in each period (with no restriction on their dependency across periods), the solutions in the deterministic and stochastic productivity cases are identical under our function form assumptions.

generated under the parameter vector Ω by $\tilde{M}_S(\Omega)$. The Method of Simulated Moments (MSM) estimator of Ω is then given by

$$\hat{\Omega}_{S,N,W} = \arg \min_{\Omega} (M_N - \tilde{M}_S(\Omega))' W_N (M_N - \tilde{M}_S(\Omega)),$$

where W_N is a symmetric, positive-definite weighting matrix.²³ Given random sampling from the population of married households with a given number of children (one or two, in our case), we have $\text{plim}_{N \rightarrow \infty} M_N = M$. The weighting matrix, W_N , is simply the inverse of the covariance matrix of M_N , which is estimated by resampling the data.²⁴ Given that the simulated moments are non-linear functions of the simulated draws so that \tilde{M}_S is biased for fixed S , for consistency of the MSM estimator we require that S also grow indefinitely large. Let the true value of the parameter vector characterizing the model be denoted by Ω_0 . Then $\text{plim}_{S \rightarrow \infty} \tilde{M}_{S,N}(\Omega_0) = M_N(\Omega_0)$. Given identification and these regularity conditions,

$$\text{plim}_{N \rightarrow \infty, S \rightarrow \infty} \hat{\Omega}_{S,N,W} = \Omega \text{ for any positive definite } W.$$

Since W_N is positive definite by construction, our estimator $\hat{\Omega}_{S,N,W_N}$ is consistent as well. We have not utilized the asymptotically optimal weighting matrix in this case due to the computational cost and issues regarding the differentiability of the objective function given the crude simulator we use. This does not seem to be a major concern since virtually all of the parameters are precisely estimated with the exception of those which we know from our earlier discussion to be tenuously identified in a data set that is the size of ours.

The moments we use include the average and standard deviation of test scores at each child age, the average and standard deviation of hours of work for mothers and fathers at each child age, and the average and standard deviation of child investment hours for mothers and fathers at each child age. In addition, we use the average and standard deviation of accepted wages and the correlation in wages across parents. We also compute a number of contemporaneous and lagged correlations between the observed labor supply, time with children, child quality, wages, and income. It is important to note that while we do not observe child inputs, labor supply, wages, and income in the same periods, our simulation method allows us to combine moments from various points in the child development process into a single estimator. A full list of the moments we utilize can be found in the Appendix B.

²³Simulation in our context is used to solve the computationally intensive integration problem. Our choice of MSM vs. an alternative simulation estimator, for example simulated maximum likelihood (SMLE) is due the greater flexibility that the MSM estimator offers in combining data from multiple sources with different sampling schemes.

²⁴We computed the M_N^g vector for each of Q resamples of the original N data points, and the covariance matrix of M_N is given by

$$W_N = \left(Q^{-1} \sum_{g=1}^G (M_N^g - M_N)(M_N^g - M_N)' \right)^{-1}.$$

The number of draws, Q , was set at 200.

4 Model Estimates

We examine the estimates of the model for the one- and two-child households in turn.

4.1 One-Child Households

4.1.1 Household Preference Parameters

Periods are in years²⁵ and the assumed planning horizon is age 16, so that $M = 16$. As discussed above, parents may continue to make child investments after this point but we do not explicitly model these investments and rely on our terminal period specification to capture the utility value of them. The annualized discount factor for the household is fixed at $\beta = 0.95$.

Table 5 presents the parameter estimates of the behavioral model in which it is assumed that parental preferences are time (i.e., child age) invariant, though we allow for population heterogeneity in preferences. The transformed parameters of the distribution are difficult to interpret, so instead we present their mean values, standard deviations, and the three correlation coefficients, which taken together describe the (estimated) first two moments of the population distribution of preferences under our parametric assumption on G . In terms of the average household preference weights, there is a virtually identical weight placed on father’s and mother’s leisure (which is approximately 0.19). The average weight applied to household consumption is 0.26, and the weight attached to child “quality” is 0.36. Thus, on average, the household values child quality (in a flow sense) more than any other “good.” Recall that the actual valuation of child quality is even greater since there is also a terminal value reward associated with it at the end of the development period.

Turning next to what the estimates imply about the dispersion of preferences, the parameter estimates yield standard deviations for the four preference parameters of between 0.085 and 0.200. The ratio of the standard deviation to the mean (the coefficient of variation) for the preference parameters varies between 0.440 and 0.619, with the relative dispersion greatest in the preference for mother’s leisure. We also estimate a strong correlation in leisure preferences across spouses, with the correlation in α_1 and α_2 estimated at 0.360. This may reflect the extent of assortative matching in the marriage market with regard to preferences in leisure. In contrast, we estimate a small negative correlation between preferences for mother’s leisure α_1 and household consumption α_3 , -0.032 , but a positive correlation between father’s leisure α_2 and consumption α_3 , 0.172. These correlations are likely not representative of the general population (including households with no children and those with more than one child) and reflect the particular sample of single-child households we use to estimate the parameters. Moreover, the difficulty of identifying the covariance matrix of the preference parameters is evident in the large standard errors

²⁵While periods are in years, the model as specified is in terms of weekly household decisions. We think of the weekly decisions being invariant over the yearly planning period.

associated with the estimated correlations. A joint test that all three are equal to zero is not rejected at conventional test sizes.

The scaling factor applied to the terminal valuation of child quality (ψ) is estimated to be 28.89. To provide some context for interpreting this number, if we were to assume the household is infinitely-lived and used the assumed discount factor of $\beta = 0.95$, the implied ψ terminal value on child quality would be $(1 - \beta)^{-1} = 20$.²⁶ Taken together, our point estimates imply that the household highly values child quality both in terms of flow utility and terminal value, where the termination date here is considered the end of this particular development process. The high value of ψ could indicate that k_{M+1} serves as an “important” initial condition for developmental processes that begin in the later teen years (though the large standard error indicates that the parameter is very imprecisely estimated).

4.1.2 Child Quality Technology Parameters

We next discuss the estimated production process for child quality, which we allow to change in a smooth parametric manner with the age of the child, as was described above. Table 6 provides the parameter estimates.

Figures 2 and 3 plot the estimated technology parameters from Table 6. We see that for active time “flow” inputs into the dynamic production process (mother’s and father’s active time), the productivity of the input changes substantially over the child development process. As expected, mother’s active time is the most productive input for young children, followed by the active time investment of the father. For young children, passive time from mothers and fathers has much lower productivity. The productivity of mother’s and father’s active time is declining with the child’s age, while the productivity of the passive time of the mother and father is relatively invariant over the development process. By the time the child reaches age 12, the estimates indicate that passive time is about as productive as active mother’s or father’s time investment. This change in the productivity of time as the child ages reflects the changing input mix of time revealed in the data. Fathers spend more time with their child as the child ages and much of this time is of a passive sort.

The declining productivity of active parental time makes intuitive sense given our model specification. Once children attain the age of 5 or 6, they typically leave the home for significant periods of time each day for formal schooling activities. This amounts to a large, probably discontinuous, shift in the child quality production process. During the period of formal schooling, the child may increasingly be subject to inputs, both good and bad, from teachers and other students, which supplant the interactions that the child previously had with the parents. From the point of view of parental inputs, their input decisions have increasingly small effects on child outcomes as they are “crowded out” by

²⁶This assumes no changes in child quality following period $M + 1$ and that the household maintains the same flow valuation of the logarithm of child quality, α_4 , in all successive periods.

these others.²⁷ While one could argue about the form of the dependence of the production process on the age of the child, it is reasonable to think that the impact of parental inputs is, in general, declining.²⁸

Figure 3 shows that the productivity of child goods expenditures (δ_{3t}) and the persistence of child quality (δ_{4t}) are increasing as the child reaches the upper age limit of our analysis. The former represents the increasing importance of child goods investments, perhaps through paid enrichment activities for the child. While we believe the latter trend may reflect a real characteristic of the development process, there is no doubt that it also reflects the ceiling effect produced by our fixed interval measure of child quality which is not age-normed.

4.1.3 Wage and Non-Labor Income Process Parameters

The parents' wage offer distribution depends on observed household characteristics, including mother's and father's age and mother's and father's education. Table 8 displays the estimated wage and income parameter estimates. We estimate that each year of schooling for mothers increases her mean wage offer by around 4.8 percent, and for fathers, by around 7.8 percent. For the fathers, we estimate a standard concave profile (with respect to age) in earnings, but we estimate a rather flat and slightly convex age earnings profile for the mothers. This is likely attributable to the limited age range of the parents in our sample, with almost all between the ages of 30 and 50. The parameter estimates imply that the mothers' and fathers' mean earnings increase by about 15 percent from age 30 to age 50. We estimate that the innovation/shock process for wage offers for mothers and fathers has a substantial positive correlation at 0.71, which we do not regard as surprising given the small number of covariates appearing in the wage functions, the possible importance of assortative marriage matching, and the fact that the mother and father are typically sharing the same labor market. We also estimate statistically significant but small cohort effects in the mean wage functions of both parents.

Nonlabor income for the household is observed for some periods but unobserved for others. Unlike the wage offer distribution, we do not face the problem of endogenous nonlabor income since we have an observation of nonlabor income for each household. The mean of the latent nonlabor income is given by μ_3^0 and does not depend on parental characteristics for reasons discussed above. The estimated standard deviation of the disturbance is large. While there is some imprecision in the estimate of μ_3^0 , the standard deviation of latent nonlabor income is quite precisely estimated.

²⁷Of course, the parents continue to have a major impact on the factor inputs through their choice of the child's schooling environment. Liu et al. (2010) focus on this important aspect of child investment decisions.

²⁸A few recent studies have pointed to the importance of the phenomenon of self-investment as the child ages (e.g., Cardoso et al. (2010)). The persistence we note in the child quality process as the child ages may be due to the child, and others, supplying inputs that are unobserved and persistent.

4.1.4 Within Sample Fit

Table 9 displays the sample fit of our simulated model to some features of the wage and income data. In general, we fit the mean and standard deviation of accepted wages and nonlabor income well.

In terms of the time allocation of the parents, the estimated model is able to fit basic patterns in time with children and labor supply. Table 10 presents the sample fit for each of the two types of parental time and the probability of working and hours worked for those who are in the labor market. The model is able to replicate the high employment rates for fathers relative to mothers, and the higher average hours of work for fathers relative to mothers. However, while the data indicate a slight fall in probability of employment for mothers as the child ages (from 0.83 to 0.79), the model indicates an increase in employment rates.

Regarding the within sample fit of time allocated to the child, we see that the model fits the lower average time fathers spend with their child. In addition, the model replicates the declining time mothers spend with the child as it ages, although the estimated model predicts a slower decrease in active mother’s time than what is observed in the data.

Figure 6 provides evidence on the sample fit of the estimated child quality process to the observed child quality measure. The estimated model is able to track the concave, increasing average level of child quality reasonably well.²⁹

For a further analysis of model fit, the reader is referred to Appendix B, where we present the list of all of the moments used to estimate the one-child model, their actual sample and model (simulation) values, the difference between these, and the p -value associated with the test of the null hypothesis that the difference between the two is zero. For the vast majority of the moments, the p -value is greater than 0.1 indicating that the difference between data and estimated simulated moments is not significant at the 10 percent level.

4.2 Two-Child Households

4.2.1 Household Preference Parameters

The third column of Table 5 contains the preference parameter estimates for the two-child model, and in column four are the bootstrapped standard errors. Recall that that the preference structure for the two-child and one-child case are not strictly nested. The two child specification contains an additional (constant) scalar parameter κ that determines how the preference of child quality is “split” between the older child (child 1) and the younger child.

²⁹In general, there are two sources of “noise” in the average level of measured child quality at each age. First, the average child quality level at each age is an aggregate of the child quality levels for our sample of heterogeneous households. Second, we are displaying the simulated *measure* of child quality using our stochastic measurement model, and the simulated measure is noisy due to the fact that the number of simulation draws is finite.

The preference parameter estimates for the two-child case are not dramatically different from what was found for one-child households. The average preference weight on mother’s leisure is slightly lower in the two-child case (0.170 versus 0.196), while the average weight attached to the father’s leisure, 0.233, is significantly larger than it was in the one-child case and is quite a bit larger than the mother’s average weight. The average weight attached to household consumption lies between the mean utility weights attached to the leisure of the mother and father. In this case, the most interesting welfare weight is that attached to total child quality ($\alpha_4 + \alpha_5$). The average weight attached to total child quality is 0.402 (= 0.185+0.217), which is significantly larger than what was found in one-child households (0.353). Our estimate of how this weight is “split” between the two children is $\hat{\kappa} = 0.460$ (= 0.185/0.402). Thus, we find a slight (though statistically significant) preference for the younger child.

