Power-sharing Agreements as
Political Risk-sharing Contracts

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August 10, 2002

*I thank Roger Myerson, Adam Przeworski, Michelle Cimato, and Michel Poitevin and seminar participants at New York University, Northwestern University and Ohio State University for invaluable comments. Very special thanks to Siddarth Sharma for superb research assistance. I am responsible for any remaining error.
Abstract

Under what conditions are long term agreements between ethnic groups self-enforcing? How can such agreements be constructed? We address these issues by analyzing the structure of long term risk-sharing contracts between minority and majority groups. We show that self-enforcement is greatly facilitated when players either value the future very highly or ex ante transfer of power is made, e.g. guaranteed share of the executive, regional autonomy, veto rights. With total or partial commitment, parties are fully insured and self-enforcing political contracts take the form of consociational democracy characterized by relatively rigid power-sharing and political immobility. With no commitment, political “insurance” is partial and self-enforcing contracts take the form of a Madisonian democracy: political power follows a Markov process and is bounded from above and below.
INTRODUCTION

International relations scholars claim that conflicts between an ethnic majority and an ethnic minority are particularly difficult to resolve. Even after a settlement is reached, persistent disparities in political and economic resources can make a long term agreement very difficult to sustain. While Walter (2002) stresses indivisibility of the stakes as the primary reason for enduring ethnic conflict, Fearon (1994) emphasizes the importance of commitment problems. To see how such problems may arise, consider for example pre-electoral negotiations between a party representing an ethnic minority and a party representing an ethnic majority. The minority might fear there is no guarantee that the majority leaders will respect the agreement after the elections. The majority party may be very cooperative before the vote and then become repressive and non-conciliatory as soon as it takes control of the coercive instruments of the state. Thus, long-term agreements to end ethnic conflicts have to be self-enforcing, i.e. provide incentives against ex post opportunism.

There is an important literature in the political economy of institutions addressing the issue of political commitment.¹ Weingast (1994) explains how this question played a role in the outbreak of the American Civil War. Weingast investigates why Southerners previously willing to accept political outcomes under the Constitution changed their minds and in what way Republicans, believed to be rather moderate on the question of the abolition of slavery, were a threat to the South. He claims that the stability of Antebellum politics was the result of an explicit set of political exchanges by which a sufficient number of Northerners joined Southerners to provide strong protection for slavery in exchange for Southerners’ cooperation on other policy dimensions. The institution which ensured Northerners’ commitment to respect the terms of the exchange was the balance rule affording each region equal representation in the Senate and hence granting each veto power over national policy making. With the break of the balance rule as well as the rise of the Republican Party which became

explicitly hostile to slavery, Northerners’ commitment ceased to be credible. This, Weingast argued, led to Southern secession and to the Civil War.

Under what conditions are post conflict settlements between minority and majority groups self-enforcing? What the general structure of such settlements ought to be? How can they be constructed? These questions have not been addressed by the political economy or the international relations literature. They are, however, quite important and relevant not only for ethnic or religious conflicts but for all types of enduring conflicts involving asymmetric players such as the Israeli-Palestinian conflict.

The aim of this paper is to fill this gap in the literature. We argue that while the players’ inability to commit creates incentives to violate the agreement, aversion to risk against uncertain political outcomes incites them to settle on stable agreements. They are likely to stick to these agreements when appropriate incentives are provided. We particularly explore self-enforcing properties of agreements involving ex ante transactions.2

Political risk-sharing

The context of the present study is a country in which a minority group and a majority group are involved in a violent conflict and are considering the possibility of a political settlement followed by the design of a long term agreement (e.g. a constitution). The constitution would create a stable and violence-free political system. We assume that the political power of each side depends on political resources that they control. The majority controls or is expected to control the central government including the country’s military, while the minority controls a small portion of the country’s territory and/or a dormant militia group capable of generating ethnic violence.

The status quo is partition:3 each group has the option to create its own state and

2 That is, bargains requiring that some transactions be made before an uncertain event takes place.
3 Donald Horowitz (1985) wrote: “if the short run is so problematical..., if it is impossible for
live off its own political and economic resources. However partition is quite risky.\textsuperscript{4} In the absence of strong outside enforcement of the new borders, violent conflict might break out again and lead to one or the other faction being wiped out. Alternatively, the territory controlled by either group may become unexpectedly very valuable due to the discovery of oil and this could enhance the power of the one group vis a vis the other. Thus, political resources could fluctuate greatly and this could lead risk-averse parties to enter in a risk-sharing relationship.

One form of a risk-sharing relationship which is the focus of this paper is a power-sharing contract mediated by periodic elections. However, ethnic politics under democracy can be risky though not as risky as partition. The majority can win more votes in a national election than anticipated and this could hurt the minority’s political interests. On the other hand, the majority’s political status could be undermined if the minority’s party unexpectedly wins more votes at the polls or if a diamond mine is discovered in the territory that it occupies.

Efficient risk-sharing requires a transfer of resources from say the majority to the minority in the event that the political power of the latter is weakened or vice-versa. Now, suppose that they agree on such a transfer. Would the majority be willing to stick to the bargain \textit{ex post}, that is after observing that the minority has had a bad draw? If not, can one design a contract which is immune against such an opportunistic behavior? The literature on long term interaction suggests that the existence of such a contract heavily depends on the discount factor, i.e. a self-enforcing contract might not exist if the players do not value the future highly. This is a serious limitation because the very existence of secession threat implies that players do not value the future of their relationship highly. Using recent advances in the theory of self-enforcing risk sharing contracts, we introduce another instrument

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\textsuperscript{4}Sambanis (1997) finds evidence suggesting that partition is positively correlated with ethnic war recurrence.
that facilitates commitment: ex ante power transfers such as guaranteed share of the executive or seats in the national assembly, veto rights, regional political autonomy.

Political risk-sharing and consociational democracy

The standard institutional response to commitment problems in ethnically divided countries is extensive power-sharing or consociational democracy. Lijphart (1969) defined consociational democracy as “a government by elite cartel designed to turn democracy with fragmented culture into a stable democracy” (p. 216). This model of democracy has been used by its proponents as an explanation of political stability in the Netherlands, Belgium and Switzerland, and has served as a normative model for constitutional engineering in ethnically divided countries. However, the consociational paradigm has been criticized for its weak empirical support in Europe and for being undemocratic since elections play little role (if at all) in allocating power between parties (Barry, 1975 and van Schedelen, 1984). Nordlinger (1972) and Horowitz (1985) have also found the model inappropriate for “deeply divided” societies in Africa, Asia and the Middle-East because it is not flexible enough to cope with changes in the balance of power between groups. The questions then become: when does consociational democracy work? Why did this model of democracy succeed in the Netherlands but fail in Lebanon?

To address these questions, we focus on the credibility of power-sharing agreements and the ability of the players to stick to the agreements ex post. We argue that unless at least one player can commit to stick to the bargain, the optimal arrangement is likely to take the form of Madisonian democracy where political power follows a Markov process and is bounded from above and below. Grand coalitions are more likely to survive if the players value the future highly or are trustworthy. These conditions are more easily met in affluent countries, such as Belgium and the Netherlands with no significant socio-economic disparities across regions or with relatively high levels of regional interdependence. In contrast, because of profound distrust between former warring factions, consociational democracy in its strong form, is less likely
to be established or to survive in post-civil war or conflict-ridden countries such as Lebanon. Thus from a political risk-sharing perspective, consociational democracy is efficient but less likely to be self-enforcing and stable. In contrast, Madisonian democracy is inefficient but more likely to be self-enforcing and stable.\(^5\)

Our approach helps expand and refine the standard literature on consociational democracy. Theorists of consociational democracy have been criticized for failing to describe and investigate power relations within the coalition government, precisely who gets what share of power, who concedes the most, or the least, and why.\(^6\) Du Toit (2000) claims that these limitations explain why we know so little about conditions under which power-sharing do in fact generate stability. In contrast with the standard literature, we formally define and illustrate the political exchange taking place between the minority and the majority groups and provide a rigorous characterization of optimal and self-enforcing power-sharing arrangements.

