A policy-based interpretation of the Shapley-Shubik index

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A quantity of obvious significance in analyzing weighted majority voting to decide policy outcomes is the probability, \( p_i \), that agent \( i \)'s ideal outcome is at the median of agent ideals on an arbitrary policy dimension. Think of this as a dimension viewed from behind a veil of ignorance about the ordering of agent ideals, or a random one-dimensional issue that might arise for decision, on which the median agent can typically achieve his/her ideal outcome under majority voting. In a multidimensional setting, \( (p_i)^n \) is the baseline probability that agent \( i \)'s ideal outcome is at the dimension-by-dimension median (DDM) of agent ideals in an arbitrary \( n \)-dimensional policy space. This is of concern, for example, to any model describing a “division-of-the-question” institutional mechanism (Kadane 1972), such as the allocation of policy jurisdictions to legislative committees (Shepsle 1979; Shepsle and Weingast 1987) or to holders of cabinet portfolios (Austen-Smith and Banks 1990; Laver and Shepsle 1996). In such models, an agent at the DDM can typically achieve his/her ideal outcomes.

It is striking that \( p_i \) is in fact identical to the Shapley-Shubik index, \( s_i \), for agent \( i \) (Shapley and Shubik 1954). To see this, note that \( s_i \) is calculated by listing all possible orders of agglomeration of agents into coalitions and counting the proportion of these for which agent \( i \) is pivotal – for which adding \( i \) turns a losing coalition into a winning one. Instead of thinking of all possible agglomeration orderings, think of all possible orderings of agent ideals on an arbitrary dimension. Self-evidently on this interpretation, \( s_i \) is also the proportion of all orderings for which \( i \) is median, which is the probability \( i \) is median in a random ordering. Thus \( p_i = s_i \), while \( (s_i)^n \) is the probability agent \( i \)'s ideal is at the DDM in an arbitrary \( n \)-dimensional policy space.

In other words the Shapley-Shubik index gives the baseline probability that agent \( i \) is median, and can typically achieve his/her ideal outcome in a majority vote, on a random one-dimensional issue. And \( (s_i)^n \) is the baseline probability s/he can typically achieve ideal outcomes under majority voting in an \( n \)-dimensional policy space with a division-of-the-question environment. In this important sense, and contrary to received wisdoms, the Shapley-Shubik index is as much about policy as about the division of a fixed set of spoils.

REFERENCES