Party Formation and Policy Outcomes
under Different Electoral Systems *

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Abstract

I introduce a simple model of representative democracy that allows for strategic parties, strategic candidates, strategic voters, and multiple districts. If the distribution of policy preferences is not too heterogeneous across and within districts, then the number of effective parties is larger under Proportional Representation than under Plurality Voting, confirming Duverger’s hypothesis, and both electoral systems determine the median voter’s preferred outcome. However, for very asymmetric distributions of preferences Duverger’s hypothesis can be reversed, and the policy outcome can differ from that of direct democracy: compared with the median voter’s preferred outcome, the policy outcome with Proportional Representation can be biased only towards the center, whereas under Plurality Voting the policy outcome can be anywhere. Strategic and sincere voting are indistinguishable in the presence of endogenous candidates and strategic parties.

Keywords: Duvergerian predictions, Electoral Systems, Incentives to Run, Voting.

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1 Introduction

In political science the most famous predictions about the effects of electoral systems are the so-called “Duverger’s law” and “Duverger’s hypothesis” (Duverger 1954). These are informal observations/predictions, which can be summarized as follows:

- **Duverger’s law**: under Plurality Voting there are forces leading the number of effective parties competing in a district to be no greater than two;

- **Duverger’s hypothesis**: under proportional systems there is a larger number of effective parties than under majoritarian systems.¹

These informal predictions are about elections in a single or unified district. Even though Duverger was writing about political parties, the subsequent literature has applied the predictions to candidates, and indeed in a single-district election the two types of political entities may be interchanged from the point of view of the Duvergerian predictions. There have been several attempts to formalize Duverger’s law,² showing that it can be derived from the rational choice of strategic voters. These formal attempts stick to the single-district world and do not distinguish between candidates and parties. One of the contributions of this paper is to provide a framework where both Duvergerian predictions can be studied even when the electorate is divided in multiple districts and candidates and parties are separate entities. The party structure as well as the type composition of the pool of candidates are endogenous and play different roles.

Any serious comparison of electoral systems in a multi-district representative democracy requires not only a careful distinction between the role of parties and that of candidates, as indicated above, but also a characterization of the interplay of strategic voters, strategic parties, and strategic candidates, within and across districts. In general, the equilibrium outcomes of representative democracy when assuming sincere voting will turn out to be almost identical to those under strategic voting, and this is because strategic parties and endogenous candidates can “substitute” for the coordination of voters’ strategies. Endogenous candidacy is necessary (see Dutta, Jackson, and Le Breton 2001) to appropriately compare

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¹For a lucid discussion and empirical evidence on the Duvergerian predictions see Cox (1997).
voting procedures, but it is also sufficient, most of the time, to determine rational outcomes even when the voters are not strategic.

Politicians (potential candidates) care both about the private benefits of being elected (e.g., “ego rents”) and about the policy outcome. Both dimensions are important for the determination of the incentives to run. The paper will show, however, that the balance between private benefits from election and policy preferences may matter for the equilibrium party structure only under sincere voting and Proportional Representation (PR). With strategic voting that balance loses its relevance, no matter what electoral system is used.

Beside the methodological innovations and the characterization results on the Duvergerian predictions and on the sincere vs. strategic voting issue, this paper also provides some simple evaluations of the most used electoral systems in terms of policy directions. In particular, the policy outcome of representative democracy under PR and Plurality Voting (PV) is compared with the outcome of direct democracy, i.e., with the median voter’s preferred policy.

The summary description of the model is as follows. For each type of policy preference in the population there is a set of politicians. There is a simple party formation stage where if politicians of different types agree on some policy compromise then they can form a heterogeneous party, otherwise all parties will simply be homogeneous sets of politicians. After the party structure is determined, the politicians decide whether to run or not (endogenous candidacy). Voting is an active stage of the game only under the strategic voting scenario. The electoral system determines a mapping from the election results (i.e., distributions of votes) to a distribution of seats in a parliament, which then determines the policy by majority rule.

The primary role of parties (homogeneous or heterogeneous) is that they provide a coordination device to voters during the elections. When sincere voting is not an equilibrium and there are many ways in which voters could vote, the party leaders help their voters to coordinate their strategies. In addition, a heterogeneous party can provide a commitment device to its politicians before elections. In contrast with the standard electoral competition models, voters know the policy preferences of all politicians, and they can believe that a

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3See Alesina (1988) and Wittman (1977) for some classical reference for electoral competition in general, and see Lizzeri and Persico (2001) for a specific electoral competition model with interesting implications for a comparison between proportional and pluralitarian systems.
politician is going to pursue a policy platform different from her own preferred one only if the announced policy platform corresponds to a policy compromise that had been agreed upon within her party.\footnote{Other papers that studied different issues related to the role of parties in rational models of representative democracy are Baron (1993), Jackson and Moselle (2001), Caillaud and Tirole (1998), and Riviere (1998). Baron (1993) views parties as coalitions of voters, each voting for one of three exogenously given candidates. Jackson and Moselle (2001) study party-like behavior in the legislature, with no explicit party formation stage. Caillaud and Tirole interpret parties as information intermediaries that select high quality candidates. Riviere (1998) views parties as a way to help candidates to share the candidacy costs.}

Equilibria are characterized for every distribution of policy preferences. Under PR there will generally be multiple candidates in each district and multiple active parties. Under PV, on the other hand, every equilibrium will display a unique running candidate in each district, and there will not be more than two active parties unless the distribution of policy preferences is very heterogeneous across and within districts. The Duvergerian predictions are so qualified by distinguishing the effect of electoral systems on incentives to run from the effects on the type composition of the set of endogenous candidates, which is what determines the number of active parties. The multiplicity of candidates and parties in India in spite of its PV system is explainable by the same forces that justify the Duvergerian predictions when the distribution of preferences is more symmetric.\footnote{For a comparison of PV systems like India and the United States, see Chhibber and Kollman (1998).}

Under PR it may matter whether voters are strategic or not. In fact, if voters are sincere and an extreme type has the relative majority of preferences in the country (in a way that it could obtain the majority of seats in the parliament), then the only way for the other types to avoid such an extreme outcome is to have only a subset of them run, so that sincere voters, who choose the closest candidates to their type, would be induced to coordinate. If the “ego rents” are small compared to the policy gains obtainable this way, some types of candidates will decide not to run in order to allow that coordination. On the other hand, if the private benefits from being elected outweigh the policy considerations, then the only way to achieve that coordination is to form a heterogeneous party at the beginning. Under strategic voting, instead, these considerations will be irrelevant, because candidates will anticipate that coordination will occur anyway at the voting stage.
As far as the policy outcome is concerned, it turns out that for not too asymmetric distributions of preferences the median voter’s preferred outcome is the unique policy outcome under both PR and PV. However, when the policy outcome of representative democracy differs from what we would have with direct democracy, under PR the policy outcome can only be more “centrist” than what the median voter wants, whereas under PV the policy outcome can be more centrist but also more extreme.

The paper is organized as follows: Section 2 describes the model; Section 3 contains the equilibrium analysis and results under both electoral systems; Section 4 highlights some robustness issues and generalizations. Section 5 concludes and emphasizes the contribution to the literature.

