Candidate Motivation and Electoral Competition

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Abstract

Standard intuition from formal models of candidate competition suggests that candidates with more flexibility will dominate those with less, and consequently almost always win elections. An implication of this tenet is that candidates motivated by winning office will outperform candidates motivated by policy outcomes. In this paper we explore this logic formally and show that it is partly true but mostly false. Surprisingly, we find that greater flexibility can in fact work against a candidate and induce endogenously a preference by voters for less flexible candidates. Applied to candidate motivation, this induced preference implies that office motivated candidates will attempt to imitate policy motivated candidates, and therefore policy motivated candidates will be victorious in a significant proportion of elections. Most significantly, we find that the presence of both types of candidates significantly affect the policy choices of each group, implying that misleading conclusions will be drawn if the standard assumption of homogeneous candidate motivation is employed.

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1 Introduction

Sparked by the seminal contribution of Downs (1957), there has been an explosion in recent decades of work on the nature of candidate competition in elections. This work has sought to determine the candidates who win elections and, more importantly, the policy positions they choose. A necessary variable in all of these models is the motivations of the candidates. However, for this variable there does not exist an assumption that is without controversy. As a result, in parallel to the work on candidate competition there has developed a debate about the true motivations of candidates. The central controversy in this debate is whether candidates are in fact only interested in gaining office or whether they are also concerned with the ultimate policy outcomes. These questions have led to almost separate but parallel literatures evolving, with each based on a particular candidate motivation, and with each claiming dominance via appeals to the behavior of real candidates.

We argue, however, that such appeals are spurious as the motivations of real political candidates cannot be inferred if only their behavior is observed. If, for example, both office and policy motivate candidates choose the same action, such as expressing a preference over policy, then they would be indistinguishable to the voters. This observation implies that the question of which type of candidates enter and win elections is unresolved. We suggest that this dilemma can be resolved formally by investigating how these different types of candidates fare in competition among each other.¹ To this end we synthesize the disparate literatures and construct a model of electoral competition in which the motivations of candidates are private information and unknown to voters, and in which the types of candidates that win office are determined endogenously.

The accepted wisdom on this question is that the answer is obvious: office motivated candidates have greater flexibility and therefore will generally defeat policy motivated candidates.² This view follows from the intuition of standard models (with only one or the other type of candidate). Office motivated candidates typically have complete flexibility in the platforms they choose and therefore can always pander to the median voter (e.g., Downs (1957)). On the other hand, policy motivated candidates are constrained in their policy platforms out of concern for policy outcomes and typically

¹This would seem reasonable as all scholars agree that both types of individual exist. They differ only on which type wins elections.
²See, for example, Calvert (1985, p. 72).
moderate their platforms by choosing a policy between their ideal point and
the median voter (e.g., Calvert (1985), Wittman (1983)). Not surprisingly,
an office motivated candidate can always, or so the logic goes, outmaneuver
a policy motivated candidate and win the election.3

In this paper we show that this intuition is partly true but mostly false.
We find that the electoral environment favors office motivated candidates but
to a lesser degree than previous intuition would suggest, and to such an extent
that policy motivated candidates are victorious in a significant proportion of
elections. Further, and most significantly, we show that the office motivated
candidates react to the presence of policy motivated candidates and choose
radically different policy platforms to those that they would choose if all
potential candidates were solely office motivated. Immediately, this finding
suggests that even though office motivated candidates are favored, an ex-ante
assumption that all candidates are office motivated produces a misrepresentation
of the competitive pressures facing these candidates and misleading
conclusions.

To understand the intuition for our results, and why the standard intuition
is incomplete, it is best to consider the motivations of voters and the
problem they face in selecting between candidates. Voters, it is reasonable
to assume, are concerned with the policies that are ultimately implemented
and not, per se, the campaign policy announcements. Therefore, voters must
attempt to infer from campaign pronouncements the policies that each can-
didate will actually implement if elected. In standard models this inference
is direct as it is assumed that candidates are bound to implement the policy
on which they campaigned. However, in a more realistic model this inference
is far more delicate. In such an environment the beliefs voters form about
policy outcomes after observing campaign platforms will vary depending on
their inferences about candidates motivations. This variation may have an
impact not only on the campaign platforms that are preferred by voters but
also the location of the campaign platforms themselves.

For example, suppose the median voter faces a choice between an office
motivated candidate with a campaign platform at her ideal point and a pol-
icy motivated candidate with a slightly divergent platform. The standard
model assumes the median voter must prefer the centrist candidate. How-
ever, as these are only campaign platforms and not actual policies, a voter

3Unless, of course, the preferred policy of the policy motivated candidate is in a neigh-
borhood of the median voter.
may prefer the divergent candidate if she believes this will lead to a more centrist final policy. There are many reasons for why this might be the case, the most notable being the incentives of candidates once in office. An office motivated candidate may be expected to chase the median voter around the policy space leading to little correlation between campaign pronouncements and final policy. On the other hand, policy motivated candidates will temper their movements towards the new median voter and may be expected to implement policies over a more centrist region with a greater degree of certainty.

Thus, even though voters are concerned solely with policy outcomes and not a candidate’s type, endogenously they develop a preference over type and favor policy motivated candidates. This preference has a profound impact on the behavior of both policy and office motivated candidates and the platforms they select. We find that this preference induces office motivated candidates to imitate policy motivated candidates, thus implying that voters (and external observers) cannot generally infer a candidate’s true type if only policy announcements are observed. This imitation also implies that the policy motivated candidates imitated by office motivated types are in fact the most attractive choice to voters and therefore they may win elections. Surprisingly, this electoral ability is not in spite of their lack of flexibility over policy, but precisely because of it. Further, the presence of office motivated candidates increases the incentives for policy motivated candidates to differentiate themselves from their competitors and choose non-centrist platforms. Counterintuitively then, the presence of office motivated candidates often leads to more policy divergence than otherwise would arise.

To formalize this intuition and incorporate an extra dimension of candidate type we build upon a model of electoral competition with incomplete information due to Banks (1990). To the best of our knowledge, this paper is the first (and still one of the few) to not explicitly bind candidates to implement their campaign platforms if elected. He assumes that each candidate’s true policy preference is private information (and, therefore, unknown to voters) and that this represents the policy position the candidate will implement if elected to office. Banks assumes that each candidate has equal flexibility in announcing platforms around his ideal point and therefore equal

\[4\]

\[5\]Note that the current median voter will generally not be the median voter subsequently, and so will not desire this variation in policy choice.

\[5\]Harrington (1993) is another example that relaxes this constraint in examining the link between reelection pressures and campaign promises.
ability to signal his true preferences to votes. We extend this framework by assuming candidates also have different levels of flexibility in making policy pronouncements, and that this ability is also private information.\footnote{See Maranell (1970) for evidence, as well as a ranking of, the different flexibility of U.S. Presidents.}

Though the heterogeneity we introduce is technically one of signalling costs, an analogy can be drawn to the true motivations of the candidates in line with the intuition presented above. In an albeit stylized way, the policy restricted types equate to policy motivated candidates and the policy free types equate to office motivated candidates as these characteristics capture their relative abilities to move about the policy space. This representation allows us to maintain generality in other features of the model and provides a unique analytic solution to what is a continuous types multidimensional signaling model.

This model bears some resemblance to work on nonspatial characteristics in candidate competition, though fundamentally different premises lead to significantly different results and intuition. Papers in this stream suppose that voters are interested in characteristics of candidates other than their policy announcements, typically taking the form of “valence factors” (Stokes (1963)). Recent examples of this literature by Aragones and Palfrey (2002) and Groseclose (2001) assume that voters exogenously award a valence advantage to one of the candidates. Therefore, a voter will only support a valence disadvantaged candidate if his policy platform is sufficiently more attractive. Implicit in this formulation is the assumption that on some deeper but unspecified level this valence advantage translates into a real benefit for the voters. In contrast, the current paper does not take this connection on faith and instead, by assuming voter preferences over the primitive policy space, shows how such an advantage may arise endogenously. Perhaps more significantly, we also explore how the possibility to affect these advantages impacts the strategies and policies of candidates.

As a final note, it is worth observing that these results have a direct application to the apparently unrelated field of social and group behavior. Bernheim (1994) develops a model of social conformity that differs from that of Banks only in that agents are individual consumers trying to win the favor of other individuals rather than candidates trying to win the favor of voters. In Bernheim’s model, agents have a preference for social status as well as consumption. Each agent has spatial preferences over a single dimension of
good type. Status is presumed to depend on these predispositions such that more centrist preferences are more highly desired. However, preferences are private information and can only be inferred from an agent’s action. Quite clearly, this formulation corresponds closely to that of Banks with preferences representing ideal points and the attractiveness of centrist choices reflecting the power of the median voter. Also like Banks, Bernheim assumes that agents have equal costs in consuming goods that do not correspond to their ideal point.

Achieving results substantively similar to those of Banks, Bernheim interprets this convergence of choice as an explanation of social conformity and the evolution of social norms. Under this interpretation, the extension we incorporate here corresponds loosely to the inclusion of “social sheep,” individuals primarily concerned with conforming and possessing status within a group. Our results then suggest that, perhaps surprisingly, the inclusion of these types in fact make conforming less desirable and leads to agents making consumption decisions more in line with their true consumption preferences and less tailored to achieving social status.

The remainder of the paper is organized as follows. The following section outlines the model and Section 3 describes equilibrium behavior under several equilibrium concepts. Section 4 explores the nature of these equilibria and draws parallels with several commonly observed regularities from real political systems. Section 5 concludes.

