The Inefficient Use of Power: Costly Conflict with Complete Information*

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Abstract

Recent work across a wide range of issues in political economy as well as American, comparative, and international politics tries to explain the inefficient use of power — revolutions, civil wars, high levels of public debt, international conflict, and costly policy insulation — in terms of commitment problems. This paper shows that a common mechanism is at work in a number of these diverse studies. This common mechanism provides a more general formulation of a type of commitment problem that can arise in many different substantive settings. The present analysis then formalizes this mechanism as an “inefficiency condition” which ensures that all of the equilibria of a stochastic game are inefficient. This condition has a natural substantive interpretation: Large, rapid changes in the actors’ relative power (measured in terms of their minmax payoffs) cause inefficiency.
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Civil wars, revolutions, litigation, strikes, economic sanctions, international conflict, and the use of power in general pose an inefficiency puzzle. Suppose that a group of actors is bargaining about how to resolve an issue or, more abstractly, about how to divide a “pie.” One or more of them can affect the outcome and possibly even impose a division through the use of some form of power – be it military, economic, legal, or more broadly political. The exercise of power, however, consumes resources, and, consequently, the pie to be divided among the bargainers before anyone tries to impose a settlement is larger than it will be afterward. As a result, there usually are divisions of the larger pie that would have given each bargainer more than it will obtain from an imposed settlement. The use of power, in other words, leads to Pareto inefficient outcomes. Why, then, do the bargainers sometimes fail to reach a Pareto superior agreement prior to the explicit use of power?

A standard explanation of inefficiency appeals to asymmetric information. Indeed, recent formal work in international relations theory on the causes of war frames the problem in terms of efficiency and focuses almost entirely on informational asymmetries. But a growing body of work across a wide range of issues in political economy as well as in American, comparative, and international politics explains inefficiency in terms of commitment problems which can arise even if the bargainers have complete information. The issue here is that bargainers are sometimes unable to commit themselves to following through on an agreement and have incentives to renege on it. These incentives may undermine the efficient outcomes. When they do, complete-information bargaining breaks down in the inefficient use of power.

For example, Acemoglu and Robinson (2000, 2001) link democratic transitions, costly coups, and revolutions to the inability of the faction in power to commit to future redistribution policies. Fearon (1998, 2002) shows that the inability of a central government

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1 See Ausubel, Cramton, and Deneckere (2002) for a recent review.
to commit to honoring a power-sharing agreement can lead to prolonged civil wars. In Alesina and Tabellini (1990) and Persson and Svensson (1989), political parties create inefficient levels of public debt because the parties cannot commit to future spending levels. Democratic decision-making in Besley and Coate (1998) may lead to inefficient outcomes if the current decisions of those in power affect the identity or preferences of future decision makers. Political parties in de Figueiredo (2002b) may impose inefficient administrative procedures in order to protect their programs from the their political opponents because competing parties cannot commit to refraining from overturning each other’s policies when in power. And, Fearon (1995, 404-08) and Powell (1999, 128-32) demonstrate that a rapidly shifting distribution of military power combined with the states’ inability to commit to an agreement can lead to war.

Despite the importance of commitment problems, we lack many general results about the basic mechanisms through which actors’ inability to commit leads to inefficient outcomes. The results we do have typically focus on specific models, as in the examples above, or the analysis demonstrates the existence of inefficient equilibria in settings where there are Pareto superior, efficient equilibria. Absent a compelling theory of equilibrium selection, inefficient equilibria that are dominated by efficient ones provide at best a weak explanation of inefficiency.

This paper shows that a common mechanism is at work in a number of the diverse studies cited above. This common mechanism affords a more general formulation of a type of commitment problem that can arise in many different substantive settings. The present analysis then formalizes this mechanism as an “inefficiency condition” which ensures that all of the equilibria of a complete-information stochastic game are inefficient. This condition has a natural substantive interpretation: Large, rapid changes in the bargainers’

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3 For example, the inefficient equilibria in infinitely repeated games are Pareto dominated by efficient ones if the players are sufficiently patient. Similarly, the inefficient equilibria are Pareto dominated by efficient ones in models of strikes (e.g., Fernandez and Glazer 1991) or, more generally, in bargaining models in which the bargainers can impose costs on each other between offers (e.g., Busch and Wen 1995, Muthoo 1999). Analogous results obtain in bargaining games in which the players can renege on or retract an accepted offer (Muthoo 1990; 1999, 194-200).
relative power cause inefficiency. More precisely, the equilibria must be inefficient even with complete information if at any time (along any efficient path) the expected per-period shift in at least one of the actors’ minmax payoffs is larger than the bargaining surplus.

The next section briefly reviews Acemoglu and Robinson (2000, 2001), de Figueiredo (2002b), Fearon (2002), and Powell (1999). This review shows that, broadly speaking, the central problem confronting the actors in these models is deciding how to divide a flow of pies in a substantive setting in which (i) the actors cannot commit to how they will divide the pies in future periods, and (ii) the payoffs the actors can lock in through the inefficient use of power varies over time.4 The review also characterizes the common commitment problem that can arise in this situation. The subsequent section formalizes this commitment problem as an inefficiency condition for stochastic games.

A Common Commitment Problem

Complete-information bargaining breaks down in costly coups in Acemoglu and Robinson (2000, 2001), in secessionist civil wars in Fearon (2002), in inefficient policy insulation in de Figueiredo (2002b), and in war in Fearon (1995) and Powell (1999) for the same basic reason. Resource constraints, the inability to commit to future transfers, and a rapidly shifting strategic environment create a situation in which every efficient path is dynamically inconsistent.

In Acemoglu and Robinson’s (2000, 2001) analysis of political transitions, a rich elite and a poor majority vie for political control of the state and the benefits that such control brings. One of these factions is in power at the start of any period, and times are either

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4 This formulation contrasts with Merlo and Wilson (1995) who also study inefficiency in a complete-information setting. In their game, the bargainers negotiate about dividing a pie the size of which varies stochastically. If, as they suggest, each period’s pie represents the present value of the expected flow of benefits, then their model can be interpreted as one in which the bargainers are dividing a flow of pies and can commit to agreements about how they will divide the future flow. Merlo and Wilson also show that the unique subgame perfect equilibrium of their game is always efficient if there are two players and transferable utility. The mechanism highlighted below can produce inefficiency even in these circumstances.
“normal” or “bad” (e.g., there is a severe economic downturn) with probabilities $1 - s$ and $s$. Whether times are normal or bad is revealed at the start of each period and is common knowledge.

When the poor are in power, they move first by setting the tax rate for that period. The rich can then accept this policy or initiate a coup. Accepting ends the period with the agreed tax policy. Launching a coup (in bad times) brings the elite to power but is also inefficient as it destroys a fraction of the economic income from that period. The elite then sets the economic policy for that period and begins the next period in power.

When the rich are in power at the start of a period, they decide the tax rate and whether or not they want to extend the franchise to the poor. (Since the median voter is assumed to be poor, this is equivalent to turning power over to the poor.) If the rich relinquish power, the poor take over, set policy for that period, and start the next period in power. If the rich retain power, the poor can accept the tax rate or launch a costly revolution which, again, destroys a fraction of that period’s income. Accepting ends the period with the agreed policy in place and the elite in power at the start of the next period. Launching a revolution effectively ends the game. The poor assume power, the rich lose everything, and the threat of a future coup is eliminated.

When one group is in power it can set a tax policy favorable to the other group as a way of trying to buy that group off and thereby avoid a coup or revolution. However, Acemoglu and Robinson identify conditions in which the poor cannot offer the rich enough to buy them off. In these circumstances, the rich always launch a coup when out of power and times are bad, and relinquish political control when in power and times are bad. Thus, the country oscillates between democratic and authoritarian regimes.