The dispersion in the preference parameters across households is similar to what was observed in the one-child case. We do find very different estimates of the linear dependence between parameters than was found for one-child households. In particular, we find a very strong positive relationship between the α ’s. It is interesting to note that, in contrast to the estimates of preference parameter correlations in the one-child case, the pairwise correlations are estimated quite precisely. This is probably partially due to the larger number of two-child households and the fact that each two-child household contains information for two children, even if there are additional production function parameters associated with the greater number of time inputs.

Finally, we note that are estimates of the terminal valuation parameters ψ_1 and ψ_2 are roughly equal in size for the two children (2.585 for the elder and 2.993 for the younger). These values differ appreciably from what was found for the case of one-child households ($\hat{\psi} = 28.887$). We believe that part of the reason is the additional data used in the estimation of the two-child households, which enables more precise estimation of virtually all model parameters due to the restrictions we have imposed on the production process and household preferences in the two-child case. In any event, the parameters ψ are added solely to improve the fit of the model; we do not view the estimates of these parameters as being of much substantive interest.

4.2.2 Child Quality Technology Parameters

Table 7 provides the parameter estimates and standard errors associated with the two-child cognitive ability production process, and Figures 4 and 5 plot the point estimates of some of the key estimated parameters. As is even more true for two-child households, the production function parameters are difficult to interpret directly, so our focus is on the figures.

Figure 4 plots the trajectories of the productivities of the active time spent for each of the parents both alone with one child and when with both children simultaneously. We see similar patterns to what was observed in the one-child family case, though it should be

noted that the productivity parameter values are substantially lower than in the one-child case; this is mainly due to the fact that there are many more time inputs in the two-child household case. We see that the productivity of active time spent with a child alone for fathers is fairly constant over the entire development period. In contrast, the productivity associated with all of the other three inputs declines markedly over this period, particularly the productivity of the mother's time alone with one child. By the end of the development period, all of these time inputs have little impact on the growth of the cognitive ability of the child.

In terms of passive time inputs, Figure 5 tells a slightly different story. Now all of these time inputs exhibit large decreases in their productivity over the development process. It is interesting to note that in terms of passive time spent with one child only, father's time is more valuable than mother's time. The passive time spent by the mother with both children is most effective at early ages, although the value of the father's time with both children overtakes the value of this type of investment by the mother when the child is 8 years of age.

4.2.3 Wage and Non-Labor Income Process Parameters

We briefly return to the results in Table 8 in order to compare the parameter estimates of the wage and nonlabor income processes for the one- and two-child households. In terms of the wage equations of the mothers, we note that the coefficient associated with mother's education is substantially larger in two-child households. Wages increase at a faster rate in the age of the mother in two-child households as well. These differences are counterbalanced by the significantly larger intercept term in one child households and the negative cohort effect in the case of two-child households.

Similar patterns are observed in the fathers' wage equation. The coefficient associated with schooling is larger, and the age relationship with wages is greater than in one-child households at all ages. Once again, the intercept is larger in one-child households and the cohort coefficient is less negative. The nonlabor income processes are similar across one- and two-child households.

4.2.4 Within Sample Fit

We return to Table 9 to assess the fit of the exogenous wage and nonlabor income processes in terms of some sample characteristics. As was true in the one-child household case, the fit is generally quite good. The model consistently reproduces the lower mean and standard deviation of mothers' wages in two-child households (though the mean is over-estimated in one-child households and under-estimated in two-child households; the reverse is true for the standard deviation of wages). The model is able to fit the wage process characteristics for fathers in two-child households extremely well, and the same is true regarding nonlabor income.

There are many sample characteristics used to estimate the many production process parameters characterizing two-child households, so we will only briefly summarize model fit here. Table 11 presents the data and model means for various time allocation decisions in the household tabulated by the age of the first child. Since households will vary in the age of the second child, we may expect to observe poorer overall fits in two-child households.

We see that the model does a poor job of fitting the labor force participation rates for women when the oldest child is aged 3-5. We believe that this is mainly due to the fact that the youngest child is not yet born in many of these households, and the model is estimated only using observations when both children are in the development process. We do see that the model fits relatively well for other age intervals, though it does over-predict mothers' participation when the child is aged 12-15.

Predicted hours of work conditional on employment are closely aligned with their sample counterparts for both mothers and fathers, with the exception of mothers with an older child 6-8 years of age. In terms of active and passive time spent with the older child by either parent, the model fits the averages quite well with the exception of active time spent with an older child of age 12-15. This is not totally surprising, since we expect that the model is somewhat inappropriate for older children who typically are spending considerable amounts of time away from home. We note that the model under-predicts active time spent with the first child by the mother and over-predicts in the case of the father. In terms of the time allocations to the second child, the active time spent with mothers of the second child aged 3-5 is under-predicted, while the active time spent with the second child aged 12-15 by fathers is over-predicted. For mothers, there is a similar finding with respect to passive time spent with second children of age 3-5. For fathers, the model fits the overall pattern of lower time spent with children relative to mothers and fits the general patterns with respect to age. However, the model has some difficulty fitting the investment times for virtually all age categories, with the most serious discrepancy seen for second children of age 9-11.

5 Comparative Statics Exercises

We now consider the predictions of the model (using the point estimates for one-child households) by performing a number of comparative statics exercises. We include exercises ranging from changing model preferences and exercises altering the resources available to the household to replicate, in a stylized fashion, policies intended to improve the level of child quality.

5.1 Preferences and Investment

We investigate the importance of modeling the child development process within a household framework by considering a number of special cases of our more general model. The

intent of the exercise is to illustrate how changes in the assumptions concerning the objectives of decision-makers dramatically alter the mapping between resource constraints and investment and consumption decisions and child outcomes. In each of these three illustrations, we modify household preferences in an extreme manner to make our points as clearly as possible. The production function estimates from our general model specification are used throughout; therefore we can think of this as a comparative statics exercise.

5.1.1 Child Quality Maximizing Preferences

In the first special case, we examine the optimal level of child inputs if it was assumed that the household *only* values child quality. The optimal allocation of inputs under these “child-quality maximizing” preferences is determined by solving the household’s choice problem after setting the weights on parental leisures and household consumption to zero, i.e., $\alpha_1 = \alpha_2 = \alpha_3 = 0$. This is equivalent to assuming the in each period t , $t = 1, \dots, M$, the household solves:

$$\begin{aligned} \max_{\tau_{1t}(a), \tau_{2t}(a), \tau_{1t}(p), \tau_{2t}(p)} & f_t(k_t, \tau_{1t}(a), \tau_{2t}(a), \tau_{1t}(p), \tau_{2t}(p), e_t), \\ \text{s.t. } & h_{jt} = T - \tau_{jt}(a) - \tau_{jt}(p), \quad j = 1, 2, \\ & e_t = w_{1t}h_{1t} + w_{2t}h_{2t} + I_t. \end{aligned}$$

Note that the under these “child quality maximizing” preferences, the household problem is strictly a static problem as the optimal allocation of inputs to maximize child quality in each period also maximizes the terminal period $T + 1$ level of child quality.

Table 12 presents the mean level of endogenous choices and final period (age 16) child quality under the baseline unrestricted model, using the heterogeneous preferences which are a component of the baseline model, and the restricted child-quality maximizing preferences (in column 2). The first row indicates that latent child quality increases significantly under the child-quality maximizing preferences. With child-quality maximizing preferences, average latent child quality increases by about 43 percent, from 13.38 to 19.20. With these preferences, the parents choose to work in the labor market substantially fewer hours than under the baseline, and the time spent with their children increases by over 300 percent in each of the four time investment categories. It is interesting to note that mothers and fathers still work under the child quality maximizing preferences and do not spend their entire time endowment on child rearing. This is because market work funds child goods expenditures, and it is optimal for the parents to continue working some hours. Even under the child-quality maximizing preferences, the relative specialization of mothers in child rearing and fathers in market work still occurs as mothers continue to spend more time on average with their children than fathers, and fathers work more than mothers. This is due to the time productivity differences between the parents and the higher wages of fathers.

Even with the lower labor income under the child-quality maximizing preferences, expenditures on children increase by approximately 300 percent with respect to the baseline. This is due to there being no household consumption expenditures under these preferences.

5.1.2 Selfish Parents Preferences

The next special case we consider is the converse of the previous one. Here we set household preferences so that there is no weight on child quality: $\alpha_4 = 0$. With these “selfish parents preferences,” in each period $t = 1, \dots, M$, the household solves the following problem:

$$\begin{aligned} & \max_{l_{1t}, l_{2t}} U(l_{1t}, l_{2t}, c_t), \\ \text{s.t. } & h_{jt} = T - l_{jt}, \quad j = 1, 2, \\ & c_t = w_{1t}h_{1t} + w_{2t}h_{2t} + I_t. \end{aligned}$$

We maintain the same distribution of preferences over leisure and consumption as estimated. These selfish parents preferences imply that parents optimally choose to spend no time with their children, i.e., $\tau_{1t}(a) = \tau_{2t}(a) = \tau_{1t}(p) = \tau_{2t}(p) = 0$ for all t , and make no expenditures on child goods, so that $e_t = 0$ for all t . As with the child-quality maximizing preferences, the household problem under the selfish parent preferences is a strictly static one.

The third column of Table 12 presents some of the results for the model estimated under these “selfish” parent preferences. The interesting aspect of this special case is the contrast of the labor supply decision with that for the baseline case. We see that for mothers, average hours in the labor market increases by 30 percent, while for fathers there is a small decline in average hours worked. The lack of time devoted to children increases the effective time endowment in this case, which should tend to increase both labor supply and leisure. However, the fact that child investment goods are no longer purchased by the household reduces the marginal utility of income, which reduces labor supply, and this latter effect is particularly important in the case of the labor supply of the fathers. The exercise illustrates that the labor supply of parents, in particular the mother, is significantly impacted by the presence of children and the child development process.

5.1.3 Technology Optimal Allocations

In the third special case we consider, the optimal level of child inputs is determined by the technology alone. This special case is intended to replicate one of the main approaches to child investment taken in the current literature, in which the production technology alone is used to draw inferences about the optimal allocation of child inputs across stages in the child’s development and across different types of inputs. Our estimate of the production

technology, which is considered as a constraint in the dynamic household welfare maximization problem, is taken from the baseline model and provides a “selection corrected” estimate of the production technology due to our modeling of endogenous input choice. We then use the estimated technology to indicate what the optimal allocation of inputs would be if a social planner were to use this technology alone in determining resource allocations. The degree to which this technologically-optimal allocation differs from the actual allocation of the household demonstrates the importance of household preferences in determining household resource allocations to child investment.

Previous empirical work estimates the technology in some manner, and then uses the parameter estimates to make inferences regarding the most efficient allocation of resources across different types of inputs and the optimal timing of inputs over the development period (see for example the widely-cited work of Cunha and Heckman (2008) and Cunha et al. (2010)). In these studies, the authors specifically interpret their production function estimates by computing the optimal combination of inputs to maximize child quality (or adult outcomes which are taken to be functions of child quality). In the policy simulations used to interpret their production technology estimates, Cunha et al. (2010) write that “Our analysis assumes that the state has full control over family investment decisions. For neither problem do we model parental investment responses to the policy or parental investment. These simulations produce a measure of the investment that is needed from whatever source to achieve the specified target.” Our exercise is an attempt to contrast this approach (using our estimated technology) with that of the solution where the household chooses the investments given its preferences and the constraints it faces.

With no resource constraint, we cannot consider the level of inputs using the estimated production technology alone (since inputs would be unbounded). Instead we define the “technologically optimal” ratio of inputs as the ratio of marginal productivities of the inputs in each period. The technologically optimal ratio of mother’s active time to father’s active time, for example, is

$$\frac{\tau_{1t}(a)}{\tau_{2t}(a)} = \frac{\delta_{1t}(a)}{\delta_{2t}(a)},$$

and the technologically optimal ratio mother’s active time to child good expenditures is given by

$$\frac{\tau_{1t}(a)}{e_t} = \frac{\delta_{1t}(a)}{\delta_{3t}}$$

The difference between the optimal ratio of inputs chosen by the household in our unrestricted baseline model and the technology optimal ratio stems from the fact that the household optimal allocation takes into account the cost of child investments from foregone parental leisure and consumption (given by α_1 , α_2 , and α_3) and the different opportunity costs of mothers’ and fathers’ time stemming from the differences in their wage offers.

Figure 9 displays the optimal ratio of mother’s active time to father’s active time with the child under (i) the baseline unrestricted household-optimal model and (ii) the technologically-optimal model. For the household-optimal model, we plot the mean input choices in our sample at each child age. In Figure 9 the technologically-optimal allocation reflects the change in productivity of mothers and fathers as the child ages (Figure 2). The technologically-optimal case allocates relatively *less* mother’s time to child development than the household optimal solution. The household optimal allocation takes into account the fact that, on average, mothers have lower wages than do fathers, and therefore the optimal allocation of time has mothers spending relatively more time with the child than do fathers. Both the technologically-optimal and the household-optimal allocations are downward sloping given that the main time varying feature of the model is the technology; mother’s time is becoming increasingly less productive relative to father’s time as the child ages.