2 The Model

Consider a representative democracy divided in three districts, indexed by \( l = 1, 2, 3 \). There are three types of citizens, identified by their position \( t_i, \ i = L, M, H \), on a unidimensional policy space (single-peaked policy preferences). To use the simplest normalization of such a policy space, Let \( t_L = 0, \ t_H = 1, \) and \( 0 < t_M \leq \frac{1}{2} \).6 In each district \( l \) there is a continuum of voters of each type.7 To avoid studying multiple cases, let’s assume that all districts have the same measure of voters, normalized to \( \frac{1}{3} \) per district. The set \( P \) of politicians (potential candidates) is exogenous.8 However, the set of actual candidates will be endogenous.

The policy outcome \( t^* \) is decided via majority rule by the elected parliament, composed of three elected members. Any politician \( k \in P \) has two potential motivations to run: first, to affect the policy outcome; second, there is a non-transferable private benefit from being elected, \( \pi \). Some interesting results will come from studying the effect of changing the relative

\[ \text{The choice to make type } M \text{ be closer to type } L \text{ than to type } H \text{ is obviously without loss of generality. It simplifies notation though: when I will have to distinguish between the closer of the two extreme types and the one further away I will be able to call them just } L \text{ and } H \text{ respectively.} \]

\[ \text{Nothing changes if one wants to use a finite number of citizens.} \]

\[ \text{In an earlier version of the paper (available upon request) the number of districts, the number of types, and the relative size of districts and types were left unspecified and general, and the set of politicians was endogenous. However, since the results for three districts, three types, and exogenous politicians are qualitatively identical, the lack of generality is not important, and the analysis becomes much easier.} \]
importance of private benefits from election and policy preferences.

Having introduced all the ingredients, let’s now turn to describe the representative democracy game.

2.1 Stage 1: Party Formation

Before the game starts, the citizens of each type $i$ are represented by a party $A_i$, which is a set of politicians taken from the set of citizens of type $i$. The exogenous set of politicians, $P = A_L \cup A_M \cup A_H$, contains for simplicity only nine members, one of each type in each district, so that $\#A_i = 3 \forall i$. Each homogeneous party $A_i$ has a leader $\lambda_i$. Since any politician of $A_i$ has the same utility function as any other citizen of type $i$, each party leader’s objective at the party formation stage is to minimize the distance between the policy outcome and the preferred policy of her type. The three leaders play a party formation game as follows.

1. First $\lambda_L$ and $\lambda_H$ simultaneously make offers to $\lambda_M$, and each of the two offers is constituted by a policy proposal $\tau_i \in [0, 1]$.

2. Then $\lambda_M$ chooses a response $r \in \{0, L, H\}$, where 0 means that no offer is accepted and $r = i$ means that the offer by $\lambda_i$ is accepted ($i = L, H$). In case of indifference, $r = 0$.9

Each profile $(\tau_L, \tau_H, r)$ determines a party structure in the following simple way:

1. If $r = 0$, then the party structure remains $\sigma_0 \equiv \{A_L, A_M, A_H\}$;

2. If $r = L$ then $\sigma_L \equiv \{A_L \cup A_M, A_H\}$;

3. If $r = H$ then $\sigma_H \equiv \{A_L, A_M \cup A_H\}$.

Thus, the endogenous number of parties is $n(\sigma_i) = 2$ for $i = L, H$ and $n(\sigma_0) = 3$. I will use the index $j$ when referring to a generic party of a party structure $\sigma$.

The corresponding vector of party positions varies as follows:

1. $\tau(\sigma_0) = (t_L, t_M, t_H)$;

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9This tie-breaking rule is consistent with any assumption that one could make about positive costs of forming heterogeneous parties.
Every politician \( k \) ending up in party \( j \) will be evaluated by voters on the basis of the endogenous \( \tau_j(\sigma) \) (which, as just described, equals \( t_k \) if \( j \) is a homogeneous party). Politicians cannot be associated to a policy platform different from their ideological one unless they belong to a heterogeneous party that agreed on a policy compromise.\(^{10}\)

### 2.2 Stage 2: Candidacy

For every outcome of stage 1, i.e., for every \( \sigma \in \{\sigma_0, \sigma_L, \sigma_H\} \) and for every \( \tau \), the nine politicians have to decide whether to run or not. For simplicity, I assume that they move sequentially, and that the politicians of type \( M \) are the first three to move; then those of type \( L \) and last those of type \( H \).\(^{11}\) The decision of politician \( k \) is denoted by \( c_k \in \{0, 1\} \), where \( 1(0) \) indicates the decision to run (not to run). The endogenous number of candidates is then \( y = \sum_{k=1}^9 c_k \). The set of endogenous candidates will be denoted by \( Y \), and the set of endogenous candidates in district \( l \) will be denoted by \( Y^l \). When all the three politicians of the three types in district \( l \) decide to run I will use the simple notation \( Y^l = P^l \).

Candidacy involves a small cost \( \epsilon \), with \( 0 < \epsilon < \pi/3 \). Note that \( \pi > 3\epsilon \) guarantees that \( y^l \geq 1 \ \forall l \) and that under PR \( y^l = 3 \) may satisfy individual rationality.

### 2.3 Stage 3: Elections

Each voter of each district \( l \) has to choose among the candidates in \( Y^l \). There is a continuum of voters in each district, with single-peaked preferences on the policy space \([0, 1]\) (with peaks

\(^{10}\)The assumption that a policy compromise \( \tau \) in a heterogeneous party \( A_i \cup A_M \ (i = L, H) \) is credible to voters is made for simplicity, and in Section 4 I will show that, given the equilibrium characterization, credibility can be easily endogenized. I will also show that the asymmetric rules of stage 1 (with the median party leader having some advantage) are a harmless simplification, and that the results would certainly extend to more symmetric party formation game forms.

\(^{11}\)The sequential choice can be substituted by a simultaneous move game, but one would then have to add some refinements to avoid multiplicities and existence problems. The choice of having the median type players move first is motivated by the desire to avoid multiple tedious cases in the main text. However, as shown in Section 4, the results do not depend on the order of play at all.
0, \ t_M, \text{ and } 1). \ The \ set \ of \ distributions \ of \ preferences \ is \ \mathcal{D} = \{\mu_i^{l} \}_{i=L,M,H; l=1,2,3} : \sum_i \mu_i^{l} = 1 \ \forall l\},
where \mu_i^{l} is \ the \ (strictly \ positive) \ fraction \ of \ voters \ of \ type \ i \ in \ district \ l.  
I \ will \ also \ use \ the \ notation \ \mu_i = \sum_l \mu_i^{l}/3 \ to \ denote \ the \ fraction \ of \ the \ country’s \ population \ who \ have \ \tau_i \ as \ most \ preferred \ policy. \ A \ specific \ distribution \ will \ be \ denoted \ by \ d \in \mathcal{D}.

I \ will \ consider \ two \ different \ scenarios: \ sincere \ voting \ and \ a \ simple \ form \ of \ strategic \ voting. \ With \ a \ continuum \ of \ voters \ sincere \ voting \ is \ actually \ an \ undominated \ Nash \ equilibrium, \ as \ any \ other \ voting \ profile, \ because \ no \ voter \ can \ ever \ be \ pivotal. \ However, \ in \ the \ presence \ of \ parties \ it \ is \ realistic \ that \ voters \ can \ coordinate \ (or \ be \ coordinated), \ and \ hence \ they \ can \ behave \ as \ a \ finite \ number \ of \ players, \ in \ which \ case \ sincere \ voting \ is \ not \ necessarily \ a \ Nash \ equilibrium \ behavior. \ In \ order \ to \ simplify \ the \ description \ of \ the \ two \ voting \ scenarios, \ I \ will \ ignore \ the \ case \ in \ which \ \mathcal{Y}^l = \emptyset \ for \ some \ l, \ since \ this \ is \ ruled \ out \ as \ a \ possibility \ by \ the \ assumption \ that \ \pi > 3\epsilon.