A dynamic commitment problem drives this oscillation. When times are bad and the poor are in power, the poor would like to buy the rich off and thereby avert a coup. To do this, the poor must promise to give the rich their certainty equivalent of launching a coup, i.e., how much the rich would expect to get were they to mount a coup. However,

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5 By assumption, launching a coup in good times is too costly (i.e., is dominated by accepting the poor’s ideal policy).
the amount that can be transferred in any one period is constrained by that period’s total output and the fact that the poor cannot set a tax rate below zero. As a result, buying the rich off requires that the poor keep taxes low for more than one period. But, the poor cannot commit to future tax rates, and, with probability $1 - s$, normal times will return in the next period. If they do, the threat of a coup will evaporate, and the poor will have an incentive to renge.\footnote{Recall that the payoffs are restricted in such a way that mounting a coup in normal times is strictly dominated.} Foreseeing this, the rich prefer to initiate a coup.

Fearon (2002) sees a commitment problem at the heart of some secessionist civil wars. His analysis grows out of an effort to explain the empirical patterns of civil wars. (Fearon and Laitin (2003) also describe some these patterns.) Fearon begins by rejecting asymmetric information as a plausible account of some types of prolonged civil wars.

\[\text{It strains credulity to imagine that the parties to a war that has been going on for many years, and that looks very much the same from year to year, can hold any significant private information about their capabilities or resolve. Rather, after a few years of war, fighters on both sides of an insurgency typically develop accurate understandings of the other side’s capabilities, tactics, and resolve. Certainly both sides in Sri Lanka (for instance) fight on in the hope that by luck and effort they will prevail militarily. But it is hard to imagine that they do so because they have some private information that makes it reasonable for them to be more optimistic about the odds than the other side is. In the absence of significant private information, why can’t they cut a deal on the basis of a more-or-less common understanding of the terms of the military stalemate (2002, 22)?}\]

The answer Fearon develops is a commitment problem arising from “a temporary shock to government capabilities or legitimacy [that] gives coup plotters or rebels a window of opportunity” (2002, 22). In his model, a government, $G$, and a rebel group, $R$, negotiate about regional control. At the start of a round (see Figure 1 which is based on Fearon’s Figure 2), the government is strong or weak with probabilities $1 - \varepsilon$ and $\varepsilon$, respectively, and whether the government is strong or weak is common knowledge. The government then makes a take-it-or-leave-it offer $c_t \in [0, 1]$ which represents the regional control that government intends to keep for itself during that period. In effect, the government proposes a division of the social pie. If the government is strong, the rebels’ only alternative
is to accept the offer and the period ends with the government’s receiving $c_t$ and the rebels’ obtaining $1 - c_t$.

If the government is weak, the rebels can accept the government’s offer or fight. Accepting ends the period and brings the government and rebels payoffs of $c_t$ and $1 - c_t$, respectively. A new period then begins with the government strong or weak with probabilities $1 - \varepsilon$ and $\varepsilon$ and the government deciding what to offer.

If the rebels fight, then the government and rebels receive payoffs $k_G$ and $k_R$ during that period. By assumption, $k_G + k_R < 1$ so that fighting is inefficient. Fighting ends in one of three ways: With probability $\alpha$, the government wins and a new period begins with the government strong or weak with probabilities $1 - \varepsilon$ and $\varepsilon$ and the government making a new offer. With probability $\beta$, the rebels prevail and the game ends with secession. Finally, the fighting results in a stalemate with probability $\gamma = 1 - \alpha - \beta$ in which case the government and rebels have to decide whether or not to keep fighting. If either of them chooses to fight, they receive the payoffs to fighting, $k_G$ and $k_R$, and the war continues for another period. As before, the government wins with probability $\alpha$, the rebels win with probability $\beta$, and a stalemate occurs with probability $\gamma$. The war continues in this way until one side of the other wins or both decide to stop fighting.
Fearon shows that in some circumstances there must be fighting in any subgame perfect equilibrium even though it is inefficient. To see the basic intuition, consider a period in which the government is weak. In order to induce the rebels not to fight, the government must concede enough to them so that they prefer these concessions to fighting. However, the rebels’ continuation payoff to fighting typically exceeds one. (This payoff includes the payoff to fighting in the current period, \( k_R \), plus the expected payoffs in subsequent periods.) This and the fact that the government can transfer no more that one to the rebels in any single period (by setting \( c_t = 0 \)) means that buying the rebels off entails a promise to transfer resources to the rebels for more than one period.

The government strictly prefers this transfer to fighting because the latter is costly. The government therefore would like to be able to commit itself to following through on this promise. But it cannot. With probability \( 1 - \varepsilon \), the government will be strong in the next period, the threat of rebellion will have disappeared, and the government’s payoff to reneging on its promised transfer will exceed its payoff to following through on it. Anticipating this, the rebel group fights while it has the chance.

A similar complete-information commitment problem arises in a very different substantive context. De Figueiredo (2002b) asks why elected officials might deliberately pursue inefficient policies. He postulates a policy environment in which changing circumstances mean that a political party, if it were sure that it would remain in power, would prefer not to lock in a rigid policy so that it could adjust its policy to changing circumstances. In other words, locking in a policy is inefficient. Nevertheless, de Figueiredo shows that a party prefers to lock its policy in when it is unlikely to remain in power.

De Figueiredo’s analysis begins with a “reciprocity” game between two political parties, \( A \) and \( B \). In this infinite game, \( A \) is in power with probability \( \gamma \) in any period and \( B \) is in power with probability \( 1 - \gamma \). During any round in which a party is in power, it implements its own policy and decides whether or not to overturn the other party’s policy (assuming that the other’s policy is still in place). A party receives one during any period

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7 Strictly speaking, Fearon shows that there are no efficient equilibria in which the government always makes the same offer. But his analysis easily extends to the case where the government can make different offers. The appendix discusses this point.
in which its policy is in place and the other party’s is not, $\beta \in (0, 1)$ during any period in which both parties’ policies are in place, and zero during any period in which its policy is not in place and the other party’s is.

If $\beta$ is larger than $\gamma$ and $1 - \gamma$ (and the discount factor is sufficiently high), then both parties prefer cooperating by not overturning the other party’s policy. Moreover, this cooperative outcome can be sustained in a subgame perfect equilibrium by the threat that should one party ever deviate by overturning the other party’s policy, then neither party will ever cooperate again.

To introduce the inefficiency puzzle, suppose that a party has an additional option when it first comes to power. It can insulate its policy by creating bureaucratic or political obstacles which make it difficult to change. For example, the party in control might create an administrative agency whose procedures are subject to judicial review. (See de Figueiredo (2002a, 2002b), de Figueiredo and Vanden Bergh (2001) for additional examples and discussion.) Formally, once a party insulates its policy, the other party cannot overturn it. Insulation, however, is costly.

If political uncertainty is low (i.e., $\gamma$ is far away from $\frac{1}{2}$), then at least one party engages in inefficient insulation. Suppose that $A$ is political weak and unlikely to hold power in general (i.e., $\gamma$ is small) but that $A$ happens to be in power in the current period. $A$ can lock its policy in place so that it obtains $\alpha < 1$ during any period in which only its policy is in place and $\alpha \beta$ during any period in which both parties’ policies are in place. (The fact that $\alpha < 1$ ensures that insulation is costly.) In order to forgo the opportunity

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8 Suppose each party always overturns the other party’s policy. If $A$ is in power it obtains one in that period and its expected payoff in all future periods is $\gamma \cdot 1 + (1 - \gamma)0 = \gamma$. Hence, $A$’s average payoff (in the limit as the discount factor $\delta$ goes to one) is $(1 - \delta)[1 + \gamma/(1 - \delta)] \rightarrow \gamma$, and $B$’s average payoff is $1 - \gamma$. Now suppose the parties cooperate by never overturning the other’s policy. Then each parties’ average payoff is arbitrarily close to $\beta$. (Because the second party to assume power obtains zero until it comes to power, its average payoff is slightly less than $\beta$ and the other party’s average payoff is slightly more than $\beta$. But these differences go to zero as the discount factor goes to one.) Both parties therefore prefer cooperation if $\beta > \max\{\gamma, 1 - \gamma\}$.

9 Political uncertainty is low if $\gamma$ is far from $\frac{1}{2}$ because one party is likely to hold power most of the time.
to lock this payoff in, A must believe that B will refrain from overturning A’s policy in future periods sufficiently often that A’s payoff to not insulating its policy is higher than its payoff to doing so.

