SAMPLE SYLLABUS

NYU | PARIS

Partial Differential Equations

MATH-UA 9263 - 4 points (IN ENGLISH)

Instruction Mode: Blended

Spring 2022

If you are enrolled in this course 100% remotely and are not a Go Local/Study Away student for NYU Paris, please make sure that you've completed the online academic orientation via Brightspace so you are aware of site specific support structure, policies and procedures. Please contact nyu.paris.academics@nyu.edu if you have trouble accessing the Brightspace site.

Syllabus last updated on: 17/12/2021

Lecturer Contact Information
Augustin Cosse
acosse@nyu.edu
Office hours: Tuesday TBA

Prerequisites
Prerequisite: MATH-UA 262 Ordinary Differential Equations with a grade of C or better or the equivalent.

Units earned
4pts

Course Details
- Lectures on Monday Wed, 7:15pm-8:30pm, Recitation on Monday 5:30pm-7:00pm
- All times are CET (Daylight Saving Time ends on October 21).
- Location: Rooms will be posted in Albert before your first class.
- Remote Participants: Your instructor will provide you with the Zoom link via NYU Classes or email.
- COVID-related details: In the interest of protecting the NYU Paris community, we are closely following CDC guidance around COVID-19 and adjusting our recommendations and policies accordingly. Your health and well-being is our top priority.
    - If you are attending in person, you will be assigned a seat on the first day and are expected to use that seat for the entire semester due to NYU COVID-19 safety protocol. Please note that you are expected to attend every class

meeting in-person; however, this may change during the drop/add period if in-person student registration increases significantly or at any point during the semester if local COVID-19 regulations require additional physical distancing.
- Additionally, in-person students will be split into cohorts who will attend sessions as indicated on Albert.

Course Description

Partial Differential Equations are ubiquitous in science and engineering and are often useful in economics and finance. Because of their many connections to several mathematical subfields, they also represent a theoretical interest on their own (e.g. the spectral theory Laplace Beltrami operators as well as the scattering theory for wave equations are intimately tied to the study of automorphic forms in number theory). PDEs have been essential to the understanding of most physical phenomena such as sound, heat, electrostatics, electrodynamics, general relativity, fluid dynamics as well as the dynamics of financial markets (Black–Scholes).

As indicated by Sergiu Klainerman in his note “PDE as a unified subject” (Visions in Mathematics - Towards 2000), it however becomes increasingly difficult to view PDEs as a subject in its own right: “The deeper one digs into the study of one PDE, the more one has to take advantage of the particular features of the equation and therefore the corresponding results may make sense only as contributions to the particular field to which that PDE is relevant. Thus each major equation seems to generate isolated islands of mathematical activity. Moreover, a particular PDE may be studied from largely different points of view by an applied mathematician, a physicist, a geometer or an analyst”

As Klainerman later indicates, there are however a few powerful general ideas that reveal useful in a variety of contexts (some of the general principles listed by Klainerman include the notion of well/ill posedness, a priori estimates and continuity arguments, regularity theory, variational methods, Energy estimates,...).

In this course, we will study the general results as well as more particular notions that are useful to the study of the most important PDEs.

We will emphasize simple models (heat flow, vibrating strings and membranes). We will cover standard topics such as the method of separation of variables, the method of characteristics, Fourier series, orthogonal functions and the Fourier transform. Non homogeneous problems will be carefully introduced including Green functions for the Laplace, heat and wave equations.

Finally, for some of the equations, we will also study how to compute numerical solutions through finite differences and possibly the finite element method.

The assignments will include (but not be limited to) paper readings, pen and paper exercises as well as numerical simulations.

Course Objective

Students who complete this course successfully will be able to:

