The Winning Card of an Underdog

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Abstract

I analyze a repeated election with imperfect information with uninformed voters. I show that the interaction between one dimensional transfers, which are observable, and a currently unobservable state of the world allows the underlying better candidate to run for austerity. In equilibrium voters are prospective, as they take the signal of a candidate as good, and the austerity of the winning candidate is bounded by the value of the unobserved state. Out of equilibrium a candidate who tricked voters would be punished retrospectively, as there is nothing else they could do. Also, if the winning candidate raised his offer he would enter the expected support of offers of the next best candidate.

The model suggests caution when interpreting political data. Observed phenomena where (i) winning candidates restrain their purse instead of being populist and (ii) incumbents have some advantage, can be consistent with the theoretical evidence presented here. Electoral data generated by a model with imperfect information can run counter to predictions with perfect information. Candidates who appear, to the naked eye of an outside observer, to be underdogs can win elections with rational agents.

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Introduction

Consider an election where candidates compete over one common observable variable and a variable unobservable to voters but not to candidates. This unobserved variable creates a wedge between the observable offers candidates can make on the observable variable and the final utility voters get. The wedge is interpreted as a valence, an advantage one candidate has over the other, as he runs consistent with a state of the world. I show that the correct candidate can use his advantage to pursue policies that are less populist than the alternative. Voters remain perfectly rational about their actions. However, whilst in the election it seems that all candidates compete in the same policy space, the interaction between observable and unobservable variables determines that a correct candidate that offers less populist policies in the observable dimensions offers a better deal, overall, for a majority. How unpopulist is the bundle the correct candidate offers is bounded by the value of the state. Moreover, competition between candidates in the political race tempers their ability to fool voters, as candidates with an advantage in the unobservable dimension have an incentive to reveal it.

Apart from its theoretical interest, the model can explain some observed puzzling stylized facts (empirical implications remain for future research). The first of these is that in elections candidates do not converge to a common position on which they overlap. Also, in many elections we see that a candidate wins by a margin not accounted for by observable variables: some because they are incumbents, others despite lacking a populist program, still others promising current sacrifice with the hope of a better future... According to the model presented here this phenomenon can take shape in different forms but they all represent the same: an inherent advantage in a variable which is unobservable to voters but which is transmitted by a candidate when he runs as an underdog. Candidates can signal their worth offering less populist policies, taking a stronger stand, than other candidates in all the observable variables they compete on.
The model takes seriously the idea that politics is about revealing information. However, it puts candidate/party competition on the center stage, not voter behavior. There are models (see Feddersen and Pesendorfer, 1997) that put voters on the center stage in the information revelation process, an idea that dates back to Condorcet. However, recent analyses have contested the ability of these to withstand party competition (see Razin, 2002).

The model relies on an extended duration of candidates/parties across time and on the memory of voters to evaluate past offers. Thus, the paper is closely related to one of John Ferejohn (Ferejohn, 1986). Here I construct a model of an election with a candidate who is the correct one at a given period in time and one who is not. The way to make sure the correct candidate is picked is by punishing the incorrect one if he were to imitate whatever behavior the correct candidate uses to differentiate himself. Contrary to Ferejohn’s model, the opposition plays a role, it is not an outside alternative picked when the incumbent deviates from the path of play. That is, in Ferejohn’s model the opposition is passive, here it is active. The dynamic nature of the game ensures that no candidate wants to win unless it is in his interest to do so. Candidates do not risk a race when they should not be winning it. It is better for them to stand by and try again in a future period when their chances are better.

Thus, the model connects to a literature that tries to understand the mechanism through which candidates are elected in the presence of imperfect information. The underlying interest of this literature is to understand whether voters are prospective or retrospective. The model presented here can be summarized in the words of an article by James Fearon “[.] Selecting Good Types versus Sanctioning Poor Performance” (Fearon, 1999), excluding the ‘versus’. Here, in equilibrium, the correct candidate signals his worth. Out of equilibrium, if an incorrect candidate were to fool voters, he would be excluded from office for a significant number of periods. Thus, rational voters are prospective in equilibrium, but punish retrospectively out of it.

The next section presents a simple example when there is an unobserved state and candidates can offer money to voters. A more general model is left for future research.

