1 Graduate Stats Class

MCMC Homework 1:


If you want to more comprehensive volume then I recommend Gelman, Carlin, Stern and Rubin. 1995. Bayesian Data Analysis. Chapman &Hall.

A) Consider two random variables $Y_1$ and $Y_2$ that are bivariate normally distributed $\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} \sim N_2( \begin{pmatrix} 5 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 & .7 \\ .7 & 1 \end{pmatrix})$.

Consider the random variable $W = 2Y_1 - 4Y_2$.

Use a MCMC method to simulate the distribution of $W$.

The following will be useful (see Greene ch3 for more details)

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \sim N_2( \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \rho \sigma_1 \sigma_2 \\ \rho \sigma_1 \sigma_2 & \sigma_2^2 \end{pmatrix})$

then $f(x_2|x_1) = N(\mu_2 + \frac{\sigma_2}{\sigma_1^2}(x_1 - \mu_1), \sigma_2^2(1 - \rho^2))$.

Be thoughtful as to how to display your results.

B) The gains above are small since you could have calculated these distributions directly. Try calculating the following.

Let $Q = \begin{cases} Y_1 & \text{if } Y_1 < 5.5 \\ 5.5 & \text{if } Y_1 \geq 5.5 \end{cases}$

Now simulate the distribution of $G = Q - 3Y_2$. 
