Informal Communication

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Abstract

The typical sender-receiver game studied in the literature assumes that the receiver is uninformed. In reality, the receiver usually relates the sender’s message to her own information obtains a signal about their truthfulness. I analyze a sender-receiver model where both agents have private information and the sender cares to be perceived as honest. If the sender’s reputation concerns are strong enough, the model predicts truthful information revelation as a unique equilibrium. This uniqueness result contrasts with the multiplicity of uninformative equilibria in cheap-talk games with an uninformed receiver. I also show that in the unique equilibrium, the sender uses (per se) extraneous information to support his recommendation for the receiver’s action. If the sender also cares about the receiver’s action, he randomizes between telling the truth and lying. Examples show that extraneous information may make communication more as well as less honest.

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1 Introduction

Most people consult with others before making important decisions. They discuss their problem with friends, and elicit opinions from various advisors or experts. The messages of a consultant (sender) rarely have direct payoffs implications, and the decision-maker (receiver) often has no way to verify the truthfulness of the information. The fact that opinions are routinely solicited and our intuition both suggest that consultants often respond truthfully to decision-makers’ inquiries. However, the cheap-talk literature establishes that when messages are costless, truthful communication is one of many possible outcomes, even if incentives are perfectly aligned.

The purpose of this essay is to answer the following questions: Why do economic agents expand resources on discussing and soliciting opinions from others who do not have strong incentives to give them accurate information? What is the role of having information about others’ information in facilitating honest communication? Why do individuals commonly augment their conclusions with information, which per se seems to be irrelevant for the others? I focus on what I call informal communication, that is, situations with incomplete contracting and unverifiable messages. In this setting, the correctness of the sender’s message cannot be verified even after the receiver implements the decision. The sender has an interest in maintaining his reputation as an honest person and may also care about the receiver’s decision, but contracting on either this decision or on messages themselves is not feasible. Here are some concrete examples involving informal communication: (a) an employer is to select one of two applicants for a job, and she has an advisor with a superior expertise to her own regarding the qualifications of both candidates, (b) a consumer wants to buy a used car, but she knows little about the construction of cars, so she wants a mechanic to estimate its current condition, (c) an attorney makes the decision to take a case; she needs information about an event from the past and speaks to potential witnesses.

In the typical sender-receiver game studied in the literature (see for example, Crawford-Sobel (1982)), the sender has a private signal, which he may convey to the uninformed receiver who has to take an action. The payoff of both players depends on
the receiver’s action and the sender’s signal. These are known as “cheap-talk” games. In some cheap-talk models, as in my games of informal communication, the sender has an interest in maintaining his reputation for honesty (see Sobel (1985)). The most significant difference between my sender-receiver games and the ones studied in this literature is that the receiver also has private information\textsuperscript{12}.

The main result of the paper is that when messages are unverifiable - or equivalently, they can be verified only after the interaction is completed - this private information, no matter how small, facilitates honest communication. More specifically, Theorem 1 asserts that if the sender is motivated only by the reputation concerns, then he reports his signal truthfully in a unique equilibrium if for every pair of the sender’s possible signals there is a pair of the receiver’s possible signals that result in the different likelihood ratios of these two signals of the sender. In the unique equilibrium, the sender reports truthfully also the payoff-irrelevant information supporting his recommendation for the receiver’s action.

In the more general case where the sender cares about the receiver’s action, my game of informal communication may have multiple equilibria. Those equilibria are usually in mixed strategies: the sender randomizes between telling the truth and lying. Hence, messages convey some information, but there are bounds on how much information gets transmitted. In this more general case, I identify situations where payoff-irrelevant information makes communication more (less) honest - that is, situ-

\textsuperscript{1}In the literature, there are two instances where the receiver’s private information plays a role. In Seidmann (1990), the receiver’s private information makes possible the existence of meaningful equilibria even when the sender’s preferences over the receiver’s actions are independent of the sender’s signal. The reason is that any message induces a distribution of the receiver’s actions across her types (private information), and the sender’s preferences over those distributions may depend on his signal, even though the preferences over single actions do not.

In Prendergast (1993), the receiver’s private information enables her to motivate the sender for collecting costly information. In his model, information is represented by correlated draws from a normal distribution, and the parties may write contracts, where the receiver rewards the sender if the reported draw is close to her private draw.

\textsuperscript{2}Section 5 contains more details on the relation of this paper to the literature on cheap-talk and the literature reputation effects.
ations where the chance of telling the truth becomes higher (lower) when the sender can support his recommendation for the receiver’s action with payoff-irrelevant information compared to when he is forced to send only the recommendation itself.

The next section contains the details of the model. Section 3 characterizes and discusses equilibria: both when the sender is motivated purely by the reputation concerns and when he also cares about the receiver’s action. Section 4 analyzes the role of payoff-irrelevant information. Finally, Section 5 discusses the relation of this paper to the literature. Proofs are relegated to Appendix.

2 Model

I study the following game: The sender S gets a signal \( s \) from set \( T_S = \{s_1, \ldots, s_m\} \). The receiver R gets a signal \( r \) from \( T_R = \{r_1, \ldots, r_n\} \). The probability that S obtains \( s = s_i \) and R obtains \( r = r_k \) is equal to \( p_{ik} \). This prior information structure can be represented by matrix

\[
P = \begin{bmatrix}
  r_1 & r_2 & \cdots & r_n \\
  p^1_1 & p^1_2 & \cdots & p^1_n \\
  p^2_1 & p^2_2 & \cdots & p^2_n \\
  \vdots & \vdots & \ddots & \vdots \\
  p^m_1 & p^m_2 & \cdots & p^m_n
\end{bmatrix}
\]

I denote rows of \( P \) by \( p_i \) and columns by \( p^k \). I assume that the prior information structure is common knowledge.

After getting a signal, the sender sends a message \( s \in T_S \) to the receiver who takes an action. It is essential that the set of messages contains all signals of the sender; the assumption that there are no other messages is made only for the sake of simplicity. The set of all possible actions \( A \) is finite with its generic element denoted \( a \). Players are allowed to use mixed strategies. A mixed strategy of the sender is denoted \( \theta^S : T_S \rightarrow [0,1] \), where \( \theta^S(s_i) \) stands for the probability that the sender with
signal $s = s_i$ sends message $s = s_j$. A mixed strategy of the receiver is denoted $\otimes: A \in T_S \in T_R$, where $\otimes(a; s_i; r_k)$ stands for the probability of taking action $a$ by the receiver with signal $r_k$ when the sender sends message $s = s_i$. There are two types of senders. With probability $\gamma$, the sender is a truth-teller, and with probability $1 - \gamma$, the sender is a strategic agent. Truth-tellers always reveal their signals honestly, and strategic senders maximize their payoffs. The payoff of a strategic sender with signal $s_i$ is a weighted sum of two components: the expected decision payoff $U_S$ and the reputation payoff $V_R$, with the weights $\frac{1}{2}$ and $\frac{1}{2}$ respectively. The decision payoff is represented by $U_S - a$ real-valued function of the receiver's action and the signals of both players. Thus, the expected decision payoff is given by

$$U_S(\frac{1}{2} \otimes s_i) = \sum_{a \in A} \sum_{j = 1}^{n} \otimes(a; s_i; r_k) \frac{1}{2} \otimes s_j j s_i) u_S(a; s_i; r_k),$$

where $\frac{1}{2} r_k j s_i)$ stands for the probability assigned by a sender with signal $s_i$ to the event that the receiver has obtained signal $r_k$. Notice that

$$\frac{1}{2} r_k j s_i) = \frac{p_k}{\prod_{j=1}^{m} p_j}.$$

The reputation payoff is assumed to be the expected posterior probability assigned by the receiver to the event that the sender is a truth-teller; it will be defined precisely later. The payoff of the receiver is represented by $U_R$ - a real-valued function of his action and the signals of both players; thus, the expected payoff of the receiver with signal $r_k$ when the sender sends message $s = s_i$ is given by

$$U_R(\frac{1}{2} \otimes r_k j s_i) = \sum_{a \in A} \sum_{j = 1}^{n} \otimes(a; s_i; r_k) (U_R(a; s_i; r_k) + (1 - \gamma) \frac{1}{2} s_j j r_k) u_R(a; s_i; r_k),$$

where $\frac{1}{2} s_j j r_k)$ stands for the probability assigned by a receiver with signal $r_k$ to the event that the sender obtains signal $s_i$. Notice that

$$\frac{1}{2} s_i j r_k) = \frac{p_k}{\prod_{j=1}^{m} p_j}.$$
Finally, let us define the reputation component of the sender's payoffs. Let $v(s_i \mid \frac{3}{4} r_k)$ denote the probability assigned to the event "$S$ is a truth-teller" by $R$ with signal $r_k$ if $S$ plays according to strategy $\frac{3}{4}$ and $S$ sends message $s_i$. Notice that

$$v(s_i \mid \frac{3}{4} r_k) = \frac{\frac{3}{4} v(s_i \mid r_k)}{\sum_{j=1}^{n} \frac{3}{4} v(s_j \mid r_k) \frac{3}{4} u(s_j ; s_i)}.$$ 