The paper is structured as follows. Section 2 presents the basic model which draws from the literature on risk-sharing contracts (Thomas and Worrall [1988], Gauthier et al [1997] and Dixit et al [2000]), followed by a discussion of the optimal power allocation under full enforceability. Section 3 discusses the structure of agreements when only one party (e.g. the majority) can commit to respect the contract and when neither party can commit. Section 4 discusses some applications and section 5 concludes.

\(^5\) This theoretical result is well in line with the empirical literature on power-sharing agreements in the context of civil war settlements: Walter wrote: “If consociational systems do not evolve, they will eventually topple. The fact that power-sharing pacts are likely to be unstable over time, therefore means that a second transition will almost certainly be need to maintain peace and stability in the long run. The ultimate challenge facing enemies in a civil war how to transform the inflexible institution structures that are necessary to convince each of them to sign a settlement in the highly tense post-war environment into a more liberal, open institutions that are necessary to bring about stability over time. (p. 168)

\(^6\) See du Toit [2001].
THE MODEL

The political environment that we consider is a pluralistic society with cleavages between (ethnic) groups. There are two active agents in this environment: an (ethnic) minority party \( m \) and an (ethnic) majority party \( M \). They have the option of partition, that is “separation agreed between the two groups involving border adjustment and population transfer”\(^7\). Besides the partition option, the groups can join their resources and create a democratic political system.

A state \( s \in \{1, 2, ..., s\} \) is identified as an election outcome or as any random shock to the parties’ political resources either under the partition/status quo or under democracy/power-sharing. States are i.i.d and finite. Party \( m \)'s political resource (asset) is \( R^s_t \) which can be seen as a measure of the political support in its group or the economic power of the minority group.\(^8\) Since the ethnic majority is also an electoral majority, its control over the central government is secured. As a result, \( M \)'s political asset is \( G \), the offices of the central government and the public administration. Thus, whereas \( M \) derives its power essentially from the electoral process, \( m \) derives its power from local political support or over economic resources in its territory.

There are infinite periods: \( t = 0, 1, .. \infty \). In each period, there are three dates, \( t_0, t_1, t_2 \). At \( t_1 \) the election outcome or some uncertain event is realized. At \( t_0 \) an ex ante transfer of power (net) from \( M \) to \( m \), \( g_t \), may be made.\(^9\) At \( t_2 \), an ex post

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\(^7\)Barry (1975) argues that in contrast to religious or class divisions, wherever the basis of division is ethnic the fundamental question may not be how the country is to be run but whether it should be run at all, whether one group should secede and become an independent state. (p. 503).

\(^8\)In South Africa, \( r^s_t \) is the level of political support enjoyed by the National Party, the party representing the interests of the white minority at the end of the Apartheid regime. It could also be a measure of the whites’ economic power. In post civil war Nigeria, \( r^s_t \) could be the oil rents or other forms of resources transferred from the territory occupied by the Ogoni minority group to the federal government.

\(^9\)Guaranteed share of the executive, e.g. presidency of the majority candidate and vice presidency for the minority candidate and veto rights for the minority party are examples of ex ante transfer of power. Such transfers have been part of most in not all experiences of (Cyprus in 1960, Lebanon in
transfer of power (net) may be made.

The minority wants to have some control over the central government in order to push for its political agenda and for this purpose it is willing to concede $r_t^s$ of its sovereignty over its region to the central government. The majority wants political or economic support from the minority ethnic group and this party is willing to concede $g_t$ units of control over the government to the minority.\textsuperscript{10} Thus, $g_t$ stands for concessions by the majority to the minority in the form of political or administrative positions and $r_t^s$ may represent concessions of minority to the majority in terms of administrative or political control over its territory. It could also represent tax revenues or mining rents transferred from the region controlled by the minority group to the central government.

A contract $\gamma_t$ is a sequence of functions \( \{g_t, \{r_t^s\}_{s \in S}\}_{t \geq 0} \) where $g_t : \mathbb{N}_{t-1} \rightarrow \mathbb{R}$ and $r_t^s : \mathbb{N}_t \rightarrow \mathbb{R}$ and \( \mathbb{N}_t \) refers to the set of histories until $t_2$. A typical element of \( \mathbb{N}_t \) is $h_t = \{s_1, s_2, \ldots, s_t\}$ and in addition $h_0 = \emptyset$.

After $M$ transfers $g_t$ and “gets back” state contingent $r_t^s$, its level of power is

$$G - g_t + r_t^s$$

Likewise the residual power of the minority party $m$ is given by

$$R_t^s + g_t - r_t^s$$

Following the standard set up in the risk-sharing literature, it is assumed that $R_t^s$ is increasing in $s$ and bounded above. That is, $0 < R^0 < R_t^s < b$ where $b \in \mathbb{R}^{++}$. In addition, $r_t^s \leq R_t^s + g_t$, $g_t \leq G + r_t^s$, $R_t^s + G < b$.

Denote by $u(.)$ and $v(.)$ two strictly increasing and concave functions representing the utility derived from power by respectively the minority and the majority. That

\textsuperscript{10}For instance the 1960 power-sharing arrangements between the Greek majority and the Turkish minority includes guarantees positions for the Turks in everything from cabinet to the civil service, the armed force, and the police, in exchange of the Turks’ s political support for the constitution.
is, $v',u' \geq 0, v'',u'' < 0$. In addition $v'(0) = u'(0) = \infty$. Both functions are assumed to be defined from a closed set to $\mathbb{R}$.

At any period $t$, the per-period payoff of party $m$ is given by:

$$u(R_t^s + g_t - r^s_t)$$

Likewise the expected utility of party $M$ is given by:

$$v(G - g_t + r^s_t).$$

For any history $h_{t-1}$, party $m$’s expected surplus from period $t$ onwards is given by:

$$U(\gamma,h_{t-1}) = E \sum_{\tau=t}^{\infty} \beta^{\tau-t} \{ u(R^s_\tau + g_\tau - r^s_\tau) - u^s_s \}$$

where $u^s_s$ is the partition utility and $\beta$ the discount factor. Note that $u^s_t = u(R^s_t)$ $\forall s,t$. Likewise, party $M$’s expected surplus from period $t$ onwards is given by:

$$V(\gamma;h_{t-1}) = E \sum_{\tau=t}^{\infty} \beta^{\tau-t} \{ v(G - g_\tau + r^s_\tau) - v^s_s \}.$$

$v^s_s = v(G,s)$ is the partition utility.

We now formally define legitimacy and consociational contractual arrangements. A political regime is legitimate if $r^s_t \geq 0$, that is if the majority secures some support or some form of political concession from the minority. A regime is illegitimate if $r^s_t = 0$. A political regime, i.e. $(\gamma,m,M)$ is oppressive if $g_t = 0$. That is, if the majority never concedes government control to the minority. A regime is of “power-sharing type” if $g_t \geq 0$. Power-sharing is “extensive” if $g_t = \bar{g}$, where $\bar{g}$ is the upper bound on all feasible pre-electoral transfers from $M$ to $m$. Finally, a political regime is stable if the contractual arrangement $\gamma$ is self-enforcing.

### Optimal Contracting with Commitment

We first analyze the benchmark case where the contractual arrangement is signed at $t = 0$ and is legally enforceable. This is the case when at any point in time, all the
terms of the contract will be respected by both parties. Party $M$ will maximize its aggregate payoff subject to the constraint that party $m$ participates. However, party $m$ will participate only if its surplus is positive, that is only if $U(\gamma, h_0) \geq 0$. So, the problem of party $M$ is

$$\gamma^f = \arg \max V(\gamma, h_0)$$

s.t.

$$U(\gamma, h_0) \geq 0$$

(1)

**Proposition 1:** The optimal contractual arrangement with commitment is such that the ratios of the benefit of one additional unit of power at time $t$ and state $s$ over the benefit of one additional unit of power at time $\tau$ and state $q$ are equal for both parties. The arrangement results in an inflexible power-sharing since the expected distribution of power is constant across time. The resulting political regime is legitimate and stable.

The optimal contract is the one which equalizes the marginal rate of substitution of parties’ residual power across states and periods. To see why this is true, suppose that the ratios of marginal benefits are different, for instance that it is 4 for the majority and 2 for the minority. This means that from time $\tau$ and state $q$ to time $t$ and state $s$, the majority puts relatively “too much” value on an additional unit of power whereas the minority puts “too little” value on this unit. As result the former will be willing to give more power while the latter will be willing to be given more power. The process will continue until the ratios of marginal benefits are equalized.