2 The Model

Consider a policy space $P \subseteq \mathbb{R}$, a closed convex interval with $|P| = 2D$, where without loss of generality we assume that the midpoint of $P$ is zero: $P = [-D, D]$. There are two candidates, A and B, whose “types”, or true policy preferences are two dimensional. With probability $q$ a candidate has a high cost of policy, hereafter referred to as a “high cost” type, and with probability $1 - q$ they have a low cost of policy, hereafter a “low cost” type. If a candidate is high cost then denote the strength of this cost by $K$, and by 0 if they are low cost. Both candidates also have true policy positions, $\alpha$ and $\beta$, that are assumed to be independent and identically distributed random variables with cumulative distribution $F(\cdot)$ and density $f(\cdot)$, where $f(x) > 0$ for all $x \in P$ and $f(\cdot)$ is symmetric about zero. These are the policies the candidates will implement if elected to office. $F$ and $q$ are common
knowledge and independent, thus a candidate’s ideal point and cost are ex ante uncorrelated. A strategy for a candidate is a function:

$$s : P \times \{0, K\} \rightarrow P$$

Where $$s_A(\alpha, k)$$ is the policy platform a candidate with ideal point $$\alpha \in P$$ and cost $$k \in \{0, K\}$$ would adopt. $$s_B(\alpha, k)$$ is similarly defined for candidate B. Denote a mixed strategy by $$\Delta s$$ with elements $$\Delta s_A(\alpha, k)$$ and $$\Delta s_B(\alpha, k)$$ which specify the set of possible actions and associated density.

There exists a finite set $$N = \{1, 2, ..., n\}$$ of voters, where $$n$$ is odd. All voters have quadratic utility over the policy space,

$$u_i(p) = - (p - p_i)^2$$

where $$p_i$$ is voter $$i$$’s ideal point. We further assume that the median voter $$v \in N$$ has an ideal point equal to the midpoint of the policy space: $$p_v = 0$$. Given beliefs $$\mu_A(\cdot)$$ concerning candidate A’s true policy position, $$i$$’s expected utility from A winning the election is

$$Eu_i(\mu_A) = - (\bar{\alpha} - p_i)^2 - \sigma^2_\alpha,$$

where $$\bar{\alpha}$$ is the mean and $$\sigma^2_\alpha$$ is the variance associated with the density $$\mu_A(\cdot)$$.

The voters select candidates (vote) after observing their announced positions. Thus, a strategy for voter $$i$$ is a function

$$r_i : P \times P \rightarrow \left\{ 0, \frac{1}{2}, 1 \right\},$$

where $$r_i(p_A, p_B)$$ is the probability that $$i$$ votes for candidate A, given that $$i$$ observes announced positions $$p_A$$ and $$p_B$$; the probability that $$i$$ votes for B is then $$1 - r_i(p_A, p_B)$$. Thus, we assume no abstention. The assumption is that voters either vote for candidate A or B with probability one, or vote for A and B with probability a half each. Let $$r(\cdot) = (r_1(\cdot), ..., r_n(\cdot))$$ summarize the voters’ strategies. For any $$(p_A, p_B) \in P \times P$$, let

$$v_A(p_A, p_B) = |\{i \in N : r_i(p_A, p_B) = 1\}|,$$

$$v_B(p_A, p_B) = |\{i \in N : r_i(p_A, p_B) = 0\}|,$$

be the number of individuals voting with probability one for A and for B, respectively.
The utility for a candidate A, given announcement positions \((p_A, p_B)\), true type \(\{\alpha, k\}\), and voter strategies \(r(p_A, p_B)\), is equal to the function \(\psi(\alpha, k, p_A)\) multiplied by the probability that A is elected. A’s utility, \(U(\alpha, k, p_A, r(p_A, p_B))\), is then equal to zero if \(v_B(p_A, p_B) \geq \frac{n+1}{2}\), \(\psi(\alpha, k, p_A)\) if \(v_A(p_A, p_B) \geq \frac{n+1}{2}\), and is

\[
\psi(\alpha, k, p_A) \cdot \left\{ \left( \frac{1}{2} \right)^{(n-v_A-v_B)} \right\} \cdot \sum_{j=\frac{n-1}{2}-v_A}^{n-v_A-v_B} \binom{n-v_A-v_B}{j} \]

otherwise. The term in brackets is simply the probability that A receives a majority of votes. The utility for B is similarly defined.

Thus candidates derive utility from winning office. For tractability we assume this function is of the form,

\[
\psi(\alpha, k, p_A) = y - k \cdot (\alpha - p_A)^2
\]

where \(p_A\) is the candidate’s ideal policy and what he will implement if elected. Thus, the utility from winning derived by high policy cost candidates is a function of the distance between, as Banks (1990) puts it, “what they say” and “what they do.” Low cost candidates do not bear any such cost. The value of \(K\) indicates the cost of announcing a platform different from \(\alpha\). Also, we assume that \(y > 0\), so that a candidate can always receive a higher payoff from some announcement, if elected, than from not being elected.

3 Equilibrium Behavior

3.1 Electoral Equilibrium

We begin by examining strategies that constitute sequential equilibrium behavior (Kreps and Wilson (1982)). In what follows we restrict attention to sequential equilibrium strategies that are symmetric with respect to candidates and the origin. That is, \(\{\alpha, k\} = \{\beta, k\} \Rightarrow s_A(\alpha, k) = s_B(\beta, k)\) and \(s_A(\alpha, k) = -s_A(-\alpha, k)\).\(^7\) This allows subscripts on candidate strategies to be dropped. As any two low cost candidates incur the same costs from a particular policy announcement, regardless of their true ideal points, we assume

\(^7\)Banks (1990) considers asymmetric strategies in only one instance and shows that some pooling equilibria exist that are not symmetric with respect to the origin. These equilibria also exist here, though they will not be considered. See Footnote 10 on page 15.
that all such candidates play the same (possibly mixed) strategy. That is, \( \Delta s(\alpha', 0) = \Delta s(\alpha'', 0) \) for all \( \alpha', \alpha'' \in P \). This restriction simplifies the specification of results without substantively changing the nature of the equilibria. We also add the assumptions that voters adopt weakly dominant strategies and mix equally if indifferent over the candidates. Combining these assumptions to the definition of sequential equilibrium we produce the following definition of an electoral equilibrium:

**Definition 1** An electoral equilibrium of the above model consists of strategies \( s^* (.) , r_i^* (.) \) and beliefs \( \mu_A^* (.) , \mu_B^* (.) \) such that

1. \( \forall \{\alpha, k\} \in \{P, \{0, K\}\}, s^* (\alpha, k) \) maximizes

   \[ q \int U(\alpha, k, s, r_i^* (s, s^* (\beta, 0))) f(\beta) d\beta + (1 - q) \int U(\alpha, k, s, r_i^* (s, s^* (\beta, K))) f(\beta) d\beta; \]

2. \( \forall i \in N, \text{ and } \forall (p_A, p_B) \in P \times P, \)

   \[ r_i^* (p_A, p_B) = \left\{ \begin{array}{ll} \frac{1}{2} & \text{as } \int u_i(\alpha) \mu_A^* (\alpha|p_A) d\alpha > \int u_i(\beta) \mu_B^* (\beta|p_B) d\beta; \\ 0 & \text{as } \int u_i(\alpha) \mu_A^* (\alpha|p_A) d\alpha < \int u_i(\beta) \mu_B^* (\beta|p_B) d\beta; \end{array} \right\} \]

3. if \( s^{*-1}(p_A) \neq \emptyset \), then \( \mu_A^* (t_A|p_A) \) is the conditional probability (relative to the priors \( f(.) \) and \( q \)) that \( \alpha \in t_A \cap \tau (p_A) \) given \( \alpha \in \tau (p_A) \) and \( \tau (p_A) = \{\alpha| \text{ for some } k, (\alpha, k) \in s^{*-1}(p_A)\} \), where \( t_A \in P \).

4. if \( s^{*-1}(p_B) \neq \emptyset \), then \( \mu_B^* (t_B|p_B) \) is the conditional probability (relative to the priors \( f(.) \) and \( q \)) that \( \alpha \in t_B \cap \tau (p_B) \) given \( \alpha \in \tau (p_B) \) and \( \tau (p_B) = \{\alpha| \text{ for some } k, (\alpha, k) \in s^{*-1}(p_B)\} \), where \( t_B \in P \).

Condition (1) states that each candidate chooses his announcement to maximize his expected utility, given the strategy of the other candidate and the strategies of voters. Condition (2) gives the weak dominance and indifference assumptions described above. Condition (2) also implies that all voters hold the same beliefs in and out of equilibrium. This is so because, in a sequential equilibrium, beliefs are assumed to be the limit of a sequence of beliefs derived using Bayes’ rule on candidate strategies which make every announcement with positive probability. Since this determines beliefs for every possible announcement, voters’ beliefs are the same all along the sequence, and hence at its limit. Conditions (3) and (4) imply that voters use Bayes’ rule to update their beliefs when an equilibrium announcement is made.

It should be noted that the voter utility does not depend directly on the candidate’s policy costs (motivation) or announced platform. Rather, voters are only concerned with the policy position that is ultimately implemented. This is also reflected in voters’ beliefs which need only be a function of candidate ideal points. In the equilibrium described later we will show that, for a given policy announcement, voters have a preference for high cost candidates over low cost candidates. However, significantly, this preference will arise endogenously as an equilibrium condition and is not imposed exogenously.

The assumption of quadratic utilities and the equilibrium condition that all voters possess the same beliefs for all possible announcements implies that we can reduce the complexity of calculating electoral equilibria. For any beliefs \( \mu_A(\cdot), \mu_B(\cdot) \), where the means of these densities \( \bar{\alpha}, \bar{\beta} \) differ, there exists a unique position \( \bar{p} \in P \) defined by

\[
\bar{p} = \frac{\bar{\alpha} - \bar{\beta}}{2} + \frac{\sigma^2_\alpha - \sigma^2_\beta}{2(\bar{\alpha} - \bar{\beta})}
\]

such that all voters with \( p_i < \bar{p} \) should vote for one candidate, and all voters with \( p_i > \bar{p} \) should vote for the other candidate. If \( \bar{\alpha} = \bar{\beta} \) then \( \bar{p} \) is undefined. If \( \sigma^2_\alpha = \sigma^2_\beta \) then all voters are indifferent between voting for A and B. If \( \sigma^2_\alpha \neq \sigma^2_\beta \) then all voters vote for the candidate with the lower variance. Thus, given beliefs \( \mu_A(\cdot), \mu_B(\cdot) \), if the median voter is not indifferent between voting for A and B, then whomever \( v \) votes for wins the election. If \( v \) is indifferent then each candidate wins with probability \( \frac{1}{2} \), whether other voters are indifferent or not. Thus, the following simplification can be made,

\[
U(\alpha, k, p_A, r(p_A, p_B)) = \psi(\alpha, k, p_A) \cdot r_v(p_A, p_B)
\]

Given strategies \( s(\cdot), r_v(\cdot) \), the probability that a candidate of type \( \{\alpha, k\} \) wins the election is then

\[
\lambda(\alpha, k) = q \int r_v(s(\alpha, k), s(\beta, 0)) f(\beta) d\beta + (1 - q) \int r_v(s(\alpha, k), s(\beta, K)) f(\beta) d\beta
\]

so that we can denote the expected utility from the strategy \( s(\cdot) \) as simply \( \lambda(\alpha, k) \cdot \psi(\alpha, k, p_A) \).