Hence, the reputation payoff of $S$ with signal $s_i$ when he sends message $s_j$, which I will call the reputation payoff to sending $s_j$, is given by

$$V(s_j \mid \frac{3}{4} s_i) = \sum_{k=1}^{N} \frac{3}{4} v(s_j \mid r_k) v(s_j \mid \frac{3}{4} r_k),$$

and the total payoff of $S$ with signal $s_i$ is given by

$$\frac{3}{4} u_s + (1 - \frac{3}{4}) V.$$

I denote this game by $G(P; \frac{3}{4})$. Except Section 4, I assume that either $\frac{3}{4} = 0$ or $p_k^i > 0$ for all $i$ and $k$, and study its Nash equilibria: $\frac{3}{4} @$. The assumption that either $\frac{3}{4} = 0$ or $p_k^i > 0$ for all $i$ and $k$ guarantee that there are no out of equilibrium information sets, and thus, Nash equilibrium can be used. The assumption has been made solely for this purpose. The analysis extends to the general case and sequential equilibria. It is important to notice that the sender’s reputation payoff to sending a message is determined by his strategy itself. Of course, the sender takes the payoffs of his messages as given when he picks up his strategy. Therefore in a Nash equilibrium of $G(P; \frac{3}{4})$, the sender’s strategy has to be the best response to both the receiver’s strategy and to itself.

Unlike traditional reputation models, my model has only one period. In order for reputation to have an effect, the value of establishing a reputation is incorporated as a component into the sender's payoff function. Then the reputation payoff may reflect either some intrinsic preference for being truthful or the sender’s expectation to benefit in the future. In the latter case, one can view my model as a reduced form of a multi-period model. Further, I assume the correctness of the sender’s messages can never be verified, even after the receiver implements her decision. I have basically in
mind situations where there is a delay between when the message is received and the information about its truthfulness becomes available. Then the sender may extract the benefits of his reputation during this delay. However, there also exist situations of interest where the receiver may literally never be able to verify messages at any later date. For example, think of a situation where an advisor recommends to an employer one of two candidates to perform a specific task.

3 Equilibria

3.1 Pure Reputation Concerns - Uniqueness of Truth-Telling Equilibrium

Consider the case of $\frac{1}{2} = 0$, i.e. the sender is motivated purely by the reputation concerns. I shall give necessary and sufficient conditions for the uniqueness of truth-telling equilibria.

Definition 1 The strategy $\theta(s_i; s_j) = 1$ for $i = 1; \cdots; m$ will be called truth-telling.

Theorem 1 (i) Truth-telling is an equilibrium.

(ii) The equilibrium is unique if and only if

$$8_{ij} = 1; \cdots; m \quad p_i \text{ and } p_j \text{ are linearly independent.} \quad (1)$$

Proof. See Appendix. ■

In other words, condition (1) means that for every pair of the sender’s signals, there is a pair of the receiver’s signals that result in different likelihood ratios of the two signals of the sender. In the interpretation, it means that different private signals of the receiver may provide different information regarding the truthfulness of different messages of the sender. Note that it is a mild condition. For example (cf Introduction): (a) An employer may have some general impression concerning

3Condition (1) slightly resembles Cremer and McLean (1988) and their necessary and sufficient condition for the extraction of full surplus in Bayesian auctions. Roughly, Cremer and McLean show
each candidate. (b) A consumer might have driven the car, and in this way, have acquired some information about how the car works. (c) Although the attorney does not know whether something has happened, she may have some information about related events or people involved. However, if the receiver is uninformed, as in cheap-talk games, every pair of rows of matrix $P$ are linearly dependent, that is condition (1) is not satisfied.

The following example illustrates Theorem 1, and will also be used to illustrate the proof of Theorem 1 and the role of extraneous information.

**Example 1**

The receiver, a friend of Mrs. Brown, would like to know if Mrs. Brown was in Princeton yesterday to visit her relative. She asks the sender if he has seen Mrs. Brown. The sender can offer one of three responses: $A_1 =$ "I have seen Mrs. Brown with an elderly woman" or $A_2 =$ "I have seen Mrs. Brown with a young man" or $N =$ "I did not see Mrs. Brown". The receiver has met the relative of Mrs. Brown's, has fifty-fifty prior that Mrs. Brown was in town yesterday, and believes that if she were in Princeton, there would be a .5 chance that the sender would have encountered her. The sender also knows that Mrs. Brown has a relative in town, and assigns probability .5 that the relative is a nephew (M) and probability .5 that it is her sister (W). There is an $\epsilon > 0$ prior chance that the sender always tells the truth; otherwise his payoff is the expected receiver's posterior assessment that he is an honest type.

The following table describes the information structure above:

<table>
<thead>
<tr>
<th></th>
<th>W</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$A_2$</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>$N$</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

that if bidders' signals regarding the valuation of the object provide also information regarding each other, then there exist an auction and its equilibrium such that the seller extracts the same surplus as she would if she knew the bidders' valuations. Of course, existence is straightforward in my result and condition (1) is necessary and sufficient for uniqueness.
0 < ° · 1=δ.

The key lemma in the proof of Theorem 1 (see Appendix, Lemma 5) asserts that for any message \( s_i \) and strategy \( ¾ \) the expectation of the probability assigned by R to S being a truth-teller, conditional on S telling the truth is not lower than that expectation conditional on S lying. Moreover, it is strictly higher unless for every signal \( r_k \),

\((F)\) the probabilities that R has signal \( r_k \) conditional on S telling the truth and conditional on S lying are equal;

that is, S' truthfulness and R's signal are independent.

Of course, the assertion and \((F)\) make sense only if message \( s_i \) is used as a lie with a positive probability. To see the main idea consider Example 1, where \( ° = 1=8 \).

I shall ...rst show that the payoå of a sender who truthfully reveals \( A_1 \) exceeds the payoå to sending \( A_1 \) of a sender with signal \( A_2 \) or \( N \). Indeed, if a sender with signal \( A_2 \) sends \( A_1 \), then with probability 1, it gets detected that he lies, i.e. he is certainly assigned probability 0 of being an honest type, whereas a sender who reveals honestly signal \( A_1 \) is assigned a positive probability of being an honest type. Consider now a sender with signal \( N \). If he sends \( A_1 \) his payoå equals the payoå of a sender who truthfully reveals \( A_1 \) if the receiver has signal \( W \); if the receiver, however, has signal \( M \), his payoå is 0. Since he assigns ...ifty chance that the receiver has \( W \), his expected payoå equals half of that for a sender who truthfully reveals \( A_1 \).

Notice now that the expectation of the probability assigned by R to S being a truth-teller, conditional on S truthfully revealing \( A_1 \) equals to the payoå of a sender who truthfully reveals \( A_1 \), and that expectation conditional on S lying equals to a weighted average of the payoås to sending \( A_1 \) of senders with \( A_2 \) and \( N \). Notice also that \((F)\) is not satisåed for \( s_i = A_1 \), because the probability that R has signal \( r_k = M \) conditional on S telling the truth equals 0 and has to be positive conditional on S lying.

Next, I derive from the key lemma and the equilibrium conditions that S' payoå is " independently of his signal. Consider again Example 1. In equilibrium, the receiver
cannot on average be misled. That is, the average payoffs of the sender equals ", where the “averaging” is being done over the three possible signals of the sender. Suppose that some signal yields the payoffs lower than ", and consider signal s, which yields the lowest payoffs. Say, s = A1. Then the senders with signals A2 and N send A1 with probability 0; otherwise, by the observation illustrating the key lemma, the payoffs of the senders with signals A2 or N would also be lower than ". Since the senders with signals A2 and N send A1 with probability 0, the payoffs of a sender with A1 cannot be lower than ", a contradiction.

The payoffs can be equal ", independently of S’ signal only if (F ) is satisfied for i = 1;:::;m and k = 1;:::;n. This set of equations yields

\[ 3\pi \Phi P = P, \]

where \( 3\pi \) represents the m \( \times \) m matrix whose entry in row i and column j equals \( 3\pi s_j; s_i \). Then linear algebra arguments imply

\[ 3\pi = I \]

i.e. P satisfies (1), I stands for the m \( \times \) m identity matrix.