But the only reason B would refrain from overturning A’s policy is that doing so would trigger a costly punishment that outweighs B’s gain from overturning A’s policy. However, the only way that A can punish B is by overturning B’s policy when A is in power. Consequently, a politically weak A will be unable to impose much punishment on B because it is unlikely to be in power very often. Indeed, if A is sufficiently weak, it cannot impose enough punishment on B to deter B from overturning A’s policies whenever B is in office. In these circumstances, A prefers to lock its policy in because B is very likely to be in power in the next period and, if so, to overturn A’s policy if it has not been insulated. Once again, inefficiency results when one actor must make concessions across multiple periods in order to buy another actor off (i.e., B must refrain from overturning A’s policy in order to induce A not to insulate). But a shifting strategic environment undermines the credibility of these promised concessions.

De Figueiredo’s conclusion that insulation is most likely to occur when political uncertainty is low contrasts with the conclusion derived from non-game theoretic work which argues that inefficient insulation is most likely to occur when political uncertainty is high (e.g., Moe 1990, 137). Moreover, he finds empirical support for this claim in his analysis of when states adopt the item veto (de Figueiredo 2002a) or an administrative procedures act (de Figueiredo and Vanden Bergh 2001). Both of these are ways for those in control of a state’s legislature to lock in or at least insulate its policies.10

The strategic environment shifts stochastically in the previous examples. Times are good or bad with probabilities 1 – s and s, the government is strong or weak with probabilities 1 – ε and ε, and A or B is in power with probabilities γ and 1 – γ. The strategic environment shifts deterministically in Powell’s (1999) study of preventive war where, nevertheless, a similar complete-information commitment problem can arise.

10 More precisely, de Figueiredo shows that fiscal conservatives who are politically weak are significantly more likely to propose a line-item veto than are fiscal conservatives who are in a strong political position, i.e., likely to remain in control of the state legislature.
In Powell’s model of states’ efforts to cope shifts in the distribution of military power, a declining state and a rising state are negotiating about revising the territorial status quo $q \in [0, 1]$. The declining state, $D$, begins the game by either proposing a revision $x_0 \in [0, 1]$ to the status quo or attacking. Attacking ends the game in a costly lottery. In this lottery, the rising state, $R$, wins all of the territory and the future benefits from that territory with probability $p_0$ and $D$ wins everything with probability $1 - p_0$. $R$’s payoff to fighting is therefore $p_0 \sum_{j=0}^{\infty} \delta^j (1 - r) + (1 - p_0) \sum_{j=0}^{\infty} \delta^j (0 - r) = (p_0 - r)/(1 - \delta)$ where $\delta$ is the states’ common discount factor and $r$ is $D$’s cost of fighting. $D$’s payoff to fighting is defined analogously.\(^{11}\)

If $D$ does not attack and makes an offer instead, then $R$ can accept, reject or fight. Accepting ends the round. $R$ and $D$ and receive payoffs $x_0$ and $1 - x_0$, $x_0$ becomes the new territorial status quo, and $D$ begins the next round by either attacking or making a new offer. If $R$ rejects $D$’s initial offer, the status quo remains in place, $R$ and $D$ receive payoffs $q$ and $1 - q$, and $D$ begins the next round by either attacking or making a new offer. Finally, $R$’s attacking in response to $D$’s offer ends the game in a costly lottery with the payoffs described above.

To formalize the shifting distribution of power between $D$ and $R$, Powell assumes that the rising state’s probability of prevailing starts out at some $p_0$. It then increases by $\Delta$ in each of the next $T$ periods (i.e., $p_t = p_0 + t\Delta$ for $0 \leq t \leq T$) after which it remains at $p_T = p_0 + T\Delta$. All of this is common knowledge.

Powell’s primary focus is on bargaining when $D$ is uncertain of $R$’s cost of fighting. But he does show that complete-information bargaining breaks down in war if there are large and rapid shifts in the distribution of power. To highlight the basic idea, observe that $D$ can lock in a payoff of $(1 - p_0 - d)/(1 - \delta)$ if it fights at the very outset of the game. By contrast, $D$’s payoff to not fighting is bounded above by $1 + \delta[1/(1 - \delta) - (p_0 + \Delta - r)/(1 - \delta)]$. The first term is the best that $D$ can do in the current period. The second term is an upper bound on $D$’s future payoffs. That is, this term is the discounted difference between

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\(^{11}\) The states are assumed to be risk neutral here for expositional ease. Powell (1999) allows the states to be risk neutral or averse.
the total flow of benefits, which is all there is to be divided between the bargainers, and how much of that total $R$ can lock in for itself by fighting in the next period. Clearly, $R$ has to get at least this much along any efficient path if it is to be induced not to fight. Therefore, the difference between all that there is to be divided and what $R$ can lock in constitutes an upper bound on what $D$ can get in the future. Consequently, $D$ strictly prefers fighting if what it can lock in by doing so is strictly greater than this upper bound on what it can get if it does not fight. In symbols, $D$ prefers to fight if 

$$\frac{1 - p_0 - d}{1 - \delta} > 1 + \delta \left[ \frac{1}{1 - \delta} - \frac{p_0 + \Delta - r}{1 - \delta} \right]$$

or, equivalently, if

$$\delta \Delta > (1 - \delta) p_0 + d + \delta r.$$ 

Hence, complete-information bargaining is sure to break down in inefficient fighting if the per-period shift in the distribution of power $\Delta$ is larger than the average amount consumed by fighting $r + d$ and if the discount factor is close enough to one.\(12\)

Less formally, $R$ would like to induce $D$ not to fight by committing itself to abiding by a territorial division that $D$ prefers to fighting. To do this, $R$ must forebear from fighting. But, $R$’s increasing military strength increases its payoff to fighting and thereby undermines its promise not to attack.

In sum, the actors in the preceding examples face the same broad strategic problem. The bargainers are trying to divide a flow of benefits in a setting in which they cannot commit to future divisions. Each actor also has the option of using some form of power to lock in a share of the flow. But the use of power is inefficient and destroys some of the flow. Finally, a shifting environment changes the amounts the bargainers can lock in.

Complete-information bargaining breaks down in each case for the same basic reason. In order to avoid the inefficient use of power, there must be an efficient allocation that each bargainer prefers at all times to engaging in costly conflict. But resource constraints mean that the transfers needed to buy off a temporarily strong bargainer (i.e., a bargainer who can lock in a high payoff in that period) must stretch across a “concession phase” lasting multiple periods. However, a rapidly shifting strategic environment also means that the expected amounts that the bargainers can lock in through the inefficient use

\(12\) Fearon (1995, 402-406) discusses the same kind of commitment problem.
of power changes enough during this concession phase to make the once-weak bargainer want to renege on the promised transfer. These shifts undermine the efficient allocations, and the bargaining breaks down.

An Inefficiency Condition

This section formalizes the common mechanism at work in the previous examples in terms of an inefficiency condition. When this condition holds, all of the equilibria of a two-actor stochastic game are inefficient. The section also shows that the inefficiency condition has three natural substantive interpretations.

The inefficiency condition applies to stochastic games which are a generalization of repeated games. In the latter, the strategic environment remains constant. The actors play the same game over and over. In a stochastic game, the strategic environment changes. The game the actors play in any period may depend on the game they played in the previous period, what they did in that period, and on additional random factors. For example, the game that the rich and poor play in Acemoglu and Robinson (2001) depends on which of them was in power in the previous period, on whether or not the out-of-power actor tried to depose the other actor, and on random fluctuations in the economy, e.g., whether times are normal or bad.

To specify some elements of a stochastic game \( \Gamma \) somewhat more formally, let \( \{ A_k \}_{k=1}^N \) denote the set of states or stage games and \( q \) be a transition function.\(^{13}\) The states define the various games the actors might play in any round. The transition function \( q(n|k, s) \) is the probability that the next state will be \( A_n \) given that the current state is \( A_k \) and that the players took actions \( s \) in \( A_k \). Play begins in \( \Gamma \) in a given state, and each actor’s payoff is the present value of the sum of its stage-game payoffs where \( \delta \) is the players’ common discount factor. When deciding what to do, the actors know the current state as well as the entire history of previous states and what the actors did in those states.