Sincere \ voting \ implies \ that \ each \ citizen \ votes \ for \ the \ candidate \ of \ the \ party \ with \ the \ closest \ \tau \ among \ those \ in \ her \ district. \ In \ other \ words, \ each \ voter \ \nu \ in \ district \ l \ casts \ her \ vote \ for \ a \ candidate \ \kappa \ such \ that 

\[ \kappa \in \arg \min_{k \in \mathcal{Y}^l} |\tau_v - \tau_k| \] 

For \ any \ set \ of \ candidates \ \mathcal{Y}, \ z_s(\mathcal{Y}) \ will \ denote \ the \ corresponding \ sincere \ voting \ profile.

As \ a \ simple \ strategic \ voting \ scenario, \ I \ will \ consider \ a \ perfect \ coordination \ environment. \ Think \ of \ the \ \sigma \ \text{ party leaders as being the players. The voters of type } i \ \text{ follow the voting recommendation of their party leader if and only if such a recommendation constitutes a best response to the recommendations made by the other party leaders. In other words, the voting recommendations of the } \sigma \ \text{ party leaders have to constitute a Nash equilibrium. In this voting game each party leader chooses a triplet } z_j = \{z_j^l\}_{l=1,2,3}, \ \text{ where each component is a recommendation to a district’s voters of the type(s) represented by the corresponding party. Formally, each component of } j\text{'s recommendation strategy is a function}

\[ z_j^l : 2^P \times D \times [0,1]^P \rightarrow P_l : \{Y,d,\{\tau_k\}_{k \in \mathcal{Y}}\} \rightarrow k \in \mathcal{Y}^l. \]

\footnote{Since } ^{12}\text{ } \mu_i^{l} \text{ is a fraction, the sum of the three fractions must be one, even though each district has measure }^{13}\text{ } \frac{1}{3} \text{ of voters.}

\footnote{If there is more than one candidate with the same } \tau \text{, the vote is given to anyone of them with equal probability.}
A strategy profile \( z^* = z_L^1, z_L^2, z_L^3, z_M^1, z_M^2, z_M^3, z_H^1, z_H^2, z_H^3 \) is a voting equilibrium given the candidate set \( Y \) iff

\[
|t(z^*(Y)) - t_i| \leq |t(z_j(Y), z^*_j(Y)) - t_i| \quad \forall i \in j, \forall j \in \sigma, \forall z_j \subset Y.
\]

\( Z^*(Y) \) will denote the set of such equilibria.\(^{14}\) A voting profile \( z^{**} \) in \( Z^*(Y) \) is Strong iff there is no coalition of parties \( C \in 2^\sigma \) such that

\[
|t(z^{**}(Y)) - t_i| > |t(z_C, z^{**}_C(Y)) - t_i| \quad \forall i \in j, \forall j \in C, \text{ for some } z_C.
\]

\( Z^{**}(Y) \) will denote the set of strong voting equilibria given \( Y \).

**Definition 1** Sincere voting is rational given the set of candidates \( Y \) only if \( z_s(Y) \in Z^*(Y) \). Sincere voting is rational if this is true for every \( Y \). Finally, sincere voting is strongly rational iff \( z_s(Y) \in Z^{**}(Y) \) \( \forall Y \in 2^P \).

Note that when \( n(\sigma) = 2 \) sincere voting recommendations are always strongly rational.

### 2.4 Electoral Systems

An electoral system determines a distribution of seats for every voting outcome: Denoting by \( v_l^j \) the number (measure) of votes obtained by candidate \( k \) in district \( l \) (with \( \sum_{k \in Y, l} v_l^k = \frac{1}{3} \)), the mapping into seats for party \( j \) is

\[
F_j : [0, 1/3]^3 \rightarrow \{0, 1, 2, 3\} : \{v_k^l\}_{k \in j, l=1,2,3} \rightarrow F_j \in \{0, 1, 2, 3\}
\]

where \( \sum_{j \in \sigma} F_j = 3 \). The distribution of votes to the candidates in \( Y \), \( \{v_k^l\}_{k \in Y, l=1,2,3} \), depends on the voting profile; hence the complete notation \( F_j^E(z(Y)) \) denotes the number of seats going to party \( j \) if the voting profile is \( z(Y) \) and the electoral system is \( E \in \{PR, PV\} \).

Among the various rules used in PR systems to transform votes into seats, the most used is the Hare quota. Recalling that each district \( l \) has a measure \( \frac{1}{3} \) of voters, the total number of votes going to party \( j \) is \( V_j = \sum_{k \in j, l=1,2,3} v_k^l \). The Hare quota rule assigns the first seat to a party \( j \) such that \( V_j \geq V_{j'} \) for every other party \( j' \); then, the second seat goes to the party with the largest remainder, where the remainder for \( j \) is \( V_j - \frac{1}{3} \) and the remainder for any

\(^{14}\)Note that in this formulation of strategic voting abstention is not allowed.
other party \( j' \) is \( V_{j'} \); the third seat, once again, goes to the party with the largest remainder after subtracting \( \frac{1}{3} \) from the total number of votes obtained by the party that got the second seat. Formally, if \( \sigma = \sigma_0, F_{j}^{PR} = 3 \) iff \( V_j - \frac{2}{3} > V_{j'} \lor j' \neq j \); \( F_{j}^{PR} = 2 \) if \( V_j - \frac{1}{3} > V_{j'} \) for some \( j' \neq j \) but \( V_j - \frac{2}{3} < V_{j'} \) for some \( j' \neq j \); \( F_{j}^{PR} = 1 \) \( \forall j \in \sigma \) iff \( \max_j V_j - \frac{1}{3} < V_{j'} \lor j' \).

The PV system is instead characterized by the following function:

\[
F_{j}^{PV} = \sum_{l} \sum_{k \in j} g_{k}^{l}
\]

where

\[
g_{k}^{l} = \begin{cases} 
1 & \text{if } v_{k}^{l} = \max_{k' \in Y_l} v_{k'}^{l} \\
0 & \text{otherwise}.
\end{cases}
\]

The number of seats going to each party depends on how many districts it wins in.

Note that the two electoral systems considered here do not satisfy candidate stability, as pointed out by Dutta, Jackson, and Le Breton (2001). This implies that any serious comparison of their implications must include endogenous candidacy, as I do in this paper.

In terms of who obtains a seat among the candidates of each party under PR, I assume for simplicity that this assignment is done by randomization among the party members who paid the candidacy cost.\(^{15}\) With PV there is no need of specifying any party assignment rule, since each seat is assigned directly to the candidate with the most votes in the corresponding district.

Given an electoral system \( E \in \{PR, PV\} \), I will denote by \( \Gamma^E_s \) the representative democracy game under sincere voting and by \( \Gamma^E_z \) the one under the strategic voting scenario.

2.5 Policy, Payoffs, and Political Outcomes

If the majority of seats is held by politicians with the same policy platform – say \( \tau_j \) – then the policy outcome is obviously \( t^* = \tau_j \). If instead the three seats are held by politicians of three different parties with different policy platforms, then pure majority rule applied to this

\(^{15}\)If one substitutes this assumption with the one that the seats obtained by a party are assigned to the candidates who received the largest number of votes, there are usually less candidates in equilibrium, but the substantive results in terms of number of active parties, policy outcome, role of strategic voting, and Duvergerian predictions are unchanged. Hence I prefer the simpler assumption.
one-dimensional bargaining space guarantees that the outcome is the policy preferred by the median of the three representatives, $t^* = t_M$.