Figure 10 displays the analysis of optimal allocations focusing on another ratio of inputs, the ratio of mother’s active time to child good expenditures. The technologically-optimal allocation of mother’s time to expenditures is several times higher than the baseline household optimal ratio. This reflects the fact that the household optimally allocates a (relatively) much greater level of expenditures to the child than would be indicated solely by technological considerations. The household optimally substitutes child goods for mother’s time. The technologically-optimal solution ignores the fact that mother’s time with the child has both an opportunity cost in terms of foregone leisure for the mother and foregone labor income from the mother’s labor supply, and therefore foregone parental consumption and child expenditures. This difference is particularly large when children are young, as the technology alone would dictate that mothers should spend substantially more time raising children than the household would optimally choose. This difference declines as the child ages and the productivity of the mother’s time with the child falls.

5.2 Policy Analysis

We conclude this section with another type of comparative statics exercise that we could possibly think of as a very simple policy experiment. In France, to cite one example, parents are given significant transfers from the government over a substantial period of time, as well as large tax breaks, for their third and fourth children. Even in the U.S., there are tax breaks given to families with children across the entire income distribution, and direct transfers to targeted households, most often those headed by single parents. The question addressed in this exercise is the impact of such transfers on child cognitive outcomes. Of course, this is a very narrow measure of the impact of such transfer policies. The French policy has as a major objective increasing fertility, a decision not modeled here. However, all such policies have as an objective increasing the welfare of children, and their cognitive ability is an important contributor to life chances and success.

The policies we study provide an amount x to a one-child family each year during

at least a portion of the development process. Motivated by some of the recent work of Cunha and Heckman (2008), in which it is argued that child investments in early years are more productive than are investments made in later years, we look at the efficacy of making monetary transfers to the household when the child is in the early stage of the development process as opposed to later years. We consider the impact on child outcomes at the end of the development process of three potential transfer patterns: (1) a \$500 transfer per week during the first half of the development process; (2) a \$500 transfer per week during the second half of the development process; and (3) a \$250 transfer per week during each week of the development process. These amounts have been selected so as to equate the total value of transfers under the three schemes.^{30,31} Our model specification, in terms of both production functions and household preferences is very different from that of Cunha and Heckman, so that there is no presumption that the effects we find will be similar.

We have examined the impact of the transfers using a “restricted” and “unrestricted” case. In the first, there is no constraint on how the transfer of x can be spent by the household. In particular, a household may spend less than x on child goods investment if it chooses to do so. In the restricted case, the household is monitored to ensure that at least x dollars are spent on the child. A simple way to enforce this without monitoring is for the planner to make direct transfers of x dollars worth of child goods to the household (assuming no resale market).

The results are presented in Table 13. The amounts are the *percentage change from baseline* for each of the transfer schemes. The first column contains the results from the unrestricted transfers case, for which we only consider the effect of a transfer of \$250 per week over the entire development period. We see that the transfers have a small impact on latent ability. However, by far the greatest impact in the column is associated with the parental labor supplies, which fall markedly. The time which is taken from labor market activity is spent on time investments with the child and on parental leisure. The reduction in earned labor income in the household is not necessarily a bad thing from the point of view of child development, because we have found that in most periods time expenditures of whatever type have a larger impact on cognitive improvements than do money expenditures. The patterns of substitution that are estimated result in relatively small net impacts on cognitive ability at the end of this development period.

The remaining columns of the table contain the results of the experiments in which we change the size and dates of the transfer and restrict all households to spend at least as much money on the child as the transfer amount the household receives. The results in column 2 are directly comparable to those in column 1 since the transfer of \$250 is received over the entire development period. We note larger average effects on child quality in the

³⁰Due to the fact that our development process consists of an odd number of years, the total number of periods of transfer in the early phase case is one year greater than the number in the late phase case.

³¹The transfer amounts are undoubtedly large, but the amounts chosen are totally arbitrary. We are interested in comparing the relative increases in child quality across transfer schemes and do not focus on the sizes of the changes, per se.

“constrained” case. This impact is largely due to the presence of households that put little weight on child quality. The requirement that \$250 dollars be spent on the child in a given week is strongly binding for a substantial proportion of households with a low preference weight on child quality and/or low household income. This constraint also is seen in the lower percentage reductions in parental labor supply, since for households whose pre-transfer level of expenditures on the child was substantially less than the amount of the transfer, there is essentially no income effect from receiving it. The other behavioral responses are also dampened, with the exception being the enormous percentage change in money expenditures on the child.

The last two columns in Table 13 contrast the impacts of the timing of transfers on child outcomes. Since the size of the transfer has doubled in the periods in which it is received, we expect that a larger share of households will be impacted by the requirement that they spend at least \$500 dollars in child investment during periods in which the transfer is received. In fact, we see this most clearly in the experiment in which transfers are made early in the development period. The percentage gain in child investment expenditures is huge, though the impact on final child quality is almost nonexistent. This is due to a number of factors, the most important probably being the fact that time investments in the child at younger ages are significantly more productive than are money investments.

The most successful transfer scheme from the point of view of increasing child quality is the one in which transfers are concentrated at the later stage of the development process. The percentage change is over twice as large as the next best transfer scheme (in this metric), which gives the household half the weekly transfer over the entire development period. Once again, there is a very large impact on child good expenditures, but the important thing here is that the relative productivity of time inputs (with respect to money inputs) is less during this stage of the development process. While there is substantially more inertia in child quality at the end of the process, it is also the case that the money expenditures on child goods at this stage have a more direct impact on the end of process level of cognitive ability due to their temporal proximity.

6 Conclusion

Using a simple dynamic production technology for child quality and a Cobb-Douglas specification of a household utility function, we employ unique data from the PSID-CDS on investments in children to recover estimates of the parameters that characterize the child development process. We view the main message of the paper to be that household time and money investments in children can only be properly understood when household preferences, production technologies, and choice sets are simultaneously considered. We found that while the average household attaches a substantial weight to child quality in its utility function, its welfare is by no means only tied to the cognitive performance of the child. In terms of the child cognitive ability production technology, we found that parental time

inputs were more valuable in producing child quality than were money expenditures on children (at least those made by the household). The value of parental time inputs decreased with the age of the child, while there was some increase in the value of money inputs as the child matured. These results are somewhat consistent with those of Cunha and Heckman (2008) and Cunha et al. (2010) in that we find that gains to investment are greatest when the child is young. However, our analysis suggests that the productivity of money investments in children (by the household) have limited impacts on child quality no matter what the stage of development.

These results are potentially important for the design of policies to increase the cognitive ability of children. As is demonstrated in the comparative statics exercises we perform using the estimated primitive parameters, changes in the nonlabor income of the household have limited impacts on the cognitive ability of the child. This result is due to the greater importance of time inputs in the growth of cognitive ability and the fact that gains in household resources are spread over parental leisure and household consumption in addition to child investment. The complex substitution patterns that are generated result in small gains in child cognitive ability.

For the most part we have focused on the case of one-child households. However, we have also estimated a two-child household extension, in which many more types of time investment patterns are considered and the household utility function is expanded to include two child quality levels instead of one. We find that the production technologies display similar patterns in the one- and two-child household cases. An interesting result from the two-child analysis is that there is not a significant relationship between birth order and final cognitive ability levels between the two siblings, even though we have allowed for different preference weights in the household utility function and given that the siblings are exposed to different resource constraints over the course of their development process (we excluded twins who would face identical resource constraints). Since our model merely conditions on the number of children in the household, we are not able to further explore the mechanism by which (initially) one-child households decide to have a second child.

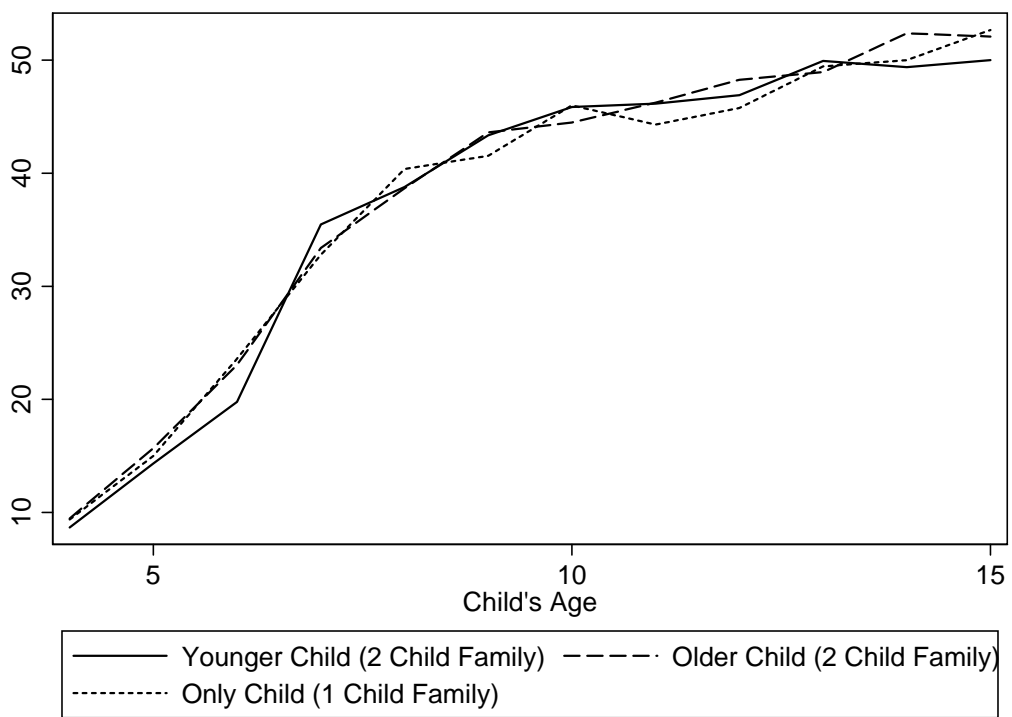
We find that both mothers' and fathers' time inputs are important for the cognitive development of their children and that their productivities decline with the age of the child. Two important factors of production, which are neglected here, are potentially capable of rationalizing this pattern. First, we do not consider formal schooling, the impact of which is likely to increase with the age of the child. Second, as in most of the literature, we do not consider the self-investment of children. We know from the CDS that children on average spend more time in self-investment (as measured by their time spent studying alone) as they mature, and it is likely that this type of investment is supplanting investment time by the parents. The incorporation of both of these factors into our modeling framework is the focus of our current research.

References

- [1] Bernal, R. (2008), “The Effect Of Maternal Employment And Child Care On Children’s Cognitive Development,” *International Economic Review*, 49, 1173-1209.
- [2] Blau, D., (1999). “The Effect of Income on Child Development”, *Review of Economics and Statistics*, 81, 261-276.
- [3] Cardoso, R. A., Fontainha, E. and Monfardini, C. (2010), “Children and Parents’ Time Use: Empirical Evidence on investment in Human Capital in France, Italy and Germany,” *Review of the Economics of the Household*, 8, 479-504.
- [4] Carneiro, P. and Heckman, J. (2003), “Human Capital Policy,” in *Inequality in America: What Role for Human Capital Policies* edited by Heckman J. and Krueger, A., Cambridge, MA: MIT Press
- [5] Cunha, F. and Heckman, J. (2008), “Formulating and Estimating the Technology of Cognitive and Noncognitive Skill Formation,” *Journal of Human Resources*, 43, 738-778.
- [6] Cunha, F., Heckman, J., and Schennach, S. (2010), “Estimating the Technology of Cognitive and Non-Cognitive Skill Formation,” *Econometrica*, 78, 883-931.
- [7] Del Boca, D., Flinn, C., and M. Wiswall (2012), “Transfers to Households with Children and Child Development, Working Paper 273, Collegio Carlo Alberto, Moncalieri, Italy.
- [8] Ermisch, J., and Francesconi, M. (2005), “Parental Employment and Children’s Welfare,” in *Women at Work: An Economic Perspective*, edited by Boeri, T., Del Boca, D., and Pissarides, C. Oxford: Oxford University Press.
- [9] Gugl E., and Welling,L. (2010), “The Early Bird Gets The Worm? Birth Order Effects In A Dynamic Family Model,” *Economic Inquiry*, 48,690-703.
- [10] Heckman J. and Masterov, D. (2007), “The Productivity Argument for Investing in Young Children,” *Review of Agricultural Economics*, 29, 446-493.
- [11] Hofferth, S., Davis-Kean, P., Davis, J., and Finkelstein, J., (1998), *User guide for the Child Development Supplement of the Panel Study of Income Dynamics*. <http://psidonline.isr.umich.edu/CDS/usergd.html>.
- [12] Liu, H., Mroz,T., and Van der Klaauw, W. (2010), “Maternal Employment, Migration, and Child Development”, *Journal of Econometrics*, 156, 212-228.