At the optimum, there is perfect risk sharing but also perfect political immobility since the expected distribution of power is stationary and remains unchanged over time. At the optimum, only $g_t - r_t^s$, the net transfer of power between parties can be determined and this transfer depends only on the current electoral outcome. The less the minority concedes, (i.e. small $r_t^s$ ), the higher is the net transfer of power to the minority. The more it concedes ( i.e. high $r_t^s$), the smaller is the net transfer
of power. The great challenge of a divided society is whether the majority is willing to give enough control of the government to the minority when the minority cannot commit to make enough concessions, or whether the minority is willing to cooperate when the majority cannot make large concessions. Finally, note that an optimal power-sharing is likely to involve some degree of political autonomy of the minority, that is $r^s_t$ needs not be equal to $R^s_t$. In fact, $r^s_t \leq R^s_t$ represents a check on the majority power.

**CONTRACTING IN THE PRESENCE OF COMMITMENT PROBLEMS**

Political power derived from partition is more volatile than power that can be derived under the contractual arrangement that we define as consociational democracy. Under partition the minority for instance can be wiped out or it can gain complete control of the political system. Under these circumstances, risk-averse parties will therefore be inclined to design a risk sharing mechanism and to derive their power from the democratic process. However while power under partition is trivially enforceable, “democratic” power may not be enforceable. For instance, if at some point in time, a party perceives that the power it can derive under partition is higher than its power under democracy, it will have an incentive to violate the contractual arrangement.\(^\text{11}\) We now investigate how parties resolve the conflict between risk sharing and self-enforcement. We aim to examine what types of actions parties design *ex ante* to surmount commitment problems or what restrictions are to be placed on the political arrangements so that they will be upheld in the future. We will first analyze the one-sided case, followed by the two-sided case.

\(^\text{11}\) Horowitz writes: “formal guarantees (to minority groups) are always going to be resented by the group on which they are imposed in the same measure as they are insisted upon by the group seeking to profit from them. Because they are resented, they are not likely to prove durable (p. 144).
One-sided Commitment

We assume that only $m$ can commit to respect the terms of the contract. That is, $M$ may renge the contract whenever it is worse off under democracy than under partition. Since this party cannot be forced to respect the terms of the agreement, it should be provided with the proper incentives. This will happen only if, at any point in time, party $m$ derives at least as much utility from respecting the contract as reneging on it. Since both parties are involved in a long term relationship, $m$’s utility from reneging depend on how $M$ reacts to this action. We assume that $M$ reverts to autarchy forever (grim strategy) if $m$ reneges on the contract. This punishment strategy will help characterize the set of feasible contracts that will be obeyed without legal enforcement.

The minority will make a credible commitment to respect the terms of the contract if the benefits of doing so outweigh the cost of reneging and “breaking” with the minority. As in section 1, the terms of the contract are described by the set $\gamma = \{g_t(h_{t-1}), r_t(h_t)\}$ for all $h_t \in \mathbb{N}_t$. Suppose the transfer of $r_t^s$ has to be made before the period $t$ election. In this case, party $m$ may have an incentive to violate this agreement if $U(\gamma, h_{t-1})$, is positive. If instead this transfer were to be made after the election and $m$ is willing to renege on this contract, the surplus from compliance is given by

$$u(G + g_t - r_t^s) - u(G + g_t) + \beta U(\gamma, h_{t-1}, s)$$
Thus, the problem of $m$ is:

$$\gamma^1 = \arg \max \quad U(\gamma, h_0)$$

s. t. $V(\gamma, h_0) \geq 0$

$$U(\gamma, h_{t-1}) \geq 0 \quad \forall t, h_{t-1}$$

$$u(G + g_t - r_t^s - v(G + g_t) + \beta V(\gamma, h_{t-1}, s) \geq 0 \quad \forall s, t, h_{t-1}$$

The first constraint is $m$’s participation constraint and the last two constraints are the self-enforcing constraints.

**Proposition 2:** Assume $M$ can commit but $m$ cannot. Then there exists an “extensive” power-sharing arrangement that is incentive compatible for the majority regardless of the discount factor $\beta$. This arrangement is efficient and leads to political immobility since the expected distribution of power remains unchanged over time.

The proposition states that if in each period the uncommitted party makes a substantial pre-electoral concession to the committed party, that is $g_t \geq \overline{g}$, then the full commitment contract can be enforced. This is true regardless of the value of the discount factor. Even when parties are not optimistic about the future of their relationship, the majority’s commitment problem can be surmounted and the optimal contract be enforced. The result indicates that ex ante transfers are substitutes for high discount factors.

The first best contract seeks efficient risk-sharing. In states with low $R^m_t$, this entails a net transfer of political resources from the majority to the minority. But when $m$ has commitment problems and can renege on the contract in any period, such a transfer may not be consistent with M’s incentive compatibility requirement. The
ex ante transfer provides an avenue for relaxing $m$’s ex post incentive constraints: by making a large enough pre-electoral transfer to $M$, $m$ ensures that it has an incentive to stay in the contract ex post (i.e. after the election or after political uncertainty is realized). If a substantial pre-electoral concession cannot be made, then the full commitment contract cannot be supported for all values of the discount factor $\beta$. If this discount factor is high enough, then player $m$ will abide by the agreement because the future benefits of such action exceed the short run gain of violating the agreement.

Next, we characterize the optimal contractual arrangement when $g_t < \overline{g}$ and the discount factor is low. This scenario captures the situation in which making “excessive” concessions to the minority is too unpopular among the ethnic majority. The following is a verbal statement of the results. A technical version is in the appendix.

**Proposition 3:** If “extensive” power-sharing is not feasible and both parties are myopic, the optimal contract can only be enforced for high discount rates. In addition, the optimal contract is such that there exists a lower bound to the minority’s power. This bound is time-independent and increasing in the majority’s electoral support.

When the discount factor is low and $\overline{g}$ is not attainable, perfect insurance, that is a fixed power sharing, is not feasible for all discount factors. However, there exists a minimum level of power which prevents the minority from violating its agreement and optimally trading off its current and future power. This result shows that either the power distribution is fixed over time or it is bounded from below. In addition, unless the discount rate is quite high, the contract induces only limited political mobility because electoral outcomes only “slightly” affect the allocation of power. The lower bound on the majority power arises because of its commitment problem and acts as the risk-sharing that the optimal contract requires. The proposition also indicates
that \( m \)’s power level \( p_t^m \) follows a Markov process where \( p_t^m \) depends on \( p_{t-1} \) and \( s \) the state realized in \( t \).

The argument remains valid even when the majority is the party with commitment problems. In this case, this party will have to make credible pre-electoral “transfers” to prove its commitment to civil order and to a stable political system.\(^{12} \) From a technical point of view this reversal of roles between the two parties means that both \( g_t \) and \( r_t^s \) take negative values.

In the process of transition to democracy, power-sharing arrangements can be seen as a gift exchange between parties. For the exchange to be efficient, the party that cannot commit to respect the terms of the contract has to be somehow disciplined. This party will have to bear all the risk of a failed transition by making some pre-electoral investments which will be lost if it behaves in an opportunistic manner.

The argument is similar to the one developed in Williamson (1983), which explains how the use of hostages could help to support efficient exchange. In this model, the hostage represents the pre-electoral investment made by party \( m \).

Fearon (1992) explains violence in the former Yugoslavia by the inability of ethnic majorities in these countries to commit not to exploit the minorities in the new state. According to the author commitment problems arise when groups interact in anarchy, that is in the absence of a third party able to guarantee and enforce contracts and the minority’s ability to create anarchy decreases with time. In such circumstances the minority will gain by fighting in the present. Fearon rightly pointed out that these problems could be solved by giving some bargaining power to minorities. This paper argues that even in the absence of a court, both parties could solve the commitment problems by designing self-enforcing contracts, that is contracts which are immune to opportunistic behavior. These arrangements arise from pre-electoral forums or roundtables between parties which then become something like an insurance market. Political stability is guaranteed by giving not just “some” leverage to the minorities

\(^{12}\) By taking such action, the minority gets rid of the means of any eventual anarchy. As a result the majority will have nothing to fear.
in the new political system, but by giving them the best deal that they could possibly
get. Furthermore the power-sharing arrangement should be effective before the elec-
tions and can be interpreted as an insurance premium. This contract will be enforced
even if, due to a low discount rate, the value of creating anarchy declines sharply over
time. Consequently even in the context of an anarchic political situation where the
minority’s ability to create anarchy decreases over time, commitment problems can
be solved and ethnic wars averted.