In Example 1, the payoffs of S can be equal ", independently of his signal only if messages A1 come only from senders who tell the truth. This in turn implies that message N also comes only from a sender who tells the truth, because if a sender with signal Ai sent N with a positive probability, then his payoffs to sending A1 would exceed ".

Finally, Example 1 illustrates how the receiver’s private information facilitates honest communication and how payoffs irrelevant information is useful and used. Both enable the receiver to discriminate truth from lie. If the receiver had not met the relative of Mrs. Brown’s, or the sender was forced to use one of two responses: Y = “I saw Mrs. Brown”, N = “I did not see Mrs. Brown”, then truthful communication could not be guaranteed. One readily checks that the game would have a continuum of equilibria, including one in which messages convey no information.
Remark 1

Consider now an arbitrary matrix $P$ whose rows may be linearly dependent. Then the rows of $P$ can be segregated into sets whose members are linearly dependent. Let $S_i = f_{s_j : p_i \text{ and } p_j \text{ are linearly dependent}}$. Then $f_{S_i : i = 1; \ldots; m}$ forms a partition of $T_S$, and if $p_i$ and $p_j$ are linearly independent for some $i$ and $j$, then the partition consists of at least two elements. Further let

$$\mathcal{G}(s_i; s_j) = \bigcap_{s_j = 2 S_j \text{ and } s_i = 2 S_i} \mathcal{G}(s_i; s_j)^a,$$

that is, $\mathcal{G}(s_i; s_j)$ stands for the probability that the sender with a signal from $S_j$ sends a message from $S_i$.

Slightly modifying the proof of Theorem 1, one can show:

Theorem 2 In every Nash equilibrium of $G(P; 0)$,

$$\mathcal{G}(s_i; s_i) = 1 \text{ for } i = 1; \ldots; m.$$

That is, the sender always sends a message from the $S_i$ that contains his signal. If the $S_i$ consists of one element, then he simply tells the truth. If, however, it consists of at least two elements, there is a continuum of equilibria that correspond to each level of information transmission from sending every member of $S_i$ with the same probability to revealing the true element of $S_i$ with probability 1.

Finally, note that there is also a puzzling implication of Theorem 1. If the sender is motivated exclusively by reputation concerns and the receiver is sure about that, then he communicates, and she fully believes even in things that initially have seemed unbelievable. It seems plausible that the receiver would rather assign to such messages a large margin of doubt, and in anticipation of that, the sender would rather avoid sending unbelievable messages.

3.2 Other Motives

In the case $\lambda > 0$, i.e. when the sender has motives other than reputation, my sender-receiver game may have several equilibria. These equilibria are usually in mixed
strategies, i.e. the sender randomizes between telling the truth and lying. Indeed, whenever a conflict of interests arises, truth-telling is no longer an equilibrium. If it were, the reputation payoffs of all messages would be equal to $\omega$, and the sender would send the message that maximizes his decision payoff. On the other hand, as the reputation concerns get stronger, the probability of telling the truth eventually approaches 1.

Proposition 3 Let $p_i$ and $p_j$ be linearly independent for any $i$ and $j$. For every $\gamma < 1$, there exists $\gamma > 0$ such that if $\gamma < \gamma$, then $\gamma(s_i; s_i) > \gamma$ for $i = 1; \ldots; m$ in every Nash equilibrium.

Proof. Briefly, it follows from Theorem 1 and continuity arguments. For the details, see Appendix.

In the interpretation, informal messages convey some information, but there are bounds on how much information gets transmitted, or individuals usually leave some margin of doubt for information acquired in informal communication. However, as the reputation concerns get stronger, this margin of doubt eventually approaches 0. Another feature of the equilibria in the general case is that the receiver’s level of trust in the sender’s message depends on her private information. In the interpretation, some individuals trust more and some trust less what the sender says. Again, as the reputation concerns get stronger, this level of trust eventually approaches 1 independently of the receiver’s private information.

4 Extraneous Information

In this section, I show how per se payoff-irrelevant information may be used for strategic reasons, and I explore whether the opportunity of augmenting messages with payoff-irrelevant information leads to more honest communication.

For each game $G(P; \gamma)$, the set of the sender’s signals can be partitioned into payoff-equivalent sets $T^q_i : q = 1; \ldots; M$. That is, if $s_i; s_j \in T^q_i$ for the same
for any $a \in A$ and $r_k \in T_R$. Of course, the interesting case is when some sets $T_S^q$ consist of more than one signal. The set $T_S^q$ containing the sender’s signal represents his payoff-relevant information. This set is what I call a conclusion or a recommendation for the receiver’s action. Each single element of $T_S^q$ represents the sender’s payoff-relevant and extraneous information. I call elements of $T_S^q$ conclusions or recommendations for the receiver’s action augmented with extraneous information.

Consider a modified version of sender-receiver game, $G_M(P; \frac{\lambda}{n})$, where the set of possible messages contains not only elements of $T_S$ but also messages $T_S^q$, $q = 1; \ldots; M$. In the interpretation, the sender is allowed to communicate only his conclusion as well as he is allowed to augment the conclusion with extraneous information. Notice that no sender has signal $T_S^q$, and so there can be out of equilibrium information sets of $G_M(P; \frac{\lambda}{n})$. Thus, I have to use the concept of sequential, instead of Nash, equilibrium.

**Corollary 4** Suppose every pair of rows of matrix $P$ is linearly independent. For every $\frac{1}{n} < 1$, there exists $\frac{\lambda}{n} > 0$ such that if $\frac{1}{n} < \frac{\lambda}{n}$, then in every sequential equilibrium of $G_M(P; \frac{\lambda}{n})$, the probability that the sender’s message belongs to $T_S$ exceeds $\frac{1}{n}$.

**Proof.** For each $T_S^q$, $q = 1; \ldots; M$, add the row with all entries equal 0 to the prior information structure $P$. Then apply Proposition 2, which readily generalizes to matrices with 0 entries and sequential equilibria.

That is, if the reputation concerns are strong enough, the sender almost surely augments his conclusion with extraneous information.

**Remark 2**

I have assumed that the sender certainly has some extraneous information supporting his conclusion. Therefore reporting just a conclusion enables the receiver to
detect that the sender is not a truth-teller. One could alternatively assume that with some probability the sender has no extraneous information supporting his conclusion. That is, one could consider the prior information structure $P$ with each conclusion $T^q_S$ represented by a row with some positive entries. Then, assuming the linear independence of every pair of rows of matrix $P$, if the sender’s reputation concerns are strong enough, the sender almost surely augments his conclusion with the extraneous information whenever he has extraneous information.

Corollary 4 captures only one reason for augmenting messages with extraneous information: the sender builds his reputation for honesty. Our intuition suggests, however, that extraneous information may also influence the receiver’s decision. Moreover, Corollary 4 does not say whether extraneous information makes communication more or less honest. To address these issues I shall compare equilibria of the following two games:

In game $G_E(P; \frac{1}{K}) := G(P; \frac{1}{K})$ the sender supports his recommendation by extraneous information, i.e. the set of possible messages equals $T_S$.

In game $G_{NE}(P; \frac{1}{K}) := G(P; \frac{1}{K})$ the sender communicates just a recommendation, i.e. the set of possible messages consists of $T^q_S$, $q = 1; \ldots; M$.

The following two examples show that the opportunity of augmenting recommendations with extraneous information may lead to more (as well as less) honest communication, in the sense that:

(a) the equilibrium probability that the sender reveals his signal honestly is higher (lower) when $G_E(P; \frac{1}{K})$ is being played than $G_{NE}(P; \frac{1}{K})$ is being played,

(b) the receiver’s payoff is higher (lower) in the $G_E(P; \frac{1}{K})$-equilibrium than in the $G_{NE}(P; \frac{1}{K})$-equilibrium.

The examples also show how extraneous information may influence the receiver’s decision.

Example 2

Recall the story about Mrs. Brown. Suppose that the receiver considers two actions: $V = \text{“to visit Mrs. Brown today”}$ and $I = \text{“to invite Mrs. Brown for}
tomorrow”. That is, the set of the receiver’s actions equals to \( f \) \( V; I \) \( g \). The receiver prefers action \( V \) if the sender has seen Mrs. Brown in Princeton yesterday, and prefers action \( I \) if the sender has not seen Mrs. Brown. The sender cares to be perceived as honest, but also cares about the receiver’s decision. Suppose that the sender wants the receiver to visit Mrs. Brown independently of his signal. More specifically, let

\[
\begin{align*}
    u_s(V; s; r) &= 1, \\
    u_s(I; s; r) &= 0
\end{align*}
\]

for all \( s \in T_S \) and \( r \in T_R \),

\[
\begin{align*}
    u_r(V; A_1; r) &= u_r(V; A_2; r) = 1 \text{ and } u_r(V; N; r) = 1, \\
    u_r(I; A_1; r) &= u_r(I; A_2; r) = 1, \quad \text{and } u_r(I; N; r) = 1
\end{align*}
\]

for all \( r \in T_R \).