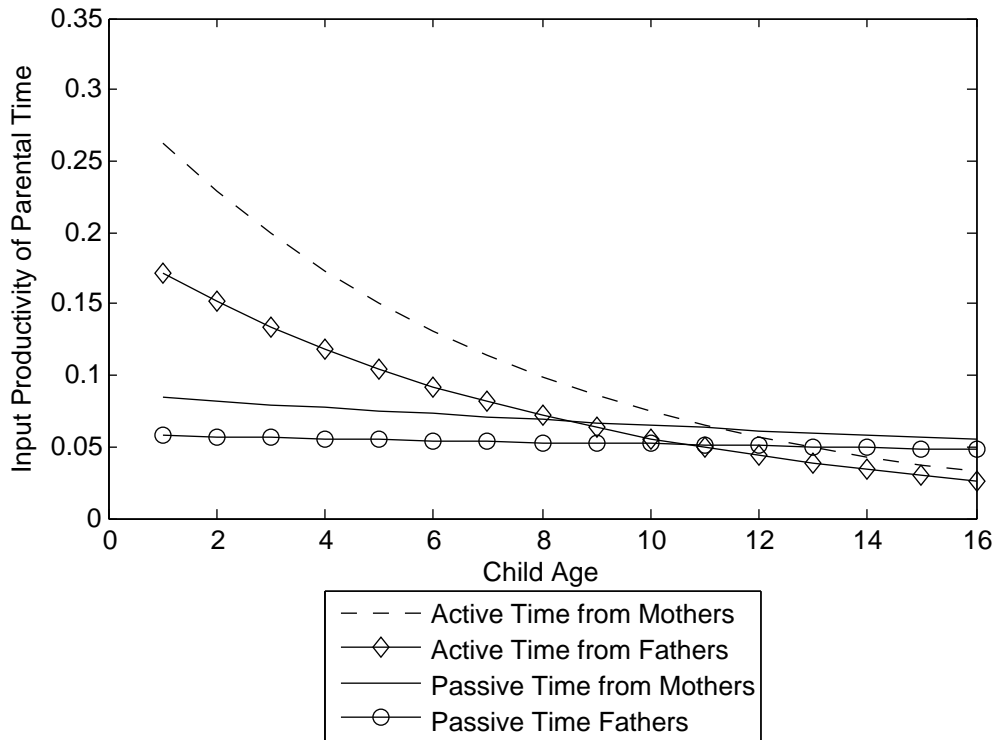
- [13] Loken, K., Mogstad, M., and Wiswall, M., (2012), "What Linear Estimators Miss: Re-Examining the Effects of Family Income on Child Outcomes", *American Economic Journals: Applied Economics*, 4, 1-35.
- [14] Lord, F. M. and Novick, M. (1968), *Statistical Theories of Mental Test Scores*. Reading MA: Addison-Wesley Publishing Company.
- [15] Pinheiro, J. and Bates, D., (2006), "Unconstrained Parametrizations for Variance-Covariance Matrices," *Statistics and Computing*, 6, 289-296.
- [16] Todd, P., and Wolpin, K. (2003), "On the Specification and Estimation of the Production Function for Cognitive Achievement," *Economic Journal*, 113, F3-F33.
- [17] Todd, P., and Wolpin, K. (2007), "The Production of Cognitive Achievement in Children: Home, School, and Racial Test Score Gaps," *Journal of Human Capital*, 1, 91-136.
- [18] Woodcock, R., and Johnson, M. (1989), *Tests of Achievement, Standard Battery [Form B]*. Chicago, IL: Riverside Publishing (Houghton Mifflin).

Figure 1: Average Child's Letter Word Score



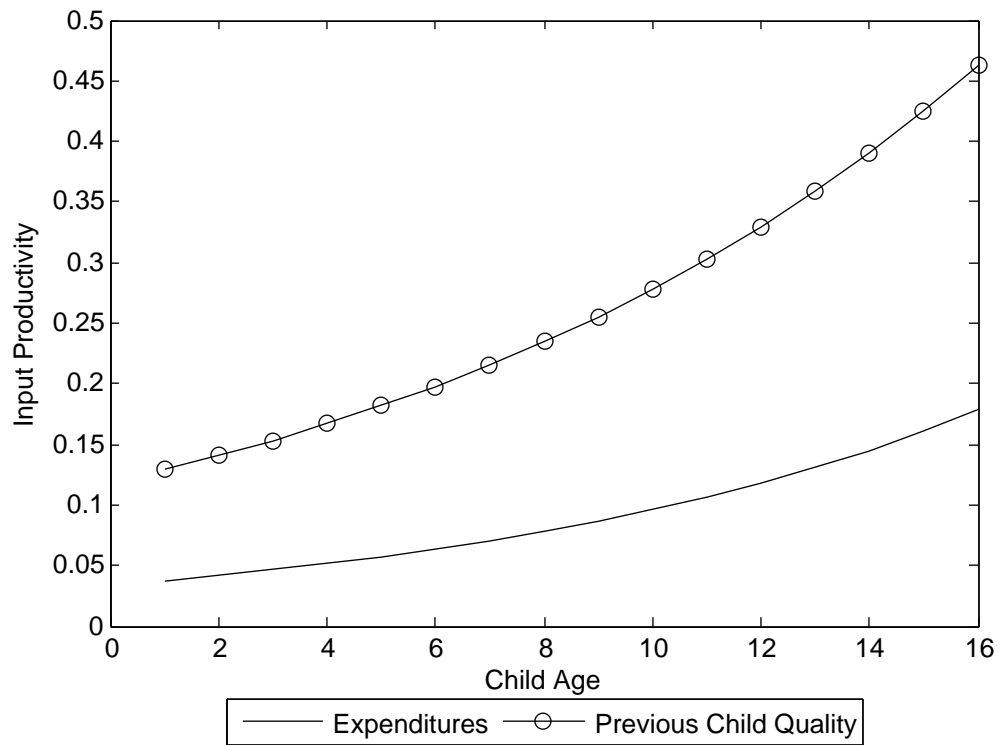
Source: PSID-CDS combined sample from 1997 and 2002 interviews and 1997, 1999, 2001, 2003 PSID core data.

Figure 2: Estimated Child Development Parameters by Child Age (1 Child Model)



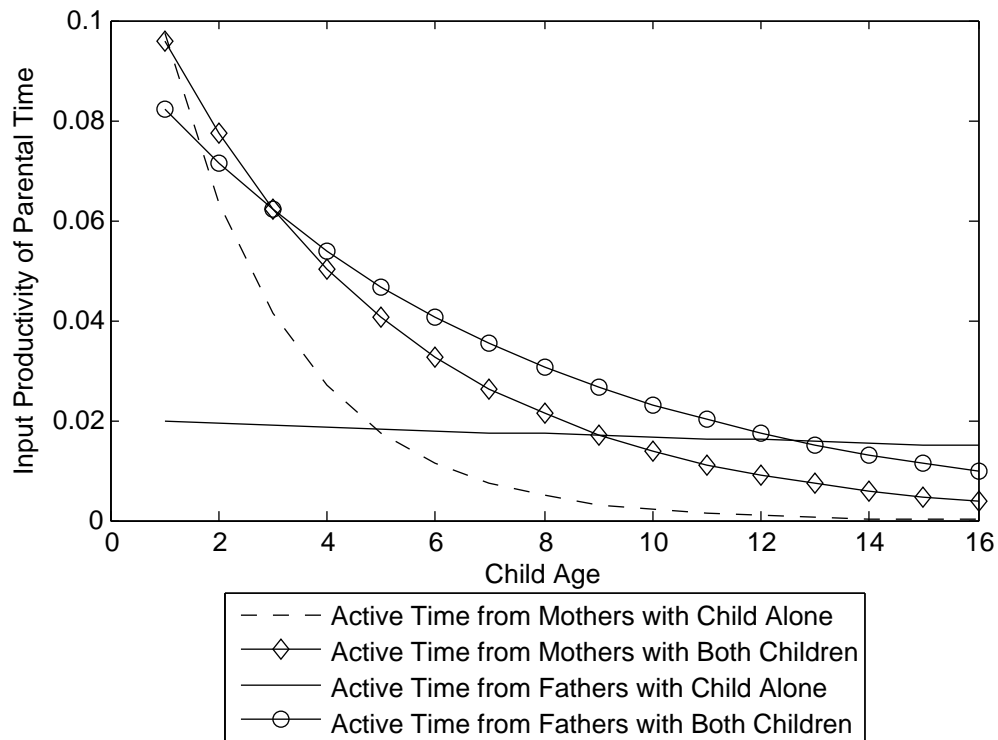
Notes: This graphs estimated parameters by child age (from Table 6).

Figure 3: Estimated Child Development Parameters by Child Age (1 Child Model)



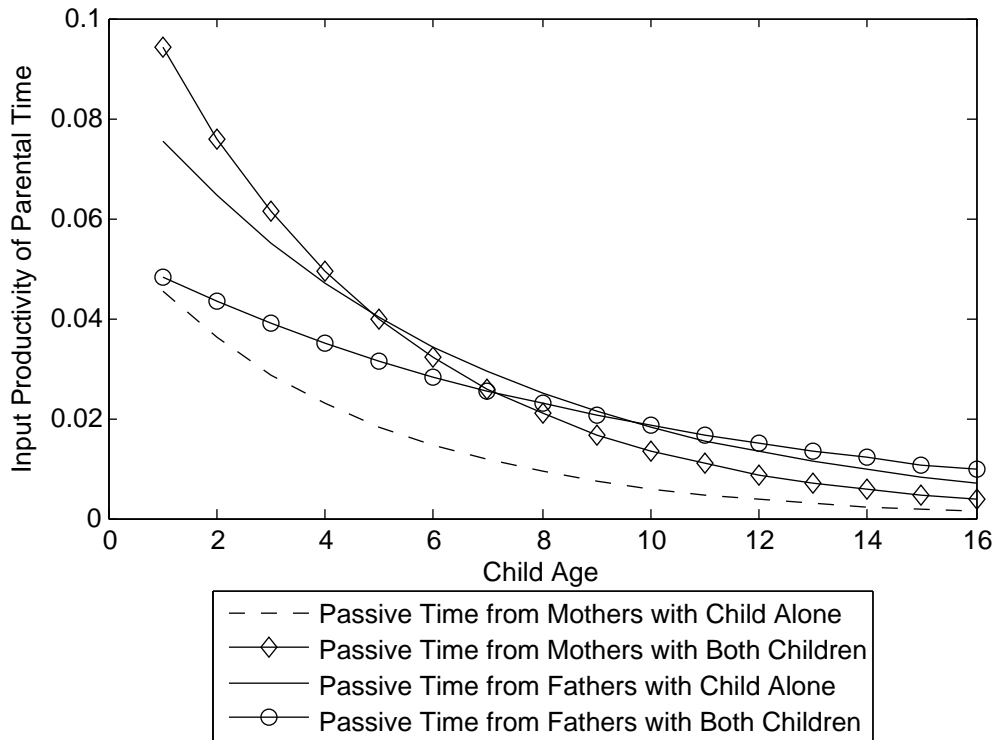
Notes: This graphs estimated parameters by child age (from Table 6).

Figure 4: Estimated Child Development Parameters by Child Age (2 Child Model)



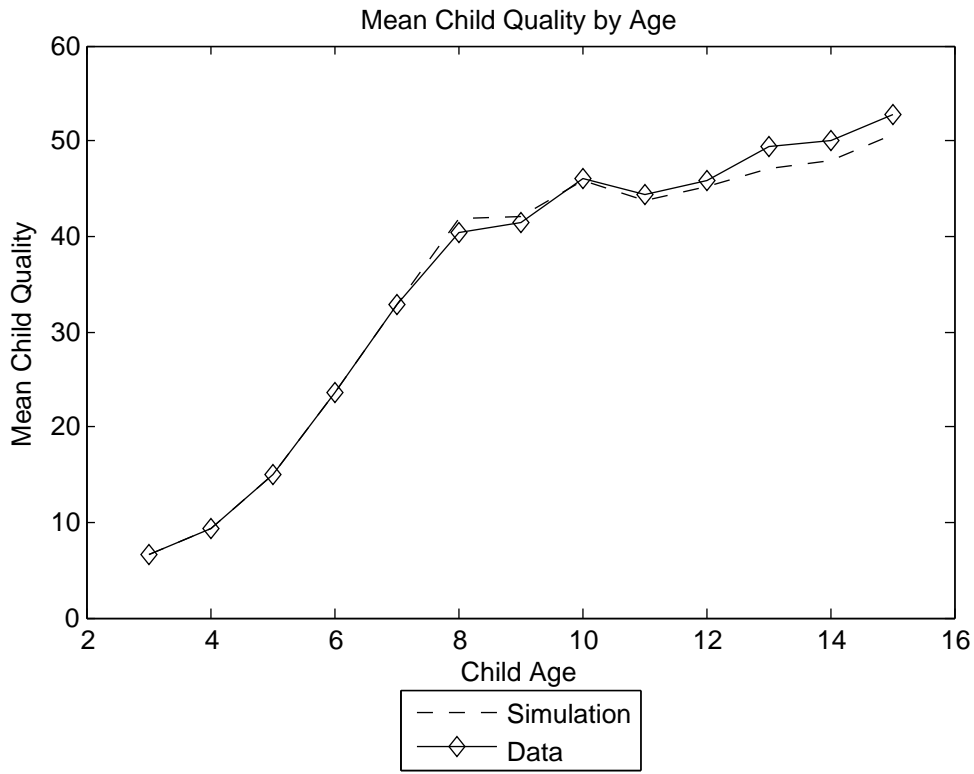
Notes: This graphs estimated parameters by child age (from Table 7).