The result in this section is based on the assumption that only one party, namely
the majority, has a commitment problem. However, as we have said earlier the
minority can also have an incentive to violate the contractual arrangement if it is
beneficial to do so. Under these circumstances the contractual arrangement that we
have just described may not be feasible. In the following lines, we study the structure
of contracts in the case when both parties have a commitment problem.

No Commitment

In the presence of bilateral commitment problems, incentives have to be designed
such that both parties will obey the contract. The optimal contract when neither
party can commit will solve the following maximization problem.

\[ \gamma^2 = \arg \max \ V(\gamma, h_0) \]

s. t. \[ U(\gamma, h_t) \geq 0 \]

\[ V(\gamma, h_{t-1}) \geq 0 \quad \forall \ t, h_{t-1} \]

\[ u(R^s + g_t - r_t^s) - u(R^s + g_t) + \beta U(\gamma, h_{t-1}, s) \geq 0 \quad \forall \ s, t, h_{t-1} \]

\[ v(G - g_t + r_t^s) - v(G - g_t) + \beta V(\gamma, h_{t-1}, s) \geq 0 \quad \forall \ s, t, h_{t-1} \]

(2)
The first two equations mean that the contract has to generate a positive surplus for both parties at any point in time and after any history. The next two mean that after any history either party has to prefer staying in the contract rather than reneging on it. The post electoral (ex post) incentive constraint for \( m \) that is, 
\[
u(R_t^m + g_t - r_t^m) - u(R_t^m + g_t) + \beta U(\gamma, h_{t-1}, s) \geq 0, \quad \text{applies when } g_t < 0 \]
and the ex post incentive constraint for \( M \), that is 
\[
v(G - g_t + r_t(s)) - v(G - g_t) + \beta V(\gamma, h_{t-1}, s) \geq 0 \quad \text{applies when } r_t^m \leq 0. \]

Note that while raising \( g_t \) will relax the majority’s ex post constraint as before, it will now tighten the minority’s ex post constraint. Hence it is not as clear cut as before how ex ante transfer helps overcome commitment problems. However, a comparison with the case in which no ex ante power transfer is allowed helps explain its role.

Suppose no ex ante transfer is allowed, that is, \( g_t = 0 \). Then if the discount factor is high enough, the first best contract (i.e. \( \gamma^f \)) remains incentive compatible. However, as we will explain later, allowing ex ante transfers will enable the efficient contract to be supported for a large range of discount factors.

There is another way in which \( g_t \) affects the optimal contract. In the absence of an ex ante transfer there exists upper and lower bounds on the contractual power levels of \( M \) in each state. These bounds are determined by the ex post self-enforcing constraints of, respectively, the majority and the minority. These bounds are restrictions on the degree of efficient risk-sharing that the contract can possibly achieve. They are time-independent (see Gauthier et al. 1997; Thomas and Worrall, 1988). Once ex ante transfers are allowed, these bounds change over time, depending on which states are realized, in such a way as to improve risk-sharing over time. Before explaining how, we characterize \( \gamma^2 \) when the discount factor is high enough and when ex ante transfers are allowed.

**Proposition 4:** Assume neither \( m \) nor \( M \) can commit. Then the optimal contract is
such that there exists a “non extensive” power-sharing arrangement that is incentive compatible for both parties as long as the discount factor $\beta$ is sufficiently high, i.e. $\beta \geq \beta^2$ for some $\beta^2 \in (0, 1)$.

This result shows that perfect risk sharing is possible even when there is a total breakdown of trust between parties. This will happen only if parties care enough about the future, that is if their discount factor is sufficiently high. To see why, suppose that the minority party is willing to create anarchy. A high discount rate makes the future cost of partition outweigh the immediate gain associated with such a situation. The minority party will gain by abiding by the terms of the contract and consequently, the efficient risk-sharing contract will be enforced.

We now characterize efficient risk-sharing under low discount rates, when $\beta < \beta^2$. In other to explain the role that ex ante transfer plays, we state the following, which is the analog of proposition 3 for the no commitment case but with $g_t = 0$. We denote by $p_t^s$ the power of the majority at time $t$ under state $s$.

**Proposition 5a:** Set $g_t = 0$, (i) for each state $s$, there exists an optimal time-invariant residual power of levels $\underline{p}^s$ and $\overline{p}^s$ such that $\underline{p}^s \leq p_t^s \leq \overline{p}^s$, $\forall t$. (ii) both bounds are increasing in states of the world, (iii) for any history $(h_{t-1}, s)$, the optimal residual power at time $t$ is such that

$$p (h_{t-1}, s) = \begin{cases} \underline{p}^s & \text{if } p^* (p_{t-1}, R_{t-1}, s) < \underline{p}^s \\ p^* (p_{t-1}, R_{t-1}, s) & \text{if } \underline{p}^s \leq p^* (...) \leq \overline{p}^s \\ \overline{p}^s & \text{otherwise} \end{cases}$$

The values of $\underline{p}^s$ and $\overline{p}^s$ represent the limits to the power of the majority party. Since power is defined as control over government, this indicates that there are “external checks” on the power of the majority. Obviously, institutional details of those checks and balances are beyond the scope of this paper. The results indicate that Madisonian democracy is a self-enforcing political arrangement when parties have commitment problems.
Now, we consider the case where ex ante transfers $g_t$ are allowed. We show in Proposition 6 (below) that, for each $g_t$, there exists optimal upper and lower bounds on residual power.

**Proposition 5b:** Assume $g_t > 0$ and $\beta < \beta^2$, the optimal value of the ex ante transfer by the majority is decreasing in its expected level of power.

Thus, when the discount factor is low, the optimal contract cannot be enforced. The second best self-enforcing contract will be such that when the majority’s expected gain from the exchange increases, its pre-electoral concession will decrease. This seems a bit surprising since one might expect the majority party to be more generous when the exchange becomes more profitable. However, the intuition of this result becomes clear when we look at how the profitability of the exchange affects the self-enforcing constraints. When the surplus is low, $M$ is almost indifferent between breaking up the relationship and obeying the contract. Consequently $M$ will be perceived as high risk and will have to pay a higher insurance premium. In other words it will have to make more substantial pre-electoral concessions in order to win the trust of its opponent.

The result may explain why extensive power-sharing proved stable in Switzerland and the Netherlands but failed in most Third World countries including Lebanon and Nigeria (Horowitz [1985]). For Weingast (1997), the main reason lies in the presence of institutions that provide incentives for mutual tolerance and in the fact that citizens want these institutions preserved. Our argument relies on two factors: (1) differences in the level of government resources that determine the value attached by the different groups to “staying together” and (2) the inability to commit to respecting the institutions that they have agreed upon. Lack of commitment to institutions requires large concessions that neither the majority nor the minority is willing to make unless they value the relationship very highly. Abundant government resources could make those concessions easier.\(^\text{13}\)

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\(^{13}\)Since government resources and economic development are highly correlated, the result echoes
We now show that for each $g_t$, there exists optimal upper and lower bounds on residual power. These bounds thus change over time. The result is derived for $s \in \{1, 2\}$. It is stated as follows:

**Proposition 6:** Suppose $s = 2$, $f(.)$ is concave and $\beta < \beta^2$. Then the expected residual power of $m$ for $t+1$ is larger than that of period $t$ if state 1 occurs in period $t$ and vice versa if $s = 2$. (ii) $M$’s power in period $t$ is smaller than $p^*(p_{t-1}, R_{t-1}, s)$ if $s = 1$ in period $t$ and vice versa if $s = 2$.

Proposition 6 (i) shows that the impact of “good news” (state 2 or more power to $m$) and bad news (state 1 or less power to $m$) is spread over periods. What matters here is the total power available for sharing, $G_t + R_t^s$. Thus in case of a bad draw (state 1) occurs, party $M$ spreads its impact over time by asking more of $m$ in the current period. That is $U_{t+1} \geq U_t$.