Let \( \Gamma_e \) denote the set of electoral equilibrium strategies in the model. In equilibrium the standard incentive compatibility condition must hold. Thus, for all \( s(\cdot), r(\cdot) \in \Gamma_e \) and all \( \{\alpha, k\} \in \{P, \{0, K\}\} \),

\[
\lambda(\alpha, k) \cdot \psi(\alpha, k, s(\alpha, k)) \geq \lambda(\alpha', k') \cdot \psi(\alpha, k, s(\alpha', k')) \quad \forall \{\alpha', k'\} \in \{P, \{0, K\}\}
\]

(1)
Equation 1 implies the following two results. These two propositions hold in the model of Banks (1990) for all candidates as they are all assumed to be high cost. The same results continue to apply here for high cost types, though not for low cost types as they do not have any signaling costs. The equilibrium behavior of these candidates is dealt with in Proposition 3. All proofs are gathered in the Appendix.

**Proposition 1** For all \(s(\cdot), r(\cdot) \in \Gamma_e\), \(s(\alpha, K)\) is monotone increasing in \(\alpha\); i.e., \(\forall \alpha, \alpha' \in P, \alpha < \alpha'\) implies \(s(\alpha, K) \leq s(\alpha', K)\).

Thus, in all electoral equilibria, high cost candidates who are “more extreme” in their true policy positions will make announcements which are (weakly) farther from the median than more moderate candidates. The next proposition shows that this implies these extreme candidates are elected (weakly) less often.

**Proposition 2** For all \(s(\cdot), r(\cdot) \in \Gamma_e\), \(\lambda(\alpha, K)\) is monotone increasing on \([-D, 0]\) and monotonic decreasing on \([0, D]\).

The low cost candidates do not have any signaling costs and so they do not face a trade-off between policy location and the probability of victory. The utility of low cost types is maximized simply when their probability of victory is maximized. Thus, in equilibrium all low cost candidates must have equal probability of victory, regardless of their true ideal points, and this probability must be at least as big as all high cost candidates. These features are summarized in the following proposition, the proof of which is obvious and has been omitted.

**Proposition 3** For all \(s(\cdot), r(\cdot) \in \Gamma_e\), \(\lambda(\alpha, 0)\) is constant \(\forall \alpha \in P\), and \(\lambda(\alpha, 0) \geq \max_{\alpha \in P} \lambda(\alpha, K)\).

In equilibrium low cost candidates may locate at more than one policy position. Thus, without signaling costs, Proposition 1 does not necessarily hold for these candidates. Given low cost candidates have complete freedom in platform choice, whereas high cost candidates are severely restricted, it may be suspected that low cost candidates will always be selected by voters. However, in subsequent results we will show that this freedom can actually prove disadvantageous as the more centrist low cost candidates are unable to
differentiate themselves from extremists, unlike the centrist high cost candidates. Consequently, in equilibrium voters develop a preference for the high cost candidates. This not only leads to high cost candidates winning a significant percentage of elections, but also induces low cost candidates to imitate high cost candidates.

Banks (1986) showed that, for the case of only high cost candidates \( (q = 0) \), the set \( \Gamma_e \) of electoral equilibria is quite large, and little of substantive value can be added to the above propositions without turning to additional equilibrium refinements. This is because the sequential equilibrium concept, and thus the concept of electoral equilibrium, places no restrictions on the beliefs of voters for out-of-equilibrium announcements. However, for the case of heterogeneous candidates \( (q > 0) \), several strong statements about the nature of equilibrium can be made with only the requirements of electoral equilibrium. These statements will now be developed.

We will say that a low cost candidate is imitating a high cost candidate if he announces an identical policy position. The implication of Propositions 3 and 2 is that low cost types can only imitate high cost types that have centrist ideal policies. The characterization of the types that can be imitated and, more significantly, the affect this has on the equilibrium behavior of the high cost types, are described in the following proposition.

**Proposition 4** For all \( s(.) \), \( r(.) \) \( \in \Gamma_e \), let \( M \) be the support of \( \Delta s(\alpha,0) \). If \( \exists \alpha' \) such that \( s(\alpha',K) \in M \) then \( \forall \alpha \in [0,\alpha'] \), \( s(\alpha,K) \in M \). Further, if \( s(\alpha,K) \) is continuously increasing for some \( \alpha \in [0,\alpha'] \) then it must be that \( s(\alpha,K) = \alpha \).

This implies that if the policy announcements of a neighborhood of imitated high cost types are not constant (i.e., not pooling) then these candidates are announcing their true ideal points. Thus, the threat of imitation reduces the pressure on high cost types to converge and allows them to announce platforms that are less costly to them. However, despite the possibility of voters observing different policy announcements, it must be that they are indifferent to all of these announcements. The alternative, of course, is that \( s(\alpha,K) \) jumps discontinuously from pool to pool. There are restrictions that can be placed on the nature of these jumps also. However, in the following section these possibilities will be eliminated by equilibrium refinements and as such we will not explore them here.

It follows from this proposition that if low cost types are imitating some high cost types but not others, then in all their possible policy announcements
they must be imitating high cost types. This is described in the following lemma.

**Lemma 1** If \( \exists \alpha', \alpha'' \in (-D, D) \) such that \( s(\alpha', K) \in M \) and \( s(\alpha'', K) \notin M \) then for all \( p \in M \) there must exist an \( \alpha \) such that \( s(\alpha, K) = p \).

Thus, even though low cost candidates incur the same costs for all policy announcements, in any electoral equilibrium that conveys information to the voters (i.e., not completely pooling) they must always imitate a high cost type. Therefore, the preference that voters develop endogenously in equilibrium for high cost types restricts completely the possible policy platforms of low cost types. This restriction is, perhaps surprisingly, not in spite of their low signaling costs but because of it.

It is also a possibility that voters are able to completely distinguish in equilibrium between high and low cost type candidates. However, the following proposition shows that such a possibility places tight restrictions on the inferences voters would be able to make within the different candidate types.

**Proposition 5** If \( \exists \alpha' \) such that \( s(\alpha', K) \in M \) then \( s(\alpha, K) \) is constant for all \( \alpha \in [0, D] \). Further, \( \min \alpha, M \geq \min \{ s(\alpha, K), 2D - s(\alpha, K) \} \).

This proposition states that for voters to be able to distinguish completely in equilibrium between high and low cost candidates, they cannot distinguish at all between candidates of different policy preferences. This suggests that equilibria will require a trade-off between separating high versus low cost candidates and moderate versus extreme candidates. Further, this implies that, for high cost types to not deviate to an announcement of a low cost type, the low cost types must only announce sufficiently extreme positions.

The logic of this proposition can also be used to show that if in equilibrium low cost types imitate any high cost types then they must imitate a set of high cost types that is measurable. To see this suppose that this set is not measurable. Then by Proposition 4 it must be that all of the low cost types pool with the high cost candidate with ideal point at zero. For voters to weakly prefer this pool over any other policy announcement it must be that all other high cost types pool together. However, if this is the case then the location of this pool must be distinctly above the imitated announcement of the zero type, thus providing an opportunity for profitable defection for types with small \( \alpha \). This characteristic is described by the following lemma.
Lemma 2 Denote by $Q'$ the set of $\alpha$ such that $s(\alpha, K) \in M$. In any electoral equilibrium $Q' = \emptyset$ or there exists an $\alpha^* > 0$ such that $[0, \alpha^*) \subseteq Q'$.

Thus, in equilibrium, low cost types must imitate an interval of high cost types with centrist ideal points, or not imitate at all.

3.2 Universally Divine Electoral Equilibrium

The previous results restricted significantly the behavior of candidates in equilibrium. However, the set $\Gamma_e$ is still too large for precise predictions and comparative statics. To further refine this set we will impose, as did Banks (1990), the requirement of universal divinity due to Banks and Sobel (1987). This refinement requires that, for every out-of-equilibrium announcement, voters decide which type of candidate is most likely to “defect” from the equilibrium and make such an announcement, and then place probability one on that type of candidate making the announcement. This refinement is more restrictive than is required for the following results. We employ it as it is both simple and widely recognized. Essentially, all that is required for the current results is that some probability is shifted from those less likely to defect to those more likely to defect.

For all $s(.) , r(.) \in \Gamma_e$, let $\theta(\alpha, k, p | s(.) , r(.) )$ be the probability of election which makes a candidate of type $\{\alpha, k\}$ indifferent between making his equilibrium announcement (and receiving his equilibrium utility) and announcing the out-of-equilibrium position $p$:

$$
\theta(\alpha, k, p) = \frac{\lambda(\alpha, k) \cdot \psi(\alpha, k, s(\alpha, k))}{\psi(\alpha, k, p)}
$$

If voters adopt a new strategy $r'(.)$ where the resulting probability of election $\lambda'(\alpha, k)$ at $p$ was greater than $\theta(\alpha, k, p)$, then this type would rather announce $p$ than $s(\alpha, k)$, thus defecting from the equilibrium. For any electoral equilibrium $s(.) , r(.) \in \Gamma_e$, we say that $\{\alpha', k'\}$ is “more likely” to defect to $p$ than $\{\alpha, k\}$ if $\theta(\alpha, k, p) > \theta(\alpha', k', p)$; that is, the set of voter strategies for which $\{\alpha', k'\}$ would want to defect is larger (by inclusion) than for $\{\alpha, k\}$. The criterion of universal divinity due to Banks and Sobel (1987) requires that, at out-of-equilibrium announcements $p$, voters should assign positive probability only to those candidate types that are most likely to defect.
Definition 2 A universally divine electoral equilibrium is an electoral equilibrium where

1) if \( s_*^{-1}(p_A) = \emptyset \), then \( \mu_A^{*}(\alpha'|p_A) > 0 \) only if, for some \( k' \), \( \{\alpha', k'\} = \arg\min_{(\alpha, k)} \theta(\alpha, k, p_A|s^*(), r^*()) \);

2) if \( s_*^{-1}(p_B) = \emptyset \), then \( \mu_B^{*}(\beta'|p_B) > 0 \) only if, for some \( k' \), \( \{\beta', k'\} = \arg\min_{(\beta, k)} \theta(\beta, k, p_B|s^*(), r^*()) \).