The payoff for voter $v$ is $-|t_v - t^*|$, and the final payoff for politician $k$ that joined party $j$ is $U_k = -c_k e - |t_k - t^*|$ if not elected and $\pi + U_k$ if elected.

Beside the policy and the corresponding citizens’ payoffs, an important political outcome of the representative democracy game is the number of active and effective parties.

**Definition 2** A party $j$ is active if and only if there is at least one running candidate of that party in the whole country, i.e., $\sum_{k \in j} c_k \geq 1$.

**Definition 3** A party $j$ is effective if and only if there is at least an elected candidate of that party in the whole country, i.e., $F_j \geq 1$.

The Reform party in the US has been active but not effective, and this distinction is important to correctly check the validity of the Duvergerian predictions in multi-district representative democracies. The Liberal party in the UK satisfies the condition for effectiveness, and so do many parties in India. I believe this is important because it is an essential feature of a multi-district polity to have minority parties that might control some districts without ever having a chance to win the majority in the whole country.

Note that with imperfect information about voters’ preferences the definition of effective party should be extended to include parties that have a significant chance to win a seat. But this paper studies a perfect information world, in which the current definition of effective party is sufficient to capture what Duverger had in mind. The analysis will also make clear that perfect information makes the characterization of equilibrium active parties practically identical to that of effective parties.

### 3 Equilibrium

Under sincere voting only stages 1 and 2 matter. A profile

$$s^* = (\{\tau_i\}_{i=L,H}; \pi(\tau_L, \tau_H); \{c_k(\sigma, \tau)\}_{k=1,\ldots, 9})$$
constitutes an equilibrium for the representative democracy game $\Gamma^E_s$ if and only if it is Subgame Perfect. The set of Subgame Perfect Equilibrium (SPE) profiles of $\Gamma^E_s$ is denoted by $S^*(E)$, which is obviously nonempty.\(^{16}\)

If voters are strategic, stage 3 becomes a real part of the game. A profile

$$\alpha^* = (\{\tau_i\}_{i=L,H}; r(\tau_L, \tau_H); \{c_k(\sigma, \tau)\}_{k=1,...,9}; \{z_j(Y)\}_{j \in \sigma})$$

collects a noncooperative equilibrium for the representative democracy game $\Gamma^E_z$ if and only if it is Subgame Perfect. The set of SPE profiles of $\Gamma^E_z$ is denoted by $S^*_z(E)$, which is also nonempty, as will be clear from the characterization of equilibria.

A noncooperative equilibrium of $\Gamma^E_z$ is a strong one if it is also robust to coalitional deviations at the voting stage, i.e., iff $z(Y) \in Z^{**}(Y)$ $\forall Y$.

### 3.1 Plurality Voting

The voting subgame requires some analysis only at the nodes where $\sigma = \sigma_0$, since otherwise sincere voting is strongly rational. For any voting Subgame with any candidates set $Y$ there always exists at least one Nash equilibrium play by the three parties that coordinate their voters. Sincere voting is rational in district $l$ given any $Y^l \neq P^l$.\(^{17}\) If $Y^l = P^l$ but district $l$ is “not pivotal” given what happens in the other districts, i.e., when the policy outcome is not affected by the voting behavior in district $l$, then every voting behavior is an equilibrium. If $Y^l = P^l$ but district $l$ is pivotal, then sincere voting can be an equilibrium behavior only if $\mu^l_i > \frac{1}{2}$ for some type $i$. If on the other hand $\mu^l_i < \frac{1}{2}$ $\forall i$, then there are only two possible equilibria: one where the median type voters help the type $L$ candidate to win rather than

\(^{16}\)After the first simultaneous proposal by the two extreme party leaders, the rest of the game is sequential with discrete choices. Thus, to see that existence is guaranteed it is enough to see that at the initial simultaneous proposal stage there are only two possibilities: (1) $\lambda_M$ is expected to accept an offer if it is close enough to $t_M$; (2) $\lambda_M$ is expected to reject all offers. In case 1 the only equilibrium is for both extreme party leaders to offer $\tau = t_M$; in case 2 all offers are equivalent and equally irrelevant.

\(^{17}\)If $y^l = 2$ then consider first the cases where the politician of type $M$ is in $Y^l$. In these cases it is clear that the voters whose type is not represented by any candidate can never lose by voting for the candidate of type $M$ (i.e., sincere voting). If $Y^l$ does not contain a candidate of type $M$, then it is equally clear that the voters of the median type can never lose by voting for the candidate of type $L$, which constitutes sincere voting since the median position is closer to $t_L$ than to $t_H$. 

11
voting sincerely,\(^{18}\) and one where the type \(L\) voters vote for the type \(M\) candidate. Formally, the former coordination equilibrium is characterized by \(z^L_M(P^l) = L\), while the latter is characterized by \(z^L_M(P^l) = M\). However, it is easy to see that only the latter is strong.

Let \(D_a \equiv \{d : \mu^l_i > \frac{1}{2}, \mu^{l'}_i > \frac{1}{2} \text{ for some } i, l, l'\}\) denote the set of distributions of preferences such that a type \(i\) has the absolute majority of preferences in at least two districts. If \(d \in D_a\) then no analysis is needed. Both sincere and strategic voting yield the same policy outcome, \(t^* = t_i\), without formation of heterogeneous parties. This implies that in such situations at least one party will never have a chance to get a seat.

**Remark 1** For every \(d \in D_a\), under Plurality Voting there are at most two effective parties and, generically, at most two active parties.\(^{19}\)

I will now demonstrate that the sincere vs. strategic voting issues are *irrelevant* also with any other distribution of preferences.

**Proposition 1** For any distribution of preferences in \(D \setminus D_a\), \(\Gamma^{PV}_s\) and \(\Gamma^{PV}_z\) have the same equilibrium outcomes:

(I) The equilibrium policy outcome is always \(t_M\) and no heterogeneous party ever forms;

(II) There is always only one running candidate per district;

(III) The running candidate of district \(l\) is of type \(i = L, H\) if and only if \(\mu^l_i > \frac{1}{2}\).

**Proof.**

- **Sincere voting.** If \(\mu^l_i > \frac{1}{2}\) for some \(i\), then of course only a candidate of type \(i\) has incentive to run (sufficiency in (III)). So (III) can be proved by showing that, whenever \(d \in D \setminus D_a\) is such that \(\mu^l_i < \frac{1}{2}\) for both \(i = L\) and \(i = H\), the unique candidate in district \(l\) is of type \(M\). To show this, note that if \(\mu^l_i < \frac{1}{2}\) for both \(i = H\) and

\(^{18}\)For example, suppose that in the other two districts the seats go to party \(M\) and party \(H\). Then if \(\mu^l_H = \max_j \mu^l_j\) there is no profitable deviation for the median type voters from the proposed voting profile where they support \(L\).