- Recognize the most important PDEs
- Understand the various types of PDEs (in particular classify a given PDE) and the nature of their solutions.
- Derive common partial differential equations from first principles based on constitutive laws and provide physical intuition for those equations.
- Understand the main solution methods for the most important PDEs (Wave, Heat, Laplace, Poisson, Transport, Burgers) and be able to apply these to a range of problems. More particularly:
    - Apply separation of variables to construct solutions to Laplace's equation, Heat equation and the wave equation in cylindrical and spherical polar coordinates in terms of Bessel functions and Legendre polynomials.
    - Apply the method of characteristics to first order PDEs such as the transport equation
    - Apply the method of characteristics to the Burgers' equation
    - Apply the method of spherical means to the wave equation
- Understand the notion of well-posedness and demonstrate well-posedness of a Cauchy problem for the wave equation
- Prove uniqueness for the boundary value problem associated to the Laplace's equation and the initial/boundary value problem associated to the heat and wave equations
- Formulate and prove the maximum principle for the heat equation and Laplace's equation
- Understand the notion of Green function and its use for the resolution of non homogeneous problems
- Understand the use of the Fourier series, Fourier transform and Laplace transform in the resolution of PDEs
- Understand the notion of Sturm-Liouville eigenvalue problem
- Apply the method of characteristics to simple first order non linear PDEs
- Understand the notions of Sobolev spaces, their properties (including some of the related inequalities) and their use in the study of linear elliptic, parabolic and hyperbolic PDEs
- Be able to prove existence and uniqueness of weak solutions for general second order elliptic parabolic and hyperbolic equations.
- Derive numerical (i.e. finite difference) solvers for the heat equation, wave equation and Laplace equation and understand the stability of the numerical scheme.
- Understand the use of the Hopf-Lax formula for the Hamilton-Jacobi equation
- Understand the use of the calculus of variations for the resolution of non linear PDEs

Assessment Components
You are expected to attend class in person or remote synchronously. Failure to submit or fulfill any required component may result in failure of the class, regardless of grades achieved in other assignments.

Required Text(s)
no required textbooks. course notes and/or handwritten notes will be provided. Possible

additional references include:

Supplemental Text(s) (not required to purchase)

The books listed below are not required for the class but are listed as additional resources for those who are interested in getting additional details/clarifications with respect to the material covered during the lectures. PDF versions can be found online for most of those books. Versions of those books will be available at the library.

- Lawrence C. Evans, Partial differential equations, Graduate Studies in Mathematics Vol. 19, American Mathematical Society, 2010
- András Vasy, Partial Differential Equations: An Accessible Route through Theory and Appli- cations, Graduate Studies in Mathematics Vol. 169, American Mathematical Society, 2015
- Walter A. Strauss, Partial Differential Equations, an Introduction, Wiley, 2008
- Sandro Salsa, Partial Differential Equations in Action: From Modelling to Theory. Springer, 2016
- Stanley J. Farlow, Partial Differential Equations for Scientists and Engineers, New York : Dover Publications 1993
- Richard Haberman, Applied Partial Differential Equations, with Fourier Series and Boundary Values Problems, Fourth Edition, Pearson 2004.

Additional Required Equipment
NA

Session 1 – 26/01/2022,
General Introduction (General presentation of the Transport, Laplace, Heat and Wave equations + derivation through the laws of physics), Classification of PDEs (E-1, V-2, Str-1)

Session 2 – 31/01/2022, 02/02/2022
First order PDEs + method of characteristics (including transport equation and Burgers' equation) E-2.1, 3.2, V-3,4, Str-1.2

Session 3 – 07/02/2022, 09/02/2022
Laplace equation + Poisson equation (fundamental solutions + Green function + harmonic functions, maximum principle, estimates on derivatives and Liouville's theorem) E-2.2. Str-6,7

Session 4 – 14/02/2022, 16/02/2022
The heat equation (Fundamental solution + homogeneous and non homogeneous Cauchy problems, Duhamel's principle) E-2.3, V-11

Session 5 – 21/02/2022, 23/02/2022

The heat equation (properties of solutions, including uniqueness on bounded domains and strong maximum principle) E-2.3, V-7

Session 6 – 28/02/2022, 2/03/2022
The wave equation (Introduction, physical interpretation, solution by spherical means) E-2.4, Str. 2.1

Session 7 – 07/03/2022, 09/03/2022
The wave equation (Kirchoff and Poisson formulas, Non homogeneous Cauchy problem, uniqueness and energy methods) E-2.4, V-7/ Revision + Midterm

Session 8 – 14/03/2022, 16/03/2022
Spring break

Session 9 – 21/03/2022, 23/03/2022
Separation of variables E-4.1, V-12, Str-4.1

Session 10 – 28/03/2022, 30/03/2022
Similarity solutions E-4.2

Session 11 – 04/04/2022, 06/04/2022
No Class on November 11
Transform methods (Part I, Introduction/reminders on the Fourier transform, convolutions, Fourier inversion, Plancherel's Theorem) E-4.3, V-8,9 Str-12

Session 12 – 11/04/2022, 13/04/2022
Transform methods (Part II, including Fourier, Radon and Laplace, application to linear constant coefficients PDEs, Bessel potentials, Heat + Wave + Schrödinger) E-4.3, V-8,9, Str-12