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1In the article Fearon himself suggests that selection and sanctioning can interact.
A Simple Example

Introduction

In this section I present a way to solve a theoretical puzzle raised when candidates’ chances are intimately related to knowing how to distribute money well. This populist incentive to buy votes is most forcefully presented in Myerson (1993).\textsuperscript{2} Myerson constructs a model in which voters only care for money and, in equilibrium, are effectively bought and the candidate who wins is he who offers the most to a particular group whilst satisfying a budget constraint. Here I construct a model where an individual voter cares for money but where, in equilibrium, he does not elect the candidate who offers the most.

Consider an election where there is an unobserved realization of a state of the world. Voters get utility out of money and electing a candidate who runs consistent with the true state. For a voter there is no difference between a candidate that runs consistent with the true state or with a ‘valence’ advantage.\textsuperscript{3} I will stick to the latter notation. If the valence were observed, the correct candidate, knowing this, could win offering an amount of money smaller than the other candidate. This would be an equilibrium as long as the difference was not greater than the value of the valence. When the valence is not observed, the correct candidate can have difficulties transmitting his advantage if the other candidate imitates his behavior. Moreover, the extreme pressure to win in politics makes it hard to believe that there is any ‘natural’ dimension, any variable left aside to allow a candidate to signal his worth to voters (e.g. through his education) or allow voters to skim him (e.g. through his ambition). Instead, in the following dynamic model I allow voters to punish a candidate who imitates the correct one when they eventually find that he is not. This ensures that the correct candidate is elected. A refinement of equilibria, based on the disincentive of incorrect candidates to win, ensures that the only equilibrium is one where the correct candidate signals his worth offering less.

It is important to mention that the results only depend on the relative advantage of a candidate or the relative benefit of one policy compared to another for a given state of

\textsuperscript{2}The model by Myerson is part of a larger literature mentioned in the conclusion.

\textsuperscript{3}A valence is any characteristic which distinguishes this candidate from others to his benefit, and in which all voters agree (see Stokes, 1963).
the world. Both candidates can have a valence (or both policies can give voters utility under both states). What is important is that one gives more utility than the other.

The Model

Consider a dynamic environment where in each period $t$ one of two states of the world is realized, $W = \{0, 1\}$, each with equal probability. Moreover, the realization of the state in each period is independent of all others. Thus

$$\Pr \left( w_t = w | w_{t-1} = w \right) = \Pr \left( w_t = w \right) = \frac{1}{2}.$$  

Distinguish the states from the policies associated to each of them, $P = \{0, 1\}$. In this environment there are two partisan political parties each of whom prefers one of the two policies no matter what. Each party has its own candidate determined prior to the realization of the state. An election takes place in each period and candidates run offering a bundle with (i) money and (ii) a policy to be implemented: $(m, p)_t$, $i = 0, 1$. Candidates get a benefit $b$ out of winning the election and face no cost out of the money they give.

In the election there is a single voter who has to pick one of the two candidates: $v_t \in \{0, 1\}$, where 0 or 1 indicates a vote for candidate 0 or 1, respectively. The voter is ex-ante completely non-partisan. He just likes money and picking the correct policy:

$$U \left( m_t, p_t, w_t \right) = m_t + u \left( p_t, w_t \right), \ \text{where} \ u \left( p_t, w_t \right) = \begin{cases} u_v & \text{if} \ p_t = w_t \\ 0 & \text{otherwise.} \end{cases}$$

When candidates and the voter look at future elections, they discount time with a common discount factor $\delta$.

Within a period $t$ the timing of the election is as follows. First, nature picks the state of the world. At this time this is always observed by candidates and parties. When there is perfect information the voter observes it now too; otherwise he observes it after the election takes place, at the end of period $t$. After this initial stage, each candidate offers a bundle to the voter if elected. The voter picks a candidate on the basis of the bundles and any information he has or can infer from the offers. Finally, the candidate implements the policy he ran for, the voter observes the state if he has not already, and payoffs are realized.
Perfect Information

For the time being assume that the realization of the state is observed by all agents at the time nature picks it. In this case, the candidate running for the policy inconsistent with it always has to offer $u_v$ more money to the voter to make him indifferent between him and the other candidate. Call the maximum amounts of money each candidate can spend in any period $\bar{m}_0$ and $\bar{m}_1$, i.e. $m_{0t} \in [0, \bar{m}_0]$ , $m_{1t} \in [0, \bar{m}_1]$. As long as these are bounded there is an equilibrium in this model.