There exists an equilibrium of \( G_{NE}(P; \frac{1}{2}) \) such that the sender reports \( Y = \text{“I saw Mrs. Brown”} \) when he has signal \( A_1 \) or \( A_2 \), and he randomizes between telling the truth and lying when he has signal \( N \). The receiver takes action \( V \) when he hears \( Y \), and takes action \( I \) when he hears \( N \). Moreover, for some parameters \( \frac{1}{2} \) and \( " \), the \( G_{NE}(P; \frac{1}{2}) \)-equilibrium has the property that the receiver who hears message \( Y \) is indifferent between taking actions \( V \) and \( I \). See Appendix for the details.

There exists an equilibrium of \( G_E(P; \frac{1}{2}) \) such that the sender reports truthfully signals \( A_1 \) and \( A_2 \), and he randomizes between telling the truth and sending \( A_1 \) or \( A_2 \) when he has signal \( N \). The receiver makes the same decisions as in the \( G_{NE}(P; \frac{1}{2}) \)-equilibrium, except when she has signal \( W \) (\( M \)) and hears message \( A_2 \) (respectively, \( A_1 \)). Then she randomizes between \( V \) and \( I \). It is shown in Appendix, that if \( \frac{1}{5} \) is small enough, then the \( G_{NE}(P; \frac{1}{2}) \)-equilibrium probability that the sender with signal \( N \) sends \( Y \) exceeds the \( G_E(P; \frac{1}{2}) \)-equilibrium probability that the sender with signal \( N \) sends \( A_1 \) or \( A_2 \). Heuristically, the reason is the possibility of taking action \( I \) by the receiver who hears extraneous information “inconsistent” with her own signal (i.e. \( A_1 \) when she has \( M \) or \( A_2 \) when she has \( W \)). This possibility creates an additional incentive for telling the truth. In \( G_{NE}(P; \frac{1}{2}) \)-equilibrium, the sender makes sure that
the action he prefers will be taken by sending message $Y$. In $G_E (P; \frac{1}{2})$-equilibrium, the action he dislikes will be taken with a positive probability, no matter which message he sends. Consequently, the probability of telling the truth by the sender with signal $N$ in the $G_E (P; \frac{1}{2})$-equilibrium is higher than that in the $G_{NE} (P; \frac{1}{2})$-equilibrium.

Example 3

Suppose now that the same matrix as in Example 2 gives the information structure. This time, the receiver considers three actions: $V$, $I$, and $; = \text{"none of } V \text{ and } I\text{"}$. Let

\[ u_S (V; s; r) = 1, \quad u_S (; ; s; r) = 0, \quad u_S (I; s; r) = -1 \]

for all $s \in T_S$ and $r \in T_R$,

\[ u_R (V; A_1; r) = u_R (V; A_2; r) = 1, \quad u_R (V; N; r) = 1 \]
\[ u_R (I; A_1; r) = u_R (I; A_2; r) = 1 \text{ and } u_R (I; N; r) = 1 \]
\[ u_R (; ; A_1; r) = u_R (; ; A_2; r) = u_R (; ; N; r) = 0 \]

for all $r \in T_R$.

There exists an equilibrium of $G_{NE} (P; \frac{1}{2})$ such that the sender reports $Y$ when he has signal $A_1$ or $A_2$, and he randomizes between telling the truth and lying the receiver when he has signal $N$. The receiver takes action $V$ when he hears message $Y$, and he takes action $I$ when he hears message $N$. Moreover, for some parameters some $\frac{1}{2}$ and $\frac{1}{2}$, the $G_{NE} (P; \frac{1}{2})$-equilibrium has the property that the receiver who hears message $Y$ is indifferent between taking action $V$ and action $I$. See Appendix for the details.

There exists an equilibrium of $G_E (P; \frac{1}{2})$ such that the sender reports truthfully signals $A_1$ and $A_2$, and he randomizes between telling the truth and sending $A_1$ or $A_2$ when he has signal $N$. The receiver makes the same decisions as in the $G_{NE} (P; \frac{1}{2})$-equilibrium, except when she has signal $W (M)$ and hears message $A_1$ (respectively, $A_2$). Then she randomizes between actions $V$ and $;$. It is shown in Appendix, that if $\frac{1}{2}$ is small enough, then the $G_E (P; \frac{1}{2})$-equilibrium probability that the sender with signal
N sends $A_1$ or $A_2$ exceeds the $G_{NE}(P; \frac{1}{2})$-equilibrium probability that the sender with signal $N$ sends $Y$. Heuristically, the reason is the possibility of taking action $V$ by the receiver who hears extraneous information "consistent" with her own signal (i.e. $A_1$ when she has $W$ or $A_2$ when she has $M$). This possibility creates an additional incentive for lying. If it happens that the sender picks up the "consistent" message, he will convince the receiver to take the action that he prefers. Consequently, the probability of telling the truth by the sender with signal $N$ in the $G_{E}(P; \frac{1}{2})$-equilibrium is lower than that in the $G_{NE}(P; \frac{1}{2})$-equilibrium.

5 Related Literature

A number of papers, beginning with Crawford and Sobel (1992), have studied "cheap-talk" games. In addition to some very important insights, cheap-talk models also provide two counter-intuitive predictions. First, cheap-talk games always have equilibria in which messages convey no information. Even in the games of pure common-interest, i.e. where the payoff functions of both agents coincide, there are equilibria where the sender communicates his true signal, as well as babbling equilibria where the sender sends each possible signal with the same probability. Second, truthful communication appears as an equilibrium in pure-conflict games, where the sender would achieve the highest payoff if his message conveyed no information.

These implausible equilibria - and especially, the impossibility of obtaining meaningful communication as a unique outcome - led to a search for refinements. The main line of research on refinements follows Farrell (1993). He, as well as Matthews, Okuno-Fujiwara and Postelwaite (1991), Rabin (1990), Rabin-Sobel (1996), Zapter (1997), define messages that are credible and should be believed. Intuitively, a message is credible if assuming that it will be believed, a sender with different information cannot benefit from sending that message. According to these papers, the receiver should believe in the sender's message whenever it is credible, because a rich common language should enable the sender to convince the receiver that he is truthful. Other papers (see Blume, Kim and Sobel (1993), and Warneryd (1993)) make
a similar point with the aid of evolutionary models. The idea is that evolutionary pressures may force populations to interpret massages in systematic ways, and consequently, they may cause one of the equilibria to prevail. My analysis shows that the sender’s reputation concerns combined with the receiver’s private information create an alternative mechanism that may prevent babbling and guarantee meaningful communication. However, there are two important differences. First, I rather focus on the question why meaningful communication may occur even if a conflict of interests arises, and I offer no insight into why no-communication outcome prevails in pure-conflict games. Second, I rather point out missing elements of cheap-talk modeling which may be responsible for counter-intuitive predictions, instead of re-ning the equilibrium concept itself.

Reputation effects are familiar from earlier work. Like Kreps and Wilson (1982), Milgrom and Roberts (1982), and Sobel (1985) in the case of honesty, there is incomplete information about the sender’s type with a fraction of senders committed to telling truth. In a broader sense, the paper belongs to the literature, where messages (or actions) reveal not only the sender’s information but also the sender’s type. Therefore the sender faces a trade-off between information transmission and building his reputation for being some type. Examples include Sobel (1985), Benabou and Laroque (1992), Brandenburger and Polak (1996), Morris (1999), Levy (2000), Ottaviani and Sorensen (1998) and (1999) (and in the cases of actions, Holmström (1999), Scharfstein and Stein (1990), Bernheim (1994), Prendergast and Stole (1996)).