Session 13 – 18/04/2022 - 20/04/2022, No Class on April 18, Make up day on Friday, April 22
Introduction to Sobolev spaces + second order elliptic equations (Existence of weak solutions + regularity, part I) E-5

Session 14 – 25/04/2022, 27/04/2022
Second order elliptic equations (Existence of weak solutions + regularity, part II) + second order hyperbolic and parabolic PDEs (Existence of weak solutions + regularity) E-6,7.1,7.2

Session 15 – 02/05/2022, 04/05/2022

Nonlinear PDEs (Intro to calculus of variations + Hamilton-Jacobi) (E-8 V- 18, Str 14)

Session 16 – 09/05/2022, 11/05/2022
Revisions + Final Exams (part I)

Session 17 – 16/05/2022, 18/05/2022
Final Exams (part II)

Classroom Etiquette
Please make you sur read and acknowledge the information regarding this section on the **NYU Paris Resources** site on Brightspace.

Suggested Co-Curricular Activities
NA

Your Lecturer

Augustin Cosse received the MS and PhD in Engineering and applied mathematics at the University of Louvain (Belgium) in 2011 and 2016 respectively. He was a visiting student at MIT between 2013 and 2014, a visiting fellow at Harvard (IACS) between 2014 and 2015 and visited the University of Chicago, Department of Statistics, between 2015 and 2016. Between 2016 and 2020, he was a postdoctoral researcher at NYU CIMS and Ecole Normale Superieure (Paris).

Academic Policies

Grade Conversion
Your lecturer may use one of the following scales of numerical equivalents to letter grades:

US Letter Grade	US numerical	French numerical	
A	94-100 or 4.0	15-20	Excellent
A-	90-93 or 3.7	14	Very Good
B+	87-89 or 3.3	13	Good
B	84-86 or 2.7	12	Good

B-	80-83 or 2.7	11	Satisfactory
C+	77-79 or 2.3	10	Sufficient
C	74-76 or 2.0	9	Sufficient
C-	70-73 or 1.7	8	Sufficient
D	65-66 or 1.0	5-7	Poor
F	below 65 or 0	1-4	Fail

Attendance Policy
Studying at Global Academic Centers is an academically intensive and immersive experience, in which students from a wide range of backgrounds exchange ideas in discussion-based seminars. Learning in such an environment depends on the active participation of all students. And since classes typically meet once or twice a week, even a single absence can cause a student to miss a significant portion of a course. To ensure the integrity of this academic experience, class attendance at the centers, or online through NYU Brightspaces if the course is remote synchronous/blended, is expected promptly when class begins. Attendance will be checked at each class meeting. If you have scheduled a remote course immediately preceding/following an in-person class, you may want to write to nyu.paris.academics@nyu.edu to see if you can take your remote class at the Academic Center.

As soon as it becomes clear that you cannot attend a class, you must inform your professor and/or the Academics team by e-mail immediately (i.e. before the start of your class). Absences are only excused if they are due to illness, Moses Center accommodations, religious observance or emergencies. Your professor or site staff may ask you to present a doctor's note or an exceptional permission from an NYU Staff member as proof. Emergencies or other exceptional circumstances that you wish to be treated confidentially must be presented to staff. Doctor's notes must be submitted in person or by e-mail to the Academics team, who will inform your professors.

Unexcused absences may be penalized with a two percent deduction from the student’s final course grade for every week's worth of classes missed, and may negatively affect your class participation grade. Four unexcused absences in one course may lead to a Fail in that course. Being more than 15 minutes late counts as an unexcused absence. Your professor is entitled to deduct points if you frequently join the class late.

Exams, tests and quizzes, deadlines, and oral presentations that are missed due to illness always require a doctor's note as documentation. It is the student's responsibility to produce this doctor's note and submit it to site staff; until this doctor's note is produced the missed

assessment is graded with an F and no make-up assessment is scheduled. In content classes, an F in one assignment may lead to failure of the entire class.

Regardless of whether an absence is excused or not, it is the student's responsibility to catch up with the work that was missed.

Final exams

Final exams must be taken at their designated times. Should there be a conflict between your final exams, please bring this to the attention of the Academics team. Final exams may not be taken early, and students should not plan to leave the site before the end of the finals period.

Late Submission of Work

(1) Work submitted late receives a penalty of 2 points on the 100 point scale for each day it is late (including weekends and public holidays), unless an extension has been approved (with a doctor's note or by approval of NYU SITE Staff), in which case the 2 points per day deductions start counting from the day the extended deadline has passed.