Let \( \Gamma_u \) denote the set of universally divine electoral equilibrium strategies. Not surprisingly, the set \( \Gamma_u \) is much smaller than the set \( \Gamma_e \). To characterize the elements of this set, we define the following family of strategies.

Definition 3 A “Cut-Point” strategy is of the following form:\(^8\); for \( k > 0 \),

1. \( \forall \alpha \in [0, \alpha'], s(\alpha, K) = \alpha \)
2. \( \forall \alpha \in [\alpha', \alpha''], s(\alpha, K) = \alpha' \)
3. \( \forall \alpha \in [\alpha'', D], s(\alpha, K) \) is strictly increasing (separating)

Several examples of this family of strategies are depicted in Figures 1, 2 and 3. We begin with some benchmark results. If \( q = 0 \) then this model collapses to that of Banks (1990) in which all candidates have high policy costs. He proved the following two results. Define \( k^* \) as the value of \( k \) which solves \( \psi(D, k, 0) = 0 \). Thus, if \( k = k^* \) then a candidate who is of the most extreme type (\( \alpha = D \)) will be indifferent between announcing the median position and losing the election.

Proposition 6 (Banks (1990)) Suppose \( q = 0 \). If \( k < k^* \), then for all \( s(,), r(,) \in \Gamma_u, s(,) = 0, \forall \alpha \in P.\(^9\)

Therefore, if the costs are sufficiently low, the only universally divine equilibrium is for all candidates to pool at the same announcement.\(^10\)

---

\(^8\)The restriction to strategies that are symmetric around zero implies that strategies for types with \( \alpha < 0 \) are defined implicitly.

\(^9\)Banks (1990) states this result with instead the weak inequality \( k \leq k^* \). However, there exist other equilibria at \( k = k^* \), though they are substantively the same as they differ only on a non-measurable set (at only one point). The variation is that it may now be the case that \( s(D, K) > 0 \). This strategy still supports an equilibrium as the extreme type is indifferent between losing with certainty and winning at zero.

\(^10\)Banks (1990) does not make the restriction that strategies be symmetric about zero in this result. In this situation he shows that there exists a continuum of equilibria in which all voters pool at a single point in some interval around the median voter. Further, he shows as \( k \) increases this interval collapses. We restrict attention to symmetric strategies as it simplifies the statement of this result as well as being used for all other results.
Figure 1: $\alpha' = 0; \alpha'' = D$

Figure 2: $\alpha' = 0; \alpha'' < D$
that this equilibrium corresponds to a Cut-Point strategy in which $\alpha' = 0$ and $\alpha'' = D$, as depicted in Figure 1, with all candidate policy platforms converging to the median. Thus, without sufficient costs, the candidates are indistinguishable to the voters who respond by picking a winner randomly. Banks’ second result is to show that if costs are instead above this threshold then there is some separation of policy platforms.

**Proposition 7 (Banks (1990))** Suppose $q = 0$. If $k > k^*$, then the unique universally divine equilibrium is of the following form:

(i) $\forall \alpha \in [0, \alpha (k)]$, $s (\alpha) = 0$

(ii) $\forall \alpha \in (\alpha (k), D]$, $s (\alpha)$ is strictly increasing (i.e. separating)

Thus, with sufficiently high costs, centrist candidates pool at the median and the more extreme candidates separate. Note that this strategy is also a Cut-Point as depicted in Figure 2 where $\alpha' = 0$ and $\alpha'' = \alpha (k)$. In this equilibrium some candidates separate from the pool and are distinguishable to the voters, though the pool remains at the median. We say that this equilibrium is ‘separating’ and ‘centrist’. These characteristics are formalized by the following definition.
**Definition 4** A Cut-Point strategy is “separating” if \( \alpha'' < D \), and “non-centrist” if \( \alpha' > 0 \).

Thus, with only high cost candidates all equilibria are centrist, though for sufficiently high costs they are also separating. It is important to note that separation implies that not all candidate types have equal probabilities of victory. We will also use the terminology of Definition 4 to describe the behavior of particular candidates. We will say that a candidate is “separating” if voters can precisely identify his true ideal point; that is, \( \{\alpha, k\} \) is separating if \( \mu(\alpha|s(\alpha, k)) = 1 \). By assumption, it is possible only for the high cost types to separate. A candidate is said to be “pooling” if he is choosing the identical policy platform as other similarly motivated candidates; that is, \( \{\alpha, k\} \) is pooling if there exists a \( \{\alpha', k\} \) such that \( s(\alpha, k) = s(\alpha', k) \). Also by assumption, it must be that all low cost candidates are pooling. Recall that a high cost candidate is said to be “imitated” if a low cost candidate announces the same policy platform. If a candidate is neither pooling nor separating (and thus he must be imitated), then we will say that he is “obscured.” Therefore, an obscured high cost candidate is distinguishable by his action from other high cost candidates, but not from imitating low cost candidates.

We now turn to the opposite extreme to that considered by Banks (1990) and suppose that all candidates are low cost. This produces the following obvious result.

**Proposition 8** Suppose \( q = 1 \). Then any \( s(\alpha, 0) \) is supportable as a universally divine electoral equilibrium.

Essentially, with only low cost candidates the game is one of cheap talk and the set of equilibrium actions cannot be pinned down at all. This extreme lack of prediction is troubling. However, when compared to the following results with both high and low cost candidates, it provides a startling benchmark that highlights how the behavior of low cost candidates is affected by the presence of high cost candidates. Remarkably, we find that in the presence of even the smallest probability of a candidate being high cost, the behavior of low cost types changes from the complete arbitrariness of Proposition 8 to being pinned down precisely. This is because in equilibrium low cost types wish to imitate high cost types, and as the freedom of movement of these candidates is much lower, so too in action is the movement of low cost types. We now develop our main results for the case of
heterogeneous candidates, beginning with the equilibrium behavior of high cost types.

**Proposition 9** Suppose \( q \in [0, 1) \). Every universally divine equilibrium requires high cost candidates to play a Cut-Point strategy.

This proposition pins down the behavior of all high cost candidates in equilibrium. As in the model of Banks (1990), all equilibria must involve high cost candidates using a cut-point strategy. However, we will see that these equilibria are not limited to the centrist strategies found by Banks, and that this leads to substantively different conclusions about candidate behavior.

Of course, any equilibrium must also specify the behavior of low cost candidates. An implication of Proposition 9 and previous results is that the low cost candidates must imitate all centrist high cost candidates who do not pool. This requirement is described in the following lemma.

**Lemma 3** Suppose \( q \in (0, 1) \) and that high cost candidates are using a Cut-Point strategy. If \( k > k^* \) then every universally divine equilibrium requires \( [0, \alpha') \subseteq \Delta s(\alpha, 0) \subseteq [0, \alpha'] \).

Proposition 3 from the previous section implies that voters are indifferent over all policy announcements in \( \Delta s(\alpha, 0) \). If these announcements are strictly preferred to an announcement of \( \alpha' \) then \( \lambda(\alpha, K) \) is discontinuous at \( \alpha' \) and candidate type \( \{\alpha', K\} \) would have the incentive to deviate and announce \( \alpha' - \varepsilon \), for some small \( \varepsilon > 0 \) (as \( \psi \) is continuous). Therefore, even if \( \alpha' \notin \Delta s(\alpha, 0) \) it must be that voters are indifferent over all announcements in the set \([0, \alpha']\). If \( \alpha' > 0 \) this requires the low cost types to mix in such a way to maintain this indifference. The following proposition shows that there always exists a \( \alpha' \) and a corresponding mixing strategy such that this indifference is achieved, and therefore that a universally divine electoral equilibrium must exist.\(^{11}\)

**Proposition 10** A universally divine equilibrium exists for all \( q \in [0, 1] \).

\(^{11}\)It is possible to characterize these equilibrium requirements quite precisely. However, they do not add significantly to the intuition of the previous results and we will not explore them in any detail here.
It follows immediately that in all such equilibria high cost candidates are playing a Cut-Point strategy and if $k > k^*$ then low cost candidates are imitating. It also follows that in any such equilibrium $\alpha'' > \alpha'$, and high cost candidates pool at the announcement $\alpha'$. If there was not a pool at $\alpha'$ then there would be a discontinuity in $\lambda$ at $\alpha'$ and types $\alpha' + \varepsilon$ would have the incentive to deviate and announce $\alpha'$.

The above results have shown that a universally divine equilibria must exist, and that the equilibrium must employ one of a unique family of Cut-Point strategies. However, the equilibrium itself is not unique and, in fact, within this family of strategies there typically exists a continuum of equilibria. These equilibria vary in the critical values $\alpha'$ and $\alpha''$, requiring the low cost types to mix in different proportions. As $\alpha'$ increases more and more low types are required to maintain indifference along the region $[0, \alpha')$, and thus the upper bound is the equilibrium when all low cost types are required in the interval $[0, \alpha')$ to maintain voter indifference (and therefore $\alpha' \notin \Delta s(\alpha, 0)$). On the other hand, as $\alpha'$ decreases the voters' expected policy from announcements in $[0, \alpha')$ increases as the probability of the candidate being low type increases. Eventually, it may be that this expectation becomes so unattractive that the median voter prefers the first separating high cost candidate. The critical point of indifference between these choices provides the lower boundary for $\alpha'$ in equilibrium. If the proportion of low cost types is not insignificant then this lower bound may be greater than zero, precluding Cut-Point equilibria with small $\alpha'$. A significant implication of this intuition is formalized in the following proposition.

**Proposition 11** Suppose $q \in [0,1)$. There exists a $k_0$ and $q_0$ such that $\forall k > k_0$ and $q > q_0$, $\alpha' = 0$ is not supportable as an universally divine equilibrium.