6 Appendix

6.1 Proof of Theorem 1

I need the following notation: 0 and 1 stand for vectors whose all coordinates are equal to 0 and 1, respectively; that is, 0=(0,...,0) 1=(1,...,1). For every vector v, the transposed vector is denoted by v^T; I denotes the identity matrix.
Let
\[ V_j(\frac{3}{n}) = \max_{i=1;:::;m} V(s_j | \frac{3}{n} s_i), \] and
\[ H_j(\frac{3}{n}) = V(s_j | \frac{3}{n} s_j). \]
denote the highest payoff that can be achieved by a strategic sender with signal \( s_j \) and a truth-teller with the same signal, respectively, given strategy \( \frac{3}{n} \). Let \( \frac{3}{n} = (\frac{3}{m} s_1; s_i; ::; \frac{3}{m} s_m), i = 1; :::; m, \) be the strategy of a sender with signal \( s_i \) represented as a row-vector, and \( \frac{3}{n} \) the same strategy represented as a column-vector. The strategy of the sender \( \frac{3}{n} \) can be represented as matrix \([\frac{3}{m} ... \frac{3}{n}]; i, = 1; :::; m \]); I will simply write that \( \frac{3}{n} = [\frac{3}{m} ... \frac{3}{n}]; \]

Given a strategy \( \frac{3}{n} \) of the sender, denote by \( s, r; s; \) and \( r \) the following random variables: the sender’s signal, the sender’s message, and the receiver’s signal, respectively. Further, denote by \( \xi \) the sender’s type; let \( \xi = 1 \) if the sender is a truth-teller, and let \( \xi = 0 \) if he is a strategic sender. Let \( P(A) \) stand for the probability of event \( A \), \( P(A | B) \) for the probability of event \( A \) conditional on event \( B \), \( E(\cdot) \) for the expected value of a random variable \( \cdot \), and \( E(\cdot | B) \) for the expected value of a random variable \( \cdot \) conditional on event \( B \).

I will prove the theorem by a sequence of lemmata.

Lemma 5 Suppose that \( s = s_i \) is used as a lie with a positive probability.

(a) If
\[ \forall_{k=1;:::;n} P(r = r_k j s = s_i & s = s_i) = P(r = r_k j s = s_i & s \notin s_i), \] then
\[ E [P(\xi = 1 j r & s = s_i) j s = s_i & s = s_i] \]
\[ = E [P(\xi = 1 j r & s = s_i) j s = s_i & s \notin s_i]. \]

(b) If (3) does not hold, then the left-hand side of (4) strictly exceeds the right-hand side.
Proof. (a)

\[ E \left[ P(\xi = 1 j r & s = s_i) j s = s_i & s = s_i \right] \]

\[ = \prod_{k=1}^{\infty} P(r = r_k j s = s_i & s = s_i)P(\xi = 1 j r = r_k & s = s_i) \]

\[ = \prod_{k=1}^{\infty} P(r = r_k j s = s_i & s \not\in s_i)P(\xi = 1 j r = r_k & s = s_i) \]

\[ = E \left[ P(\xi = 1 j r & s = s_i) j s = s_i & s \not\in s_i \right] \]

(b) Let

\[ x : = \prod_{j \notin i} \frac{p_j \xi \mathbf{I}^T}{x \xi \mathbf{I}^T}, \quad c := \frac{p_i \xi \mathbf{I}^T}{x \xi \mathbf{I}^T}, \text{ and } a := p_i \xi. \]

Then \( c + a = p_i \), \( a \xi \mathbf{I}^T = 0 \). Observe that (3) can be reformulated as

\[ p_i = c. \]

Indeed,

\[ P(r = r_k j s = s_i & s = s_i) = \frac{P(r = r_k & s = s_i & s = s_i)}{P(s = s_i & s = s_i)} = \frac{p_k^{[\xi] + (1 - \xi)\mathbf{I}(s_i; s_i)}}{p_i \xi \mathbf{I}^T [(1 - \xi) \mathbf{I}(s_i; s_i)]} = \frac{p_k}{p_i \xi \mathbf{I}^T}, \]

and similarly,

\[ P(r = r_k j s = s_i & s \not\in s_i) = \frac{P \prod_{j \notin i} \mathbf{I}(s_i; s_i) p_j}{\prod_{j \notin i} \mathbf{I}(s_i; s_i) p_j \xi \mathbf{I}^T} = \frac{x_k \xi \mathbf{I}^T}{x \xi \mathbf{I}^T}. \]

Thus, if (3) does not hold, then \( a \not\in 0 \). Without loss of generality, I can assume that \( a = (a_1, \ldots, a_l, 0, \ldots, 0, a_{l+1}, \ldots, a_n) \) where \( a_1, \ldots, a_l > 0 \) and \( a_{l+1}, \ldots, a_n < 0 \).

Let \( P(t), t \geq 0 \), stand for the matrix obtained from \( P \) by replacing \( p_i \) with \( c + ta \). Further, denote by \( E^t(s = s_i) \) and \( E^t(s \not\in s_i) \), respectively, the expected values corresponding to \( E \left[ P(\xi = 1 j r & s = s_i) j s = s_i & s = s_i \right] \) and \( E \left[ P(\xi = 1 j r & s = s_i) j s = s_i & s \not\in s_i \right] \) when \( P \) is replaced with \( P(t) \). I shall show that

\[ \frac{E^t(s = s_i)}{a} > 0, \quad (5) \]
for every $t \in [0;1]$.

Indeed, observe that

$$E^t(s = s_i) = \sum_{k=1}^{\infty} P^t(r = r_k \mid \sigma = s_i \land s = s_i) P^t(\xi = 1 \mid r = r_k \land \sigma = s_i) =$$

$$= \sum_{k=1}^{\infty} \frac{P^t(r = r_k \land \sigma = s_i \land s = s_i)}{P^t(\sigma = s_i \land s = s_i)} \frac{P^t(\xi = 1 \land r = r_k \land \sigma = s_i)}{P^t(r = r_k \land \sigma = s_i)} =$$

$$= \sum_{k=1}^{\infty} \frac{(\alpha_k + t_a_k)}{\alpha \xi t^T} \frac{(\alpha_k + t_a_k)}{\alpha \xi t^T} + \frac{(\alpha_k + t_a_k)(1 - \theta^\top s_i)}{\alpha \xi t^T}.$$

Put

$$r := \frac{1_i - \theta^\top s_i}{\alpha \xi t^T}, \quad R = \frac{1}{\alpha \xi t^T},$$

and rewrite the previous equality as

$$E^t(s = s_i) = R \sum_{k=1}^{\infty} \frac{(\alpha_k + t_a_k)^2}{(\alpha_k + t_a_k) + rx_k}.$$

Then

$$\frac{\partial E^t(s = s_i)}{\partial} = R \sum_{k=1}^{\infty} \frac{6_i}{4_i} \frac{1}{1 + \frac{\alpha_k + t_a_k}{rx_k}} + \frac{2}{3}.$$
Since \( \frac{\alpha_k + t_k}{r_{x_k}} > \frac{c}{r} \) for \( k = 1; \ldots; l \) and \( \frac{\alpha_k + t_k}{r_{x_k}} < \frac{c}{r} \) for \( k = q; \ldots; n \),

\[
\min_{k=1;\ldots;l} \frac{1}{1 + \frac{\alpha_k + t_k}{r_{x_k}}} > \max_{k=q;\ldots;n} \frac{1}{1 + \frac{\alpha_k + t_k}{r_{x_k}}},
\]

and thus, taking into account that \( \prod_{k=1}^{n} a_k = i \prod_{k=q}^{n} a_k \),

\[
\frac{\partial E(t)}{\partial t} > 0.
\]

Observe that

\[
E^t(s \in s_i) = \sum_{j \in i} P(s = s_j \& s = s_i \& s \in s_i) \cdot E[P(\hat{z} = 1| r \& s = s_i) \& s = s_i \& s \in s_j]
\]

\[
= \sum_{j \in i} p_{j} \left( \prod_{k=1}^{\frac{3q}{4} s_i} (\alpha_k + t_k)^{n_k} \cdot \prod_{k=1}^{\frac{1}{4} s_i} (\alpha_k + t_k)^{n_k} \cdot \prod_{k=1}^{\frac{3q}{4} s_i} (\alpha_k + t_k)^{n_k} \right) \cdot \prod_{k=1}^{\frac{1}{4} s_i} (\alpha_k + t_k)^{n_k}
\]

\[
= dR \sum_{k=1}^{\frac{3q}{4} s_i} \frac{a_k}{(\alpha_k + t_k) + r_{x_k}}
\]

and so

\[
\frac{\partial}{\partial t} E_t(s \in s_i)
\]

\[
= dR \sum_{k=1}^{\frac{3q}{4} s_i} \frac{a_k}{1 + \frac{\alpha_k + t_k}{r_{x_k}}} - 2
\]

\[
\cdot \sum_{k=1}^{\frac{1}{4} s_i} \frac{a_k}{1 + \frac{\alpha_k + t_k}{r_{x_k}}}
\]

\[
= dR \sum_{k=1}^{\frac{3q}{4} s_i} \frac{a_k}{1 + \frac{\alpha_k + t_k}{r_{x_k}}} - 2
\]

\[
+ dR \sum_{k=q}^{n} \frac{a_k}{1 + \frac{\alpha_k + t_k}{r_{x_k}}}.
\]
Since $\frac{c_k + t_{ak}}{r_x_k} > \frac{c}{r}$ for $k = 1, \ldots, l$ and $\frac{c_k + t_{ak}}{r_x_k} < \frac{c}{r}$ for $k = q, \ldots, n,$

$$\max_{k=1,\ldots,l} \frac{1}{1 + \frac{c_k + t_{ak}}{r_x_k}^2} < \min_{k=q,\ldots,n} \frac{1}{1 + \frac{c_k + t_{ak}}{r_x_k}^2},$$

and thus

$$\frac{\partial (E_t(s \notin s_i))}{\partial t} < 0.$$

Finally, notice that $E^0(s = s_i) = E^0(s \notin s_i)$ by virtue of (a) for $P(0),$ and so

$$E [P(\xi = 1 | r & s = s_i) j s = s_i \& s = s_i] = E^1(s = s_i) > E^0(s = s_i) =$$

$$= E^0(s \notin s_i) > E^1(s \notin s_i) = E [P(\xi = 1 | r & s = s_i) j s = s_i \& s \notin s_i]$$

by (5) and (6). $\blacksquare$

In the next lemma, I list these implications of Lemma 5 that I need in the sequel.