Figure 5: Estimated Child Development Parameters by Child Age (2 Child Model)



Notes: This graphs estimated parameters by child age (from Table 7).

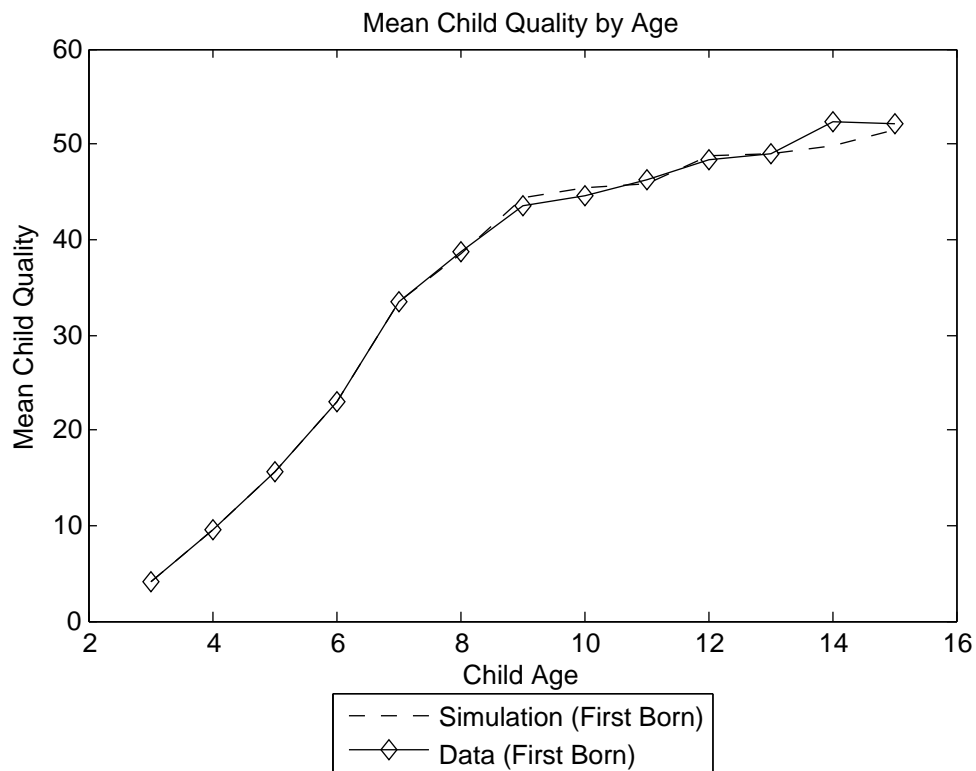
Figure 6: Simulated and Actual Average Letter Word Score by Child Age (1 Child Family)



Notes: Data is actual data from sample of intact households (mother and father present in household) with one child. Simulated is the model prediction at estimated parameters given above.

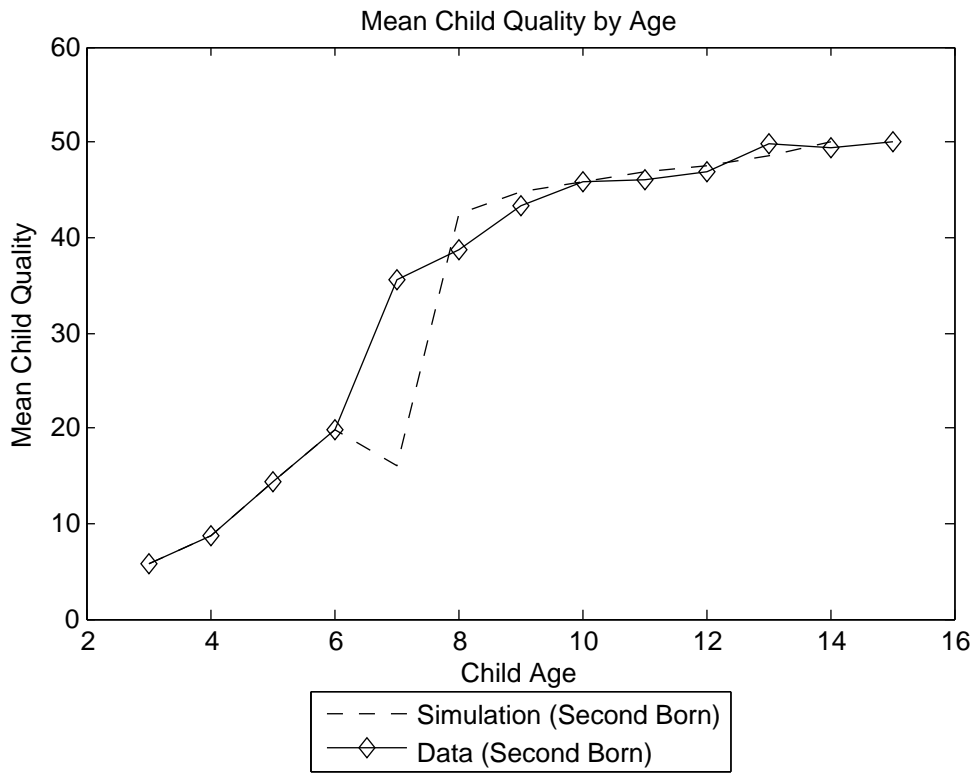
Source: PSID-CDS combined sample from 1997 and 2002 interviews and 1997, 1999, 2001, 2003 PSID core data.

Figure 7: Sample Fit of Average Child's Letter Word Score (2 Child Family, First Born)



Source: PSID-CDS combined sample from 1997 and 2002 interviews and 1997, 1999, 2001, 2003 PSID core data.

Figure 8: Sample Fit of Average Child's Letter Word Score (2 Child Family, Second Born)



Source: PSID-CDS combined sample from 1997 and 2002 interviews and 1997, 1999, 2001, 2003 PSID core data.

Figure 9: Optimal Ratio of Mother's and Father's Time with Child under Different Modeling Assumptions

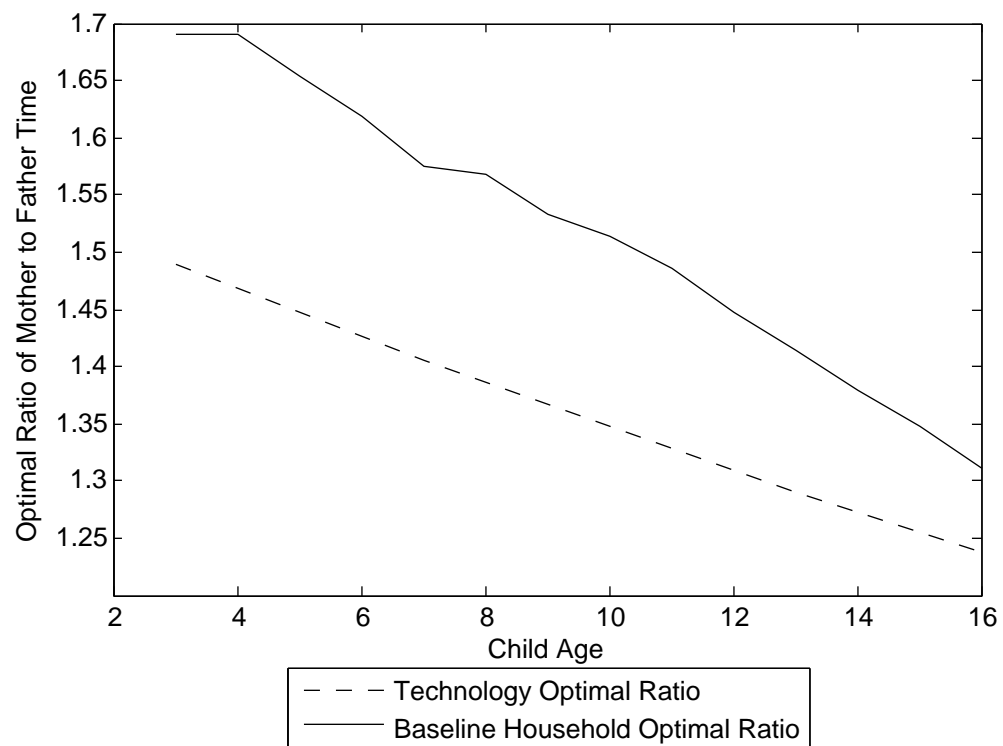


Figure 10: Optimal Ratio of Mother's Time with Child and Child Expenditures under Different Modeling Assumptions

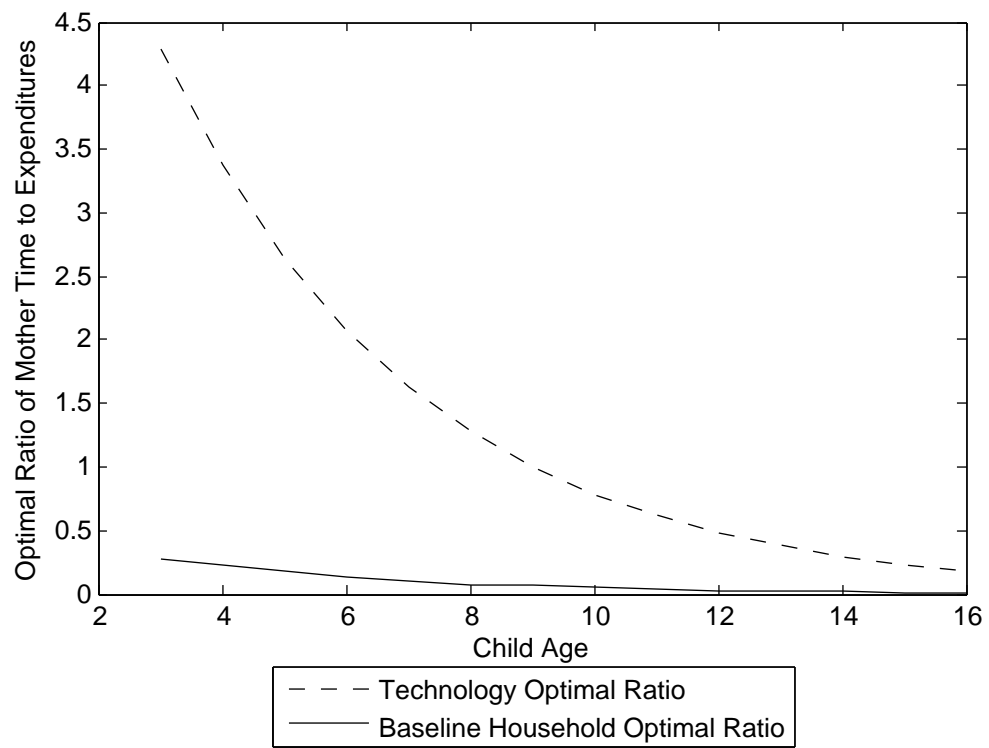


Table 1: Descriptive Statistics

1997 PSID-CDS				
	One Child		Two Child	
	Mean	Std.	Mean	Std.
Mother's age	34.78	6.33	33.83	5.40
Father's age	37.28	8.20	36.10	6.38
Mother's education	13.50	2.22	13.67	2.19
Father's education	13.55	2.23	13.54	2.79
Birth Spacing			2.73	1.18
			Older	
Child's age	6.32	2.97	7.77	2.45
Fraction Male	0.495	0.502	0.470	0.501
Mean Letter Word raw score	23.91	16.61	32.64	14.45
Median LW raw score	21		37.5	
Minimum LW raw score	1		4	
Maximum LW raw score	55		55	
			Younger	
Child's age			5.04	2.26
Fraction Male			0.515	0.502
Mean Letter Word raw score			20.46	14.95
Median LW raw score			13	
Minimum LW raw score			1	
Maximum LW raw score			57	
1996-2002 PSID				
	One Child		Two Child	
	Mean	Std.	Mean	Std.
Mother's Work Hours per Week	32.19	16.65	26.91	18.37
Father's Work Hours per Week	43.79	11.21	45.35	11.30
Mother's Hourly Wage	15.38	10.24	14.86	8.56
Father's Hourly Wage	19.52	11.89	23.07	16.74
Non-Labor Income per Week	105.41	213.28	142.17	217.08

Notes: Sample of intact households (mother and father present in household) with one or two children. The top panel statistics are for the year 1997 from the 1997 PSID-CDS. Work hours, wages, and non-labor income statistics are averaged over all years of PSID data.