\(^{19}\)Under sincere voting there is a non generic case where all the three parties are active, namely when some type \(i\) has the absolute majority in two districts and the other two types tie in the third district. Under strategic voting there is not even this non generic case.
\(i = L\), then the politician of type \(M\) runs and is sure to win unless both the other
two politicians run. But the one of them with less preferences has no incentive to
run since she would lose anyway. Given this, even the extreme type with the relative
majority of preferences decides not to run, anticipating that, if she did so, all the
other votes (absolute majority, composed of the votes of the median type voters plus
those of the other extreme type voters) would go to the median candidate. Hence the
median candidate runs uncontested even if there are very few people with the median
preference. (II) follows immediately. Moreover, in \(D \setminus D_a\) the median party always
obtains at least a seat if \(\sigma = \sigma_0\), and no other party can obtain more than two seats by
construction, hence majority rule implies \(t^* = t_M\), which implies \(r = 0\) for every pair
of offers \((\tau_L, \tau_H)\), \(\to (I)\).

- **Strategic voting.** Even though it is still obvious that \(\mu_i^l > \frac{1}{2}\) is sufficient to have \(i\) as
  a unique running type in district \(l\), with strategic voting one wonders whether this
  should remain necessary for \(i = L, H\). To see this, simply note that if \(\mu_i^l < \frac{1}{2}\ \forall \ i\),
  then the politician of type \(M\) always runs, anticipating that the type \(L\) politician will
  then decide to stay out because she knows that if she entered the type \(H\) would then
  optimally stay out and make the median type win anyway.

  QED.

Proposition 1 shows that if \(d \in D \setminus D_a\) then in any equilibrium: the policy outcome is
the one preferred by the median type; there are only three running candidates; and in each
district the equilibrium running candidate is of some extreme type if and only if it has the
absolute majority of preferences in that district. These results, combined with Remark 1,
imply the following corollary result:

**Corollary 1** Under Plurality Voting there are three active (and effective) parties if and only
if the distribution of preferences is in the set \(D_b\), which denotes the set of all distributions
such that \(\mu_i^l > \frac{1}{2}\) in district \(l\), \(\mu_i^{l'} > \frac{1}{2}\) in district \(l'\), and \(\mu_i^{l''} < \frac{1}{2}\) in district \(l''\), \(i = L, H\).\(^{20}\)

This summary result constitutes a realistic qualification of Duverger’s Law, in contexts
of multiple districts. It says that Duverger’s law continues to hold unless the districts are

\(^{20}\)This ignores the non-generic case mentioned in footnote 19.
so heterogeneous that each party contains the median voter of one district. It is realistic because it combines the strong intuition of Duverger’s law for a unified electorate with the observation that countries like India continue to have many active and effective parties even though they use PV.

Another important corollary of the above analysis comes from noting that in each district the unique running candidate is always of the same type as the median voter of that district. In fact, if \( \mu_i^l > \frac{1}{2} \) for \( i = H \) or \( i = L \), then the median voter of district \( l \) is of type \( i \) (\( t_{ml}^l = t_i \)); otherwise the median voter of district \( l \) is of type \( M \) (\( t_{ml}^l = t_M \)). Hence, Proposition 1(III) guarantees that the running candidate of district \( l \) is always of type \( t_{ml} \), the median voter’s position of the district. Therefore the policy outcome for the whole country is the median of the median voters’ positions of the three districts, which I denote by \( t_{mm} \).

**Corollary 2** Under Plurality Voting the policy outcome is always the median of the median voters’ positions of the three districts, \( t_{mm} \).

These two corollaries will be recalled for the major comparative results in Section 3.3. I conclude this section on PV by stressing the irrelevance of the voting assumptions for the equilibrium characterization:

**Remark 2** Even though sincere voting strategies are not necessarily rational, the equilibrium voting behavior under PV is always sincere. In fact, \( Y^l = P^l \) cannot happen in any district in any equilibrium when candidacy is endogenous.

### 3.2 Proportional Representation

With Proportional Representation it is convenient to study sincere voting and strategic voting separately. Recall that \( \mu_i = \sum_i \mu_i^l / 3 \).

**Proposition 2** Consider the game \( \Gamma_{PR} \).

(I) If \( \pi - \epsilon < t_M \), then \( \sigma^* = \sigma_0 \) in every equilibrium.

(II) If \( \pi - \epsilon \geq t_M \), then:
(X) \( n(\sigma^*) = 2 \) is possible only if the distribution of preferences is such that
\[
\max_{i=L,H} \mu_i < \frac{1}{2} \quad \text{and} \quad \max_{i=L,H} \mu_i - \frac{1}{3} > \mu_M
\]  

(XX) There exists \( \pi \) such that, for every \( \pi > \pi^* \), (1) is also sufficient.

(III) The equilibrium policy outcome is \( t_M \) unless an extreme party \( i = L, H \) has \( \mu_i > \frac{1}{2} \) and \( \mu_i - \frac{1}{3} > \min_{i'} \mu_{i'} \).

Proof.

(I) Assume \( \sigma^* = \sigma_0 \) and consider first the set \( D_1 \) of distributions of preferences such that \( \mu_i - \frac{1}{3} < \mu_{i'} \) for \( i = L, H, i' \neq i \). In these cases the policy outcome is always \( t_M \). By construction, then, \( \lambda_M \) has no incentive to accept any offer when the distribution of preferences is in \( D_1 \).

Consider now the set \( D_2 \) of distributions of preferences such that \( \mu_L = \max_i \mu_i < \frac{1}{2} \) and \( \mu_L - \frac{1}{3} > \mu_i \) for some \( i \). In these cases party \( A_L \) obtains the majority of seats if \( \sigma = \sigma_0 \) and \( Y = P \). If no politician of type \( H \) becomes a candidate at stage 2, however, the Hare quota guarantees that the median candidates grab the majority of seats. But then, if \( \pi - \epsilon < t_M \) the politicians of type \( H \) receive more utility from changing the policy outcome from \( t_L = 0 \) to \( t_M \) (by not running) than from a seat, hence indeed decide not to run in any continuation equilibrium of \( \sigma_0 \). Hence no incentive once again for \( \lambda_M \) to accept offers.

When \( \mu_H = \max_i \mu_i < \frac{1}{2} \) and \( \mu_H - \frac{1}{3} > \mu_i \) for some \( i \) – call this set of distributions of preferences \( D_2' \) – the incentive argument just made for the distributions in \( D_2 \) applies to the type \( L \) politicians \emph{a fortiori}, since \( 1 - t_M > t_M \).

Finally, for any distribution in the set \( D_3 \) such that \( \mu_i > \frac{1}{2} \) for some extreme party \( i \) and \( \mu_i - \frac{1}{3} > \min_{i'} \mu_{i'} \), such a party obtains two seats no matter what the others do at any stage, hence, once again, no incentives to accept offers to form heterogeneous parties.

(II) Let \( \pi - \epsilon > t_M \).

(X) I need to show that whenever (1) does not hold \( \sigma^* = \sigma_0 \). First of all, if the first inequality in (1) is the only one to be reversed, then we are in \( D_3 \), and heterogeneous parties are useless no matter what \( \pi \) is. Second, if the second inequality of (1) is reversed, then, regardless of what happens to the first inequality, there are two subcases: (A) \( \max_{i=L,H} \mu_i - \frac{1}{3} < \mu_{i'} \forall i' \); (B)
max_i=L,H μ_i - \frac{1}{3} > \min_i=L,H μ_i. Subcase (A) falls in D_1, where we know that heterogeneous parties will never be formed; Subcase (B) falls in D_2 or D_2'. In this subcase one of the extreme parties (A_H in D_2 and A_L in D_2') would not get any seat even if \sigma = \sigma_0 and Y = P, so \pi can be as high as you want and would never matter: the politicians of that type would still decide not to run, hence, anticipating that, no incentives for \lambda_M to accept any offers.