Lemma 6 Suppose that $s = s_i$ is used as a lie with a positive probability.

(a) There exists a constant $c_i$ such that if

$$c_i p_i \notin \frac{3/4}{j \in i} p_j,$$

then $V(s \notin s_i) < V(s \notin s_i)$ for some $j \notin i$ such that $3/4 s_i; s_j > 0.$

(b) If (7) does not hold, then either $V(s \notin s_i) < V(s \notin s_i)$ for some $j \notin i$ such that $3/4 s_i; s_j > 0,$ or $V(s \notin s_i) = V(s \notin s_i)$ for every $j \notin i$ such that $3/4 s_i; s_j > 0.$

Proof. It follows from the following observations:

(i) (7) coincides with (3) if one takes $c_i$ equal to $c$ from the proof of Lemma 5;

(ii)

$$V(s \notin s_i) = E [P(\xi = 1 | r & s = s_i) j s = s_i \& s = s_i];$$

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(iii) there exist $w_j$, $j \notin i$, such that $\prod_{j \notin i} w_j = 1$, and
\[ w_j V(s_j \nmid s_i) = E \left[ P(\xi = 1 \mid r \& s = s_i) \mid s = s_i \& s \notin s_i \right]; \]
more specifically,
\[ w_j = \frac{p^{3/4} s_i; s_j P_j \cdot 1^T}{p^{3/4} s_i; s_j P_j \cdot 1^T}. \]

Lemma 7 In every equilibrium, the expected payo$\grave{s}$ of the sender equals " independently of his signal:
\[ V_j(\nmid s_i) = " \text{ for } j = 1; \cdots; m. \]

Proof. In equilibrium,
\[ "p_i \cdot 1^T H_i(\nmid s_i) + \sum_{j=1}^{m} (1 \nmid s_i) P_j \cdot 1^T 3/4 s_i; s_j V(s_j \nmid s_i) = "p_i \cdot 1^T. \quad (8) \]

Indeed,
\[ "p_i \cdot 1^T H_i(\nmid s_i) + \sum_{j=1}^{m} (1 \nmid s_i) P_j \cdot 1^T 3/4 s_i; s_j V(s_j \nmid s_i) = \]
\[ \sum_{k=1}^{m} \left( \prod_{j=1}^{3/4 s_i; s_j} \frac{p_j \cdot 1^T 3/4 s_i; s_j}{p_j \cdot 1^T 3/4 s_i; s_j} \right) + \]
\[ \sum_{k=1}^{m} \left( \prod_{j=1}^{3/4 s_i; s_j} \frac{p_j \cdot 1^T 3/4 s_i; s_j}{p_j \cdot 1^T 3/4 s_i; s_j} \right) \frac{p_j \cdot 1^T 3/4 s_i; s_j}{p_j \cdot 1^T 3/4 s_i; s_j} \]
\[ = \sum_{k=1}^{m} \frac{p_j \cdot 1^T 3/4 s_i; s_j}{p_j \cdot 1^T 3/4 s_i; s_j} \frac{p_j \cdot 1^T 3/4 s_i; s_j}{p_j \cdot 1^T 3/4 s_i; s_j} \]
\[ = \prod_{k=1}^{m} \frac{p_j \cdot 1^T 3/4 s_i; s_j}{p_j \cdot 1^T 3/4 s_i; s_j} \]
Since every strategic sender can mimic the behavior of an honest sender with the same signal, \( V_i(\frac{3}{4}) \), \( H_i(\frac{3}{4}) \) for every \( i = 1; \ldots; m \). Consider the set \( A \) of those \( i \) that \( V_j(\frac{3}{4}) \) for every \( j = 1; \ldots; m \). Say, \( V_i(\frac{3}{4}) \) \( V \) if \( i \in A \). By virtue of Lemma 6, 
\[
\frac{3}{4}s_i; s_j) = 0 \text{ for every } j \notin A \text{ and } i \in A. \tag{9}
\]
Further notice that \( H_i(\frac{3}{4}) \), \( V_i(\frac{3}{4}) \) for any \( i \in A \). Indeed, if \( \frac{3}{4}s_i; s_j) = 0 \) for every \( i = 1; \ldots; m \), then \( H_i(\frac{3}{4}) = 1 \); if \( \frac{3}{4}s_i; s_j) > 0 \) for some \( j \), then by Lemma 6, there is also such a \( j \) that \( V_i(\frac{3}{4}) \cdot V_j(\frac{3}{4}) = V(\frac{3}{4}s_j; s_j) \cdot V(\frac{3}{4}s_j; s_j) = H_i(\frac{3}{4}) \). Therefore \( H_i(\frac{3}{4}) = V_i(\frac{3}{4}) = V \) for every \( i \in A \). Since \( V(\frac{3}{4}s_j; s_j) = V_i(\frac{3}{4}) \), \( V_i(\frac{3}{4}) = V(\frac{3}{4}s_j; s_j) \) for every \( j \) such that \( \frac{3}{4}s_i; s_j) > 0 \), Lemma 6 yields \( V(\frac{3}{4}s_j; s_j) = V(\frac{3}{4}) \) \( V \) whenever \( \frac{3}{4}s_i; s_j) > 0 \). Thus by (8) and (9),
\[
X'_{i \in A} p_iq_i^T = X_{i \in A} p_iq_i^T + X_{j \in A} (1 - p_j)q_j^T_{i \in A} \frac{3}{4}s_i; s_j) = #
\]
\[
= V_{i \in A} p_iq_i^T + V_{j \in A} (1 - p_j)q_j^T_{i \in A} \frac{3}{4}s_i; s_j) \cdot #
\]
\[
= V_{i \in A} p_iq_i^T + V_{j \in A} p_jq_j^T(1 - p_j)_{i \in A} \frac{3}{4}s_i; s_j) .
\]
and so \( V \). In particular, \( H_i(\frac{3}{4}) \), \( " \) for \( i = 1; \ldots; m \) and \( V(\frac{3}{4}s_j; s_j) \), \( " \) whenever \( \frac{3}{4}s_i; s_j) > 0 \).

Thus, by Lemma 6, \( H_i(\frac{3}{4}) \), \( " \) for every \( i = 1; \ldots; m \) as well as \( V(\frac{3}{4}s_j; s_j) \), \( " \) whenever \( \frac{3}{4}s_i; s_j) > 0 \). This in turn yields \( H_i(\frac{3}{4}) = " \) for every \( i = 1; \ldots; m \) as well as \( V(\frac{3}{4}s_j; s_j) = " \) whenever \( \frac{3}{4}s_i; s_j) > 0 \) by virtue of (8).

Remark 3

If \( \text{rank} P = m \), then the uniqueness of the truth-telling equilibrium follows from Lemmata 6(a) and 7. Indeed, if \( \frac{3}{4}s_i; s_j) > 0 \) for some \( j \in i \), then there is also such
Lemma 8  In every equilibrium

\[ \frac{3}{4} \Phi P = P. \]

Proof. By Lemmata 6(a) and 7

\[ c_i p_i = \sum_{j \neq i} \left( \frac{3}{4} \sigma_i; s_j \right) p_j \]

for some constant \( c_i \). Thus,

\[ V(\sigma_i; s_i) = \frac{X^n}{\prod_{k=1}^{i} p_i \cdot \prod_{k=1}^{i} p_j^{\gamma} + p_i(1 - \gamma) \cdot \frac{3}{4} \sigma_i; s_i) + (1 - \gamma) c_i p_i^{\gamma} \]

\[ = \frac{1}{\prod_{k=1}^{i} p_i \cdot \prod_{k=1}^{i} p_j^{\gamma}} \cdot \frac{X^n}{\prod_{k=1}^{i} p_j^{\gamma} + (1 - \gamma) \left[ \frac{3}{4} \sigma_i; s_j \right] + c_i}. \]

If \( \frac{3}{4} \sigma_i; s_j > 0 \), then \( V(\sigma_i; s_i) = \) by Lemma 7. Suppose that \( \frac{3}{4} \sigma_i; s_j = 0 \).