Proposition 6 (ii) in conjunction with proposition 5, shows how ex ante payment improves risk-sharing. When $g_t = 0$, bounds to political power are time-invariant. Thus Proposition 6 (ii) implies that at $t$ the expected power for the next period $(t+1)$ is $\pi_1^1 \underline{p}^1 + (1 - \pi_1^1) \overline{p}^2$; this is the same in all periods. Now when ex ante transfers are allowed every level of $g_t$ that arises in any period under $\gamma^2$ defines upper and lower bounds on political power $\overline{p}^\gamma(g_t)$ and $\underline{p}(g_t)$ for that period. By proposition 6(ii) then the expected level of power in period $t+1$ at period $t$ is $\pi_1^1 \underline{p}^1 (g_{t+1}) + (1 - \pi_1^1) \overline{p}^2 (g_{t+1})$.

These optimal bound decrease with $g_t$ (an extension of proposition 5).

To show this formally, suppose that state 1 occurs in period $t$. Then $U_{t+1} \geq U_t$ (by proposition 6(i)). This implies that $g_{t+1} \geq g_t$ (by Proposition 5 (iv)), which implies that $\underline{p}^1$ and $\overline{p}^2$ are lower in period $t+1$ compared to period $t$. Thus the expected power level is lower in period $t+1$ than the expected power in period $t$. Thus, if a bad state occurs, current and future expected power are reduced (and vice versa). A succession of poor election results will lower expected residual power to $m$ from the Przeworski et al (2000) empirical result showing that democracy never collapse in countries with GDP over $2,800.$
contract. Thus $\gamma^2$ improves risk-sharing and \textit{ex ante} transfers help self-enforcement even when no party can commit.

The optimal contract is characterized by partial insurance since bad electoral performance by a party leads to less power for that party and good electoral performance leads to more power. Moreover a succession of bad (good) performances will reduce (increase) power even further. In other words, parties bear more risk than they do in the full enforcement case. As in Proposition 5a, the arrangement has a Madisonian flavor since it induces upper and lower bounds on political control over the government, that is checks and balances. Thus, from a risk-sharing perspective, Madisonian democracy is second best, i.e. a self-enforcing but inefficient political institution.

Riker (1982) argues that the role of democratic institutions is to restrain the power of the majority and of elected officials. The protection of the minorities against exploitation comes from the fact that political majorities are in general unstable and transitory. “The majority is not likely to tyrannize over people who may themselves be on top after the next elections.” (p 234). The model of democracy that we consider in this paper is different from a Madisonian democracy in one very important aspect. The majority is not temporary or unstable. As a result, elections provide absolutely no external check to prevent the exploitation of the minority. Restraints on the majority will arise not from the electoral process but from direct interaction between parties, from the willingness of the majority to stabilize its hold on the political system and to prevent rebellions or secession. These restraints arise as optimal solutions to commitment problems, to a lack of trust between parties.

\textbf{AN EMPIRICAL TEST}

Case studies on the settlements of ethnic wars suggest that power-sharing tend to have limited scope and to allow for a positive role for the electoral process. This observation is well in line with Proposition 5-6 particularly because the no commitment
assumption is obviously valid for former warring factions.)\textsuperscript{14} We thus perform the following empirical test, derived from Proposition 5-6:

Hypothesis: Peace agreements that include a limited power-sharing pacts are more likely to prove stable than those agreements that do not.

The main dependent variable is level democratic stability two to five years after the civil war. To measure democratic stability, we use the Polity 98 political regime score, both 2 and 5 years after the end of conflict (Jaggers and Gurr 1999). Polity 98 data sets provide regime data for 232 countries starting as early as 1800 and ending in 1998. The data measure countries on a democratic scale from 0 to 10 and an authoritarian scale from 0 to 10. As a test of the robustness of these results, we also used Alvarez, Chebuib, Limongi, Przeworski (ACLP) measure of political regimes. The coding of the variables employs a minimalist definition of democracy: countries are scored as democratic if the chief executive and/or the legislature is elected, if there is more than one party and some degree of political alternation.

Following the standard procedure in the empirical literature in conflict analysis, I use as control variables a dummy for ethnic wars (ethnic War), log of deaths and displacements (Cost of War), the duration of the war (War Duration), the number of hostile factions (Number of Factions), and the level of economic development (Development), Natural Resource dependence. The data were drawn from Doyle and Sambanis (2000) and Wantchekon and Jensen (2002).

\textsuperscript{14}For instance, one of the warring factions in Mozambique, the RENAMO, would demobilize its forces only partially before the 1996 elections unless the United Nations created a monitoring police force, just in case there was a need to return to civil war, as was the case in Angola, where the elections were considered not free and fair. Meanwhile, during the election, the government still had two mechanized divisions with tanks protecting the capital, Maputo. Thus, despite the agreement between the warring factions to make peace and create democratic institutions, there might still be serious concerns about regime stability. See Walter (2002) for further empirical justification of the no commitment assumption.
The main independent variable is “power sharing” is coded as a 1 by Walters 2000 if any one of the following three conditions hold, and 0 otherwise: (1) there is a political pact where “a settlement offer the combatants guaranteed positions in the new government at the level of cabinet or above, or a specific quota of power in at least one of the main branches of government.” 15(Walters 2000, 62-63), (2) if there is a military pact where factions are guaranteed representation in the military (3) if there is a territorial pact that provides a region with some type of political autonomy. 16 The results are presented in Table I

Insert Table I here

In all four models presented in the Table I, consistent with our hypothesis, weak power sharing arrangements have a large positive effect on democratic stability. This result is significant at the 0.10 level in three of the regressions, and at 0.05 level in one regression.

CONCLUDING REMARKS

This paper uses insights from the theory of self-enforcing risk sharing contracts to discuss the rationality of power-sharing arrangements. When there is limited enforceability of these arrangements, restrictions are derived such that the arrangement will in fact be upheld in future periods. Many of these restrictions are consistent with observed political institutions in South Africa, Mozambique and elsewhere. We point out for instance the critical importance of timing in the application of the contract.

A major restriction in the structure of the model developed in this paper concerns

15For example the agreement between the FRELIMO and RENAMO included: (1) standing orders in parliament to give all parties a vice-president of the assembly (with their rank determined by their party’s size in parliament) and a share of committee chairs, (2) special status for Dhlakama as leader of the opposition (though nothing had been openly declared, he was expected to be given the salary of a minister in addition to other economic incentives and (3) the right to appoint three advisors to the governor of each province.

16See Walters (2000, 64) for detailed examples.
the asymmetry between parties. Consequently, given its demographic weight, the minority is expected to lose any elections to be held in the future. Even if this assumption reflects the realities of countries such as Switzerland, South Africa, Sri Lanka or Russia, it does not apply to all types of democracies. If our model were to capture key features of these political systems, parties should have a fairly equal chance of winning the elections. In this case Dahl (1956) claims that the main protection against the exploitation of minorities comes from the fact that majorities are unstable and transitory. We intend, in future works, to explore the issue of commitment and constitutional stability in the more general setting of a Madisonian democracy as well as to provide a formal perspective to Dahl’s conjectures.

REFERENCES


APPENDIX

Proof of Proposition 1:

We solve for $\gamma_f$ neglecting the constraint $V(\gamma; h_0) \geq 0$. This is because the reservation level of $m$ (the first constraint) is zero, implying that a contract satisfying both constraints trivially exists.

The Lagrangian for problem (3.1) is

$$L = V(\gamma, h_0) + \lambda U(\gamma, h_0)$$

Given the form of $V(.)$ and $U(.)$ the only choice variable is $n_{st}$, the net transfer from $M$ to $m$. Hence the First-Order Conditions (FOC) is:

$$\frac{u'(R^q_t + n^q_{st})}{u'(R^s_t + n^s_{st})} = \frac{v'(G - n^q_{st})}{v'(G - n^s_{st})}$$

for all $q, s \in S$ and for all $t$. Q.E.D.

Note that the optimal contract only specifies the net transfer $g_t + r^s_t = n^s_{st}$. Also, as one should expect in all optimal risk-sharing contracts, $n^s_{st}$ is increasing in $R^s_t$ (i.e. $m$ gets more in states where it gets high political resources.)