Source: PSID-CDS combined sample from 1997 and 2002 interviews and 1997, 1999, 2001, 2003 PSID core data.

Table 2: Parent's Labor Supply by Child Age

Fraction Working > 0 Hours						
Child Age	One Child		Younger Child		Older Child	
	Mother	Father	Mother	Father	Mother	Father
3	0.750	0.937	0.651	0.977	–	–
4-5	0.821	0.982	0.781	0.979	0.750	0.979
6-8	0.822	0.985	0.792	0.971	0.712	0.975
9-11	0.882	0.961	0.783	0.992	0.796	0.984
12-15	0.835	0.987	0.891	0.957	0.833	0.978

Average Hours Working						
Child Age	One Child		Younger Child		Older Child	
	Mother	Father	Mother	Father	Mother	Father
3	26.38	44.38	23.53	44.98	–	–
4-5	37.63	44.58	24.48	45.76	35.19	44.91
6-8	38.44	45.69	25.96	45.02	32.64	46.26
9-11	38.08	44.46	28.02	45.26	32.31	46.43
12-15	39.83	43.13	35.76	47.52	36.36	46.33

Notes: Sample of intact households (mother and father present in household) with one or two children.

Source: PSID-CDS combined sample from 1997 and 2002 interviews and 1997, 1999, 2001, 2003 PSID core data.

Table 3: **Parent's Time with Child by Child Age**

Active Time (Avg.)						
Child Age	One Child Families		Two Child Families			
	Mother	Father	Younger Child		Older Child	
			Mother	Father	Mother	Father
3	29.29	16.90	23.19	13.20	–	–
4-5	21.37	11.08	17.64	8.40	17.46	10.78
6-8	16.47	12.11	11.06	6.95	13.03	8.70
9-11	15.72	8.59	8.63	6.30	10.50	7.40
12-15	12.30	8.93	5.61	3.50	8.11	5.80

Passive Time (Avg.)						
Child Age	One Child Families		Two Child Families			
	Mother	Father	Younger Child		Older Child	
			Mother	Father	Mother	Father
3	12.45	5.16	17.99	5.50		
4-5	13.22	6.37	20.10	8.12	16.93	8.28
6-8	9.47	8.07	11.10	6.07	16.68	6.96
9-11	10.88	8.08	7.08	4.84	9.69	5.22
12-15	15.22	13.19	5.59	5.57	7.18	5.35

Notes: Sample of intact households (mother and father present in household) with one or two children. Child age for two child families is the age of either the younger or the older child.

Source: PSID-CDS combined sample from 1997 and 2002 interviews and 1997, 1999, 2001, 2003 PSID core data.

Table 4: **Joint Time Allocation of Parents**

Younger {passive,active,none}	Older {passive,active,none}	Mother's Time	Father's Time
active	-	4.49	2.38
passive	-	4.08	1.90
-	active	1.20	1.22
-	passive	1.87	1.73
active	active	11.45	7.09
active	passive	2.45	0.93
passive	active	1.86	1.16
passive	passive	10.72	4.65

Notes: Sample of intact households (mother and father present in household) with two children.

Source: PSID-CDS combined sample from 1997 and 2002 interviews and 1997, 1999, 2001, 2003 PSID core data.

Table 5: Preference Parameter Estimates

	1 Child		2 Child	
	Estimate	SE	Estimate	SE
Mean of α_1	0.196	(0.011)	0.170	(0.0062)
Mean of α_2	0.194	(0.0096)	0.233	(0.0075)
Mean of α_3	0.257	(0.016)	0.194	(0.0074)
Mean of α_4	0.353	(0.015)	–	
Mean of α_4 (Child 1)	–		0.185	(0.0040)
Mean of α_5 (Child 2)	–		0.217	(0.013)
Std. of α_1	0.121	(0.012)	0.084	(0.0049)
Std. of α_2	0.085	(0.010)	0.094	(0.0049)
Std. of α_3	0.093	(0.012)	0.095	(0.0078)
Std. of α_4	0.200	(0.015)	–	
Std. of α_4 (Child 1)	–		0.119	(0.0052)
Std. of α_5 (Child 2)	–		0.139	(0.0090)
Correlation of α_1 and α_2	0.360	(0.142)	0.764	(0.048)
Correlation of α_1 and α_3	-0.032	(0.158)	0.777	(0.048)
Correlation of α_2 and α_3	0.172	(0.194)	0.984	(0.014)
	Terminal Payoff to Child Quality			
ψ	28.89	(6.61)	–	
ψ_1 (Child 1)	–		2.58	(0.268)
ψ_2 (Child 2)	–		2.99	(0.393)

Table 6: **Technology Parameter Estimates (1 Child Families)**

	Estimate	SE
Mother's Active Time intercept	-1.33	(0.054)
Mother's Active Time slope	-0.139	(0.0030)
Father's Active Time intercept	-2.47	(0.016)
Father's Active Time slope	-0.029	(0.0033)
Mother's Passive Time intercept	-1.76	(0.029)
Mother's Passive Time slope	-0.125	(0.0023)
Father's Passive Time intercept	-2.86	(0.038)
Father's Passive Time slope	-0.012	(0.0054)
Child Expenditures intercept	-3.27	(0.037)
Child Expenditures slope	0.104	(0.0058)
Last Period's Child Quality intercept	-2.047	(0.027)
Last Period's Child Quality slope	0.085	(0.0068)

Table 7: **Technology Parameter Estimates (2 Child Families)**

	Estimate	SE
Mother's Parameters		
Active Child Alone intercept	-2.33	(0.0090)
Active Child Alone slope	-0.430	(0.0002)
Passive Child Alone intercept	-3.09	(0.0030)
Passive Child Alone slope	-0.227	(0.0003)
Active Both Children intercept	-2.35	(0.0056)
Active Both Children slope	-0.215	(0.0005)
Active Child 1, Passive Child 2 intercept	-3.58	(0.0038)
Active Child 1, Passive Child 2 slope	-0.236	(0.0003)
Passive Child 1, Active Child 2 intercept	-3.99	(0.0031)
Passive Child 1, Active Child 2 slope	-0.252	(0.0006)
Passive Both Children intercept	-2.36	(0.0035)
Passive Both Children slope	-0.216	(0.0002)
Father's Parameters		
Active Child Alone intercept	-3.93	(0.0021)
Active Child Alone slope	-0.018	(0.0003)
Passive Child Alone intercept	-2.58	(0.0051)
Passive Child Alone slope	-0.158	(0.0009)
Active Both Children intercept	-2.50	(0.0034)
Active Both Children slope	-0.141	(0.0003)
Active Child 1, Passive Child 2 intercept	-4.12	(0.0024)
Active Child 1, Passive Child 2 slope	-0.159	(0.0003)
Passive Child 1, Active Child 2 intercept	-4.23	(0.0044)
Passive Child 1, Active Child 2 slope	-0.225	(0.0004)
Passive Both Children intercept	-3.03	(0.0031)
Passive Both Children slope	-0.107	(0.0008)
Child Expenditures intercept	-1.83	(0.0039)
Child Expenditures slope	0.0017	(0.0008)
Last Period's Child Quality intercept	-1.873	(0.0046)
Last Period's Child Quality slope	0.112	(0.0012)

Table 8: **Wage and Income Parameter Estimates**

	1 Child		2 Child	
	Estimate	SE	Estimate	SE
Mother's Log Wage Offer				
μ_1^0 (Intercept)	1.4195	(0.072)	1.057	(0.014)
μ_1^1 (Mother's Education)	0.049	(0.0038)	0.070	(0.0003)
μ_1^2 (Mother's Age)	0.0044	(0.0003)	0.0068	(0.0001)
μ_1^3 (Mother's Age Sq x 1000)	0.161	(0.043)	0.225	(0.018)
μ_1^4 (Mother's Year of Birth x 1000)	0.076	(0.045)	-0.138	(0.013)
σ_1 (Standard Deviation of Innovation)	0.047	(0.0141)	0.185	(0.018)
ρ_{12} (Correlation with Father's Wage Shock)	0.710	(0.017)	0.753	(0.012)
Father's Log Wage Offer				
μ_2^0 (Intercept)	1.3694	(0.073)	1.12	(0.018)
μ_2^1 (Father's Education)	0.081	(0.0039)	0.102	(0.0005)
μ_2^2 (Father's Age)	0.0081	(0.0003)	0.0091	(0.0001)
μ_2^3 (Father's Age Sq x 1000)	-0.014	(0.049)	0.235	(0.0160)
μ_2^4 (Father's Year of Birth x 1000)	-0.0050	(0.031)	-0.134	(0.0090)
σ_2 (Standard Deviation of Innovation)	0.731	(0.094)	0.738	(0.039)
Latent Non-Labor Income				
μ_3^0 (Intercept)	-14.12	(36.61)	-32.14	(29.61)
σ_3 (Standard Deviation of Innovation)	376.16	(32.67)	352.30	(25.42)

Table 9: **Sample Fit for Wages and Income**

	1 Child		2 Child	
	Data	Simulated	Data	Simulated
Avg. Mother's Wage	15.38	16.62	14.86	13.34
Std. Mother's Wage	10.24	9.59	8.56	9.23
Avg. Father's Wage	19.52	18.42	23.07	23.79
Std. Father's Wage	11.89	10.72	16.74	15.68
Avg. Non-Labor Income	142.17	142.17	122.08	122.10
Std. Non-Labor Income	216.81	216.81	194.62	194.61
Fraction with 0 Non-Labor Income	0.621	0.621	0.633	0.658

	Data	Simulated	Data	Simulated
Avg. Mother's Wage (Mother's Age < 30)	14.08	13.23	9.84	10.42
Avg. Mother's Wage (Mother's Age \geq 40)	16.74	18.50	16.62	16.10
Avg. Father's Wage (Father's Age < 30)	14.13	14.10	12.35	13.56
Avg. Father's Wage (Father's Age \geq 40)	20.44	19.50	27.27	29.22

Notes: Data is actual data from sample of intact households (mother and father present in household) with one or two children. Simulated is the model prediction at estimated parameters given above.

Source: PSID-CDS combined sample from 1997 and 2002 interviews and 1997, 1999, 2001, 2003 PSID core data.

Table 10: **Sample Fit of Mother and Father’s Time Allocation by Child Age (1 Child Families)**

Probability Work > 0 Hours				
	Mother		Father	
Child Age	Data	Simulated	Data	Simulated
3-5	0.806	0.784	0.986	0.980
6-8	0.822	0.859	0.985	0.978
9-11	0.882	0.874	0.961	0.982
12-15	0.835	0.935	0.987	0.989
Hours Worked if Work (Avg.)				
	Mother		Father	
Child Age	Data	Simulated	Data	Simulated
3-5	34.44	38.71	46.18	45.02
6-8	32.43	37.61	48.31	44.94
9-11	33.86	37.30	43.29	44.55
12-15	28.65	36.32	45.18	44.72
Active Time (Avg.)				
	Mother		Father	
Child Age	Data	Simulated	Data	Simulated
3-5	25.56	22.34	14.16	13.31
6-8	16.48	16.49	12.11	10.39
9-11	15.72	12.70	8.59	8.40
12-15	12.30	14.94	8.93	10.74
Passive Time (Avg.)				
	Mother		Father	
Child Age	Data	Simulated	Data	Simulated
3-5	12.82	10.03	5.73	6.31
6-8	9.47	10.29	8.07	6.92
9-11	10.88	11.01	8.08	7.84
12-15	15.22	19.44	13.19	15.26

Notes: Data is actual data from sample of intact households (mother and father present in household) with one child. Simulated is the model prediction at estimated parameters given above.

Source: PSID-CDS combined sample from 1997 and 2002 interviews and 1997, 1999, 2001, 2003 PSID core data.