Lemma 1 Suppose $\bar{m}_0, \bar{m}_1 \in \mathbb{R}_+$. Then there is an equilibrium.

Also, when both candidates face a similar bound, the correct one is a sure winner.

Lemma 2 If $w_t = 1$ then in all equilibria candidate 1 is a sure winner iff $\bar{m}_1 + u_v > \bar{m}_0$.

This case is extremely simple. However, once we introduce an unobserved state of the world, it will be useful to bear in mind. For now, note that when $w_t = 1$ and $\bar{m}_1 + u_v > \bar{m}_0$ in equilibrium there is nothing that the incorrect candidate can do to win. Moreover, the correct candidate can exploit this difference to his advantage. He can offer a bundle to the voter that gives him more utility than he could ever get out of the incorrect candidate, but which offers less money.

Lemma 3 If $w_t = 1$ the correct candidate can offer less money than the other, $m^*_1 + u_v > m^*_0$.

For simplicity assume, from now on, that the budget constraints of the two candidates are common

$$(m_{it}, p_{it}) \in [0, \bar{m}] \times P.$$ 

Imperfect Information

Consider now the same model but where the state of the world is only observable to parties and their candidates at the beginning of period $t$. The voter only observes the state at the end of the period. Note that, even if within a period $t$ the game is one of imperfect information, the voter can ‘threat’ the candidates for future periods based on the realization of the state.
If information is not revealed, the voter is indifferent

When no information is revealed, the voter has no basis on which to condition his pick. However, is there some strategy he could use which could be of some benefit? That is, if he sticks to a candidate is he more likely to pick the correct one or not? Our assumption of independence of states across time rules this possibility out. Any of the two candidates is ex-ante equally likely to be the correct candidate, so sticking to one candidate in particular brings no advantage. However, it does not bring any disadvantage either.

**Lemma 4** When no information is revealed in the game, the voter is indifferent between sticking to a candidate or switching randomly with any probability between the two, when they offer the same amount of money.

**Proof.** Call $\alpha$ the probability that the voter votes for candidate 1 ($1 - \alpha$ being the probability of voting for candidate 0). The claim is that when no information is revealed the voter is indifferent between any $\alpha \in [0, 1]$.

If a voter picks candidate 0 his expected payoff when the state of the world in the previous period was $w$ is

$$m_{0t} + [u_v \cdot \Pr (w_t = 0/w_{t-1} = w) + 0 \cdot \Pr (w_t = 1/w_{t-1} = w)] = m_{0t} + \frac{1}{2} u_v,$$

instead, his expected payoff from picking candidate 1 is

$$m_{1t} + [0 \cdot \Pr (w_t = 0/w_{t-1} = w) + u_v \cdot \Pr (w_t = 1/w_{t-1} = w)] = m_{1t} + \frac{1}{2} u_v.$$

When the voter sets probability $\alpha$ for candidate 1, the problem of picking the appropriate $\alpha$ becomes

$$\max_{\alpha \in [0, 1]} \left[ m_{0t} + \frac{1}{2} u_v \right] + (1 - \alpha) \left[ m_{1t} + \frac{1}{2} u_v \right].$$

As, by assumption, $m_{0t} = m_{1t} = m$ this expression is independent of $\alpha$.
Revealing information

Clearly, if there were a channel through which the correct candidate could transmit the voter the state of the world, both would be better off than when the voter knows nothing about it. However, no candidate has an outside technology to transmit that information reliably. The only way candidates can reveal information is by their actions. The problem is that there is an asymmetry between the incentives of the two candidates to reveal their information. The correct candidate is better off if he can communicate the state, whilst the incorrect one is worse off. Thus, the problem, from the side of the strategy of the correct candidate, is that he might very well be imitated by the incorrect candidate to make sure that no information is revealed. That is, there is no ‘natural’ variable through which he can differentiate himself without being imitated.

Fortunately for them, as after the race the true state is revealed, voters can decide to punish an incorrect candidate who runs as if nature were by his side. That is, as the channel through which information can be transmitted depends on the absence of noise of the incorrect candidate, he can be punished in future periods if he runs as a correct one. This threat of not being elected is credible. Once the channel has been destroyed by the deviation of one player, the voter is clueless about the state and is indifferent between picking any of the two candidates, as Lemma 4 has shown. As there is no relation between present and future realizations, the voter can threat a candidate not to elect him a number of periods if he tricks him. This threat, which never comes into being, ensures the channel of information remains uninterrupted by the temptations a candidate might entertain when he is the incorrect one.