To see this, suppose the contrary. That is $R^q_t > R^s_t$ but $n^q_{st} > n^s_{st}$, then

$$u'(R^q_t + n^q_{st}) < u'(R^s_t + n^s_{st})$$

since $u''(.) < 0$. This implies that $v'(G - n^q_{st}) < v'(G - n^s_{st})$ (from the F.O.C), which implies that $G - n^q_{st} > G - n^s_{st}$. This means that $n^q_{st} < n^s_{st}$, a contradiction.■
**Proposition 2:** Denote the first best optimal contract by $\gamma^f$ and by $n^{f0}$ the net transfer from $M$ to $m$ in state 0 and let $\overline{g}$ be the upper bound on a possible ex ante transfer from $M$ to $m$. If $\overline{g} \geq n^{f0}$ then $\gamma^f = \gamma^1$.

Proof

Denote by $\gamma^1$ the solution to problem 2. Since $\gamma^f = \arg\max U (\gamma, h_0)$ subject to $V (\gamma, h_0) \geq 0$, $\gamma^f$ provides at least as much net expected utility as $\gamma^1$. Since $\gamma^f$ is stationary, $U (\gamma^f, h_{t-1}) \geq 0, \forall t, h_{t-1}$. It remains to check that $\gamma^f$ satisfies the third constraint, which is party $M$'s ex post self-enforcing constraints.

Suppose that we set $g_t = \overline{g}$, for all $t$. We know that $M$ makes the largest ex ante transfer under $\gamma^f$ in state 0: That is, $n^{f0} \geq n^{fs}$ for all $s$. Now $r^s_t = g_t - n^{fs}$ (by definition). In addition by stationary of $\gamma^f$, we have $r^s_t = \overline{g} - n^{fs} \geq n^{f0} - n^{fs} \geq 0 \forall s, t$ (since $\overline{g} \geq n^{f0}$). Hence all ex post constraints in Problem 2 are also satisfied by $\gamma^1$.

**Proposition 3** Suppose that $\overline{g} < n^{f0}$, (i) then there exists $\beta^1$ such that for all $\beta \in (\beta^1, 1]$, $\gamma^f = \gamma^2$ (the optimal contract is the first best.), (ii) fix $\beta \in (0, \beta^1)$, then $\gamma^f$ involves the maximal ex ante payment in all periods; i.e. $g_t = \overline{g}, \forall t$, (iii) there exists an optimal time-invariant lower bound $p^s$ to the majority's residual power under $\gamma^1$ in each state. (iv) this lower bound $p^s$ is increasing in each state.

Proof:

(i) A shown above, all ex ante constraints in problem 2 are satisfied by $\gamma^f$. For all $\gamma^f$ to satisfy party M's ex post self-enforcing constraint,

$$ v \left( p^{fs} \right) - v (G - g_t) + \frac{\beta}{1 - \beta} E_q \left\{ v (p^{fq}) - v (G) \right\} \geq 0, \forall s \quad (3) $$
We may set \( g_t = \overline{g} \) to make the constraint less binding and then adjust \( r^s_t \) to make \( p^s_t = p^f_s \) for all \( s \) and \( t \). Then (3.1) requires that
\[
\beta \geq \frac{v(G - \overline{g}) - v(p^f_s)}{v(G - \overline{g}) - v(p^f_s) + E_q\{v(p^f) - v(G)\}}
\]

If \( \beta \) satisfies (3.1) for \( s = 0 \) then it will satisfy (3.1) for all \( s \) since party M's ex post constraint is the most binding under \( \gamma^f \), in state 0. Hence
\[
\beta_1 = \frac{v(G - \overline{g}) - v(p^0)}{v(G - \overline{g}) - v(p^f_s) + E_q\{v(p^f) - v(G)\}}
\]

and (3.1) is satisfied by all \( \beta \in [\beta_1, 1] \). Note that since \( E_q\{v(p^f) - v(G)\} \geq 0 \), \( \beta_1 \leq 0 \).

(ii)

Suppose the optimal contract is such that there exists some history \( h_{t-1} \) with \( g_t(h_{t-1}) \neq \overline{g} \). If \( g_t \) is set at \( \overline{g} \), adjusting \( r^s_t \) such that \( p^s_t \) remains unchanged for all \( s \), then ex ante constraints are satisfied. But since \( v(G - g_t) \) is now lower, ex post self-enforcing constraints in period \( t \) are relaxed.

In what follows, \( g_t \) is set at \( \overline{g} \), for all \( t \).

Define \( \Omega(h_{t-1}) \) to be the set of contracts satisfying the majority’s ex post self-enforcing constraints following history \( h_{t-1} \).

As in Gauthier et al (1997), we define a Pareto frontier attainable by an efficient self-enforcing contract:
\[
g(U_t) = \max_{\gamma \in \Omega(h_{t-1})} \{V(\gamma; h_{t-1}) \text{ s.t. } U(\gamma; h_{t-1}) \geq U_t\}
\]

Note that \( g(.) \) is time-invariant since all the constraints defining \( \Omega(h_{t-1}) \) and the functions \( U(\cdot; h_{t-1}) \) and \( V(\cdot; h_{t-1}) \) are forward-looking.

Since \( v(.) \) is concave and \( \Omega(h_{t-1}) \) is convex, following history \( h_{t-1} \), the set of \( U_t \) such that a self-enforcing contract for \( M \) exists is some compact interval \( I_t \) in \( \mathbb{R} \);
\[
I_t = [-K, \overline{U}]
\]
where \(-K\) is the discounted surplus of \( m \) when it pays out to \( M \) in total power in all states and all time periods. Clearly, such a contract is self-enforcing for \( M \). Also, there exists some upper bound \( \overline{U} \) on some surplus \( M \) can concede to...
m in any self-enforcing contract. Because \( u(.) \) is continuous and \( \beta \in (0,1) \) the dominated convergence theorem implies that \( \bar{U} \) is actually obtained by some self-enforcing contract. Hence \( I_t \) is closed.

\( g(.) \) is decreasing. It is strictly concave because \( u(.) \) and \( v(.) \) are strictly concave while \( \Omega \) is convex. Also, it is continuously differentiable almost everywhere [see Gauthier et al (1997) for proof].

Following any history, the optimal contract \( \gamma^1 \) must necessarily lie on the Pareto frontier; otherwise it would be possible to replace it by an efficient contract (i.e. a contract on the Pareto frontier) that dominates it. This implies that \( g(.) \) satisfies the following recursive equation (Problem 3)

\[
g(U_t) = \max_{\{p_t, U_{t+1}\}} \left\{ v(p_t^s) - v(G) + g(U_{t+1}^s) \right\}
\]

\[
s.t. \quad g(U_{t+1}^s) \geq 0, \forall s
\]

\[
v(p_t^s) - v(G) + g(U_{t+1}^s) \geq 0
\]

\[
E_s \left\{ v(p_t^s) - v(G) + g(U_{t+1}^s) \right\} \geq 0
\]

We denote by \( \beta \pi^s \alpha^s \), \( \pi^s \theta^s \) and by \( \Psi \) the Lagrangian associated with the first, second and third constraint respectively. The first two constraints are self-enforcing for M and the last constraint ensures dynamic consistency.

The first-order conditions (FOCs) for the Problem 3 with respect to \( p_t^s, U_{t+1}^s \) are, respectively

\[
(1 + \theta^s) v'(p_t^s) - \Psi u'(G + R_t^s - p_t^s) = 0 \quad (3.2)
\]

\[
(1 + \alpha^s + \theta^s) g'(U_{t+1}^s) + \Psi = 0 \quad (3.3)
\]

The envelop condition is:

\[
g'(U_t) = -\Psi \quad (3.4)
\]

**Lemma 1:** Party m’s expected surplus is decreasing over time (i.e. \( U_{t+1} \leq U_t \))

**Proof:** From (3.3) and (3.4) we get
\[(1 + \alpha^s + \theta^s) g' (U^s_{t+1}) = g' (U_t) \quad (3.5)\]

Since \(1 + \alpha^s \geq 0\) and \(g\) is decreasing and concave, then \(U^s_{t+1} \leq U_t\), for all \(s\).