In order to specify the inefficiency condition, let \( M_j(k) \) be \( j \)'s minmax payoff for the

\[^{13}\text{Abusing the definitions but greatly easing the exposition, I use “stage game” and “state” synonymously. See Friedman (1986, 124-25) for a complete description of a stochastic game.}\]
two-player stochastic game starting in state $A_k$. That is, $j$ can assure itself of an expected payoff of $M_j(k)$ starting from state $k$. It is important to emphasize that this payoff is not the minmax payoff of the stage game $A_k$. $M_j(k)$ is the minmax payoff of the continuation game starting in state $k$. Consequently, $j$’s payoff in any subgame perfect equilibrium starting from state $A_k$ must be at least as large as $M_j(k)$.

Now consider an efficient path, i.e., a pair of strategies $e = (e_1, e_2)$ such that the expected payoffs to following these strategies are Pareto optimal in the stochastic game. If either player has an incentive to deviate from $e$, then $e$ is not subgame perfect. And, a player is sure to have an incentive to deviate if there exists a state along $e$ at which that player’s minmax payoff is strictly greater than its payoff to continuing along $e$. If, moreover, such a state exists along every efficient path, then there are no efficient equilibria. The inefficiency condition ensures that this is the case by finding an upper bound on a player’s payoff to continuing along an efficient path and then requiring this upper bound to be strictly less than the player’s minmax payoff.

To specify an upper bound on $j$’s continuation payoff to following $e$ starting at $A_k$, let $B_k$ be the maximum expected flow of future benefits starting in $A_k$. Loosely, $B_k$ is all that there is to be divided between the two players starting from $A_k$. It is the present value of the expect flow of pies. ($B_k$ is defined formally in the proof of Proposition 1.)

Now observe that the other player, $i$, can guarantee that it will obtain a certain amount of this flow. Let $a_k^i$ be a lower bound on $i$’s payoff starting in $A_k$ given that the players are following an efficient path. (This payoff and the expectation discussed below also are defined formally in the proof of Proposition 1.) Player $i$ must get at least this much in the current round if play follows $e$. This actor can then assure itself of getting at least its minmax payoff $M_i(n)$ if the next stage game is $A_n$. Hence, $i$ ’s expected future payoff starting at $A_k$ is bounded below by the (discounted) value of $i$’s expected minmax payoff $E_k[M_i(n)]$ where the expectation is based on what is known at $A_k$. Thus, $i$ is sure to receive at least $a_k^i + \delta E_k[M_i(n)]$ starting at $A_k$.

Putting all of this together, $j$’s continuation payoff to following $e$ starting from $A_k$

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14 Generalizing the inefficiency condition to more than two players is straightforward.
is bounded above by $B_k - [a_{ik}^j + \delta E_i[M_i(n)]]$. Roughly, this is the difference between all there is to be divided if the actors play efficiently and what $i$ can assure itself. Now define the inefficiency condition for actor $j$ to be

$$M_j(k) > B_k - [a_{ik}^j + \delta E_i[M_i(n)]] .$$

When this holds, $j$ will have an incentive to deviate from $e$ at $A_k$ and, consequently, the efficient path $e$ cannot be an equilibrium.

In practice, a closely related condition often turns out to be easier to use than (1). An action in state $k$ is conditionally dominated if, starting from that state, any strategy that puts positive weight on that action is strictly dominated. For example, trying to depose the faction in power in normal times is conditionally dominated in Acemoglu and Robinson. By construction, launching a coup or revolution in good times is so costly that the out-of-power faction is always strictly better off if it waits for bad times before acting. Let $M'_j(A_k)$ be $j$’s minmax payoff starting in $A_k$ given that the other player $i$ does not play conditionally dominated strategies. Then the analogue of condition (1) is:

$$M'_j(k) > B_k - [a_{ik}^j + \delta E_k[M'_i(n)]]$$

These conditions lead immediately to:

**Proposition 1:** A two-actor stochastic game $\Gamma$ has no efficient subgame perfect equilibria if for each efficient path $e$ there exists a state $A_k$ at which inefficiency condition (1) or (1') holds for one of the players.

**Proof:** The first step is to define the bounds $B_k$ and $a_{ik}^j$ and the expected minmax payoff $E_k[M_i(n)]$. Let $\Sigma$ be the set of strategy profiles of $\Gamma$ and $V^j(\sigma|h_k)$ be $j$’s continuation payoff if play follows $\sigma \in \Sigma$ and starts from state $A_k$ after history $h_k$. Then define the expected flow of benefits to be the maximum of the sum of the two players’ continuation values: $B_k = \max \{V^j(\sigma|A_k) + V^i(\sigma|A_k) : \sigma \in \Sigma\}$ where this maximum clearly depends only on $A_k$ and not on the history leading up to that state. If, moreover, the game has transferable utility, then the sum of the bargainers’ utilities in each period is the size of
that period’s pie. \( B_k \) in this case is simply the present value of the expected flow of pies.

As for \( a^i_k \), let \( a^i_k(s) \) be \( i \)'s payoff in \( A_k \) if the players take actions \( s \). Take \( \mathcal{E} \) to be the set of efficient paths in \( \Gamma \). Also let \( h_k \) be any history leading to state \( A_k \), \( H_k \) be the set of all such histories, and \( \pi(h_k|\sigma) \) be the probability of \( h_k \) given that play follows \( \sigma \in \Sigma \). Then, \( a^i_k \) is the minimum payoff \( i \) receives at \( A_k \) along any efficient path. In symbols, \( a^i_k = \min \{ a^i_k(e|h_k) : e \in \mathcal{E}, h_k \in H_k, \text{ and } \pi(h_k|e) > 0 \} \).\(^{15}\)

Finally, to specify \( E_k[M_i(n)] \) let \( e(h_k) \) be the actions the players take at \( A_k \) following \( h_k \) given that they are playing according to \( e \). Then, the \( i \)'s expected minmax payoff starting in the next period given what is known in the current period is simply \( E_k[M_i(n)] \equiv \sum_{n=1}^N q(n|k, e(h_k)) M_i(n) \).

The rest of the proof follows directly from these definitions. Player \( j \) will have an incentive to deviate from any \( e \in \mathcal{E} \) if there exists a state \( A_k \) such that \( M_j(A_k) > V^j(e|h_k) \) for some \( h_k \). Condition (1) ensures that this is case. Let \( \sigma^* \) be a strategy profile that maximizes the flow of benefits starting from \( A_k \), i.e., \( \sigma^* \) satisfies \( B_k = V^j(\sigma^*|A_k) + V^i(\sigma^*|A_k) \). Because \( \sigma^* \) maximizes \( V^j(\sigma|A_k) + V^i(\sigma|A_k) \), it follows that \( V^j(\sigma^*|A_k) + V^i(\sigma^*|A_k) \geq V^j(e|h_k) + V^i(e|h_k) \). This along with \( V^i(e|h_k) \geq a^i_k + \delta E_k[M_i(A_n)] \) and condition (1) imply:

\[
M_j(k) > B_k - [a^i_k + \delta E_k[M_i(n)]] \\
> V^j(e|h_k) + V^i(e|h_k) - [a^i_k + \delta E_k[M_i(n)]] \\
> V^j(e|h_k).
\]

Hence, there will be no efficient equilibria if (1) holds for all \( e \in \mathcal{E} \). An analogous argument shows that there are no efficient equilibria if (1') holds. \( \blacksquare \)

In order to develop substantive interpretations for (1) and, implicitly, (1'), it will be useful to relax these conditions slightly. If necessary, normalize the payoffs so that each player always receives at least zero in any period along any efficient path. This means

\(^{15}\) In the examples above and in applied work more generally, \( a^i_k \) is usually quite easy to determine. A looser lower bound that leads to a somewhat looser version of (1) is to normalize the player’s utilities so that all of the payoffs in every stage game are nonnegative and then take \( a^i_k = 0 \).
Clearly, condition (1) holds and the game has no efficient equilibria if $M_j(k) > B_k - \delta E_k[M_i'(n)]$. Rewriting this inequality yields two expressions that have natural substantive interpretations:

$$M_j(k) + \delta E_k[M_i(n)] > B_k$$  \hspace{1cm} (2)

$$\delta E_k[M_i(n)] - M_i(k) > B_k - [M_j(k) + M_i(k)]$$  \hspace{1cm} (3)

The left side of condition (2) is the amount that $j$ can assure itself or “lock in” starting in state $k$ plus the (discounted) expected value of what $i$ can lock in the next period if the actors follow the efficient path at $k$. Condition (2) simply says that the sum of these lock-ins exceeds the total amount there is to be divided. When this is the case, it is impossible to satisfy both players’ claims on the flow of benefits.