Table 11: **Sample Fit of Mother and Father’s Time Allocation by Child Age (2 Child Families)**

Probability Work > 0 Hours				
Child 1’s Age	Mother		Father	
	Data	Simulated	Data	Simulated
3-5	0.771	0.589	0.967	0.996
6-8	0.712	0.721	0.975	0.994
9-11	0.796	0.818	0.984	0.992
12-15	0.833	0.881	0.978	0.997
Hours Worked if Work (Avg.)				
Child 1’s Age	Mother		Father	
	Data	Simulated	Data	Simulated
3-5	30.10	31.37	47.82	44.96
6-8	36.71	28.93	48.73	46.15
9-11	28.94	29.52	46.05	46.24
12-15	28.55	28.09	46.83	46.27
Child 1 Active Time (Avg.)				
Child 1’s Age	Mother		Father	
	Data	Simulated	Data	Simulated
3-5	18.14	17.93	10.56	10.10
6-8	13.02	13.49	8.70	8.99
9-11	10.50	9.99	7.40	8.05
12-15	8.11	6.47	5.80	7.17
Child 1 Passive Time (Avg.)				
Child 1’s Age	Mother		Father	
	Data	Simulated	Data	Simulated
3-5	16.99	17.56	8.26	8.50
6-8	16.68	13.78	6.97	7.47
9-11	9.69	10.42	5.22	6.58
12-15	7.18	6.81	5.35	5.68
Child 2 Active Time (Avg.)				
Child 2’s Age	Mother		Father	
	Data	Simulated	Data	Simulated
3-5	19.95	16.59	10.30	9.41
6-8	11.18	10.84	6.94	8.04
9-11	8.60	7.26	6.18	7.19
12-15	5.86	4.53	3.43	6.45
Child 2 Passive Time (Avg.)				
Child 2’s Age	Mother		Father	
	Data	Simulated	Data	Simulated
3-5	19.31	15.95	7.27	8.79
6-8	11.02	11.31	6.13	7.33
9-11	7.12	7.90	4.95	6.37
12-15	5.47	5.07	5.81	5.36

Notes: Data is actual data from sample of intact households (mother and father present in household) with two children. Simulated is the model prediction at estimated parameters given above. Child 1 is the first born child in the family.

Source: PSID-CDS combined sample from 1997 and 2002 interviews and 1997, 1999, 2001, 2003 PSID core data.

Table 12: **Optimal Decisions with Alternative Preferences**

	Level at Baseline	Child Quality Maximizing Preferences	Selfish Parent Preferences
Mean Latent Child Quality (Age 16)	13.38	19.20	0
Mean Hours Work (Mother)	33.25	16.38	43.30
Mean Hours Work (Father)	44.02	40.29	43.31
Mean Active Time w/ Child (Mother)	15.66	52.60	0
Mean Active Time w/ Child (Father)	10.38	38.43	0
Mean Passive Time w/ Child (Mother)	13.96	43.02	0
Mean Passive Time w/ Child (Father)	10.29	33.28	0
Mean Leisure (Mother)	49.13	0	68.70
Mean Leisure (Father)	47.31	0	68.69
Mean Child Expenditures / 1000	0.436	1.211	0
Mean Household Consumption / 1000	1.11	0	1.69
Mean Utility /1000	0.0632	0.0091	0.056

Notes: Child Quality Maximizing Preferences set the preference weight on parental leisure and consumption to 0: $\alpha_1 = \alpha_2 = \alpha_3 = 0$. Under these preferences, the household then maximizes the level child quality, and consumption $c_t = 0$ for all t . Selfish Parent Preferences set $\alpha_4 = 0$, and the household puts no weight on child quality. With these preferences, all child inputs equal 0 for all t . Mean Latent Child Quality (Age 16) is the latent value of child quality at the end of age 16 or the start of period $t = 17$, k_{17} .

Table 13: Counterfactual Simulations: Untargeted and Targeted Transfers

	(1)	(2)	(3)	(4)
	Untargeted Trans.	Targeted Child Goods Transfer		
	\$250 in	\$250 in	\$500 in	\$500 in
	Non-Labor Income	Child Goods	Child Goods	Child Goods
	All Ages	All Ages	Ages 3-9 Only	Ages 10-15 Only
	Percent Change from Baseline			
Mean Latent Child Quality (Age 16)	1.61	4.628	0.024	9.42
Mean Hours Work (Mother)	-15.12	-10.36	-4.29	-10.69
Mean Hours Work (Father)	-12.62	-7.55	-3.78	-6.92
Mean Active Time w/ Child (Mother)	6.13	4.85	2.55	4.64
Mean Active Time w/ Child (Father)	8.22	6.17	3.81	5.32
Mean Passive Time w/ Child (Mother)	5.86	5.63	1.93	7.17
Mean Passive Time w/ Child (Father)	7.13	6.58	2.62	8.09
Mean Leisure (Mother)	5.32	3.01	1.17	2.88
Mean Leisure (Father)	8.53	4.33	2.14	3.62
Mean Child Expenditures / 1000	4.90	28.77	51.61	26.10
Mean Household Consumption / 1000	6.88	4.16	2.28	3.42
Mean Utility /1000	6.13	4.41	2.86	3.73

Notes: All values are the percentage change from the baseline values given in prior tables. Experiment (1) provides \$250 in non-labor income per week to all households at all child ages. Experiment (2) provides a subsidy of \$250 in non-labor income for the household and sets child goods expenditures at a minimum level of \$250 at all child ages. Experiment (3) provides a subsidy of \$500 and sets child goods expenditures at a minimum level of \$500 for child ages 3-9 only; children at older ages receive no subsidy. Experiment (4) provides a subsidy of \$500 and sets child goods expenditures at a minimum level of \$500 for child ages 10-15; children at younger ages receive no subsidy. Mean Latent Child Quality (Age 16) is the latent value of child quality at the end of age 16 or the start of period $t = 17$, k_{17} .

A Appendix: Model Solution for the Two-Child Case

We describe the model solution for the two-child case which uses a fine disaggregation of time investment expenditures by the parents during the phase in which the household is investing in the cognitive development of both children. For each child, time $t + 1$ child quality is determined by the current level of child quality, k_t^b , for child $b = 1, 2$, parental time investments in the child, and expenditures on the child, all of which are made when the child is age t . We allow for various types of parental time investments in the children, which vary on their intensity of interaction (passive (p) versus active (a)), whether a sibling is present, and the intensity of the interaction with the sibling if the sibling is present. Our model then distinguishes between time the parents spend with each child separately and time the parents spend with both children present. At any given instant, each parent ($j = 1$ mother and $j = 2$ father) can make one of eight different types of time investments:

1. $\tau_{j,t}(a, 0)$ (active time of parent j with first born alone)
2. $\tau_{j,t}(0, a)$ (active time of parent j with second born alone)
3. $\tau_{j,t}(p, 0)$ (passive time of parent j with first born alone)
4. $\tau_{j,t}(0, p)$ (passive time of parent j with second born alone)
5. $\tau_{j,t}(a, a)$ (active time with first born and active time second born jointly)
6. $\tau_{j,t}(p, p)$ (passive time with first born and passive time with second born jointly)
7. $\tau_{j,t}(a, p)$ (active time with first born and passive time with second born jointly)
8. $\tau_{j,t}(p, a)$ (passive time with first born and active time with second born jointly)

The input demand equations follow closely the one-child case. The time inputs from parent j are

$$\tau_{j,t}(a, 0) = (T - h_{jt}) \frac{\beta \eta_{1,t+1} \delta_{j,t}(a, 0)}{\alpha_j + \Delta_{j,t}}$$

$$\tau_{j,t}(p, 0) = (T - h_{jt}) \frac{\beta \eta_{1,t+1} \delta_{j,t}(p, 0)}{\alpha_j + \Delta_{j,t}}$$

$$\tau_{j,t}(0, a) = (T - h_{jt}) \frac{\beta \eta_{2,t+1} \delta_{j,t}(0, a)}{\alpha_j + \Delta_{j,t}}$$

$$\tau_{j,t}(0, p) = (T - h_{jt}) \frac{\beta \eta_{2,t+1} \delta_{j,t}(0, p)}{\alpha_j + \Delta_{j,t}}$$

$$\begin{aligned}
\tau_{j,t}(a, a) &= (T - h_{jt}) \frac{\beta(\eta_{1,t+1}\delta_{j,t}(a, a) + \eta_{2,t+1}\delta_{j,t}(a, a))}{\alpha_j + \Delta_{j,t}} \\
\tau_{j,t}(p, p) &= (T - h_{jt}) \frac{\beta(\eta_{1,t+1}\delta_{j,t}(p, p) + \eta_{2,t+1}\delta_{j,t}(p, p))}{\alpha_j + \Delta_{j,t}} \\
\tau_{j,t}(a, p) &= (T - h_{jt}) \frac{\beta(\eta_{1,t+1}\delta_{j,t}(a, p) + \eta_{2,t+1}\delta_{j,t}(p, a))}{\alpha_j + \Delta_{j,t}} \\
\tau_{j,t}(p, a) &= (T - h_{jt}) \frac{\beta(\eta_{1,t+1}\delta_{j,t}(p, a) + \eta_{2,t+1}\delta_{j,t}(a, p))}{\alpha_j + \Delta_{j,t}}
\end{aligned}$$

where

$$\begin{aligned}
\Delta_{j,t} &= \beta[\eta_{1,t+1}(\delta_{j,t}(a, 0) + \delta_{j,t}(p, 0)) + \eta_{2,t+1}(\delta_{j,t}(0, a) + \delta_{j,t}(0, p))] \\
&+ (\eta_{1,t+1}\delta_{j,t}(a, a) + \eta_{2,t+1}\delta_{j,t}(a, a)) + (\eta_{1,t+1}\delta_{j,t}(p, p) + \eta_{2,t+1}\delta_{j,t}(p, p)) \\
&+ (\eta_{1,t+1}\delta_{j,t}(a, p) + \eta_{2,t+1}\delta_{j,t}(p, a)) + (\eta_{1,t+1}\delta_{j,t}(p, a) + \eta_{2,t+1}\delta_{j,t}(a, p))
\end{aligned}$$

The latent labor supply terms (12) can then be found by substituting for the new values of $\Delta_{1,t}$ and $\Delta_{2,t}$ given here. $\Delta_{3,t}$ remains the same.

B Appendix: List of Moments used in Estimation of 1 Child Model

Child's Age 3-5

- 1) Avg. Mother's Work Hours
- 2) Std. Mother's Work Hours
- 3) Avg. Father's Work Hours
- 4) Std. Father's Work Hours
- 5) Avg. Mother's Active Time
- 6) Std. Mother's Active Time
- 7) Avg. Father's Active Time
- 8) Std. Father's Active Time
- 9) Avg. Mother's Passive Time
- 10) Std. Mother's Passive Time
- 11) Avg. Father's Passive Time
- 12) Std. Father's Passive Time
- 13) Prob. Mother's Work Hours > 0
- 14) Prob. Father's Work Hours > 0
- 15) Prob. Both Work Hours > 0

Child's Age 6-8

- 16) Avg. Mother's Work Hours
- 17) Std. Mother's Work Hours
- 18) Avg. Father's Work Hours
- 19) Std. Father's Work Hours
- 20) Avg. Mother's Active Time
- 21) Std. Mother's Active Time
- 22) Avg. Father's Active Time
- 23) Std. Father's Active Time
- 24) Avg. Mother's Passive Time
- 25) Std. Mother's Passive Time
- 26) Avg. Father's Passive Time
- 27) Std. Father's Passive Time
- 28) Prob. Mother's Work Hours > 0
- 29) Prob. Father's Work Hours > 0
- 30) Prob. Both Work Hours > 0

Child's Age 9-11

- 31) Avg. Mother's Work Hours
- 32) Std. Mother's Work Hours
- 33) Avg. Father's Work Hours
- 34) Std. Father's Work Hours
- 35) Avg. Mother's Active Time
- 36) Std. Mother's Active Time

- 37) Avg. Father's Active Time
- 38) Std. Father's Active Time
- 39) Avg. Mother's Passive Time
- 40) Std. Mother's Passive Time
- 41) Avg. Father's Passive Time
- 42) Std. Father's Passive Time
- 43) Prob. Mother's Work Hours > 0
- 44) Prob. Father's Work Hours > 0
- 45) Prob. Both Work Hours > 0
- Child's Age 12-15
- 46) Avg. Mother's Work Hours
- 47) Std. Mother's Work Hours
- 48) Avg. Father's Work Hours
- 49) Std. Father's Work Hours
- 50) Avg. Mother's Active Time
- 51) Std. Mother's Active Time
- 52) Avg. Father's Active Time
- 53) Std. Father's Active Time
- 54) Avg. Mother's Passive Time
- 55) Std. Mother's Passive Time
- 56) Avg. Father's Passive Time
- 57) Std. Father's Passive Time
- 58) Prob. Mother's Work Hours > 0
- 59) Prob. Father's Work Hours > 0
- 60) Prob. Both Work Hours > 0
- 61) Avg. LW Score, Age 8
- 62) Avg. LW Score, Age 9
- 63) Avg. LW Score, Age 10
- 64) Avg. LW Score, Age 11
- 65) Avg. LW Score, Age 12
- 66) Avg. LW Score, Age 13
- 67) Avg. LW Score, Age 14
- 68) Avg. LW Score, Age 15
- 69) Avg. Mother's Wage
- 70) Std. Mother's Wage
- 71) Avg. Father's Wage
- 72) Std. Father's Wage
- 73) Corr. in Mom's Work Hours and Dad's Work Hours
- 74) Corr. in Mom's Wage and Mom's Work Hours
- 75) Corr. in Dad's Wage and Dad's Work Hours
- 76) Corr. in Dad's Wage and Mom's Work Hours