If also \( \frac{3}{4} \sigma_i; s_j = 0 \) for every \( j \neq i \), then \( V(\sigma_i; s_i) = 1 \) which cannot happen in equilibrium. If \( \frac{3}{4} \sigma_i; s_j > 0 \) for some \( j \neq i \), then by Lemmata 6 and 7, again \( V(\sigma_i; s_i) = \) . Solving the equation for \( c_i \), I obtain \( c_i = \gamma - \frac{3}{4} \sigma_i; s_i \), so

\[ p_i = \sum_{j=1}^{X^n} \frac{3}{4} \sigma_i; s_j p_j, \]

or equivalently \( \frac{3}{4} \Phi P = P. \)

Definition 2  Let \( \frac{3}{4} \) be an equilibrium strategy of the sender.

(i) A set \( A \in A \) \( 1 \leq 1; \ldots; m \) is called separable if \( \frac{3}{4} \sigma_i; s_j = 0 \) for every \( j \neq A \) and \( i \in A \).

(ii) A separable set \( A \) is called minimal if no proper subset of \( A \) is a separable set.
In words, it means that a sender with a signal $s_j$, $j \not\in A$, never sends a signal $s_i$, $i \in A$; notice, however, that the definition allows a sender with a signal $s_i$, $i \in A$, to send a signal $s_j$, $j \not\in A$. The next lemma lists these properties of separable sets that I need in the sequel.

Lemma 9  
(a) $A = \{1; \ldots; m\}$ is a separable set.  
(b) Every separable set contains a minimal separable set.  
(c) If $\frac{3}{4}s_i; s_i < 1$ for some $i = 1; \ldots; m$, then there is a separable set $A$ such that $\frac{3}{4}s_i; s_i < 1$ for every $i \in A$.  
(d) If $\frac{3}{4}s_i; s_i < 1$ for every $i \in A$ and $A$ is a separable set, then $A$ contains at least two elements.

Proof.  (a) and (b) are straightforward. To obtain (c) put $A = \{1; \ldots; m: \frac{3}{4}s_i; s_i < 1\}$. (d) follows from Lemma 7. Indeed, otherwise the equilibrium payoff of the sender with the only signal $s_i$, $i \in A$, would be greater than $\gamma$.

For any set $A \subseteq \{1; \ldots; m\}$ denote by $m_A$ the number of elements of $A$. Given a strategy $\frac{3}{4}$ let $\frac{3}{4}_A$ stand for the matrix obtained from $\frac{3}{4}$ by deleting all row and columns corresponding to signals $s_i$, $i \not\in A$, and let $P_A$ denote the matrix obtained from $P$ by deleting all rows corresponding to signals $s_i$, $i \not\in A$.

Lemma 10 If $A$ is a minimal separable set and $\frac{3}{4}s = s_j; s_i < 1$ for every $i \in A$, then

$$\text{rank}(\frac{3}{4}_A \cdot I) \cdot m_A \cdot 1.$$  

Proof. I shall show that either $\gamma = 1$ is not an eigenvalue of $\frac{3}{4}_A$, or the space of eigenvectors corresponding to the eigenvalue $\gamma = 1$ is 1-dimensional. Namely, each eigenvector must have all coordinates equal.

Suppose $v$ is an eigenvector corresponding to $\gamma = 1$. Without loss of generality I can assume that the first coordinate of $v$ is the largest coordinate, $v_1 \geq v_i$ for $i = 1; \ldots; m_A$. Suppose to the contrary that $v_1 > v_i$ for some $i$, and let $i^\ast$ be the
smallest such an i. Then each of the rst i coordinates of v ∈ A is not greater than
\[ X_i^{A_{s_1}} = \sum_{j=1}^{3/4s_1} v_j. \]

Since \( P_{i=1}^{m_A} 3/4s_i; s_i \cdot 1 \) and \( 3/4s_i; s_i \) 0 for every i, this expression can be equal to \( v_1 \) only if either \( v_1 = v_i \) for every i or \( 3/4s_i; s_i = 0 \) for every j, i \( \leq i \leq i \). The former case is excluded by assumption. In the latter case, I obtain a contradiction with the minimality of A; its proper subset obtained by removing \( i = 1; \cdots; i = 1 \) form A is a separable set as well.

Now I can prove Theorem 1.

Proof. (a) and the “only if” part of (b) are straightforward. To show “if” part suppose that \( 3/4s_i; s_i < 1 \) for some i. Take a minimal separable set A with \( 3/4s_i; s_i < 1 \) for every i \( \geq 2 \). By Lemma 9 such a set exists and contains at least two elements. Represent \( f s_1; \cdots; s_m \) as \( S^1 \cup S^2 \), where \( S^1 = f s_i : i \geq 2 \) Ag and \( S^2 = f s_i : i \leq Ag \). By Lemma 10, \( \text{rank}(S^1 \cup S^2) \geq m_A \) 1. Therefore \( \text{rank}(P_{A \setminus I}) \geq m_A \) 1 ranking \( 3/4s_i; s_i \) = 1 which completes the proof of “if” part of (b) as \( P_A \) consists of at least two rows of P.

6.2 Proof of Proposition 2

Proof. Let \( \mathcal{NE}(P; 1/4) \) denote the set of all Nash equilibria of \( G(P; 1/4) \). One readily checks that \( \mathcal{NE}(P; 1/4) \) is an upper hemi-continuous correspondence with respect to \( 1/4 \). Thus Proposition 2 follows from Theorem 1.

6.3 Examples to Section 4

Example 2. Let \( " = 1\Downarrow 4 \) and \( \Downarrow = 1\Rightarrow 2 \). The following strategies form an equilibrium of \( G_{\mathcal{NE}}(P; 1/4) \): S sends \( Y \) when he has signal A1 or A2. If he has signal N, he sends N with probability \( 3/4\mathcal{NE} = 1\Downarrow \) and Y with probability \( 1 \Downarrow 3/4\mathcal{NE} \). R takes action V whenever she hears Y, and she takes I whenever she hears N. The sender’s payo is
sending $N$ coincides with the reputation payoffs and equals to

$$
(1 + \frac{1}{2} + (1 + \frac{1}{2})^{\frac{3}{2}}) = (1 + \frac{1}{2}).
$$

His payoffs to sending $Y$ equals to the sum of his action payoffs $\frac{1}{2}$ and his reputation payoffs

$$
(1 + \frac{1}{2} + (1 + \frac{1}{2})^{\frac{3}{2}}) = (1 + \frac{1}{2}).
$$

Hence, $S$ is indifferent between sending $Y$ and $N$. If $R$ hears $Y$, she assigns probability

$$
\frac{1}{2} + \frac{1}{2} = \frac{2}{3}
$$

that $S$ has signal $A1$ or $A2$. Thus, she is indifferent between actions $V$ and $I$. $R$ obviously prefers action $I$ whenever she hears $N$.