To construct this threat we need to define $\delta^*$ and $N^*$ in the following way.

**Condition 5** Let $\delta^* > 0$ and $N^* > 0$ be such that

$$
\frac{1}{2} \frac{b\delta^*}{1 - \delta^*} \geq b + \frac{1}{2} \frac{b (\delta^*)^{N^*+1}}{1 - \delta^*}.
$$
If this condition holds an incorrect candidate would not deviate and run as a correct one, when he discounts the future at $\delta^*$ and is not elected for $N^*$ periods. The left hand side represents his expected payoff if he lets the other candidate win today. Instead, the right hand side represents the payoff if he runs as the correct candidate today, and is out of office thereafter for $N^*$ periods. Thus, for such a $\delta^*$ and $N^*$, the voter can threaten the incorrect candidate with a punishment so severe that he would never want to deviate in the first place.

Note that there are lower bounds on $\delta^*$ and $N^*$. These are given by the case when $\delta^* = 1$ (that is, if candidates value the future as much as the present), as then $N^*$ need not be greater than 2, and when $N^* = \infty$ (that is, if a candidate is punished for ever after he deviates when he is the incorrect one), then $\delta^*$ need not be greater than $\frac{2}{3}$. So we know candidates cannot discount the future at a rate smaller than two-thirds, $\delta^* \in \left(\frac{2}{3}, 1\right)$, and the appropriate punishment belongs to the interval $N^* \in (2, \infty)$.

**Strategies and an equilibrium**

A candidate’s strategy is simple: the policy he runs for is predetermined by the party that supports him prior to the realization of the state. However, the amount of money he can offer is not. So his strategy collapses to only a map on $m_t$.

In principle a voter should consider both the amount of money and the policy offered by the candidate to determine his beliefs. However, a candidate runs for the latter irrespective of what he might have observed. Thus, the voter knows that candidates can only strategize on the amount of money they offer. Thus, both the beliefs and the voting decision of the voter can only depend on the amount of money offered by the candidates.

Let $m_{it} : w_t \to [0, \bar{m}]$ be the strategy of each candidate $i = 1, 2$ upon observing state $w_t$. Let $\varphi_t : (m_{0t}, m_{1t}) \to [0, 1]$ be the belief function giving the voter’s probability assessment that the type-1 candidate is running consistent with the true state after observing monetary transfer offers $m_{0t}$ and $m_{1t}$. The strategy of the voter is a mapping $v_t : (m_{0t}, m_{1t}) \to \{0, 1\}$. 
The point of this model is to show, in a stylized setting, that the voter can interpret the correct candidate if he offers less. Thus, I claim the following.

**Lemma 6** If $\delta \in [\delta^*, 1)$ and $N \geq N^*$; that is, if the voter can punish candidates sufficiently harsh, then there is a Perfect Bayesian Equilibrium in which

if $p_{t-1} = w_{t-1}$ the assessment for $t$ is

$$
m^*_1 \in \begin{cases} 
\{m - u_v + a\} & \text{if } w_t = 1 \\
\{m - u_v + a, m\} & \text{if } w_t = 0, \text{ where } a > 0
\end{cases}
$$

$$
m^*_0 \in \begin{cases} 
\{m - u_v + a\} & \text{if } w_t = 0 \\
\{m - u_v + a, m\} & \text{if } w_t = 1, \text{ where } a > 0
\end{cases}
$$

$$
\varphi^*_t = \begin{cases} 
1 & \text{if } m^*_t = \arg \max \{\min \{m^*_0, m^*_1\} + u_v, \max \{m^*_0, m^*_1\}\} \\
0 & \text{otherwise,}
\end{cases}
$$

$$
v^*_t = \begin{cases} 
1 & \text{if } m^*_t = \arg \max \{\min \{m^*_0, m^*_1\} + u_v, \max \{m^*_0, m^*_1\}\} \\
0 & \text{otherwise,}
\end{cases}
$$

that is, the winning candidate offers less money than the loser. Otherwise,

if $p_{t-1} \neq w_{t-1}$ the assessment for the following $s = t, \ldots, t + N$ periods is

$$
m^*_1 \equiv m^*_0 \equiv m = \bar{m}
$$

$$
\varphi^*_s = \frac{1}{2}
$$

$$
v^*_s = \begin{cases} 
1 & \text{if } v^*_{t-1} = 0 \text{ and } m^*_1 \geq m^*_0 \\
0 & \text{if } v^*_{t-1} = 1 \text{ and } m^*_0 \geq m^*_1
\end{cases}
$$