Next, we show that \(\alpha^s = 0\), \(\forall s\).

If \(\alpha^s > 0\), then \(g (U^s_{t+1}) = 0\) (complementary slackness). Since \(U^s_{t+1} \leq U^s_t\), this implies that \(g (U_t) = 0\) (since \(gt(.) < 0\)). But this contradicts (3.4) and implies that \(\alpha^s = 0\), \(\forall s\).

Now using \(\alpha^s = 0\), (3.2) and (3.3) implies that
\[
\frac{v' (p^s_t)}{u' (G + R^s_t - p^s_t)} = -g' (U^s_{t+1}) \quad (3.6)
\]

The self-enforcing constraint for state \(s\) is:
\[
v (p^s_t) - v (G - G) + \beta g (U^s_{t+1}) = 0 \quad (3.7)
\]

Now there exists a maximum value of \(U^s_{t+1}\) say \(\overline{U}_s\) such that both (3.6) and (3.7) are satisfied. The corresponding level of power is \(\overline{p}^s\). Then (3.6) and (3.7) implicitly define a lower bound on \(m\)'s residual power in any state (from 3.6) and 3.7)
\[
v (p^s_t) - v (G - G) + \beta g (U^s_{t+1}) = 0 \quad (3.8)
\]

Since the LHS of the ex post constraint (3.7) is increasing in \(p^s_t\), (3.7) is satisfied for \(p^s_t \geq \overline{p}^s\).

Since the Pareto-frontier is time independent so is \(\overline{p}^s\). This proves proposition 3 (iii).

To prove 3 (iv) we simply totally differentiate (3.6) and (3.7) at \(U^s_{t+1} = \overline{U}^s\) and \(p^s_t = \overline{p}^s\). This gives us \(\frac{dp^s_t}{dR^s_t} \geq 0\).

(v) From (3.2), (3.4) and (3.6) we get
\[
(1 + \theta^s) \frac{v' (p^s_t)}{u' (G + R^s_t - p^s_t)} = \frac{v' (p_{t-1})}{u' (G + R^s_{t-1} - p_{t-1})}
\]
Suppose \( p^* (p_{t-1}, R_{t-1}, s) \geq \xi^s \). This is satisfied when \( p^*_t = p^* (.) \) and \( \theta^s = 0 \). Now suppose that \( p^* (p_{t-1}, R_{t-1}, s) < \xi^s \). Then this expression is satisfied when \( p^*_t = \xi^s \) and \( \theta^s > 0 \) (see definition of \( p^* () \) and the fact \( \theta^s > 0 \Rightarrow p^*_t = \xi^s \).)

**Proposition 4:** When neither party can commit, \( \exists \beta^2 \in (0, 1) \) such that \( \gamma^f = \gamma^f \) for all \( \beta \in [\beta^2, 1] \).

First, note that \( \gamma^f \) is stationary and hence satisfies the first two constraints (i.e. the ex ante constraints of Problem 4). \( \gamma^f \) will be self-enforcing if there exists values of \( g \) and \( \beta \) such that both ex post constraints (the last two constraints of Problem 4 are satisfied.)

Since \( n^{f_0} \) is the highest net transfer made by \( M \) to \( m \) under \( \gamma^f \), party \( m \)'s ex post constraints will be satisfied if its ex post constraint for \( S \) is satisfied. Thus, to find a \( (g, \beta) \) pair with the smallest \( \beta \) among all such pairs that satisfy all ex post constraints, we need to solve

\[
\min_{\{\beta, g\}} \beta \\
\text{subject to } v (p^{f_0}) - v (G - g) + \frac{\beta}{1 - \beta} E_q \{ v (p^{f_q}) - v (G) \} \geq 0, \\
u (G + R^{S} - p^{f_S}) - u (R^{S} + g) + \frac{\beta}{1 - \beta} E_q \{ u (G + R^{q} - p^{f_q}) - u (R^{q}) \} \geq 0.
\]

First-order conditions show that these constraints are binding, and so the solution is at their intersection. Since \( E_q \{ v (p^{f_q}) - v (G) \} \geq 0 \) and \( E_q \{ u (G + R^{q} - p^{f_q}) - u (R^{q}) \} \geq 0 \), we have \( \beta^2 < 1 \).

Note: when no ex ante transfer is allowed \( (g_t = 0) \), then \( \beta^2 \) will have to satisfy both constraints. This implies that \( \beta^2 \) will be the higher of the two \( \beta_s \) that solve these constraints with equality. Since \( g \) is no longer a choice variable in the minimization problem stated above, \( \beta^2 \) when \( g = 0 \) will in general be higher than \( \beta^2 \) when \( g \geq 0 \).

**Proposition 5:** Set \( g_t = 0 \), (i) for each state \( s \), there exists optimal time-invariant residual power levels \( \underline{p}^s \) and \( \overline{p}^s \) such that \( \underline{p}^s \leq p^*_t \leq \overline{p}^s \), \( \forall t \). (ii) both bounds are increasing in states of the world, (iii) for any history \( (h_{t-1}, s) \), the optimal residual power at time \( t \) is such that
\[ p(h_{t-1,s}) = \begin{cases} 
\xi^s & \text{if } p^*(p_{t-1}, R_{t-1}, s) < \xi^s \\
p^*(p_{t-1}, R_{t-1}, s) & \text{if } \xi^s \leq p^*(\cdot) \leq \bar{p}^s \\
\bar{p}^s & \text{otherwise} 
\end{cases} \]

(iv) assume \( g_t > 0 \) and \( \beta < \beta^2 \), the optimal value of the ex ante payment is decreasing in its expected surplus.

The proof of 5(i), 5(ii) and 5(iii) is analogous to that of Proposition 3. It may be found in Thomas and Worrall (1988).

5(iv) The FOCs for problem 6 are:

wrt \( g_t \):
\[-\sum_s \pi^s v' (G - g_t + r^s_t) - \sum_s \pi^s \theta^s \left( v' (G - g_t + r^s_t) - v' (G - g_t) \right) + \sum_s \pi^s (\lambda^s + \Psi) u'(R^s_t + g_t) - \sum_s \pi^s \lambda^s u'(R^s_t + g_t) = 0 \quad (6.1)\]

wrt \( r^s_t \):
\[-\pi^s v' (G - g_t + r^s_t) (1 + \theta^s) - \pi^s (\lambda^s + \Psi) u'(R^s_t - r^s_t + g_t) = 0, \forall s \quad (6.2)\]

wrt \( U_{t+1}^s \):
\[(1 + \alpha^s + \theta^s) f' (U_{t+1}^s) + \lambda^s + \phi^s + \Psi = 0, \forall s \quad (6.3)\]

and the envelop condition,
\[ f' (U_t) = -\Psi \quad (6.4) \]

**Lemma 2:** If \( f(.) \) is concave, then it is continuously differentiable (see Gauthier et al, 1997)

**Lemma 3:** For both \( m \) and \( M \), there exists a state \( s_i \) in which either agent’s ex post constraint is binding.

Proof: we know that when \( \beta < \beta^2 \), at least one ex post constraint binds (or else \( \gamma^2 \) is inefficient). Adding up (6.2) to (6.1), we get
\[-\sum_s \pi^s \{ \lambda^s u'(R^s_t + g_t) - \theta^s v' (G - g_t) \} = 0 \quad (6.5)\]

Since at least one of the \( \lambda^s \) or \( \theta^s \) is non zero, (6.5) implies that there exists a state \( s^1 \) for which \( \theta^s > 0 \) (implying that \( M \)'s ex post constraint is binding) and there exists \( s^2 \) for which \( \lambda^{s^2} > 0 \) (implying that \( m \)'s ex post constraint is binding).

We continue with the proof of proposition 5(iv)

By the theorem of the Maximum, the solution to problem 6 is continuous over the state variable \( U_t \) where \( f (\bar{U}) = 0 \).

