Measuring the Racial Fairness of Redistricting

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## Summary of Court Action, 2001-02

<table>
<thead>
<tr>
<th>Category</th>
<th>House</th>
<th>Senate</th>
<th>Congressional</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plan Challenged</td>
<td>25</td>
<td>27</td>
<td>14</td>
</tr>
<tr>
<td>Impasse</td>
<td>5</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Rejected/Corrected</td>
<td>8</td>
<td>7</td>
<td>1</td>
</tr>
</tbody>
</table>
Criteria for Contesting a Plan

• Population Equality
• Compactness and Contiguity
• Partisan Fairness
• Racial Fairness (Voting Rights Act)

Goal: A consistent and practical measure of the racial fairness of a proposal redistricting plan.
Our Definition of Racial Fairness

Our measure of racial fairness is based on the probability that a given voter will cast a decisive ballot in an election. This is a measure of voting power.

A redistricting plan is fair if it equalizes, as much as possible, the voting power of individuals from different racial/ethnic groups in the state as a whole.
Talk Outline


2. Our Proposed Alternative Measure: Statistical Details and Potential Problems

3. Example from Texas Congressional Redistricting

4. Conclusions
Current Measures of Racial Fairness

There is no generally accepted standard. Some suggested alternatives:

- Use neutral redistricting criteria
- Maximize minority representation
- “No retrogression”
- Proportional representation
Current judicial practice is given by a 3 prong test in *Gingles* case:

1. The minority group is geographically compact so that it could constitute a majority of single member district.

2. The minority group is political cohesive.

3. The majority votes significantly as a bloc to prevent minority candidates of choice from winning.

As a matter of practice, an expert witness is used to show that there is “racial bloc voting” using ecological inference. Upon finding racial bloc voting, districts that contain “sizable” minority voters are created.
Normative Problems with Current Practice

- It is based on group versus individual rights claims.
- It is a notion of fairness based on equal outcomes instead of equal opportunity.
Practical Problems with Current Practice

• How do we systematically identify candidate of choice? Race of candidate or candidate preferred by majority/plurality of a group?

• How do we handle group heterogeneity?

• How do we count multi-ethnic districts?

• The procedures are ad hoc without using standard statistical procedures.
## Estimated Voting Behavior in Texas District 23 (Rep. Bonilla)

<table>
<thead>
<tr>
<th>Year</th>
<th>Hispanics</th>
<th>Anglos</th>
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</thead>
<tbody>
<tr>
<td>1992</td>
<td>0.695 (0.068)</td>
<td>0.146 (0.016)</td>
</tr>
<tr>
<td>1994</td>
<td>0.567 (0.065)</td>
<td>0.220 (0.014)</td>
</tr>
<tr>
<td>1996</td>
<td>0.623 (0.068)</td>
<td>0.115 (0.017)</td>
</tr>
<tr>
<td>1998</td>
<td>0.600 (0.070)</td>
<td>0.161 (0.016)</td>
</tr>
<tr>
<td>2000</td>
<td>0.654 (0.069)</td>
<td>0.216 (0.015)</td>
</tr>
</tbody>
</table>
Estimated Voting Behavior in Texas District 29 (Rep. Green)

<table>
<thead>
<tr>
<th>Year</th>
<th>Hispanics</th>
<th>Anglos</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992</td>
<td>0.643</td>
<td>0.430</td>
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<td></td>
<td>(0.074)</td>
<td>(0.018)</td>
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<td>1994</td>
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<td>0.502</td>
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<td>(0.076)</td>
<td>(0.017)</td>
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<tr>
<td>1996</td>
<td>0.745</td>
<td>0.566</td>
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<tr>
<td></td>
<td>(0.072)</td>
<td>(0.017)</td>
</tr>
<tr>
<td>1998</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2000</td>
<td>0.891</td>
<td>0.552</td>
</tr>
<tr>
<td></td>
<td>(0.071)</td>
<td>(0.018)</td>
</tr>
</tbody>
</table>
An Alternative Method

Let $N_i$ be the number of voters in district $i$ and let $v_i$ be the Democratic share of the two party vote in $i$. If $N_i$ is even then the probability on average that a given voter is decisive is:

$$P_i \equiv \Pr[\text{vote is decisive in district } i]$$

$$= \Pr[N_i v_i = 0.5 N_i]$$

$$= \Pr[v_i = 0.5]$$

$$\approx \frac{f_{v_i}(0.5)}{N_i},$$

where $f_{v_i}$ is the probability density function of $v_i$. 
If $N_i$ is large we get a similar expression:

$$P_i \equiv \Pr[\text{vote is decisive in district } i]$$

$$= \Pr[N_i v_i = 0.5(N_i - 1)]$$

$$= \Pr[v_i = (0.5 - \frac{0.5}{N_i})]$$

$$\approx f_{v_i}(0.5) \frac{N_i}{N_i}.$$

Assuming $N_i$ is large, then the difference between the two cases is negligible.
We want to summarize these \( P_i \)'s across the entire district map in order to be able to rank them. A natural way that we will consider is to construct the weighted average across the districts:

\[
\overline{P} = \frac{\sum_{i=1}^{T} N_i P_i}{N}
\]

where \( T \) is the number of districts in the plan, \( N_i \) is the size of the voting population in the district \( i \), and \( N \) is the total population in the state.
The (average) probability that a black resident of the state casts a decisive ballot is then:

$$\overline{P}^B = \sum_{i=1}^{T} \frac{N_i^B P_i}{N^B}$$

where $N_i^B$ is the number of blacks of voting age population in district $i$ and $N^B$ is the total number of blacks of voting age in the state. And the similar expression for a white resident is:

$$\overline{P}^W = \sum_{i=1}^{T} \frac{N_i^W P_i}{N^W}.$$
We are now in a position to define what we mean by racial fairness.

We will call a redistricting map \textit{racially fair} if:

\[
\overline{P}^B = \overline{P}^W.
\]

As a secondary criteria, we may prefer plans that maximize the probability of a voter casting a decisive ballot.
Estimation

In order to actually estimate our voting power measure, we need to estimate the distribution of votes across the districts, $f_{v_i}$. We do this using a model of legislative elections developed by Gelman and King (1994).

More formally, our probability model comes from a random component linear regression of Democratic vote in district $i$ in year $t$, $v_{i,t}$ on a set observable regressors, $X_{i,t}$. 
The model, therefore is,

\[ v_{i,t} = X_{i,t} \beta_t + \gamma_i + \varepsilon_{i,t}, \]

where \( \beta_t \) is a vector of \( k \) parameters that must be estimated from the data, and \( \gamma_i \) and \( \varepsilon_{i,t} \) are independent error terms. We further assume that both error terms are normally distributed with mean zero and variances, \( \sigma_\gamma \) and \( \sigma_\varepsilon \). The result of the regression are the estimates of \( \{ \beta, \sigma_\gamma, \sigma_\varepsilon \} \), as well as the estimated covariance matrix, \( V \), from which standard errors are calculated.
Since the vote shares are Normally distributed, we can write

\[ P_i \approx \frac{1}{N_i} \phi(0.5|X_{i,t}^{hyp} \beta_t + \delta_i^{hyp}, \sigma^2 + \sigma^2) , \]

where \( \phi(y|\mu, \tau^2) \) is the Normal density function, \( X_{i,t}^{hyp} \) are hypothetical values of the observable regressors corresponding to a district plan and \( \delta_i^{hyp} \) is a constant chosen to set the “partisan swing.” Once we have estimates of the parameters, we can draw \( M \) simulations from the approximate posterior distribution in order to estimate \( P_i \) and hence \( P^B \) and \( P^W \).
The Good & Bad with Our Alternative

Good:
• (Practical) It can be estimated from available data using standard statistical techniques.
• (Normative) It is based on equal opportunity.

Bad:
• (Strategic Voting) Our measure assumes non-strategic behavior on the part of voters.
• (Primaries) We measure voting power in general election, but what if meaningful decisions are made in primaries.
• (Data) What about small legislatures or congressional delegations.
2001 Texas Congressional Redistricting

- Senate had slim Rep. majority and House is controlled by Dems.
- Neither chamber makes headway on redistricting following 2000 Census, but House committee reports out a bill (HB 722).
- First a trial in state court, but this plan is vacated by State Supreme Court.
- U.S. District court draws a plan for congressional districts.
Estimated average probability of casting a decisive ballot in Texas Congressional elections for a person of voting age for a selected set of plans considered in the state court case.
Estimated average probability of a casting a decisive ballot in Texas Congressional elections for a Hispanic of voting age for a selected set of plans from the state court case.
Estimated average probability of a casting a decisive ballot in Texas Congressional elections for a White of voting age for a selected set of plans from the state court case.
Estimated ratio of the average probabilities of a casting a decisive ballot by a Hispanic person to a randomly selected person of voting age in Texas Congressional elections for a selected set of plans from the state court case.
Estimated ratio of the average probabilities of a casting a decisive ballot by a Hispanic person to a White person of voting age in Texas Congressional elections for a selected set of plans from the state court case.
Conclusions

- The current method for measuring the racial fairness of redistricting plans are at best inadequate.
- Measuring racial fairness using voting power seems more promising and practical.
- Future work: What does this framework say about majority-minority districts? How does this measure relate to other characteristics of a redistricting plan?