The second interpretation is more dynamic. The left side of (3) is the expected shift in $i$’s minmax payoff. In a rough sense this shift measures how much more powerful $i$ will become and, implicitly, how much weaker $j$ will become. The right side is the size of the bargaining surplus, i.e., the difference between what there is to be divided, $B_k$, less the sum of what each player can assure itself. Thus, (3) holds and there are no efficient equilibria when the expected shift in one of the player’s minmax payoff is larger than the bargaining surplus. Less formally, large, rapid changes in the bargainers’ relative power (measured by shifts in their minmax payoffs) cause inefficiency.

Shifts of this kind are what drive the inefficiency in Acemoglu and Robinson (2000, 2001), Fearon (2002), de Figueiredo (2002b), and Powell (1999). (The appendix establishes the relationship between these examples and conditions (1) and (1’) more formally.) The rich launch a costly coup when times are bad and the cost of deposing the opposition is low, because normal times are likely to return in the next period and the rich will be weak. The rebels fight when the government is weak in Fearon because, with high probability, the government will be strong in the next period. A weak party that happens to find itself in office in de Figueiredo insulates its policies because it is likely to be out of office in next period. And a declining state in Powell fights if it will be much
weaker in the next period. These changes alter the players’ minmax payoffs and result in the inefficient use of power.

As discussed in the previous section, these shifts undermine an actor’s ability to buy the other actor off by limiting the amount that the former can credibly commit to transferring to the latter. For example, the poor in Acemoglu and Robinson (2001) cannot credibly commit to lower taxes long enough to dissuade the rich from mounting a coup. A third substantive interpretation of condition (1) highlights the role that the actors’ limited ability to make transfers plays in the commitment problem.

To develop this interpretation, assume that maximizing the sum of the current and future benefits also maximizes the current benefits. That is, maximizing the total flow of pies does not require accepting a smaller pie in the current period. (All of the models discussed above satisfy this “separability” condition which is formalized in the proof of Corollary 1). This means that the maximum flow of current and future benefits $B_k$ equals the maximum benefits there are to be had in the current period, $C_k$, plus the discounted maximum flow of future benefits $F_k$, i.e., $B_k = C_k + \delta F_k$.

Corollary 1 below shows that condition (1) implies

$$M_j(k) > \pi^i_k + \delta[F_k - E_k[M_i(n)]]$$

where $\pi^i_k$ is the maximum payoff $j$ can achieve in state $A_k$ along any efficient path. Put another way, this is the most that $i$ can transfer to $j$ in state $A_k$ on an efficient path. The term in brackets is the difference between the future flow of benefits and what $i$ can assure itself in the future. This difference is therefore an upper bound on what $i$ can credibly promise to transfer to $j$ in the future. Were $i$ to try to transfer more than this, then $i$’s future payoff in some state $n$ would be less than its minmax payoff $M_j(n)$, thus giving $i$ an incentive to renege. Hence, condition (4) can be interpreted as saying that $j$’s minmax payoff in $k$ is larger than the amount the $i$ can transfer to $j$ in the current period plus what it can credibly promise to transfer in the future given the expected shifts in $i$’ power as measured by its minmax payoff.

**Corollary 1:** If the maximum flow of benefits is separable in as described informally
above and formally below, then condition (1) implies condition (4).

Proof: The first step is to formalize the separability assumption. Define $C_k$ to be the maximum of the sum of the actors’ payoffs in state $A_k$: $C_k \equiv \max \{a_k^i(s) + a_k^j(s) : s \in S_k\}$ where $S_k$ is the set of action profiles in $A_k$. $F_k$ is simply the maximum of the expected sum of the actors’ future payoffs given the present state is $A_k$: $F_k \equiv \max \left\{ \sum_{n=1}^N q(n|k, \sigma) [V^j(\sigma|A_n) + V^i(\sigma|A_n)] : \sigma \in \Sigma \right\}$. Clearly, $B_k \leq C_k + \delta F_k$. The separability condition requires that this condition hold with equality, i.e., $B_k = C_k + \delta F_k$.

Now define the upper bound on $j$’s payoff at $A_k$ along any efficient path $e$ as $\bar{a}_k^j \equiv \max \left\{ a_k^j(e(h_k)) : e \in \mathcal{E}, h_k \in H_k, \text{ and } \pi(h_k|e) > 0 \right\}$. Substituting for $B_k$ in condition (1) then gives $M_j(k) > C_k + \delta F_k - [a_k^i + \delta E_k[M_i(n)]]$.

The proof is complete if $C_k - a_k^i \geq \bar{a}_k^i$. To see that this is so, let $e^*$ be an efficient path that gives $j$ its maximum $\bar{a}_k^j$ in state $A_k$. Then, the maximum sum of the players’ payoffs in state $A_k$ is at least as large as what $e^*$ gives them whenever they are in $A_k$. In symbols, $C_k \geq \bar{a}_k^i + a_k^j(e^*(h_k))$ given $\pi(h_k|e) > 0$. But $a_k^i$ is the minimum that $i$ obtains at $A_k$ along any efficient path. So, $a_k^j(e^*(h_k)) \geq a_k^i$ as long as $\pi(h_k|e) > 0$. Hence, $C_k \geq \bar{a}_k^i + a_k^i$ or $C_k - a_k^i \geq \bar{a}_k^i$. ■

Finally, it is interesting to consider the efficient equilibria in infinitely repeated games in light of the inefficiency conditions. As the folk theorem shows (Fudenberg and Maskin 1986), there are always efficient equilibria in an infinitely repeated game if the actors do not discount the future too much. The existence these equilibria means that the inefficiency conditions must not hold in an infinitely repeated game. Why not?

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16 In some cases a technicality must be overcome. In Powell (1999), for example, there is a game-ending move in some of the stage games, e.g., one of the countries can go to war. The payoffs to this move include the payoffs obtained in the current period plus the flow payoffs from unmodeled future periods. This future flow may be large compared to the players’ per-period payoffs from other actions in the stage game. Hence, maximizing the sum to the actors’ payoffs in this stage game does not correspond to maximizing the sum of the actors’ “current” payoffs which $C_k$ is intended to represent. To finesse this issue, each stage game $A_k$ with game-ending action profiles $\{s_k^r\}_{r=1}^R$ should be replaced with a stage game $A'_k$ and a set of null games $G_1, \ldots, G_R$ such that playing $s_k^r$ in $A'_k$ results in a null state $G_r$. Each player has only one move in $G_r$, the state remains $G_r$ thereafter, and the payoff to playing $s_k^r$ in $A'_k$ and $G_j$ is the average of the payoff to playing $s_k^r$ in $A_k$. 

18
In a repeated game, there is a single stage game, say $A$, which is repeated infinitely often. This means that the strategic environment is completely stable in the sense that the continuation game is always the same, namely, an infinite repetition of $A$. There are no large, rapid shifts that can undermine efficiency in repeated games.

To put this point more formally, let $m_j$ be $j$’s minmax payoff in the two-actor stage game $A$ where the payoffs have been normalized so that they are all nonnegative. Then, $j$’s minmax payoff in the continuation game is the present value of having $m_j$ in every period. This means $M_j = m_j/(1 - \delta)$. Moreover, $j$’s expected minmax payoff in the continuation game starting in the next period is also $M_j$, because the continuation game never changes. As a result, condition (3) becomes $-(1 - \delta)M_1 > B - (M_1 + M_2)$ where $B$ is the maximum flow of future benefits. But, the fact that each player must get at least as much as its minmax payoff in equilibrium implies that the maximum flow of benefits must be at least as large as the sum of the minmax payoffs, i.e., $B \geq M_1 + M_2$. Hence, (3) never holds in the unchanging strategic environment of an infinitely repeated game.