This result implies that the centrist equilibria of Banks (1990) may still exist for small $q$. That is, if we add only a small fraction of low cost types who pool at zero, then voters still prefer a selection out of this pool to any separating type and the equilibrium survives. However, if too many low cost types are added (i.e., $q$ increases) then this preference will eventually break down, and so too the equilibrium.\footnote{To formalize these critical values, define $\hat{k}$ as the value of $k$ that solves $\psi(\alpha^{ce}, \hat{k}, 0) = 0$, where $\alpha^{ce}$ is voters' certainty equivalent policy platform with respect to a random draw of low cost types. Therefore, if $K > \hat{k}$ then a neighborhood of types around type $\{\alpha^{ce}, K\}$ will never pool at zero. Thus, for all $K > \hat{k}$ there exists a $\hat{q}(K)$ such that $\forall q > \hat{q}(K)$ the} Thus, for large enough $q$, and costs $k$,
all universally divine electoral equilibria will involve a non-centrist Cut-Point strategy. An example of this strategy was given in Figure 3.

We can use this result to explore the comparative statics of the equilibria as parameters vary. Here we will focus on the cost of signalling for high cost types. Banks (1990) showed that as \( k \to \infty \), \( \alpha'' \to 0 \) and all types separated (recall, with \( q = 0 \) it must be that \( \alpha' = 0 \)). However, we can see here that with a positive fraction of low cost types, this result cannot hold as, for small enough \( \alpha'' \), voters must prefer the first separating type to a selection out of the pool of imitated types. This intuition is summarized in the following lemma, which follows immediately from Proposition 11.

**Lemma 4** Suppose \( q > 0 \) and that \( k \to \infty \). Then there exists a \( \hat{\alpha} > 0 \) such that in all universally divine electoral equilibrium \( \alpha' > \hat{\alpha} \), and \( \alpha'' \) is arbitrarily close to \( \alpha' \).

Thus, as costs increase, all high cost candidates announce policies arbitrarily close to their true policy preference, as was the case in Banks (1990) for \( q = 0 \). However, due to the imitation of low cost types, there is not complete separation and the electorate does not approach full informativeness.

### 4 Equilibrium Characteristics

Natural questions to ask of a formal model are how the equilibria relate to observation and whether the results are robust to model perturbation. In this section we investigate the nature of equilibria and note several interesting features. We explore the consistency of the equilibria with several empirical regularities and find quite significant support. Firstly, however, we will address the question of robustness.

A criticism of this work may be that by restricting analysis to only two types of costs, and one of them being zero, we have produced an overly restricted, and perhaps special case, result. It is true that the assumption that one type of candidate has zero costs simplifies the analysis. Though, it does so with a considerable benefit. In a model with a single sender, Stamland (1994, 1999) shows how the standard monotonicity condition of signaling games may fail when the sender’s type is multidimensional. As Cut-Point strategy in which \( \alpha' = 0 \) (i.e., pool at zero) does not constitute a universally divine electoral equilibrium.
a result multiple equilibria exist that survive the D1 refinement (Cho and Kreps (1987)), and D1 does not ensure Pareto Dominance. Stamland goes on to construct an equilibrium that satisfies D1 and conveys some but not all information, and in which types with low costs pool with high cost types that have a higher quality (using the language of the signaling literature; a higher quality is analogous here to a more centrist policy preference).

In contrast, with the current model we are able to solve analytically a multi-dimensional signaling problem and produce an essentially unique equilibrium prediction that, we hope, provides robust insight into the nature of candidate competition among differentially motivated candidates. We conjecture that with a richer type space the same intuition holds and equilibria exist in which voters generate endogenously a preference for high cost candidates and that this induces low cost types to imitate them.

We turn now to the equilibria. A non-exhaustive list of interesting equilibrium characteristics is the following.

1. As announced policy becomes more extreme, voters become more certain of the actual policy that will be implemented.

2. Candidates that announce more extreme policy platforms are more likely to be policy motivated (i.e., high cost) and be firm in their policy (i.e., deviate less from their announcement to the policy they implement). This relationship and that of point 1 are closely related and appear regularly in the empirical literature (see Blomberg and Harrington (2000) for an explicit consideration).

3. More extreme candidates don’t necessarily become less likely to win (until a critical point is reached). Therefore, the candidate announcing nearest the median doesn’t always win and observationally the median voter result fails. However, the logic of the median voter theorem still applies. The median voter is still pivotal but as her preferences are over implemented policy and not campaign pronouncements, her (weakly) preferred candidate may not be closest to her ideal point in campaign platform.

4. If $\alpha' > 0$ then measure zero candidates converge on the median voter (see Figure 3). The selection of non-centrist platforms is another frequently observed phenomenon. For the case of the U.S. this is shown by Alesina and Rosenthal (1995, chapter 2).
5. In evaluating two centrist candidates, voters will be unsure whether the campaign announcement of the more centrist candidate represents his true policy preference or whether he is just being opportunistic. Equilibrium requires this uncertainty to exactly balance such that the median voter is indifferent between these candidates. This relationship would seem to be consistent with how real voters react to campaign announcements, and also explain why we don’t see a ‘race towards the middle’ in real elections (even when we suspect the candidates of being low cost) as voters would not be fooled by such convergence.\textsuperscript{13}

6. To some degree Calvert (1985) interprets his results as a robustness test on the classic Downsian model. He shows that as the degree of candidates’ policy motivation approaches zero the candidates converge on the median voter, implying that convergence is robust to perturbations of candidate motivation. In the current environment, however, robustness of the classic model is not quite so evident. We find that even as the percentage of high cost candidates approaches zero (i.e., $q \to 1$) the equilibrium is pinned down and non-centrist platforms prevail. Then, at the limit, the set of equilibria explode and any set of announcements can occur in equilibrium as the game is one of pure cheap talk.

7. Despite their policy inflexibility, high cost (policy motivated) candidates often win, even beating low cost candidates. Surprisingly, they are not required to sacrifice completely their policy preferences in order to do so (i.e., by jumping to the median voter).

The results of this paper have several common features with the models of hierarchies developed in a series of papers by Harrington (1998, 1999, 2000) and may be best seen as their complement.\textsuperscript{13} In an explicit electoral

\textsuperscript{13} This intuition may shed some light on the disastrous Goldwater presidential candidacy of 1964. To the dismay of many, Goldwater mapped out a very extreme policy stance and then spent much of the campaign ignoring these issues and focusing on principles and character. His claim was that he followed his principles whereas his opponent, President Johnson, did not. As Wildavsky (1965, p. 393) noted, “It was not so much his principles but the belief that he stuck to them that counted most with his supporters.” Interpreted through the prism of our model, Goldwater’s strategy may have been to convince voters he was a preferred high cost type and that the more centrist Johnson was a low cost type. Success in this task would then allow him to overcome his policy extremism. His failure, therefore, was not necessarily because he was not as centrist as Johnson, but because he either was just too extreme or he failed to convince voters of his more desirable motivation.
setting, Harrington (2000) shows how the electoral selection process may produce higher level office holders who appear to be something they are not. Harrington assumes that all candidates are office motivated and chase the median voter, and that voters have a preference for stability in policy. Essentially, the candidates that then are successful in elections and move to higher offices are those that are simply fortunate enough to have a stable electorate requiring little policy deviation. This implies that these higher office holders have exhibited policy consistency and appear to be policy motivated despite their true motivation solely being to win office. Harrington (1998) broadens this model and allows for agent heterogeneity, though not strategic behavior. He shows why under similar conditions rigid agents will outperform flexible agents and dominate higher positions in a hierarchy. Harrington (1999) allows for strategic behavior but restricts candidates to be homogeneous. He shows that candidates who have a preference for higher positions will be policy consistent at lower levels but then act opportunistically upon reaching higher levels.

5 Conclusion

The motivations of political candidates are a crucial foundation in the operation of political systems. Motivations impact directly not only the behavior of candidates during an election campaign but also when in office. In this paper we have attempted to provide insight on the question of candidate motivations by exploring a model of electoral competition with candidates of heterogeneous motivation. Consistent with accepted wisdom, we find that candidates with policy flexibility are favored in elections. However, policy constrained candidates are not always defeated and, most significantly, their presence has a significant impact on the behavior of all other candidates.

Within the boundaries of an electoral campaign we have identified an incentive for imitation among political actors in order to influence an audience. It would seem reasonable to conclude that this incentive is not confined to election campaigns and in fact permeates throughout other pockets of the political world. Exploring these possibilities in settings such as the legislature and judiciary would seem a potentially profitable direction for future work.
6 Appendix

6.1 Proof of Proposition 1

Identical to the proof for Proposition 1 in Banks (1990), with simply a change in notation to allow for the richer type space.

6.2 Proof of Proposition 2

Suppose not: thus $\exists \alpha_1, \alpha_2$ such that $\alpha_1 < \alpha_2$ and $\lambda (\alpha_1, K) < \lambda (\alpha_2, K)$. Therefore, $Eu_v (\mu (s(\alpha_1, K))) < Eu_v (\mu (s(\alpha_2, K)))$. Define the following: $T (\alpha_i) = \{\alpha \in P | s(\alpha, K) = s(\alpha_i, K)\}$. By Proposition 1, it must be that $s(\alpha_2, K) \in \Delta s(\alpha, 0)$ and thus $Eu_v ([0, D_1, 2f(\cdot)]) > Eu_v (T (\alpha_1), 2f(\cdot)) > Eu_v (T (\alpha_2), 2f(\cdot))$. However, as $Eu_v (T (0), 2f(\cdot)) > Eu_v ([0, D_1, 2f(\cdot))$ it must be that $Eu_v (\mu (s(0, K))) > Eu_v (\mu (s(\alpha_2, K)))$, violating Proposition 3 (though its statement follows in the paper its proof is independent).

6.3 Proof of Proposition 4

Define $Q'$ such that $\forall \alpha \in Q', s(\alpha, K) \in M$, where by assumption $Q'$ is non-empty. Suppose $0 \notin Q'$, then $s^{-1}(s(0, K)) = \{[0, \alpha'], K\}$ for some $\alpha' < \min \{\alpha | \alpha \in Q'\}$ and $Eu_v ([0, \alpha', \cdot]) > Eu_v ([0, D, \cdot]), Eu_v (\alpha, \cdot), \forall \alpha \in Q'$, which implies $Eu_v ([0, \alpha', \cdot]) > Eu_v (\mu (M))$, where $\mu (M)$ are the median voter’s beliefs after observing some $\alpha \in M$. As this violates Proposition 3 it must be that $0 \in Q'$.