- 77) Corr. in Mom's Wage and Dad's Hours
- 78) Corr. Mom's Work Hours (t-1), Mom's Active Time (t)
- 79) Corr. Dad's Work Hours (t-1), Dad's Active Time (t)
- 80) Corr. Dad's Work Hours (t-1), Mom's Active Time (t)
- 81) Corr. Mom's Work Hours (t-1), Dad's Active Time (t)
- 82) Corr. Dad's Wage (t-1), Mom's Active Time (t)
- 83) Corr. Dad's Wage (t-1), Dad's Active Time (t)
- 84) Corr. Mom's Wage (t-1), Mom's Active Time (t)
- 85) Corr. Mom's Wage (t-1), Dad's Active Time (t)
- 86) Corr. Mom's Wage (t-1), Mom's Passive Time (t)
- 87) Corr. Dad's Wage (t-1), Dad's Passive Time (t)
- 88) Corr. Mom's Work Hours (t-1), Mom's Passive Time (t)
- 89) Corr. Dad's Work Hours (t-1), Dad's Passive Time (t)
- 90) Corr. Dad's Work Hours (t-1), Mom's Passive Time (t)
- 91) Corr. Mom's Work Hours (t-1), Dad's Passive Time (t)
- Average Father's Wage for Sub-Groups:
- 92) Father \geq 16 years of schooling
- 93) Mother \geq 16 years of schooling
- 94) Father = 12 years of schooling
- 95) Mother = 12 years of schooling
- 96) Father \geq 40 years of age
- 97) Mother \geq 40 years of age
- 98) Father $<$ 30 years of age
- 99) Mother $<$ 30 years of age
- Average Mother's Wage for Sub-Groups:
- 100) Father \geq 16 years of schooling
- 101) Mother \geq 16 years of schooling
- 102) Father = 12 years of schooling
- 103) Mother = 12 years of schooling
- 104) Father \geq 40 years of age
- 105) Mother \geq 40 years of age
- 106) Father $<$ 30 years of age
- 107) Mother $<$ 30 years of age
- Average Father's Work Hours for Sub-Groups:
- 108) Father \geq 16 years of schooling
- 109) Mother \geq 16 years of schooling
- 110) Father = 12 years of schooling
- 111) Mother = 12 years of schooling
- 112) Father \geq 40 years of age
- 113) Mother \geq 40 years of age
- 114) Father $<$ 30 years of age

- 115) Mother < 30 years of age
Average Mother's Work Hours for Sub-Groups:
- 116) Father >= 16 years of schooling
117) Mother >= 16 years of schooling
118) Father = 12 years of schooling
119) Mother = 12 years of schooling
120) Father >= 40 years of age
121) Mother >= 40 years of age
122) Father < 30 years of age
123) Mother < 30 years of age
124) Mother's Active Time, Mother >= 16 years of schooling
125) Mother's Active Time, Mother >= 16 years of schooling
126) Father's Active Time, Father >= 16 years of schooling
127) Father's Active Time, Father >= 16 years of schooling
128) Corr. LW score (97), LW score (2002)
129) Corr. Dad Active Time (97), LW Score (2002)
130) Corr. Mom Work Hours (96), LW Score (2002)
131) Corr. Dad Work Hours (96), LW Score (2002)
132) Corr. Mom Work Hours (98), LW Score (2002)
133) Corr. Dad Work Hours (98), LW Score (2002)
134) Corr. Mom Wage (96), LW Score (2002)
135) Corr. Dad Wage (96), LW Score (2002)
136) Corr. Mom Wage (98), LW Score (2002)
137) Corr. Dad Wage (98), LW Score (2002)
Prob. of LW Score (2002) conditional on LW score (97) in [0,25]
138) Prob. Lw Score (2002) in [26,33]
139) Prob. Lw Score (2002) in [34,41]
140) Prob. Lw Score (2002) in [42,49]
141) Prob. Lw Score (2002) in [50,57]
Prob. of LW Score (2002) conditional on LW score (97) in [26,33]
142) Prob. Lw Score (2002) in [34,41]
143) Prob. Lw Score (2002) in [42,49]
Prob. of LW Score (2002) conditional on LW score (97) in [34,41]
144) Prob. Lw Score (2002) in [42,49]
145) Prob. Lw Score (2002) in [50,57]
Prob. of LW Score (2002) conditional on LW score (97) in [42,49]
146) Prob. Lw Score (2002) in [42,49]
147) Prob. Lw Score (2002) in [50,57]
Prob. of LW Score (2002) conditional on LW score (97) in [50,57]
148) Prob. Lw Score (2002) in [50,57]

Moment	Data	Simulated	Diff.	P-Value of Diff.
1.00	34.44	35.93	-1.50	0.41
2.00	13.18	19.40	-6.22	0.09
3.00	46.18	41.39	4.79	0.86
4.00	16.66	19.79	-3.12	0.27
5.00	25.56	23.28	2.29	0.81
6.00	14.73	17.13	-2.41	0.12
7.00	14.16	14.73	-0.57	0.36
8.00	11.79	11.04	0.75	0.68
9.00	12.82	10.67	2.15	0.91
10.00	10.48	7.83	2.66	0.98
11.00	5.73	7.06	-1.32	0.11
12.00	6.90	5.28	1.62	0.89
13.00	0.81	0.70	0.11	0.95
14.00	0.99	0.97	0.02	0.86
15.00	0.79	0.68	0.12	0.96
16.00	32.43	34.91	-2.48	0.33
17.00	17.63	20.38	-2.75	0.20
18.00	48.31	41.18	7.13	0.99
19.00	13.57	18.61	-5.04	0.08
20.00	16.48	17.47	-0.99	0.26
21.00	9.03	13.33	-4.30	0.00
22.00	12.11	11.51	0.61	0.68
23.00	8.23	8.73	-0.49	0.35
24.00	9.47	11.21	-1.74	0.05
25.00	6.63	8.51	-1.87	0.02
26.00	8.07	7.77	0.30	0.60
27.00	7.11	5.85	1.26	0.94
28.00	0.82	0.80	0.02	0.69
29.00	0.98	0.96	0.02	0.98
30.00	0.81	0.76	0.04	0.82
31.00	33.86	34.49	-0.63	0.46
32.00	11.62	21.05	-9.43	0.00
33.00	43.29	40.76	2.53	0.78
34.00	14.26	18.28	-4.02	0.15
35.00	15.72	13.53	2.19	0.89
36.00	10.64	10.53	0.10	0.52
37.00	8.59	9.35	-0.75	0.21
38.00	6.31	7.09	-0.79	0.12
39.00	10.88	12.14	-1.26	0.16
40.00	8.69	9.43	-0.74	0.19
41.00	8.08	8.88	-0.79	0.28
42.00	8.75	6.74	2.01	0.98
43.00	0.88	0.84	0.04	0.85
44.00	0.96	0.96	0.00	0.54
45.00	0.85	0.80	0.05	0.88
46.00	28.65	33.48	-4.84	0.20
47.00	17.75	20.52	-2.77	0.24
48.00	45.18	41.06	4.11	0.90
49.00	14.21	17.59	-3.38	0.22
50.00	12.30	14.89	-2.59	0.04

Moment	Data	Simulated	Diff.	P-Value of Diff.
51.00	9.52	8.29	1.24	0.79
52.00	8.93	11.16	-2.23	0.03
53.00	8.16	6.08	2.08	0.97
54.00	15.22	20.14	-4.92	0.00
55.00	11.06	11.55	-0.48	0.31
56.00	13.19	16.14	-2.94	0.06
57.00	11.30	9.22	2.08	0.99
58.00	0.84	0.93	-0.09	0.03
59.00	0.99	0.98	0.01	0.76
60.00	0.84	0.91	-0.07	0.08
61.00	40.37	41.96	-1.59	0.09
62.00	41.54	42.16	-0.62	0.28
63.00	46.00	46.11	-0.11	0.47
64.00	44.30	43.95	0.35	0.58
65.00	45.77	45.37	0.40	0.63
66.00	49.44	47.34	2.10	0.90
67.00	50.00	48.18	1.82	0.81
68.00	52.67	50.64	2.02	0.94
69.00	15.38	17.25	-1.87	0.01
70.00	10.24	9.53	0.71	0.71
71.00	19.52	18.72	0.80	0.78
72.00	11.89	10.73	1.16	0.75
73.00	-0.01	0.04	-0.04	0.30
74.00	-0.10	0.09	-0.19	0.02
75.00	0.01	0.16	-0.15	0.01
76.00	-0.11	-0.02	-0.08	0.16
77.00	0.01	0.10	-0.09	0.03
78.00	-0.21	-0.18	-0.03	0.40
79.00	-0.02	-0.06	0.04	0.62
80.00	0.06	-0.03	0.09	0.81
81.00	0.01	-0.12	0.12	0.92
82.00	-0.03	0.01	-0.05	0.29
83.00	-0.05	-0.03	-0.02	0.41
84.00	-0.12	-0.03	-0.10	0.12
85.00	0.01	0.01	0.00	0.52
86.00	-0.12	0.05	-0.17	0.05
87.00	-0.04	0.01	-0.05	0.23
88.00	-0.22	0.05	-0.27	0.00
89.00	-0.29	0.12	-0.41	0.00
90.00	0.08	0.15	-0.07	0.35
91.00	0.06	0.13	-0.07	0.19
92.00	27.49	23.38	4.11	0.93
93.00	25.40	21.26	4.14	0.93
94.00	16.30	16.15	0.16	0.57
95.00	18.22	18.11	0.11	0.53
96.00	20.44	19.77	0.67	0.65
97.00	22.09	20.20	1.89	0.81
98.00	14.13	14.4476	-0.31	0.42
99.00	15.44	16.11	-0.67	0.27
100.00	19.03	19.62	-0.58	0.37

Moment	Data	Simulated	Diff.	P-Value of Diff.
101.00	21.06	20.11	0.95	0.69
102.00	13.58	16.22	-2.64	0.01
103.00	14.49	16.02	-1.52	0.10
104.00	16.47	18.54	-2.06	0.07
105.00	16.74	19.40	-2.66	0.04
106.00	14.58	13.05	1.53	0.80
107.00	14.08	13.30	0.78	0.78
108.00	43.80	45.60	-1.80	0.22
109.00	42.46	41.62	0.84	0.66
110.00	43.92	38.94	4.98	1.00
111.00	42.91	40.88	2.03	0.85
112.00	42.34	43.04	-0.70	0.34
113.00	42.34	43.02	-0.68	0.37
114.00	43.34	34.55	8.79	1.00
115.00	45.18	36.71	8.47	1.00
116.00	32.19	24.46	7.74	0.98
117.00	32.75	31.41	1.34	0.67
118.00	29.95	32.11	-2.16	0.24
119.00	32.99	28.50	4.49	0.89
120.00	32.96	33.81	-0.85	0.37
121.00	33.30	36.01	-2.72	0.18
122.00	31.08	21.38	9.69	1.00
123.00	33.03	20.21	12.82	1.00
124.00	15.47	15.87	-0.40	0.44
125.00	17.49	17.69	-0.20	0.46
126.00	12.01	11.09	0.92	0.75
127.00	12.62	12.33	0.28	0.58
128.00	-0.09	0.43	-0.52	0.00
129.00	-0.12	0.43	-0.54	0.00
130.00	-0.03	-0.08	0.05	0.67
131.00	-0.02	-0.09	0.07	0.73
132.00	0.05	-0.04	0.09	0.77
133.00	0.15	-0.04	0.19	0.98
134.00	0.18	0.00	0.18	0.96
135.00	0.38	0.01	0.37	1.00
136.00	0.00	0.02	-0.02	0.45
137.00	0.22	0.01	0.21	0.93
138.00	0.02	0.02	0.00	0.56
139.00	0.18	0.11	0.08	0.98
140.00	0.22	0.33	-0.11	0.01
141.00	0.08	0.09	-0.01	0.37
142.00	0.01	0.01	-0.00	0.44
143.00	0.08	0.04	0.04	0.99
144.00	0.10	0.09	0.02	0.72
145.00	0.07	0.08	-0.00	0.44
146.00	0.02	0.06	-0.04	0.06
147.00	0.17	0.08	0.09	1.00
148.00	0.03	0.03 77	-0.00	0.47