(XX) Let \overline{\pi} be the value of \pi such that \pi/3 - \epsilon > t_M. Consider first the values of \pi > \overline{\pi}. Let \mu_H = \min_i=L,H μ_i. When (1) holds, party A_H must be able to obtain a seat if \sigma = \sigma_0 and Y = P.\footnote{To see that this must be the case, note that given (1) the shares of votes for A_L and A_M together when Y = P cannot sum to more than \frac{2}{3}, and the remaining \frac{1}{3} (or more) for A_H must be greater than \mu_M, so A_H must get a seat.} Hence if \pi > \overline{\pi} the politicians of type H would run, and this creates the incentive for \lambda_M to accept some offers and to form a heterogeneous party. Given that the party leaders maximize the utility of the citizens of their type, \lambda_H has indeed an incentive to make an offer. So n(\sigma^*) = 2 for every s^* \in S^* for every distribution of preferences satisfying (1).\footnote{If (1) holds but \max_i=L,H μ_i = \mu_H, an identical argument goes through. The only difference is that for that case the relevant lower bound is \overline{\pi} such that \pi/3 - \epsilon = 1 - t_M.} However, (1) is not a sufficient condition for intermediate values of \pi such that \pi - \epsilon > t_M but \pi/3 - \epsilon < t_M, since for these values there are individual incentives to run only if the seat can be obtained without having a candidate in every district.

(III) Note that even when there are incentives to accept offers the offer stage is competitive, and hence in any equilibrium both extreme parties must offer \tau = t_M. \hfill QED.

Proposition 2 shows that, unless the distribution of preferences is so skewed on one side that the median and the other extreme party cannot find any effective coordination strategy (neither at the party formation nor at the candidacy stage), the policy outcome is the one desired by the median type. In terms of the equilibrium party structure, I have shown that when the private benefits from holding office are sufficiently low with respect to the policy gains that can be obtained by a politician of any extreme party by not running, there is no incentive to form heterogeneous parties. The reason is that in this case the median type politicians expect that if the outcome is t_L when everybody runs then type H politicians will strategically decide not to run. On the other hand, when the private benefits from holding office are sufficiently high, no such strategic incentive can be expected, hence the median
type has to accept to form a heterogeneous party in order to obtain the median outcome.

I will now show that under strategic voting strategic candidacy becomes less important, and hence the value of $\pi$ loses its relevance.

**Proposition 3** Consider the game $\Gamma^R_z$. Item (III) of Proposition 2 extends. (I) and (II) are instead replaced by the following:

(I) $n(\sigma^*) = 2$ is possible in a noncooperative equilibrium only if the distribution of preferences is such that

\[
\max_i \mu_i = \mu_H < \frac{1}{2} \quad \text{and} \quad \mu_H - \frac{1}{3} > \mu_i \quad \text{for some } i; \tag{2}
\]

(II) (2) is sufficient to have $n(\sigma^*) = 2$ in a noncooperative Equilibrium if $\forall Y \in \{Y : F_H(z_s(Y)) \geq 2\}$ voters play the Nash equilibrium most favorable to type $L$.

**Proof.**

(I) I need to show that whenever (2) does not hold $\sigma^* = \sigma_0$ in every equilibrium of $\Gamma^R_z$. If $\mu_H > \frac{1}{2}$ and $\mu_H - \frac{1}{3} > \mu_i$ for some $i$, then we are in $D_3$, as defined in the proof of Proposition 2. For these distributions of preferences there is no way to take away the majority of seats to party $A_H$, hence (1) every continuation equilibrium of $\sigma_0$ has the same policy outcome and (2) sincere voting is rational. Therefore heterogeneous parties are useless. If $\mu_H - \frac{1}{3} < \mu_i \ \forall i$, then, regardless of what happens to the first inequality, we are in $D_1$, as defined in the proof of Proposition 2. Hence the voters of type $M$ expect $t_M$ as outcome if they vote sincerely. No other type of voters has a positive benefit from voting strategically either. Thus the sincere voting profile is rational and no other equilibrium outcome is possible when $\sigma = \sigma_0$. Hence heterogeneous parties will never be formed.

(II) Suppose that the distribution of preferences is such that (2) holds. Then if $\sigma = \sigma_0$ sincere voting is not rational given $Y = P$, since the voters of type $L(M)$ could profitably deviate by voting for candidates of type $M(L)$. So, if $\sigma = \sigma_0$ and $Y = P$ there are two continuation equilibria: (1) Voters of type $L$ vote for candidates of type $M$ and everybody else votes sincerely; (2) Voters of type $M$ vote for candidates of type $L$ and everybody else votes sincerely. If the continuation equilibrium is (1), then, like with sincere voting, no incentive at stage 1 to form a heterogeneous party; on the other hand, if the expected
continuation equilibrium is (2), then $\lambda_H$ would have incentive to deviate and offer $\lambda_M$ to form a party, with an offer that $\lambda_M$ would accept. The only equilibria of $\Gamma^{PR}_z$ that are compatible with (2) have both extreme parties compete to have $\lambda_M$ accept the offer, hence the party structure has a heterogeneous party. However, note that, again because of the competition at the offer stage, the policy outcome remains $t_M$. Note also that when (2) holds with $L$ instead of $H$, the voters of the median party always vote sincerely because the other small party they could vote for in order to defeat the relative majority party has the more distant policy position. Hence no incentive to form heterogeneous parties. QED.

**Proposition 4** In $\Gamma^{PR}_z$ strong equilibria always exist, and the unique equilibrium party structure in a strong equilibrium is $\sigma^* = \sigma_0$, for all distributions of preferences.

**Proof.** As shown in the proof of Proposition 3, whenever sincere voting is not rational there are two types of Nash equilibria. The continuation voting equilibrium where $z'^L(P^l) = M$ and everybody else vote sincerely is robust to coalitional deviations at the voting stage, hence strong equilibria always exist that imply $\sigma^* = \sigma_0$. To see that such a party structure is also the unique one compatible with a strong equilibrium, note that if the other Nash equilibrium with $z'_M(P^l) = L$ is the continuation voting profile, then the voters of type $M$ and $H$ can profitably deviate by following a deviating recommendation by their party leaders to vote all for type $M$ candidates if $Y^l = P^l$. QED.

Proposition 3 and 4 highlight an important difference between $\Gamma^{PR}_z$ and $\Gamma^{PR}_s$. Under sincere voting the equilibrium party structure depends on the distribution of preferences and on the relative size of private benefits and policy gains by not running, whereas under strategic voting only the expectations of voters’ behavior matter. When $A_H$ has the relative majority of preferences but not the absolute, then the other two parties can do better than suggesting their voters to vote sincerely. Both the coordination of all their voters on the median candidates and the coordination on the other small party’s candidates are equilibria in this case, but only the former is strong. In terms of the sincere vs. strategic voting issue, the result is less sharp than under PV, and is summarized as follows:

**Remark 3** Sincere voting is rational under Proportional Representation unless (2) holds.
All the analysis of this section implies the following corollary in terms of number of active and effective parties:

**Corollary 3** Under Proportional Representation there are three active and effective parties unless \( \mu_i > \frac{1}{2} \) and \( \mu_i - \frac{1}{3} > \mu_{i'} \) for some \( i \) and \( i' \).