Suppose now that $Q'$ is not convex and there exists $\alpha_1 < \alpha_2 < \alpha_3$ such that $\alpha_1, \alpha_3 \in Q'$ but $\alpha_2 \notin Q'$. Define by $\alpha^{ce} \in [0, D]$ the certainty equivalent that satisfies $Eu_v ([0, D], 2f(\cdot)) = Eu_v (\alpha^{ce}, 1)$. It follows that the median voter is indifferent between the implementation of this certainty equivalent and a random draw over all ideal points with a density $2f(\alpha)$. If $\alpha_3 \leq \alpha^{ce}$ then $Eu_v (\mu (s(\alpha_2, K))) > Eu_v (\mu (s(\alpha_3, K)))$ where $\mu (s(\alpha, K))$ are the beliefs formed by the median voter after observing the policy platform of a high cost candidate with ideal point $\alpha_i$. This inequality violates Proposition 3. If $\alpha_3 > \alpha^{ce}$ then, as low cost candidates are playing strategies independent of their types, $Eu_v (\mu (s(\alpha_1, K))) \neq Eu_v (\mu (s(\alpha_3, K)))$ unless $s(\alpha_1, K) = s(\alpha_3, K)$. The first inequality violates Proposition 3, and the

\[ ^{14} \text{These terms are defined in the proof of Proposition 4 that follows.} \]
second inequality, combined with $\alpha_2 \notin Q'$, violates Proposition 1. Therefore, $Q'$ must be convex.

Suppose for all $\alpha \in (\alpha_4, \alpha_5) \subset Q'$, $s(\alpha, K)$ is continuously increasing, and that $s(\alpha', K) < \alpha'$ for some $\alpha'$. Consider the case when $\alpha'$ deviates and imitates the type $(\alpha'', K)$, where $\alpha'' = \frac{s(\alpha', K) + \min[\alpha', s(\alpha, K)]}{2}$. As $Q'$ is convex and by Proposition 3, $\lambda(\alpha', K) = \lambda(\alpha'', K)$ and $\psi(\alpha', K, s(\alpha', K)) < \psi(\alpha', K, s(\alpha'', K))$. Therefore the deviation is profitable and this can’t occur in equilibrium. The possibility that $s(\alpha', K) > \alpha'$ for some $\alpha'$ is ruled out analogously. Consequently, if $s(\alpha, K)$ is continuously increasing on $Q'$ it must be that $s(\alpha, K) = \alpha$.

6.4 Proof of Lemma 1

If $\exists \alpha', \alpha''$ such that $s(\alpha', K) \in M$ and $s(\alpha'', K) \notin M$, then by Proposition 4 there exists an $\alpha' < 1$ such that $s(\alpha, K) = s(0, K)$ for all $\alpha \in [0, \alpha']$. Suppose the lemma is not true and there exists a $p \in M$ such that for no $\alpha$ does $s(\alpha, K) = p$. Then, if $\alpha' > 0$ it must be that $E_{u_v}(\mu(s(0, K))) > E_{u_v}([0, D], 2f(\cdot))$, which violates Proposition 3. If $\alpha' = 0$ then $E_{u_v}(\mu(s(0, K))) = E_{u_v}([0, D], 2f(\cdot)) < E_{u_v}(\mu(s(\varepsilon, K)))$, for some small $\varepsilon$, unless $s(\alpha, K)$ is constant for all $\alpha \in (0, D)$. As this inequality violates Proposition 3 it must be that $s(\cdot)$ is constant and $E_{u_v}(\mu(s(0, K))) = E_{u_v}([0, D], 2f(\cdot)) = E_{u_v}(\mu(s(\varepsilon, K)))$. This implies that $\lambda(\alpha, K)$ is constant for all $\alpha \in [0, D)$. As it must be that $s(\varepsilon, K) - s(0, K) > \gamma > 0$ there exists a $\varepsilon$ small enough such that $\psi(\varepsilon, K, s(0, K)) > \psi(\varepsilon, K, s(\varepsilon, K))$ and a profitable deviation exists.

6.5 Proof of Proposition 5

Suppose now that $Q'$ is empty. If $s(\alpha, K)$ is not constant then $s^{-1}(s(0, K)) = \{[0, \alpha'], K\}$ for some $\alpha' < 1$ by Proposition 1 and $E_{u_v}([0, \alpha'], ) > E_{u_v}([0, D], 2f(\cdot))$, thus violating Proposition 3. Further, if $\min_\alpha M < \min \{s(\alpha, K), 2D - s(\alpha, K)\}$ then $\exists \alpha$ such that $\psi(\alpha, K, \min_\alpha M) > \psi(\alpha, K, s(\alpha, K))$. As $\lambda(\alpha, k) = \frac{1}{2}$ for all $\alpha$ and $k$, this deviation is profitable, violating the equilibrium requirements.
6.6 Proof of Lemma 2

Suppose \( Q' \) is a singleton. By Proposition 4, \( Q' = 0 \). If \( s(\alpha, K) \) is not constant for all \( \alpha \in (0, D) \) then there exists an \( \alpha_1 \) such that \( Eu_v(\mu(s(\alpha_1, K))) > Eu_v([0, D], 2f(\cdot)) \), thus violating Proposition 3. Therefore, for all \( \alpha \in (0, D) \) set \( s(\alpha, K) = \kappa > s(0, K) \). For \( \alpha_2 < \frac{\kappa}{2} \), \( \psi(\alpha_2, K, s(0, K)) > \psi(\alpha_2, K, \kappa) \). Thus, by Proposition 3, a deviation by type \( \alpha_2 \) to \( s(0, K) \) is profitable and \( Q' \) can’t be a singleton. By Proposition 4 it must therefore be measurable and connected.

6.7 Proof of Proposition 9

Let \( s(\cdot) \) be any strategy satisfying incentive compatibility (Equation 1), and define

\[
\theta(\alpha, k, p) = \frac{\lambda(\alpha, k) \cdot \psi(\alpha, k, s(\alpha, k))}{\psi(\alpha, k, p)}
\]

Since in equilibrium \( \lambda(\cdot) \cdot \psi(\cdot) \) is continuous in \( \alpha \), \( \theta(\cdot) \) will be as well and it will be differentiable everywhere except at the jump discontinuities of \( s(\cdot) \). For low cost types, \( \theta(\alpha, 0, p) = \lambda(\alpha, 0) \) and \( \frac{\partial \theta}{\partial \alpha} = 0 \). For high cost types,

\[
\frac{\partial \theta(\alpha, K, p)}{\partial \alpha} = \left\{ \psi(\alpha, K, p) \cdot \left[ +\lambda(\alpha, K) \cdot \left( \frac{\partial \psi(\alpha, K, s(\alpha, K))}{\partial \alpha} + \frac{\partial \psi(\alpha, K, s(\alpha, K))}{\partial s} \cdot \frac{\partial s}{\partial \alpha} \right) \right] - \frac{\psi(\alpha, K, p) \cdot \lambda(\alpha, K) \cdot \psi(\alpha, K, s(\alpha, K))}{[\psi(\alpha, K, p)]^2} \right\}
\]

If \( s \) is separating at \( \alpha \) then \( \lambda(\alpha, K) = (1 - q) 2 (1 - F(\alpha)) \) and \( \frac{\partial \lambda}{\partial \alpha} = -(1 - q) 2 f(\alpha) \). This is because a separating type \( \{\alpha, K\} \) must be defeated by all \( \alpha' < \alpha \) and defeat all \( \alpha' > \alpha \) (of other high cost candidates). Also, Proposition 3 implies they are beaten by all low cost types. Incentive compatibility implies that utility is maximized for type \( \{\alpha, K\} \) at announcement \( \alpha \). Thus, the first order necessary condition for an equilibrium is that \( s(\cdot) \) satisfy

\[
\frac{\partial \left[ (1 - q) 2 (1 - F(\alpha')) \cdot \psi(\alpha, K, s(\alpha', K)) \right]}{\partial \alpha'} \bigg|_{\alpha' = \alpha} = 0
\]

or

\[
-(1 - q) \cdot \psi(\alpha, K, s(\alpha, K)) + \frac{\partial \psi(\alpha, K, s(\alpha, K))}{\partial s} \cdot \frac{\partial s(\alpha, K)}{\partial \alpha} \cdot (1 - F(\alpha)) = 0
\]
Combining these relationships, (3) can be simplified to\(^{15}\)

\[
\frac{\partial \theta(\alpha, K, p)}{\partial \alpha} = \lambda(\alpha, K) \left\{ \frac{\psi(\alpha, K, p) \cdot \left( \frac{\partial \psi(\alpha, K, p)}{\partial \alpha} \cdot k(\alpha, K) \right)}{[\psi(\alpha, K, p)]^2} - \frac{\partial \psi(\alpha, K, p)}{\partial \alpha} \cdot \psi(\alpha, K, s(\alpha, K)) \right\}
\]

(4)

While if \(\{\alpha, K\}\) pools then \(\frac{\partial \alpha}{\partial \alpha} = 0\) and \(\frac{\partial s}{\partial \alpha} = 0\), so Equation 4 holds for these types also. For high cost candidates that are obscured, it must be that \(\psi(\alpha, K, s(\alpha, K)) = y\) and \(\frac{\partial s}{\partial \alpha} = 0\). Therefore,

\[
\frac{\partial \theta(\alpha, K, p)}{\partial \alpha} = -\frac{\partial \psi(\alpha, K, p)}{\partial \alpha} \cdot \lambda(\alpha, K) \cdot \psi(\alpha, K, p) / [\psi(\alpha, K, p)]^2
\]

(5)

which is signed by \(-\frac{\partial \psi(\alpha, K, p)}{\partial \alpha}\).