Suppose there is a marginal increase in \( U_t \) but no change in \( g_t \). We show a contradiction. From the envelop condition (6.4), \( f'' (U_t) dU_t = -d\Psi \). Suppose first that
Consider all ex post constraints that were binding before the marginal increase in \( U_t \) and remain so, so that residual power remains unchanged. If the constraint was binding for \( m \) (which implies non-binding for \( M \) which implies that \( M \) does not change), then (6.1) and (6.2) requires that \( d\lambda^s = d\Psi \). Conversely, if the constraint was binding for \( m \), then

\[
- S_s \pi_s v(G - g_t) u(R_s + g_t - r_s^t) d\theta_s = - u(R_s + g_t - r_s^t) d\Psi
\]

Since \( d\Psi = 0 \), this is a contradiction if \( g_t \) remains unchanged.

Suppose now that \( f(U_t) = 0 = d\Psi dU_t \). Again, residual power changes only in unconstrained states, in which FOCs imply

\[
u'(p_t^s) \frac{u'(R_t^s + g_t + r_t^s)}{v'(p_t^s)} - u'(R_t^s + g_t) \Psi.
\]

Substituting these changes into (6.5):

\[
- \sum_s \pi^s \left\{ v'(G - g_t) \frac{u'(R_t^s + g_t - r_t^s)}{v'(p_t^s)} - u'(R_t^s + g_t) \right\} d\Psi = 0
\]

Since \( d\Psi \neq 0 \), this is a contradiction if \( g_t \) remains unchanged.

Suppose now that \( f'(U_t) = 0 = \frac{d\Psi}{dU_t} \). Since the LHS of the above decreases with \( p_t^s \), and since \( p_t^s \) is changing with \( U_t \), this implies that \( d\Psi dU_t = 0 \), a contradiction. Hence, \( g_t \) is monotonic in \( U_t \) in the range \([0, \overline{U}]\). Suppose that \( U_t = \overline{U} \). Then only \( M \) has a binding constraint. From Proposition 3 (ii), we know that \( m \) makes the maximal ex ante transfer, implying that \( g_t > 0 \). Similarly, \( g_t < 0 \) at \( U_t = 0 \). Monotonicity of \( g_t \) in \( U_t \) then implies that it is increasing in \( U_t \).

**Proposition 6:** Suppose \( s = 2 \), \( f(.) \) is concave and \( \beta < \beta^2 \). Then the expected residual power of \( m \) for \( t + 1 \) is larger than that of period \( t \) if state 1 occurs in period \( t \) and vice versa if \( s = 2 \). (ii) \( M \)'s power in period \( t \) is smaller than \( p^s (p_{t-1}, R_{t-1}, s) \) if \( s = 1 \) in period \( t \): and vice versa if \( s = 2 \).

By Lemma 3, we know that there exist binding states for each agent. Suppose that \( \lambda^2 > 0 \) and that \( \theta^1 > 0 \), then \( \lambda^1 = 0 \) and \( \theta^2 = 0 \). From FOC (6.2) and the envelop condition (6.4) we get

\[
(1 + \theta^1) \frac{v'(p_t^1)}{w'(R_t^1 + G - p_t^1)} = - f'(V_t)
\]

\[
\frac{v'(p_t^1)}{w'(R_t^1 + G - p_t^1)} - \lambda^2 = - f'(V_t) \cdot \lambda^2 \Rightarrow p_t^1 > p_t^2 \text{ (since } u() \text{ and } v() \text{ are concave)}.
\]
and $\lambda^2, \theta^1 > 0$.

But $p^1_t = \overline{p}^1$ while $p^2_t = \overline{p}^2$ (since $M$’s ex post constraint binds in state 1 while $m$’s binds in state 2). This implies that $\overline{p}^1 > \overline{p}^2$, contradicting Proposition 5(ii). Hence $\lambda^1 > 0, \theta^2 > 0 \Rightarrow \lambda^2 = \theta^1 = 0$.

(i) From (6.3) we get that for state 1

$$(1 + \alpha^1) f' (U^t_{t+1}) + \lambda^1 + \phi^1 = f' (V_t)$$

If $\alpha^1 > 0$, then $U^t_{t+1} = \overline{U} \geq U_t$ (complementary slackness, plus definition of $\overline{U}$)

if $\alpha^1 = 0$, then the above expression is

$$f' (U^t_{t+1}) + \lambda^1 + \phi^1 = f' (U_t)$$

which implies that $U^t_{t+1} > U_t$.

Similarly, we can show that $V^2_{t+1} \leq V_t$.

Thus $U^2_{t+1} \leq U_t \leq U^1_{t+1}$. Note that since this implies that $U^1_{t+1} > 0$, we have $\phi^1 = 0$; also since $f (U^2_{t+1}) > 0$, we have $\alpha^2 = 0$. ■

(ii)

$0 \leq U_t \leq \overline{U}$. FOC in period $t - 1$ imply

$$\frac{v' (p_{t-1})}{u' (G + R_{t-1} - p_{t-1})} = -f' (U_t).$$

Thus,

$$(1 + \theta^2) \frac{v' (p^1_t)}{u' (G + R^1_t - p^1_t)} = \frac{v' (p^1_t)}{u' (G + R^1_t - p^1_t)} - \lambda^1 = \frac{v' (p_{t-1})}{u' (G + R_{t-1} - p_{t-1})}$$

This implies that

$$\frac{v' (p^2_t)}{u' (G + R^2_t - p^2_t)} < \frac{v' (p_{t-1})}{u' (G + R_{t-1} - p_{t-1})} < \frac{v' (p^1_t)}{u' (G + R^1_t - p^1_t)}$$ (since $\theta^2, \lambda^1 > 0$.)

These inequalities prove Proposition 6 (ii) (by definition of $p^* (p_{t-1}, R_{t-1}, s)$.) ■
Table I: Power Sharing Agreements and Democratic Stability

<table>
<thead>
<tr>
<th>Variable</th>
<th>Polity 2 year</th>
<th>Polity 5 year</th>
<th>ACLP 2 year</th>
<th>ACLP 5 year</th>
</tr>
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<tr>
<td>Lagged Democracy</td>
<td>0.430***</td>
<td>0.427***</td>
<td>1.276**</td>
<td>2.631***</td>
</tr>
<tr>
<td></td>
<td>(3.810)</td>
<td>(3.802)</td>
<td>(2.518)</td>
<td>(3.110)</td>
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<tr>
<td>Ethnic War Dummy</td>
<td>-3.657***</td>
<td>-3.843**</td>
<td>0.963**</td>
<td>1.691***</td>
</tr>
<tr>
<td></td>
<td>(-2.576)</td>
<td>(-2.532)</td>
<td>(2.159)</td>
<td>(2.611)</td>
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<tr>
<td>Cost of War</td>
<td>0.014</td>
<td>0.174</td>
<td>0.002</td>
<td>0.097</td>
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<tr>
<td></td>
<td>(0.037)</td>
<td>(0.449)</td>
<td>(0.014)</td>
<td>(0.526)</td>
</tr>
<tr>
<td>Duration of War</td>
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<td>-0.007</td>
<td>0.000</td>
<td>0.003</td>
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<tr>
<td></td>
<td>(0.163)</td>
<td>(-0.384)</td>
<td>(0.103)</td>
<td>(0.585)</td>
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<td>Number of Factions</td>
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<td>-1.174</td>
<td>0.542</td>
<td>1.061**</td>
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<td></td>
<td>(-0.516)</td>
<td>(-0.351)</td>
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<td>Number of Factions Sq</td>
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<td>(0.330)</td>
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<td>0.003***</td>
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<tr>
<td></td>
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<td>(2.875)</td>
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<td>(-0.744)</td>
<td>(-0.186)</td>
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<tr>
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<td>-1.073*</td>
<td>0.530**</td>
<td>0.914***</td>
</tr>
<tr>
<td></td>
<td>(1.741)</td>
<td>(-1.920)</td>
<td>(2.208)</td>
<td>(3.974)</td>
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<tr>
<td><strong>Power Sharing</strong></td>
<td><strong>2.812</strong></td>
<td><strong>3.151</strong></td>
<td><strong>-1.259</strong></td>
<td><strong>-1.300</strong></td>
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<td>(1.800)</td>
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<td>(-1.803)</td>
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<td>-6.583**</td>
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<tr>
<td></td>
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<td>(1.218)</td>
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<td>(-2.424)</td>
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<td>58</td>
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<td>0.45</td>
<td>0.36</td>
<td>0.57</td>
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Conclusion:
-Powering sharing pacts increase democracy (weakly significant)