The following strategies form an equilibrium of $G_E(P; \frac{1}{2})$:

(i) $S$ reveals truthfully signals $A1$ and $A2$,

(ii) $S$ with signal $N$ reveals his signal truthfully with probability $\frac{3}{4}$, and he sends $\overline{A1}$ and $\overline{A2}$ with probability $(1 + \frac{3}{4}) = \frac{2}{3}$ each,

(iii) $R$ takes action $V$ whenever she hears $\overline{A1}$ ($\overline{A2}$) and her own signal is $W$ ($M$),

(iv) $R$ takes action $V$ with probability $\varpi$ and action $I$ with probability $1 + \varpi$ when she hears $\overline{A1}$ ($\overline{A2}$) and her own signal is $M$ ($W$),

(i) $R$ takes action $I$ whenever she hears $N$, provided $\frac{3}{4}$ and $\varpi$ satisfy

$$
(1 + \frac{1}{2} + (1 + \frac{1}{2})^{\frac{3}{2}}) = \frac{1}{2} + \frac{1}{2} + \varpi
$$

$$
(1 + \frac{1}{2} + (1 + \frac{1}{2})^{\frac{3}{2}}) = \frac{1}{2} + \frac{1}{2} + \varpi
$$

$$
(1 + \frac{1}{2} + (1 + \frac{1}{2})^{\frac{3}{2}}) = \frac{1}{2} + \frac{1}{2} + \varpi
$$

provided $\frac{3}{4}$ and $\varpi$ satisfy

$$
(1 + \frac{1}{2} + (1 + \frac{1}{2})^{\frac{3}{2}}) = \frac{1}{2} + \frac{1}{2} + \varpi
$$

$$
(1 + \frac{1}{2} + (1 + \frac{1}{2})^{\frac{3}{2}}) = \frac{1}{2} + \frac{1}{2} + \varpi
$$

$$
(1 + \frac{1}{2} + (1 + \frac{1}{2})^{\frac{3}{2}}) = \frac{1}{2} + \frac{1}{2} + \varpi
$$

$$
(1 + \frac{1}{2} + (1 + \frac{1}{2})^{\frac{3}{2}}) = \frac{1}{2} + \frac{1}{2} + \varpi
$$

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Equation (10) guarantees that R with signal \( M \) (or \( W \)) is indifferent between taking \( V \) and \( I \) when she hears message \( A_1 \) (or, respectively, \( A_2 \)), and equation (11) guarantees that S with signal \( N \) is indifferent between sending each of the three messages.

Notice also that \( \theta = 1 \) and \( \frac{3}{4} \) satisfy (10) and (11) for \( \theta = 0 \), and for \( \theta \) sufficiently small, there exist unique \( \frac{3}{4} \) and \( \theta \) satisfying (10) and (11). Notice further that by continuity, \( \theta < 1 \). Suppose that \( \frac{3}{4} \) satisfies (11) for \( \theta = 0 \),

\[
\frac{1}{8} \theta ^{2} < \frac{1}{4} \theta ^{2} \left( 1 - \frac{3}{4} \right) \left( 1 - \frac{3}{4} \right) = \frac{1}{16},
\]

and so R with signal \( M \) prefers action \( I \) when she hears message \( A_1 \), a contradiction with \( \theta < 1 \). That is, \( \frac{3}{4} \) satisfies (11).

It is obvious that the receiver's payoff is higher in the equilibrium of \( G_E(P; \frac{3}{4}) \) than in the equilibrium of \( G_{NE}(P; \frac{3}{4}) \).

Example 3. Let \( \gamma = 1 = 4 \), and \( \frac{1}{2} = 3 = 13 \). One can check that the following strategies form an equilibrium of \( G_{NE}(P; \frac{3}{4}) \): S sends \( Y \) whenever he has signal \( A_1 \) or \( A_2 \). If he has signal \( N \), he sends \( \overline{N} \) with probability \( \frac{3}{4} \) and \( Y \) with probability \( \frac{1}{2} \). R takes action \( I \) whenever she hears \( Y \), and she takes \( I \) also when she hears \( \overline{N} \). Note that although R takes \( I \) when she hears \( Y \), she is indifferent between taking \( V \) and \( I \). Indeed, she assigns probability

\[
\frac{1}{2} \left( \frac{1}{2} \right) \left( 1 - \frac{3}{4} \right) = \frac{1}{4}
\]

that S has signal \( N \).

The following strategies form an equilibrium of \( G_E(P; \frac{3}{4}) \):

(i) S reveals truthfully signals \( A_1 \) and \( A_2 \),

(ii) S with signal \( N \) reveals his signal truthfully with probability \( \frac{3}{4} \), and he sends \( \overline{A_1} \) and \( \overline{A_2} \) with probability \( \frac{1}{2} \) each,

(iii) R takes action \( I \) whenever she hears \( \overline{A_1} \) (\( \overline{A_2} \)) and her own signal is \( M \) (\( W \)),

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(iv) \( R \) takes action \( V \) with probability \( \varpi \) and action \( I \) with probability \( 1 - \varpi \) when she hears \( \overline{A}T \; (\overline{A}2) \) and her own signal is \( W \; (M) \).

(iv) \( R \) takes action \( I \) whenever she hears \( \overline{N} \),

provided \( \frac{3}{4} \tilde{E} \) and \( \varpi \) satisfy

\[
\frac{1}{8} + \varpi = 2 \left( 1 - \frac{1}{4} \right) \left( \frac{1}{2} \right) \frac{1}{2} \left( \frac{3}{4} \right) \tilde{E} \tag{12}
\]

\[
\begin{align*}
\frac{1}{2} \left( 1 - \frac{1}{2} \right) + \left( 1 - \frac{1}{2} \right) \left( \frac{3}{4} \right) \tilde{E} \tilde{N} & = \frac{1}{2} \varpi + \frac{1}{2} \left( 1 - \varpi \right) + \frac{1}{2} \left( 1 - \varpi \right) \tilde{N} \\
+ \left( 1 - \frac{1}{2} \right) \left( \frac{3}{4} \right) \tilde{E} \tilde{N} & = \frac{1}{2} \left( 1 - \varpi \right) + \frac{1}{2} \left( 1 - \varpi \right) \tilde{N} \\
\end{align*}
\tag{13}
\]

Equation (13) guarantees that \( S \) with signal \( N \) is indifferent between sending each of the three messages, and equation (12) guarantees that \( R \) with signal \( W \; (M) \) is indifferent between taking \( V \) and \( I \); when she hears message \( \overline{A}T \) (or, respectively, \( \overline{A}2 \)).

Notice \( \frac{3}{4} \tilde{E} = 1 \Rightarrow \varpi = 0 \) satisfy (13) and (12) for \( \varpi = 0 \), and for \( \varpi \) sufficiently small, there exist unique \( \frac{3}{4} \tilde{E} \) and \( \varpi \) satisfying (13) and (12). Notice further that by continuity, \( \varpi < 1 \). Suppose that \( \frac{3}{4} \tilde{E} \) \( \neq \frac{3}{4} \tilde{N} \). Then, since \( \frac{3}{4} \tilde{N} \) satisfies (12) for \( \varpi = 0 \),

\[
\frac{1}{8} + \varpi > 2 \left( 1 - \frac{1}{4} \right) \left( \frac{1}{2} \right) \frac{3}{4} \tilde{E} \tilde{N} \tag{14}
\]

and so \( R \) with signal \( W \) prefers action \( V \) when she hears message \( \overline{A}T \), a contradiction with \( \varpi < 1 \). That is, \( \frac{3}{4} \tilde{E} < \frac{3}{4} \tilde{N} \).

Let \( U_{NA} \) and \( U_A \), respectively, stand for the ex ante utility of \( R \) in the \( G_{NE}(P; \frac{1}{2}) \) and \( G_{E}(P; \frac{1}{2}) \) equilibria defined above. One readily checks that

\[
U_{NA} = \frac{1}{2} \left( 1 - \frac{1}{4} \right) \frac{3}{4} \tilde{E} \tilde{N} \left( 1 - \frac{1}{4} \right) \frac{1}{2} \frac{1}{2} \left( \frac{3}{4} \right) \tilde{E} \tag{14}
\]

\[
U_A = \frac{1}{2} \left( 1 - \frac{1}{4} \right) \frac{3}{4} \tilde{E} \tilde{N} \left( 1 - \frac{1}{4} \right) \frac{1}{2} \frac{1}{2} \left( \frac{3}{4} \right) \tilde{E} \tilde{N} \left( 1 - \frac{1}{4} \right) \frac{1}{2} \frac{1}{2} \left( \frac{3}{4} \right) \tilde{E} \tilde{N} \tag{14}
\]

\[
= \frac{1}{2} \left( 1 - \frac{1}{4} \right) \frac{3}{4} \tilde{E} \tilde{N} \left( 1 - \frac{1}{4} \right) \frac{1}{2} \frac{1}{2} \left( \frac{3}{4} \right) \tilde{E} \tilde{N} \left( 1 - \frac{1}{4} \right) \frac{1}{2} \frac{1}{2} \left( \frac{3}{4} \right) \tilde{E} \tilde{N} \tag{14}
\]
Recall that $\gamma < 1$. If $\gamma > 0$, then $R$ must be indifferent between taking action $V$ and action $A$; when she hears $A_1$ and her own signal is $W$. That is,

$$
\mu \frac{1}{8} + \nu = 2 \frac{1}{4} (1 - \gamma) \left( \frac{1}{2} \gamma \right)
$$

and so $U_A = \frac{1}{2}'' + \frac{1}{2} \gamma (1 - \gamma)$ by (14). If $\gamma = 0$, then $U_A = \frac{1}{2}'' + \frac{1}{2} \gamma (1 - \gamma)$ follows directly from (14). This yields $U_A > U_{NA}$ as $\gamma < \gamma^E$.

7 References


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