### 3.3 Comparisons

The comparison of Corollary 1 and Corollary 3 gives an exhaustive qualification of the Duvergerian predictions. When the distribution of preferences is not too heterogeneous across and within districts, then Duverger’s hypothesis is confirmed. For example, for any distribution of preferences in the neighborhood of \( d : \mu_l^i = \frac{1}{3} \ \forall i, \forall l \), there is only one active and effective party with Plurality and three active and effective parties under PR. This conforms with the observation that, while in countries like the US and UK using Plurality there is always a majority party, in countries using PR we almost always see the formation of coalitional governments.

However, if the distribution of preferences is sufficiently heterogeneous, things may go the opposite way, as in the following example. Consider the distribution

\[
\begin{align*}
\mu_L^1 &= 0.6; \quad \mu_L^2 = \mu_L^3 = 0.01; \\
\mu_H^2 &= 0.6; \quad \mu_H^1 = \mu_H^2 = 0.02; \\
\mu_M^2 &= 0.97; \quad \mu_M^1 = 0.38; \quad \mu_M^3 = 0.39.
\end{align*}
\]

In this example there are three active and effective parties in any equilibrium with Plurality, whereas there are only two active parties under PR.

All the results on the number of effective parties that help to qualify the extendibility of Duverger’s hypothesis to multi-district representative democracies can be summarized, as long as \( \mu_i < 0.77 \), in the following table.\(^{23}\)

---

\(^{23}\)**Legenda**: In the table DH obviously means Duverger’s hypothesis and \( D_b \) is the set of distributions of policy preferences defined in Corollary 1. On the rows I distinguish between number of effective parties when the distribution of policy preferences is not in \( D_b \) and when it is. The conjunction “or” refers to the fact that there are subsets of those two cases where the equilibrium number of effective parties differ, and not to the
It might be of some interest to compare the policy outcomes of the two electoral systems studied in this paper against the benchmark of direct democracy. Note that under PR having the absolute majority of preferences is not a sufficient condition to obtain two seats. For example, one party might well have 52 percent of the preferences, but if the other two parties have 24 each, then the Hare quota will assign one seat to each party, hence the policy outcome is $t_M$, which is not the median voter’s preferred outcome in this case. The relationship between the policy outcome under PR and the median voter’s preferred policy is summarized by the following remark:

**Remark 4** The equilibrium policy outcome of $\Gamma_{s}^{PR}$ and $\Gamma_{z}^{PR}$ is the one preferred by the median voter unless the following two conditions hold: $\max_{i=L,H} \mu_i > \frac{1}{2}$ and $\max_{i=L,H} \mu_i - \frac{1}{3} < \mu_{i'} \forall i'$. 

Given Corollary 2, the set of parameters generating an outcome different from the median voter’s preferred policy under PV is very different:

**Remark 5** The equilibrium policy outcome of $\Gamma_{s}^{PV}$ and $\Gamma_{z}^{PV}$ is the one preferred by the median voter unless

- either
  
  (1) $\mu_i > \frac{1}{2}$ in two districts and $\mu_i < \frac{1}{2}$ for some $i \in \{L, H\}$;

  or

  (2) $\mu_i > \frac{1}{2}$ for some $i \in \{L, H\}$ but $\mu_i' > \frac{1}{2}$ in only one district.

From the two remarks above it should be noted that the type of situations where the outcome of direct democracy does not coincide with the outcome of representative democracy possibility of different equilibrium numbers of effective parties for a given distribution of policy preferences. Finally, the distributions where $\mu_i > 0.77$ are not included in the table because in those cases there can be a unique dominating effective party in both systems.
are qualitatively different under the two electoral systems. Under PR it may happen that the policy outcome is centrist even when the median voter is actually of one of the two extreme types. The opposite cannot happen. On the other hand, with PV the deviations can also go the other way: it may happen that the policy outcome is one extreme even though the median voter is of the median type or even of the opposite extreme type. So it is fair to say that PR always yields moderate outcomes (sometimes even too moderate), whereas under PV the majority of preferences in the country is irrelevant.\footnote{Under PV an extreme outcome is possible even when the median voter is of the opposite extreme party and the latter has the majority of preferences in the country. This is clear when there are only two big parties, like in the last American presidential elections, but it extends to situations with more parties.}

4 Robustness and Generalizations

An assumption of the model is that policy compromises agreed upon in a heterogeneous party are perfectly credible to voters. This credibility property can actually be obtained endogenously. Note that the only equilibrium policy compromise that can be made is on \( \tau = t_M \). This implies that I could have assumed that at stage 1 the extreme party leaders can simply propose to withdraw their politicians instead of proposing a policy compromise. The extreme party leaders would be willing to make the withdrawal proposals in the same circumstances where they propose a compromise in the current model, and the results would therefore be absolutely identical, with no credibility assumption.

One could note that the current formulation of the game makes it infeasible for a heterogeneous party \( A_L \cup A_H \) to form. A more indirect way to obtain the exact same restriction in equilibrium is to assume that every heterogeneous party has an exogenous probability of break down after the elections, and that the latter is increasing in the distance among the policy positions of the party members. In other words, I could assume that the credibility of a policy compromise agreed upon by two types is decreasing in the ideological distance between the two types. With an assumption like this it is possible to make stage 1 a simultaneous or sequential move party formation game where \( A_L \cup A_H \) is feasible but never arises in equilibrium. As argued in the previous paragraph, it is not difficult to make the compromises that can happen in this model credible, whereas it is impossible to imagine how \( A_L \cup A_H \)
would obtain a useful credible compromise, since all the politicians of that heterogeneous party would be extreme. So, if I assume either (1) that proposals are only policy proposals but credibility decreases with ideological distance, or (2) that the party leaders can also propose to withdraw their politicians to make a compromise credible, then stage 1 can be made a more symmetric game where every coalition is feasible, and it is easy to show that all the results of the paper extend (calculations available upon request). The fact that in all the generalizations of the model that I can think of the equilibrium heterogeneous parties (if any) are always “connected” coalitions (i.e., only coalitions of adjacent types) is not surprising, and is consistent with real world observations.\footnote{To see other intuitive justifications for considering only coalitions of adjacent types in general, see Axelrod (1970).}

One could also ask whether the results of this paper are robust to changes in the order of play at stage 2. The clear answer is as follows.

**Proposition 5** The results for $\Gamma_{PV}^z$ remain true for every order of play at stage 2. The results for $\Gamma_{PV}^s$ remain true for every order of play at stage 2 in terms of policy outcome but there is one case in which $n(\sigma^*) = 2$, namely when the order of play is $L, M, H$.

*Proof.*

1. **Strategic voting.** Let $\sigma = \sigma_0$. Consider any district $l$ in which $\mu_i^l < \frac{1}{2}$ $\forall i$.

   Consider first the order $L, M, H$. In this case the type $M$ politician runs no matter what the type $L$ one does, since she anticipates that after her move the type $H$ politician will decide not to run, in order to avoid the useless (or even harmful if the district is pivotal) expenditure of the candidacy cost. Consider now the order $L, H, M$. In this case if the type $L$ politician decides to run, the type $H$ politician optimally decides to stay out, since there is no continuation equilibrium of $Y^l = P^l$ nor of $Y^l = \{L, H\}$ where the type $H$ candidate wins. Hence the type $L$ politician stays out, since the continuation equilibrium of her running has the type $M$ politician run and win. The type $M$ politician still runs uncontested.