**Proof.** By construction of $\delta^*$ and $N^*$. □
Other equilibria and a refinement

The equilibrium above is one of many. There are Perfect Bayesian Equilibria in which the condition of the lemma holds and yet the correct candidate offers more to be elected.

Example 7 If $\delta \in [\delta^*, 1)$ and $N \geq N^*$ then the assessment for $t$

$$m_{1t}^* - m_{0t}^* = \begin{cases} a & \text{if } w_t = 1 \\ -a & \text{if } w_t = 0, \text{ where } a > 0, \end{cases}$$

$$\varphi_t^* = \begin{cases} 1 & \text{if } m_{1t} = \max \{m_{0t}, m_{1t}\} \\ 0 & \text{otherwise}; \end{cases}$$

$$v_t^* = \begin{cases} 1 & \text{if } m_{1t} = \max \{m_{0t}, m_{1t}\} \\ 0 & \text{otherwise} \end{cases}$$

if $p_{t-1} = w_{t-1}$, and the same assessment as in Lemma 6 if $p_{t-1} \neq w_{t-1}$, is a Perfect Bayesian Equilibrium.

The multiplicity of such equilibria relies on the fact that the incorrect candidate never wants to win or tie. Any belief which creates a dichotomy between what is expected from the correct and the incorrect candidates creates the appropriate response from both, as the incorrect candidate always wants to lose and it is never worth for the correct candidate to tie.

However, now assume that candidates face an increasing utility cost from the money they offer $c(m_t)$, with $c'(m_t) > 0$, even if it does not come from their own pockets. This cost is always smaller than the benefit of winning, $c(\hat{m}) < b$. Now, whilst the incorrect candidate remains indifferent among all responses expected from him, as long as he loses, the correct candidate does not. After all, the bundle he offers in equilibrium is taken by the voter, and that bundle involves money and now he faces an increasing cost for that money. Thus, even if the correct candidate prefers to win to lose or tie, he prefers to win spending little money to more. Different equilibrium payoffs become ranked for the correct candidate.
In this game there is a measure observable to all the participants of what would be spending more compared to less for the correct candidate: the amount of money the other (incorrect) candidate offers. That is, it is observable to all that when the winning candidate is offering more than the loser he is spending more than if the opposite were true.

Thus, if a voter were to observe a deviation from the equilibrium play, the only candidate who could have an incentive to deviate, if it were understood, would be the correct candidate. He would be deviating to avoid offering more than the other candidate to win, hoping that it is interpreted as such. The set of equilibria that provide no incentives to deviate, if a deviation is understood by the voter, are those that satisfy the Cho-Kreps Intuitive Criterion (Cho and Kreps, 1987).

Proposition 8 If \( \delta \in [\delta^*, 1) \), \( N \geq N^* \) and \( c'(m_t) > 0 \), but \( c(\bar{m}) < b \) then the set of Perfect Bayesian Equilibria remains the same, but only those such that

\[
m^*_t - m^*_0 = \begin{cases} 
-a & \text{if } w_t = 1 \\
   a & \text{if } w_t = 0, \text{ where } a > 0.
\end{cases}
\]

satisfy the Cho-Kreps Intuitive Criterion.

Proof. As \( c'(m_t) > 0 \), the correct candidate prefers equilibria where he offers less money to more. This rank comparison of equilibrium payoffs for the correct candidate, together with the indifference of the incorrect candidate among them, justifies interpreting a deviation as only possibly coming from the correct candidate. \( \square \)

Whilst the cost \( c(m_t) \) has no bite to determine equilibria, it provides a ranking among equilibria: between having to offer more to less money to win, the correct candidate always prefers offering less. With an appropriate equilibrium refinement, all equilibria but that in Lemma 6 are ruled out.
Conclusion

In one dimension get opposite observation than Myerson (1993): winning candidate is not only buying votes.

References