We now seek to sign 4 for all high cost candidates and compare to low cost candidates in order to determine the type that minimizes 2. Note that for separating segments of types, say \((\alpha^\prime, \alpha^\prime^\prime)\), it must be that \(s(\alpha, K) \leq \alpha\) for all \(\alpha \in (\alpha^\prime, \alpha^\prime^\prime)\). If not then suppose type \(\{\alpha, K\}\) deviates by imitating type \(\{\tilde{\alpha}, K\}\), where \(\tilde{\alpha} = \min \left[ \alpha, \frac{s(\alpha, K) + s(\alpha^\prime, K)}{2} \right]\). By Proposition 2, and because \(\psi(\alpha, K, s(\alpha, K)) < \psi(\alpha, K, s(\alpha^\prime, K))\), this deviation is profitable and \(s(.)\) can’t support an equilibrium.

Firstly, suppose that \(s(0, K) = x > 0\). If \(s^{-1}(0) = \emptyset\) then consider the deviation to \(p = 0\). For separating and pooling types, \(\frac{\partial \psi(\alpha, K, s(\alpha, K))}{\partial \alpha} > \frac{\partial \psi(\alpha, K, p)}{\partial \alpha} < 0\) and \(\psi((\alpha, K, s)) > \psi((\alpha, K, p))\), therefore \(\frac{\partial \theta}{\partial \alpha} > 0\). For obscured types, \(\frac{\partial \psi(\alpha, K, 0)}{\partial \alpha} < 0\) and \(\frac{\partial \theta}{\partial \alpha} > 0\) also. As \(\psi(0, K, x) < \psi(0, K, 0), \theta(0, K, 0) < \lambda(0, K) = \lambda(\alpha, 0) = \theta(\alpha, 0, p)\), and thus universal divinity implies voters believe with probability one that the deviator is type \(\{0, K\}\). As this leads to certain victory and no costs, the deviation is profitable and so in equilibrium it must be that \(s^{-1}(0) \neq \emptyset\) and some type announces zero.

If \(s(0, K) > 0\) then, by Proposition 1, \(s(\alpha, K) \geq s(0, K) > 0\) and it must be low cost types announcing zero. Consider again the deviation to \(p = 0\). By Proposition 3 \(\lambda(\alpha, 0) \geq \lambda(0, K)\), and as \(\psi(0, K, 0) > \psi(0, K, x)\), the deviation is profitable and thus in equilibrium it must be that \(s(0, K) = 0\).

If \(s(\alpha, K) = 0\) for all \(\alpha\) then candidates are using a cut-point strategy, so we will suppose that this is not the case. By Lemma 2, it must be that

\(^{15}\)Note that the \(q\) terms cancel out and leave the exact same expression as in Banks (though he has several typos in his statements).
there exists an $\alpha$ such that high cost candidates with ideal points in $[0, \alpha]$ are imitated and that $\alpha > 0$. Suppose the interval of types, $[0, \alpha_1)$ is obscured, and thus, by Proposition 4, that $\forall \alpha \in [0, \alpha_1), s(\alpha, K) = \alpha$. Further, suppose that types $(x_1, x_2)$ are separating (ignoring for the moment the behavior of type $x_1$), which implies $\lambda(\alpha, K)$ is discontinuous at $x_1$. Therefore, as $\varepsilon \to 0^+$, $\psi(x_1 + \varepsilon, K, s(x_1 - \varepsilon, K)) \to y$ and $\lambda(x_1 - \varepsilon, K) \psi(x_1 + \varepsilon, K, s(x_1 - \varepsilon, K)) > \lambda(x_1 + \varepsilon, K). \psi(x_1 + \varepsilon, K, s(x_1 - \varepsilon, K))$ and the deviation to $x_1 - \varepsilon$ is profitable. Thus, in equilibrium $s(\alpha, K)$ can’t jump from obscured to separating, it can only jump to pooling. A similar analysis proves that this pool must be at the announcement $x_1$ and that $\forall \alpha \in (x_1, x_2), \lambda(\alpha, K) = \lim_{\varepsilon \to 0} \lambda(x_1 - \varepsilon, K)$. Thus, by Proposition 1, $s(x_1, K) = x_1$.

Suppose the interval of types $[\alpha_1, \alpha_2)$ is pooling, where $\alpha_1 = 0$ or $[0, \alpha_1)$ are obscured. If the interval of types $(\alpha_2, \alpha_2 + \beta)$ is obscured then $s(\alpha_2 + \varepsilon, K) = \alpha_2 + \varepsilon$ for all $\varepsilon \in (0, \beta)$. Consider the candidate with ideal point at $\alpha_2 - \omega$ and the deviation to $\alpha_2 + \omega$ for $\omega > 0$. By Proposition 3, $\lambda(\alpha_2 - \omega, K) = \lambda(\alpha_2 + \omega, K)$, and as $\psi(\alpha_2 - \omega, K, \alpha_2 + \omega) > \psi(\alpha_2 - \omega, K, s(\alpha_2 - \omega, K))$ for small enough $\omega$, the deviation is profitable and an equilibrium strategy can’t jump from pooling to obscured. Note also that a jump from separating can’t be made to obscured (as $Q'$ is convex). Therefore, any obscured types must be located only in the most centrist interval of types.

Suppose the out-of-equilibrium announcement $p \in (s(\alpha_1, K), s(\alpha_2, K))$, where $s(\alpha_2, K) < \alpha_2$ (by Proposition 1 low cost types can’t announce $p$ in equilibrium) is observed and consider the following partition:

(i) For obscured types, who must be only in $[0, \alpha_1)$, $\frac{\partial \psi(\alpha, K, p)}{\partial \alpha} > 0$, and therefore $\frac{\partial \theta(\alpha, K, p)}{\partial \alpha} < 0$.

(ii) For $\alpha \in [\alpha_1, p], \frac{\partial \psi(\alpha, K, p)}{\partial \alpha} > 0 > \frac{\partial \psi(\alpha, K, s(\alpha, K))}{\partial \alpha}$. Therefore, $\frac{\partial \theta(\alpha, K, p)}{\partial \alpha} < 0$.

(iii) For $\alpha \in (p, \alpha_2), \frac{\partial \psi(\alpha, K, p)}{\partial \alpha} > \frac{\partial \psi(\alpha, K, s(\alpha, K))}{\partial \alpha}$ and $\psi(\alpha, K, p) > \psi(\alpha, K, s(\alpha, K))$. Therefore, $\frac{\partial \theta(\alpha, K, p)}{\partial \alpha} < 0$.

(iv) For $\alpha > \alpha_2$, $0 > \frac{\partial \psi(\alpha, K, s(\alpha, K))}{\partial \alpha}$ and $\psi(\alpha, K, s(\alpha, K)) > \psi(\alpha, K, p)$. Therefore, $\frac{\partial \theta(\alpha, K, p)}{\partial \alpha} > 0$.

If $s(\alpha_2, K) \geq \alpha_2$ then step (iii) is not required and (iv) still holds (recall that for $s$ to be separating it must be that $s(\alpha, K) < \alpha$. As $\psi(\alpha_2, K, p) > \psi(\alpha_2, K, s(\alpha_1, K))$ and $\lambda(\alpha, 0) = \theta(\alpha, 0, p)$, for all deviations $p \in (s(\alpha_1, K), s(\alpha_2, K))$, universal divinity requires beliefs to be $\mu(\{\alpha_2, K\} | p) = 1$. Suppose that type $\{\alpha_2, K\}$ is pooling with other high cost types $(\alpha_2, \alpha_3)$. If $\{\alpha_2, K\}$ deviates and announces $p = s(\alpha_2, K) - \varepsilon$, then $\lambda(\alpha_2, K | p - \delta > \lambda(\alpha_2, K)$ for some
\( \delta > 0 \). As \( \psi(\alpha_2, K, p) \to \psi(\alpha_2, K, s(\alpha_2, K)) \) as \( \varepsilon \to 0 \), for small enough \( \varepsilon \) the deviation by type \( \{\alpha_2, K\} \) is profitable. Thus, if high cost types \( (\alpha_1, \alpha_2) \) pool then high cost types \( (\alpha_2, \alpha_3) \) must separate. In the proof of Proposition 10 it will be shown that deviations by type \( \{\alpha_2, K\} \) in this case aren’t profitable.

Suppose then that the interval of high cost types \( (\alpha_2, \alpha_3) \) is separating. These candidates are, of course, not imitated and thus a jump cannot be to obscured types. Further, if a jump is to another separating segment then \( \psi \) would be discontinuous and \( \lambda \) continuous, therefore \( \psi, \lambda \) would be discontinuous, which violates the equilibrium condition. Thus, the jump must be to a pooling interval, which we will assume is at \( m \). Consider an out-of-equilibrium announcement \( p \in (\alpha_3, m) \) and again partition the type space (recall that \( s(\alpha, K) < \alpha \) if separating).

(i) For obscured types, \( \frac{\partial \psi(\alpha, K, p)}{\partial \alpha} > 0 \), and therefore \( \frac{\partial \psi(\alpha, K, p)}{\partial \alpha} < 0 \).

(ii) For \( p > \alpha > s(\alpha, K) \), \( \frac{\partial \psi(\alpha, K, p)}{\partial \alpha} > 0 > \frac{\partial \psi(\alpha, K, s(\alpha, K))}{\partial \alpha} \). Therefore, \( \frac{\partial \theta(\alpha, K, p)}{\partial \alpha} < 0 \).

(iii) For \( \alpha > p > s(\alpha, K) \), \( 0 > \frac{\partial \psi(\alpha, K, p)}{\partial \alpha} \) and \( \psi(\alpha, K, p) > \psi(\alpha, K, s(\alpha, K)) \). Therefore, \( \frac{\partial \theta(\alpha, K, p)}{\partial \alpha} < 0 \).

(iv) For \( \alpha > \alpha_3 \), \( 0 > \frac{\partial \psi(\alpha, K, s(\alpha, K))}{\partial \alpha} \) and \( \psi(\alpha, K, s(\alpha, K)) > \psi(\alpha, K, p) \). Therefore, \( \frac{\partial \theta(\alpha, K, p)}{\partial \alpha} > 0 \).