**Conclusion**

A common mechanism is at work across a very wide range of recent work in American, comparative, and international politics. In each case, a temporarily weak actor wants to induce its adversary to refrain from an inefficient use of power, e.g., launching a coup, starting a civil war, insulating its policy, or attacking. Because the use of power consumes resources, avoiding its use saves resources and means that there is enough for the weaker actor to buy its adversary off. But resource constraints mean that the transfers needed to accomplish this will take several periods to complete. However, the strategic environment is shifting sufficiently fast that the temporarily weak actor is likely to become strong enough during this concession phase that it will renege on its promised transfers. This undermines the credibility of these transfers and leads to the inefficient use of power. In short, large, rapid shifts in relative bargaining power can lead to bargaining break downs even if there is complete-information.

This common mechanism provides a unifying perspective on these analyses and a
more general formulation of a fundamental strategic problem that can cause breakdowns and the inefficient use of power in very diverse substantive settings. Seeing the basic mechanism driving this inefficiency more clearly also poses a challenge for future work. The models discussed above and the more general condition (1) "black box" these shifts, e.g., the government is either strong or weak with probabilities $1 - \varepsilon$ and $\varepsilon$ in Fearon (2002). Opening up this blackbox and specifying the microfoundations for these changes is an important task for future work.

It is, for example, relatively easy to see how the distribution of power can shift quickly and dramatically in the context of legislative bodies and a two-party system. Electorally weak parties that happen to capture a majority of seats acquire the significantly greater powers of the majority party. Insofar as these parties are likely to lose the next election and be in the minority for a substantial period, there will to be a large and rapid shift in the expected distribution of power. Moreover, the direct empirical evidence in support of this mechanism is strongest in this substantive setting (e.g., de Figueiredo 2002a, de Figueiredo and Vanden Bergh 200).

It it is less clear how the distribution of power can change so rapidly in other contexts. In Powell’s model, for example, rapid shifts in the distribution of military power lead to war. But he suggests that the shifts in the distribution of military power due to differential rates of economic growth are empirically too small to account for war through this mechanism, although he also recognizes that it is extremely hard to identify plausible parameter values in such spare models (1999, 133).

The mechanism defined in condition (1) is a formal result. It shows that complete-information bargaining breaks down for the same fundamental reason in a number of seeming unrelated games that have been used to study an important and diverse set of substantive issues. But even if a specific model satisfies condition (1), whether this mechanism is at work in actual cases depends on how well the model represents these cases. One way to advance our understanding of these cases and of the general mechanism is to begin to elaborate the microfoundations underlying these shifts in order to compare them to what appears to be happening on the ground.
Appendix

This appendix formalizes the relation between conditions (1) and (1') and the inefficiency conditions derived in the equilibrium analyses in Acemoglu and Robinson (2000, 2001), Fearon (2002), de Figueiredo (2002b), and Powell (1996). Because (1) and (1') are based on minmax payoffs, they might be much looser than or not very closely related to the equilibrium conditions. Were this case, it would indicate that the mechanism described in (1) and (1') was not the source of the inefficiency in those examples. If, by contrast, there is little or no slack between the equilibrium conditions and (1) or (1'), then this mechanism is capturing the source of the inefficiency in those examples.

The latter turns out to be the case. The slack between condition (1') and the equilibrium conditions in Acemoglu and Robinson (2001) is due solely to their assumption that taxation induces a dead-weight loss. Were there no such loses, Acemoglu and Robinson’s “general results would not be altered” (2001, 941) and there would be no slack. Moreover, condition (1) is identical to the equilibrium condition needed to guarantee inefficiency in Fearon (2002) and, in the limit as the discount factor goes to one, in Powell (1999). The slack between (1) and the equilibrium condition in de Figueiredo’s game is due to the nonlinearity of the Pareto frontier and disappears as this frontier becomes linear.

The inefficiency in Acemoglu and Robinson (2001) arises when the poor are in power (i.e., the regime is democratic) and times are bad. In these circumstances, the poor may or may not be able to buy off the rich through lower taxes and thereby prevent an inefficient coup. Whether or not the poor can prevent a coup depends on the cost of launching a coup as well as the values of other parameters. To simplify the analysis, Acemoglu and

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17 Space limitations make it impossible to repeat these analyses here, so the present discussion presumes that readers can refer to them.

18 This close relationship between the inefficiency condition based on minmax payoffs and the equilibrium conditions is actually not very surprising. In applied work, one often simplifies the analysis by constructing models in which the most severe punishments one actor can impose on another are part of a subgame perfect equilibrium, e.g., defecting in every round in a prisoner’s dilemma. When this is done, little will be lost in looking at minmax payoffs of continuation games rather than the incentive compatiblity constraints on equilibrium payoffs in continuation games.
Robinson focus on Markov perfect equilibria and derive a condition sufficient to ensure that there will be costly coups in any Markov perfect equilibrium. However, this condition does not guarantee that there will always be coups in non-Markov equilibria, for the rich can punish the poor more severely in a non-Markov equilibrium if the poor renege on a promised level of taxation. This harsher punishment means that the poor can credibly commit to lower taxes in a non-Markov setting and this makes it easier to sustain efficient equilibria. Hence, the first step in comparing condition (1) to the sufficient equilibrium condition in Acemoglu and Robinson’s game is to extend their analysis to the non-Markov case. To ease the analysis, we also focus on the case in which there is no dead-weight loss to taxation. (Acemoglu and Robinson (2001, 914) assume a dead-weight loss to taxation as a matter of convenience in order to avoid corner solutions.)

The first step in the analysis is to identify the worst equilibrium for each player, i.e., the equilibrium that gives each player its lowest equilibrium payoff. Because this is the worst equilibrium, an efficient allocation can be supported in equilibrium if and only if both players are deterred from deviating from this allocation by the threat to revert to this worst equilibrium. Hence, a condition sufficient condition to ensure inefficiency in the non-Markov case is that at least one of the players prefers to deviate from every efficient allocation even when threatened in this way.

To construct the worst equilibrium, observe first that the rich and poor actors in Acemoglu and Robinson’s game are really aggregates of identically behaved individuals. Let $R$ and $P$ denote these aggregate actors, and $r$ and $p$ denote rich and poor individuals. Now consider the following strategies: The rich, $R$, set a tax rate of zero whenever they are in power and mount a coup whenever they are out of power and times are bad. The poor, $P$, set their optimal tax rate $\tau_m$ of one hundred percent whenever they are in power and launch a revolution whenever they are out of power and the times are bad. (By assumption, the tax revenues are redistributed through transfers the size of which cannot depend on whether an individual is rich or poor. The poor, therefore, maximize their net income by taxing everything away and redistributing it evenly across the population.)

These strategies constitute a subgame perfect equilibrium. It is also clear that this is
the worst equilibrium for both players. Call this worst equilibrium $Z$, and let $Z^j(\sigma)$ for $j = p$ or $r$ denote individual $j$’s payoff in the continuation game starting in state $\sigma$ given that $R$ and $P$ play according to $Z$.

Now consider a path along which $P$ sets a tax rate of $\tau(\sigma|\pi)$ where $\sigma$ is the current economic state, i.e., whether times are normal or bad, and $\pi$ is the history of economic states leading up to $\sigma$. This path can be supported in equilibrium if and only if the following strategies are subgame perfect: $P$ offers $\tau(\sigma|\pi)$ and $R$ does not mount a coup if the current state is $\sigma$ and the history of previous states is $\pi$. If either player ever deviates from these actions in any period, $P$ and $R$ start playing according to $Z$. Let $V^j(\sigma|\pi)$ for $j = r$ or $p$ be $j$’s continuation payoff to playing according to these strategies starting in state $\sigma$ after history $\pi$.