(2) Without an approved extension, written work submitted more than 5 days (including weekends and public holidays) following the submission date receives an F.

(3) Assignments due during finals week that are submitted more than 3 days late (including weekends and public holidays) without previously arranged extensions will not be accepted and will receive a zero. Any exceptions or extensions for work during finals week must be approved by Academic Affairs (nyu.paris.academics@nyu.edu).

(4) Students who are late for a written exam have no automatic right to take extra time or to write the exam on another day.

(5) Please remember that university computers do not keep your essays - you must save them elsewhere. Having lost parts of your essay on the university computer is no excuse for a late submission.

Academic Honesty/Plagiarism

As the University's policy on "Academic Integrity for Students at NYU" states: "At NYU, a commitment to excellence, fairness, honesty, and respect within and outside the classroom is essential to maintaining the integrity of our community. By accepting membership in this community, students take responsibility for demonstrating these values in their own conduct and for recognizing and supporting these values in others." Students at Global Academic Centers must follow the University and school policies.

NYU takes plagiarism very seriously; penalties follow and may exceed those set out by your home school. Your lecturer may ask you to sign a declaration of authorship form, and may check your assignments by using TurnItIn or another software designed to detect offences against academic integrity.

The presentation of another person’s words, ideas, judgment, images, or data as though they were your own, whether intentionally or unintentionally, constitutes an act of

plagiarism. It is also an offense to submit work for assignments from two different courses that is substantially the same (be it oral presentations or written work). If there is an overlap of the subject of your assignment with one that you produced for another course (either in the current or any previous semester), you MUST inform your professor.

For guidelines on academic honesty, clarification of the definition of plagiarism, examples of procedures and sanctions, and resources to support proper citation, please see:

NYU Academic Integrity Policies and Guidelines

NYU Library Guides

Inclusivity Policies and Priorities

NYU's Office of Global Programs and NYU's global sites are committed to equity, diversity, and inclusion. In order to nurture a more inclusive global university, NYU affirms the value of sharing differing perspectives and encourages open dialogue through a variety of pedagogical approaches. Our goal is to make all students feel included and welcome in all aspects of academic life, including our syllabi, classrooms, and educational activities/spaces.

Attendance Rules on Religious Holidays
Members of any religious group may, without penalty, excuse themselves from classes when required in compliance with their religious obligations. Students who anticipate being absent due to religious observance should notify their lecturer AND NYU SITE's Academics Office in writing via e-mail one week in advance. If examinations or assignment deadlines are scheduled on the day the student will be absent, the Academics Office will schedule a make-up examination or extend the deadline for assignments. Please note that an absence is only excused for the holiday but not for any days of travel that may come before and/or after the holiday. See also University Calendar Policy on Religious Holidays

Pronouns and Name Pronunciation (Albert and Zoom)
Students, staff, and faculty have the opportunity to add their pronouns, as well as the pronunciation of their names, into Albert. Students can have this information displayed to faculty, advisors, and administrators in Albert, Brightspace, the NYU Home internal directory, as well as other NYU systems. Students can also opt out of having their pronouns viewed by their instructors, in case they feel more comfortable sharing their pronouns outside of the classroom. For more information on how to change this information for your Albert account, please see the Pronouns and Name Pronunciation website.

Students, staff, and faculty are also encouraged, though not required, to list their pronouns, and update their names in the name display for Zoom. For more information on how to make this change, please see the Personalizing Zoom Display Names website.

Moses Accommodations Statement
Academic accommodations are available for students with documented and registered disabilities. Please contact the Moses Center for Student Accessibility (+1 212-998-4980 or

mosescsd@nyu.edu) for further information. Students who are requesting academic accommodations are advised to reach out to the Moses Center as early as possible in the semester for assistance. Accommodations for this course are managed through NYU Paris.

Bias Response

The New York University Bias Response Line provides a mechanism through which members of our community can share or report experiences and concerns of bias, discrimination, or harassing behavior that may occur within our community.

Experienced administrators in the Office of Equal Opportunity (OEO) receive and assess reports, and then help facilitate responses, which may include referral to another University school or unit, or investigation if warranted according to the University's existing Non-Discrimination and Anti-Harassment Policy.

The Bias Response Line is designed to enable the University to provide an open forum that helps to ensure that our community is equitable and inclusive.

To report an incident, please contact one of the following:

- Online using the Web Form (link)
- Email: bias.response@nyu.edu
- Phone (NY): +1 (212) 998-2277
- Office of the Director, NYU Paris: +33 1 53 92 50 80