   Now consider the case in which the type $H$ politician moves first. If she decides to run, then either the type $M$ politician runs and wins (i.e., when $z_L^l(P^l) = M$), or the other
politician runs and wins (when $z^1_M(P^H) = L$). Therefore the type $H$ politician actually decides not to run, leaving once again the type $M$ politician as unique equilibrium runner.

2. Sincere voting. Let $\mu^i_1 < \frac{1}{2} \forall i$. Without loss of generality, assume $\mu^i_H > \mu^i_L$. In addition, assume $\mu^i_H > \mu^i_M$ as well.\(^{26}\) Consider first the order $L, M, H$. If $\sigma = \sigma_0$, then the type $L$ politician runs, since she expects the type $M$ politician to stay out in order to obtain $t_L$ instead of $t_H$. But then if this is the expected order of play, $\lambda_M$ will accept offers at stage 1, and since at least one other party leader has incentive to make an offer, in equilibrium both extreme parties will compete and the policy outcome will once again turn out to be $t_M$. Consider now the order $L, H, M$ and assume $\sigma = \sigma_0$. Suppose that the type $L$ politician decides to run. Then the type $H$ politician optimally decides to stay out, since she anticipates that if she decided to run then the type $M$ politician would stay out because it would be useless given $\mu^i_H > \mu^i_M$. But then, given that in the continuation equilibrium of her decision to run the type $L$ politician expects the type $H$ one to stay out, she must also expect that as a result the type $M$ politician will actually decide to run, and hence the optimal decision for the type $L$ politician is not to run, leaving the type $M$ politician to be the unique equilibrium runner. With the other two orders, $H, M, L$ and $H, L, M$, it is clear that the optimal decision for the type $H$ politician is not to run, since she expects that in the continuation equilibrium of her running one of the other two would enter and win. Hence, once again, only the type $M$ politician runs in equilibrium. \(\text{QED.}\)

Under Proportional Representation the generalization goes through as smoothly as in Proposition 5. Like for Plurality Voting, it is only with sincere voting that the order of play may change the equilibrium characterization. When the type $L$ politicians move first the relative dimension of private benefits and policy gains do not matter for their decision to run. Hence, once again, if the order of play is expected to be $L, M, H$ then a heterogeneous party has to form at stage 1 regardless of $\pi$. However, Proposition 3 can be shown to extend word by word, since the order of play at stage 2 never enters the arguments in its proof.

\(^{26}\)The reason for looking only at this case is that if $\mu^i_M$ is greater than the other two then the order of play obviously doesn’t matter, and no proof is required.
All the above suggests that with strategic voting the order of play at stage 2 is totally irrelevant, whereas with sincere voting there may be some differences in terms of equilibrium party structures but without affecting the main results on the qualification of the Duvergerian predictions and on the comparison with direct democracy.

Another generalization that is worth discussing concerns the number of policy preferences, or types. The only change to the model that is needed to allow for \( T > 3 \) types is at stage 1. I would let the party leaders move sequentially, and each party leader would have to make a proposal concerning (1) a heterogeneous party (or electoral coalition) and (2) a policy compromise within that proposed coalition. Then, as mentioned at the beginning of this section, it would be enough to assume that the credibility of a policy compromise is decreasing in the heterogeneity of the members of a coalition to obtain exactly the same results of this paper, in terms of the qualification of the Duvergerian predictions as well as for the comparison with direct democracy and the role of strategic voting.\(^{27}\)

### 5 Concluding Remarks

The first authors to study representative democracy with endogenous candidates are Osborne and Slivinsky (1996) and Besley and Coate (1997). Osborne and Slivinsky only have sincere voting, whereas in Besley and Coate’s model citizens are strategic both at the candidacy stage and when they have to vote. In contrast with these citizen-candidate models, where parties are missing, in this paper parties play an important role both \textit{ex ante} as commitment devices for politicians and during the elections, where they coordinate voters’ strategies. Besley and Coate only consider first-pass-the-post elections, whereas the framework proposed here accommodates a wide range of electoral systems, and indeed the impact of the electoral system on equilibrium behavior and outcomes can be studied. The importance of conducting a comparison of electoral systems in the presence of endogenous candidacy can be stressed by referring the reader to the recent paper of Dutta, Jackson, and Le Breton (2001): They show that a comparative analysis of voting procedures often needs to be done taking into account the endogenous incentives to run, because most well known voting mechanisms are not...\(^{27}\)In earlier versions of this paper I had a general number of types, seats, and districts, so some of the generalizations of this kind are available upon request.
not candidate stable.

Beside these methodological issues, the model presented in this paper may be appreciated for the characterization results. As pointed out in the introduction, the formal models on Duverger’s law all focused on strategic voting with fixed candidates in a unified district.28 This paper has extended the analysis to multi-seat/multi-district national elections with parties connecting the candidates of the various districts, and Duverger’s hypothesis has also been checked. Moreover, this model shows that the same phenomena can be explained by the strategic behavior and incentives of politicians, rather than voters. In reality both voters and politicians are strategic, so it is likely that the phenomena first identified by Duverger are due to all of this. The Duvergerian predictions have been shown to hold in a representative democracy with multiple districts as long as the distribution of policy preferences is not too heterogeneous across districts, whereas if the distribution of policy preferences is very asymmetric, then the number of effective parties can be larger under Plurality Voting than under Proportional Representation.29

Under most distributions of policy preferences the choice of an electoral system is irrelevant for the policy outcome, since the median voter’s preferred outcome is the equilibrium outcome of both Proportional Representation and Plurality Voting, both with sincere and strategic voters. However, there is a positive measure of distributions of policy preferences (quite heterogeneous within and across districts) such that the policy outcome differs from that of direct democracy. Under Plurality Voting the policy outcome may be more “extreme” than that desired by the median voter, when an extreme party has the absolute majority of preferences in the majority of districts but does not have the absolute majority of preferences in the country. It can also be more “centrist” than what the median voter wants, when an extreme party has the absolute majority of preferences in the whole country but the absolute majority only in one district. On the other hand, under Proportional Representation the policy outcome can only be more “centrist” than the one desired by the median voter, never more “extreme”. So the two electoral systems may determine outcomes different from direct democracy, but if they do, they often do so in opposite directions.


29For a discussion of other political effects of electoral systems, see Taagepera and Shugart (1997).
The existing empirical results (summarized in Persson and Tabellini (1999)) show that within the set of representative democracies the electoral systems don’t seem to have any significant impact on the size of government. This paper suggests, consistently with Persson and Tabellini’s findings, that when policy preferences are not too different from district to district the outcome preferred by the median voter prevails regardless of the institutional and strategic differences between the two types of representative democracy studied. The model is probably too stylized to give very precise empirically testable predictions, but one speculation that I would venture to make is the following: Over time policy preferences change and swing, so departures from the median voter’s theorem are likely in the long run; since this model suggests that the deviations from the direct democracy benchmark are always in the moderate direction for PR but in all directions under PV, I would expect the variance of policy outcomes to be higher in a time series for a country using PV than in a time series for a country using PR. This conjecture on the second moment is not incompatible with the insignificant differences found in terms of average policies, but its empirical relevance will have to be checked in future work.

Austen-Smith (2000) cites some empirical evidence confirming his result that under PR the equilibrium tax rate is higher. This evidence contrasts with the results of Persson and Tabellini. However, trying to resolve this inconsistency between sets of empirical observations is beyond the scope of this paper.

Cukierman and Spiegel (1998) look at the same comparison between representative and direct democracy in terms of policy outcome, but they work with Proportional Representation only.
References