Following Acemoglu and Robinson’s notation, take $h_j$ to be individual $j$’s capital stock. The fraction of the population that is poor is $\lambda$, so the total capital stock, $h$, satisfies $h = (1 - \lambda)h_r + \lambda h_p$. Times are normal and bad with probabilities $1 - s$ and $s$, respectively, and a unit of capital yields an income of one in normal times and an income of $a < 1$ in bad times. Consequently, setting a tax rate of $\tau$ in normal times yields a revenue of $\tau[(1 - \lambda)h_r + \lambda h_p]$ and a net transfer to a poor person of $\Delta^p(\tau, n) \equiv \tau[(1 - \lambda)h_r + \lambda h_p] - \tau h_p = \tau(1 - \lambda)(h_r - h_p)$. The net transfers $\Delta^r(\tau, n)$, $\Delta^p(\tau, b)$, and $\Delta^r(\tau, b)$ are defined analogously where the balanced budget requirement implies $(1 - \lambda)\Delta^r(\tau, \sigma) + \lambda \Delta^p(\tau, \sigma) = 0$ in state $\sigma$.

This implies that the poor’s payoff starting in a bad state following history $\pi$ is:

$$V^p(b|\pi) = ah_p + \Delta^p(\tau(b|\pi), b) + \beta [(1 - s)V^p(n|\pi') + sV^p(b|\pi')]$$

where $\pi' = \{\pi, b\}$ and $\beta$ is the common discount factor. The first two terms on the right side of the equation are $p$’s payoff in the current period and the third term is its discounted expected continuation payoff. Thus, the continuation value of the aggregate actor $P$ is:

$$V^P(b|\pi) = \lambda [ah_p + \Delta^p(\tau(b|\pi), b)] + \beta [(1 - s)V^P(n|\pi') + sV^P(b|\pi')]$$
where individual and aggregate payoffs are related by $V^P(\sigma|\pi) \equiv \lambda V^P(\sigma|\pi)$ for state $\sigma$.

Because the rich and poor simply divide each period’s income as long as there is no coup, the aggregate income of the poor plus the aggregate income of the rich equals the present value of the expected flow of income:

$$V^P(b|\pi) + V^R(b|\pi) = ah + \frac{\beta [(1-s)h + sah]}{1-\beta}.$$  

Combining the previous expressions and using $h = (1-\lambda)h_r + \lambda h_p$ and $(1-\lambda)\Delta^r(\tau, b) + \lambda \Delta^p(\tau, b) = 0$ gives:

$$V^R(b|\pi) = (1-\lambda)ah_r + (1-\lambda)\Delta^r(\tau(b|\pi)) + \frac{\beta(1-s+sa)h}{1-\beta} - \beta [(1-s)V^P(n|\pi') + sV^P(b|\pi')] .$$

The net transfers to the rich are negative if the poor set a positive tax rate, so $\Delta^r(\tau(b|\pi))$ is bounded above by zero. Incentive compatibility at the aggregate level (which is equivalent to the individual level) also means that $V^P(n|\pi') \geq Z^P(n)$ and $V^P(b|\pi') \geq Z^P(b)$ where $Z^P(\sigma) \equiv \lambda Z^p(\sigma)$. Hence,

$$V^R(b|\pi) \leq (1-\lambda)ah_r + \frac{\beta(1-s+sa)h}{1-\beta} - \beta [(1-s)Z^P(n) + sZ^P(b)] .$$

But incentive compatibility also requires the aggregate payoff of the rich to be at least as large as their payoff to deviating and getting $Z$ instead, i.e., $V^R(b|\pi) \geq Z^R(b)$. So a condition sufficient to ensure that there are no efficient equilibria is:

$$Z^R(b) > (1-\lambda)ah_r + \frac{\beta(1-s+sa)h}{1-\beta} - \beta [(1-s)Z^P(n) + sZ^P(b)] . \tag{A1}$$

This is the condition needed to ensure inefficiency based on an equilibrium analysis of the game. It is also identical to condition (1'). The left side of (1') is $R$’s minmax payoff in conditionally undominated strategies. This excludes the possibility that either player would try to depose the other in normal times as these are conditionally dominated. $P$, therefore, minmaxes $R$ (in conditionally undominated strategies) by setting a tax rate of one hundred percent whenever the poor are in power and launching a revolution whenever
the poor are out of power and times are bad. This gives a rich individual \( r \) and the rich \( R \) minmax payoffs of \( Z^r(b) \) and \( Z^R(b) \) starting from bad times. It also means that the left side of (1’) is \( Z^R(b) \).

As for the right side of (1’), the total flow of benefits starting from bad times is just \( B_b = ah + \beta[(1 - s)h + sah]/(1 - \beta) \). The worst that the poor can do in bad times along an efficient path is what they would received if they paid no taxes and therefore received no transfers, i.e., \( \lambda ah_p \). As for their expected minmax payoff starting in the next period, \( R \) minmaxes \( P \) (in conditionally undominated strategies) by setting a tax rate of zero whenever the rich are in power and launching a coup whenever the rich are out of power and times are bad. This means that \( P \)’s expected minmax payoff before the state is revealed is \( (1 - s)Z^P(n) + sZ^P(b) \). Condition (1’) then gives:

\[
Z^R(b) > ah + \frac{\beta[(1 - s)h + sah]}{1 - \beta} - \lambda ah_p - \beta \left[(1 - s)Z^P(n) + sZ^P(b)\right]
\]

which reduces to (A1).

Turning to the relation between condition (1) and Fearon’s (2002) equilibrium conditions, his Proposition 3 establishes equilibrium conditions which ensure that there exists no efficient subgame perfect equilibrium in which the government offers the same \( c^* \) in each period. This, however, does not guarantee the absence of an efficient equilibria as it may be possible to sustain an equilibrium in which the government makes different offers depending on whether it is strong or weak. More specifically, suppose the government offers \( c \) when strong and \( c' \) when weak with \( c' < c \). (Recall that if the government offers \( c \), its payoff and the rebels’ payoffs are \( c \) and \( 1 - c \), respectively.) Then, the analogues of Fearon’s incentive compatibility constraints (11) and (12) are:

\[
c + \frac{\delta [(1 - \varepsilon) c + \varepsilon c']} {1 - \delta} \geq 1 + \delta V^P_G \tag{11'}
\]

\[
(1 - c') + \frac{\delta [(1 - \varepsilon)(1 - c) + \varepsilon(1 - c')]} {1 - \delta} \geq V^W_R \tag{12'}
\]

The left side of 11’ is the government’s continuation payoff to receiving \( c \) when strong.
and $c'$ when weak starting from a state in which the government is strong. The right side is the an upper bound on how well the government can do if it deviates. It can get one in the current period and then the discounted continuation payoff $V^P_G$. This continuation payoff is the government’s expected payoff going into a peace period given that the government reverts to a strategy of never offering anything and the rebels fight at every opportunity. Similarly, the left side of $12'$ is the rebels’ continuation payoff to being offered $c$ when the government is strong and $c'$ when it is weak starting from a state in which the government is weak. The right side $12'$ is the rebel’s continuation payoff to the fighting. (See Fearon (2002) for a more detailed specification of $V^P_G$ and $V^W_R$. The notation used here is consistent with his.)

By adding his incentive compatibility constraints 11 and 12, Fearon shows that an efficient subgame perfect equilibrium exists for a constant offer of $c^*$ only if $V^W_R + \delta V^P_G \leq \delta/(1-\delta)$. Adding 11' and 12' gives:

$$c - c' + \frac{\delta}{1-\delta} \geq V^W_R + \delta V^P_G > \frac{\delta}{1-\delta}$$

When these inequalities hold, there is an efficient equilibrium involving different offers but no efficient constant-offer equilibrium. But, $c - c'$ is bounded by one. So, the equilibrium condition needed to ensure that there are no efficient equilibria in Fearon’s model even if the government can make different offers in different conditions is $V^W_R + \delta V^P_G > 1/(1-\delta)$.