As \( \psi(\alpha_3, K, p) > \psi(\alpha_2, K, s(\alpha_3, K)) \) and \( \lambda(\alpha, 0) = \theta(\alpha, 0, p) \), for all deviations \( p \in (s(\alpha_3, K), m) \), universal divinity requires beliefs to be \( \mu(\{\alpha_2, K\} | p) = 1 \). Thus, for small \( \delta \), a deviation by type \( \{\alpha_3, K\} \) to \( s(\alpha_3, K) + \delta \) is profitable as his type is again fully revealed (so \( \lambda(\alpha_3, K) \) is unchanged) and \( \psi(\alpha_3, K, p) > \psi(\alpha_3, K, s(\alpha_3, K)) \). Hence, a separating interval of types must continue all the way to the boundary at \( \alpha = D \), and thus in any universally divine electoral equilibrium high cost candidates must play a Cut-Point strategy.

### 6.8 Proof of Lemma 3

As \( k > k^* \) it must be that \( s(D, K) > 0 \), and therefore, by Proposition 5, \( Q' \) is non-empty. Denote by \( \alpha^* \) the maximal value such that \( [0, \alpha^*) \subseteq Q' \), where \( \alpha^* > 0 \) by Lemma 2. Suppose \( \alpha^* \neq \alpha' \). It must be that high cost types with \( \alpha \in (\alpha^*, \alpha') \) are separating and that \( \lambda(\alpha, K) \) is discontinuous at \( \alpha^* \). Consider the candidate with type \( \{\alpha^* + \varepsilon, K\} \) and the deviation to \( \alpha^* \). As \( \varepsilon \to 0 \), \( \psi(\alpha^* + \varepsilon, K, \alpha^*) \to \psi(\alpha^* + \varepsilon, K, \alpha^* + \varepsilon) \) and for small \( \varepsilon \), it must
be that $\lambda (\alpha^*, K), \psi (\alpha^* + \varepsilon, K, \alpha^*) > \lambda (\alpha^* + \varepsilon, K), \psi (\alpha^* + \varepsilon, K, \alpha^* + \varepsilon)$ and incentive compatibility is violated. Thus, $[0, \alpha') \subset \Delta s (\alpha, 0)$.

For $\Delta s (\alpha, 0) \subset [0, \alpha']$ it must be that $Q' = [0, \overline{\alpha}]$, where $\overline{\alpha} > \alpha''$, as high cost candidates are using a Cut-Point strategy. So consider type $\{\alpha'' + \delta, K\}$. The proof of Proposition 9 showed that for small $\delta > 0$ it must be that $s (\alpha'' + \delta, K) < \alpha'' < \alpha'' + \delta$. As $\alpha'' + \delta \in Q'$, it must be that $\lambda (\alpha'' + \delta, K) = \lambda (\alpha'', K)$. Given $\psi (\alpha'', K, \alpha') < \psi (\alpha'', K, s (\alpha'' + \delta))$, candidate of type $\{\alpha'', K\}$ has incentive to deviate and imitate type $\{\alpha'' + \delta, K\}$. Thus, in equilibrium it must be that $\Delta s (\alpha, 0) \subset [0, \alpha']$.

### 6.9 Proof of Proposition 10

Suppose, firstly, that high cost candidates are using a Cut-Point strategy and consider possible out of equilibrium deviations. Possible deviations are $\forall p \in (\alpha', s (\alpha'', K))$ and $p > s (D, K)$. Consider these in turn. From the proof of Proposition 9, beliefs after any deviation $p \in (\alpha', s (\alpha'', K))$ must be that $\mu (\{\alpha'', K\} | p) = 1$. As the probability of victory for this type is unchanged and $\psi (\alpha'', K, p) < \psi (\alpha'', K, s (\alpha'', K))$, this deviation isn’t profitable.

Consider instead deviations such that $p > s (D, K)$. If type $\{D, K\}$ is separating in equilibrium then $\lambda (D, K) = 0$ and therefore, $\forall p > s (D, K), \theta (D, K, p) = 0 < \theta (\alpha, K, p)$ for all $\alpha < D$. Thus, after this deviation beliefs must be $\mu (\{D, K\} | p) = 1$, which implies a zero probability of victory. This makes type $\{D, K\}$ indifferent between the equilibrium strategy and deviating, and all other types strictly worse off. Thus, the equilibrium is sustained. Suppose instead that type $\{D, K\}$ is pooling. By Proposition 9 this pool must be at $\alpha'$ and voters are indifferent over all equilibrium announcements. If $\alpha' > 0$ then indifference is violated as $Eu_v (0, .) > Eu_v (\alpha^c, 1) > Eu_v (\alpha', .)$. Therefore, if $\{D, K\}$ pools it must be that $\alpha' = 0$ (the pool is at the median and there are no obscured types). Consider all $p > 0$ that are not played in equilibrium (Proposition 5 implies that such points exist) and partition the space as follows for high cost types:

(i) For $\alpha > p$, $\frac{\partial \psi (\alpha, K, p)}{\partial \alpha} > 0 > \frac{\partial \psi (\alpha, K, s (\alpha, K))}{\partial \alpha}$. Therefore, $\frac{\partial \theta (\alpha, K, p)}{\partial \alpha} < 0$.

(ii) For $p > \alpha$, $0 > \frac{\partial \psi (\alpha, K, p)}{\partial \alpha} > \frac{\partial \psi (\alpha, K, s (\alpha, K))}{\partial \alpha}$ and $\psi (\alpha, K, p) > \psi (\alpha, K, s (\alpha, K))$. Therefore, $\frac{\partial \theta (\alpha, K, p)}{\partial \alpha} < 0$.

For all such $p$, $\lambda (\alpha, 0) = \theta (\alpha, 0, p)$. If $p < 2D$ then $\psi (\alpha_3, K, p) > \psi (\alpha_2, K, s (\alpha_3, K))$ and universal divinity requires beliefs to be $\mu (\{D, K\} | p) = 1$. This implies the probability of winning the election is zero and for
that the Cut-Point strategy with parameters $CE® > ®$ decrease). Consequently, as an event with expected value less than the average is added, the average must increase. For all $\alpha$, $k \in \{0, K\}$, which is a Cut-Point strategy in which $\alpha' = 0$ and $\alpha'' = D$. By the definition of $k^*$, $\psi(\alpha, k) \geq 0$ for all types, there exist no profitable out-of-equilibrium deviations, and there are no deviations that imitate other types. Therefore, a universally divine equilibrium exists.

Consider now the strategy of the low cost types. Let the low cost types mix over $[0, \alpha')$, with density $g(.)$. Voter beliefs after observing an announced position $\alpha \in [0, \alpha')$, therefore, will be a weighted average over type $\{\alpha, K\}$ and $\alpha^\infty$ (as this is equivalent to a random selection of a low cost type). Denote the certainty equivalent for any observed announcement $\alpha$ by $CE\alpha$. Choose, if possible, $g(.)$ such that $CE\alpha_1 = CE\alpha_2$ for all $\alpha_1, \alpha_2 \in [0, \alpha')$. That is, such that voters are indifferent over all such announcements. Clearly, for any $q$, such a distribution must exist for any $\alpha'$ in some interval $[0, \bar{\alpha})$. Further, for small enough $\alpha'$, $g(.)$ can be chosen such that $CE\alpha$ is arbitrarily close to $\alpha^\infty$ for all announcements in $[0, \alpha')$. Thus, for small $\alpha'$, $EU_v([\alpha', \alpha'), 2f(.) < EUv(CE\alpha)$ for all $\alpha \in [0, \alpha')$.

For all $\alpha \in [0, \alpha')$, if $CE\alpha > \alpha'$ then, for a fixed $q$, $\frac{\partial CE\alpha}{\partial \alpha'} < 0$ (as an event with expected value less than the average is added, the average must decrease). Consequently, as $\frac{\partial EU_v(\alpha', \alpha')}{\partial \alpha'} > 0$, there exists by continuity an $\hat{\alpha}'$ such that $EU_v([\hat{\alpha}', \alpha'), CE\alpha) < \alpha^\infty$, for all $\alpha \in [0, \hat{\alpha}')$. Therefore, we claim that the Cut-Point strategy with parameters $\hat{\alpha}'$ and $\hat{\alpha}''$, and corresponding...
density \( g(.) \), constitute a universally divine electoral equilibria.

Out-of-equilibrium deviations were dealt with above, so consider now in-equilibrium-deviations. Low cost types are maximizing their probability of victory for all announcements in \( \Delta s(\alpha, 0) \), and therefore have no incentive to deviate (Proposition 3 is satisfied). High cost types with \( \alpha \in [0, \hat{\alpha}'] \) are maximizing their probability of victory and announcing their ideal points. Thus, there exist no profitable deviations for these types. High cost types with \( \alpha \in (\hat{\alpha}', \hat{\alpha}'') \) do not profit from deviating to any \( \alpha_1 \in [0, \hat{\alpha}') \) as the probability of victory is unchanged yet \( \psi(\alpha, K, \hat{\alpha}') > \psi(\alpha, K, \alpha_1) \). Type \( \{\hat{\alpha}'', K\} \) is indifferent between the announcements \( \hat{\alpha}' \) and \( s(\hat{\alpha}'', K) \). Therefore, types \( \alpha \in (\hat{\alpha}', \hat{\alpha}'') \) and \( \alpha > \hat{\alpha}'' \) strictly prefer their equilibrium announcements (as the separating segment is determined according to incentive compatibility).

6.10 Proof of Proposition 11

If \( \alpha' = 0 \) then the first separating type is a function of costs and the percentage of types, denote this critical value by \( \alpha'(k, q) \). Further, define \( k' \) to be the value of \( k \) that solves \( \psi(\alpha', K, 0) = 0 \). Therefore, for \( K > k' \), there exists an \( \varepsilon > 0 \) such that, for all \( q \), \( \alpha''(K, q) < \alpha^{ce} - \varepsilon \). Therefore, in equilibrium, \( CE|_{\alpha=0} < \alpha^{ce} \) (all low cost types locate at zero) and, consequently, \( \frac{\partial CE|_{\alpha=0}}{\partial q} > 0 \) and \( CE|_{\alpha=0} \to \alpha^{ce} \) as \( q \to 1 \) (as events with expected value greater than the average are added). Hence, there exists a \( q' \) such that \( \forall q > q', \alpha''(k', q') \), and the equilibrium does not exist as voters prefer the first separating type to the pooling type (thus violating, for example, Propositions 3 and 2).

References


33