This requirement is identical to condition 1 above. The rebels’ minmax payoff starting in a period in which the government is weak is what they obtain by fighting in every period and is $V^W_R$. Expressing this in the notation used in condition (1) gives $M_{R}(\text{weak}) = V^W_R$. Since the pie to be divided in each period is one, $B_k = 1/(1-\delta)$. The minimum $R$ can get in any period in which is weak along any efficient path is zero. Finally, if the government and rebels reach an efficient allocation when the government is weak, the next period will be what Fearon calls a peace period and the governments’ expected minmax payoff entering that period is $V^P_G$. This leaves $E_w(M_G) = V^P_G$. Condition 1 then gives $V^W_R > 1/(1-\delta) - 0 - \delta V^P_G$, which is identical to the equilibrium condition.

Inefficiency condition (1) and the equilibrium condition needed to ensure inefficiency
in de Figueiredo (2002b) are identical if the Pareto frontier is linear, i.e., if $\beta = \frac{1}{2}$. To establish this, take $\beta = \frac{1}{2}$ and assume without loss of generality that $A$ is the weaker party (i.e., $\gamma \leq \frac{1}{2}$). To specify the equilibrium conditions leading to inefficient insulation, recall that a party only has the option of insulating the first time it comes to power and consider the subgame in which $B$ comes to power in the first round and, therefore, before $A$. This subgame is outlined in Figure 2.

Evaluating $A$'s decisions, suppose $A$ is deciding whether to insulate when it first comes to power, i.e., $A$’s decision at (ii) in Figure 2. If $A$ insulates, it obtains $\alpha$ in the current period plus an expected continuation payoff of $\delta[\alpha \gamma + \frac{1}{2} \alpha (1 - \gamma)]/(1 - \delta)$. If $A$ does not insulate, $B$’s unique best response is to overturn $A$'s policies whenever possible. (With $\beta = \frac{1}{2}$, there are no gains to cooperating on not overturning each other’s policies.) This leaves $A$ with a payoff of one whenever it is in power and zero whenever it is out of power. This yields $1 + \frac{\delta \gamma}{1 - \delta}$. $A$, therefore, weakly prefers to insulate at (ii) if:

$$\alpha \left[1 + \frac{\delta[\gamma + \frac{1}{2}(1 - \gamma)]}{1 - \delta}\right] \geq 1 + \frac{\delta \gamma}{1 - \delta}. \quad (A2)$$

De Figueiredo’s equilibrium analysis assumes that $A$ either insulates or not regardless of whether $B$ has ever been in office before. But, $A$ knows whether $B$ has held power before – and, therefore, whether $B$ will have the option of insulating when it next comes to power. $A$, therefore, can condition its decision on this information. This affects the equilibrium conditions at which a party insulates but not de Figueiredo’s general conclusions.
or, equivalently, if $\alpha \geq \alpha_{A|N} \equiv [1 - \delta(1 - \gamma)]/[1 - \delta(1 - \gamma)(1 - \beta)]$. Similarly, $A$ weakly prefers to insulate at (iii) if $\alpha \geq \alpha_{A|I} \equiv 1 - \delta(1 - \gamma)$. Clearly, $\alpha_{A|I} < \alpha_{A|N}$.

Now consider $B$’s decision at (i) if $A$ overturns $B$’s policy whenever possible. Assume further that $\alpha_{A|I} < \alpha < \alpha_{A|N}$ which means that $A$ only insulates if $B$ did. Then $B$ weakly prefers to insulate if $\alpha \geq \alpha'_{B} \equiv [1 - \delta(1 - \gamma)][1 - \delta \gamma]/[1 - \delta + \frac{1}{2}\delta(1 - \gamma)]$. Algebra then shows that $\alpha'_{B} > \alpha_{A|N}$. $B$, therefore, prefers not to insulate if $\alpha \in (\alpha_{A|I}, \alpha_{A|N})$. This implies that all the equilibria in subgame (i) are inefficient only if $\alpha > \alpha_{A|N}$.

A similar analysis of the subgame in which $A$ comes to power in the first round demonstrates that there will be insulation here only if $\alpha > \alpha_{B|N}$. The fact that $\gamma \leq \frac{1}{2}$ then gives $\alpha_{A|N} < \alpha_{B|N}$. Hence, the equilibrium condition needed to ensure inefficiency is that $\alpha > \alpha_{A|N}$ or, equivalently, that $A2$ holds strictly.

To compare this to (1), consider node (ii) where $A$ has come to power for the first time and $B$ did not insulate when it had the chance. Let $M_{A}$ denote $A$’s minmax payoff starting from this state. As for the upper bound on $A$’s continuation payoff defined by the right side of (1), observe that, because $B$’s policy is in place when $A$ comes to power at (ii), the maximum of the sum of the two players’ continuation payoffs is one in each period. This gives $B_{k} = 1/(1 - \delta)$. Moreover, the minimum $B$ can get at (ii) if $A$ plays efficiently (i.e., does not insulate) is zero. And, $B$’s expected minmax payoff starting in the next period is what it obtains if each player always overturns the other’s policy: $(1 - \gamma)/(1 - \delta)$. Condition (1) then becomes:

$$M_{A} > \frac{1}{1 - \delta} - \frac{1 - \gamma}{1 - \delta} > 1 + \frac{\delta \gamma}{1 - \delta} \quad (A3)$$

Proposition 1 shows that there are no efficient equilibria whenever $A3$ holds. Consequently, the only difference between $A2$ and $A3$ can occur if there are no efficient equilibria ($\alpha_{A|N} < \alpha$) and $A3$ still does not hold. But $\alpha > \alpha_{A|N}$ and $A2$ imply that $A$’s best reply to being minmaxed is to insulate. So, $M_{A} = \alpha[1 + \delta[\gamma + \frac{1}{2}(1 - \gamma)]/(1 - \delta)]$. Thus, $A3$ is identical to $A2$ if $\beta = \frac{1}{2}$, and the slack between these conditions is due to the nonlinearity
of the Pareto frontier that arises if $\beta > \frac{1}{2}$.\footnote{If $\beta > \frac{1}{2}$, there are are gains from cooperation and equilibria in which the parties do not always overturn each others policies. This increases $A$’s potential payoff if $A$ does not insulate at (ii), and this raises $\alpha_A|N$. The maximum of the sum of the two players continuation payoffs also increases to $B_k = 2\beta/(1 - \delta)$. This makes comparing (1) and the equilibrium conditions with $\beta > \frac{1}{2}$ very complicated and tedious.}

As shown above, bargaining breaks down in war in Powell’s model if

$$
\frac{1 - p_0 - d}{1 - \delta} > 1 + \delta \left( \frac{1}{1 - \delta} - \frac{p_0 + \Delta - r}{1 - \delta} \right)
$$

This inequality is equivalent to condition (1): The left side is the declining state’s minmax payoff which it can obtain by attacking. Since the size of the pie is one in every period, $B_k = 1/(1 - \delta)$. The rising state’s minimum payoff in any period along any efficient path is zero. And, the rising state’s minmax payoff in the next period is what it can obtain by fighting at that time: $(p_0 + \Delta - r)/(1 - \delta)$.

Turning to the equilibrium condition, there is a status quo distribution of territory $q$ in Powell’s model, i.e., the rising state controls $q \in [0, 1]$ of the territory at the outset of the game. This means that the declining state obtains a payoff of $1 - q$ if does not fight in the first period. Thereafter the declining state keeps the rising state indifferent between fighting and accepting the risings state’s offer in each period. Thus, the declining state’s continuation payoff if it does not fight in the first period is $1 - q + \delta[1/(1 - \delta) - (p_0 + \Delta - r)/(1 - \delta)]$ where $(p_0 + \Delta - r)/(1 - \delta)$ is the rising state’s payoff to fighting in the next period when its probability of prevailing has increased to $p_0 + \Delta$. This implies that the equilibrium condition that yields fighting is:

$$
\frac{1 - p_0 - d}{1 - \delta} > 1 - q + \delta \left( \frac{1}{1 - \delta} - \frac{p_0 + \Delta - r}{1 - \delta} \right)
$$

The slack between this equilibrium condition and (1) is due solely to payoffs in the initial period and this difference goes to zero as the discount factor goes to one.
References


