Contracting without Commitment: Economic Transactions in the Political Economy of States and Mafias*

Ethan Bueno de Mesquita† Catherine Hafer‡

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Abstract
Mafias play a critical role as alternative providers of contract enforcement and revenue protection in transition economies. However, the mafia can also be used as a tool of extortion, creating a commitment problem between firms: firms are better off if no one hires the mafia, but, because of the possibility of extortion, each firm finds it individually rational to do so. Strengthening the government relative to the mafia by increasing tax revenue offers a means by which the firms can mitigate this commitment problem. The effectiveness of this tactic is limited by the agency problem between the firms and the government, which chooses to pocket tax revenue under threat of electoral sanction or to spend it on law enforcement. Analysis of a model incorporating these features demonstrates the necessity of the government’s seeking out and confronting the mafia, rather than relying on reports from threatened firms, in order to limit the mafia’s role in the economy. The level of government corruption is weakly increasing in the tax rate, but government spending on law enforcement is not monotonic in tax rate, first increasing to a plateau and then decreasing. If we admit the possibility that the mafia and the government may collude to the extent that the government is bribed not to seek a confrontation actively, then the tax rate is weakly higher and the mafia is weakly more pervasive than in the case where no such collusion is possible.

1 Introduction
In the existing literature, mafias are often viewed as benign competitors of the state, supplying comparable services at comparable terms to sustain an underground economy (Schelling 1984; Gambetta 1993; Grossman 1995; Johnson et al. 1997; Hay and Shleifer 1998; Skaperdas 2001; Varese 2001; Alexeev et al. 2003); other times mafias are treated as an extortionist or bandit (Konrad and Skaperdas 1998).† There is some truth to both accounts. While the

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†Assistant Professor of Political Science, Washington University, St. Louis, e-mail: ebuenode@arts.wustl.edu.

‡Assistant Professor of Politics, NYU, e-mail: catherine.hafer@nyu.edu

1Some view the mafia and the state as analagous, recognizing not only that the mafia may provide valuable services but also that the state may be corrupt or extortionary itself (Tilly 1985, Konrad and Skaperdas 1999).
enforcement of contractual arrangements and property rights is one of the essential public goods that governments typically provide, governmental weakness or corruption create an opportunity for mafias to compete successfully with governments to provide these services. Gambetta and Reuter (1995) conclude that mafias often provide real direct benefits, including “the enforcement of a variety of allocation agreements among independently owned firms, with racketeer income being payment for service.” Systematic surveys of Russian shopkeepers on their interactions with rackets confirm this conclusion (Frye and Zhuravskaya 2000). Nonetheless, the emergence of a strong mafia, and the dependence of the economy on its services, carries with it the risk of extortion, since a mafia is generally not accountable to the firms under its protection.2

In order to better understand why mafias emerge and persist, in spite of their obvious liabilities, we present a model of the relationship between three key types of players: governments, mafias, and economically productive agents (firms). As in earlier work, the mafia is an alternative provider of contract enforcement and revenue protection. However, the current model departs from the literature in two principal respects: first, the mafia can serve as a tool of extortion against firms that do not pay it, and second, the government is to some extent accountable to the firms, who choose whether or not to keep it in office and what taxes to levy against themselves in order to fund it. Moreover, we explicitly model the government’s choices of both whether or not to combat mafia dominance of the political economy through law enforcement and whether to invest tax revenues in these efforts or to expropriate them. These features of the model allow us to address a central question behind the emergence of mafias: why do (elite) economic actors sometimes prefer patronizing the mafia to strengthening the state? Franchetti, a 19th century observer of the effects of the Sicilian mafia, writes “…it is the upper classes that... allow [the mafia] to survive. If the upper classes wished to destroy such industry, they would have both the means and the moral authority to do so” (quoted in Bandeira 2003).

The fact that firms may hire the mafia to extort other firms, and not just to enforce contractual agreements between them, creates a commitment problem between the firms. The firms must each choose whether to depend on the government for contract enforcement or to hire the mafia for a fee. If only one firm hires the mafia, that firm may use the mafia to extort money from the other firm. If both firms hire the mafia, however, neither can extort the other.3 The mafia, in this case, runs a protection racket, collecting fees from both firms. Thus the firms prefer the outcome in which neither hires the mafia to the outcome in which both do, but neither can credibly commit to not hiring the mafia. In capturing the incentives of the classic prisoners’ dilemma, this part of our model is similar in spirit to that of Bandeira (2003).4

The firms, acting as voters, endogenously choose how well to fund the government through

2The existence of a mafia may lead to economic inefficiency in other ways, as well. Its need to hide its identity and its actions limits its ability to operate efficiently relative to a legitimate state (Smith and Varese 2001, Baccara and Bar-Issac 2005). There is also evidence that it undermines competition, creating numerous monopolies (Braguinsky 1999).

3For conditions under which a mafia will not double cross a firm in a repeated game framework see Dixit (2004), chapter 4.

4Bandeira uses a common-agency model to demonstrate that mafia profits from protection services are greater when land is more fragmented and finds historical support for this result.
taxation. They are willing to pay taxes because law enforcement can mitigate the commit-
ment problem. Government law enforcement can potentially make illegal activity so costly
that it drives the mafia out of business. Even when the mafia is not entirely eliminated,
law enforcement can reduce the fees the mafia is able to demand in a protection racket by
decreasing the risk of expropriation that a firm faces should it refuse to hire the mafia. Of
course, the weakening of the mafia comes at a price to the firms—taxation. Moreover, the
firms also have to worry about government corruption—the government may expropriate
resources rather than spend them on law enforcement. The firms use electoral incentives to
try to solve this moral hazard problem.\footnote{For another view of why firms might be unwilling to fund a government when mafia enforcement of contracts is an option, see Sonin (2002).}

These trade-offs drive several key results of our model. We demonstrate the government
investment in law enforcement is not monotonic in tax revenue. For low revenue levels, the
electoral incentives are sufficient to insure that the government invests all of the revenue
into law enforcement activities. Hence, for low levels of taxation, increasing taxation results
in higher levels of law enforcement, a higher probability of government success against the
mafia, and a smaller proportion of the firms’ after-tax profits going to the mafia. However,
as government revenue grows, the temptation to be corrupt also grows, and (if the benefits
of office remain the same) electoral incentives are no longer adequate to prevent government
corruption. The government skims a larger and larger proportion of revenues as taxation
grows. For intermediate levels of taxation, the level of investment in law enforcement and
the probability of government success against the mafia plateau, but for high rates of taxa-
tion, government corruption becomes so extensive that investment into law enforcement, in
absolute terms, decreases, and hence the government’s ability to defeat the mafia actually
decreases as well.

Ironically, the proximate cause of the decrease (in absolute terms) in law enforcement
is the reduced presence of the mafia. While the amount that the mafia is able to demand
from the firms is decreasing the level of law enforcement, the mafia itself remains ubiquitous
at low and intermediate levels of taxation. But for sufficiently high levels of taxation and
sufficiently punitive measures by the government, the mafia becomes more rare, choosing
on some occasions to exit the market rather than to risk a confrontation with the govern-
ment. Because the mafia is less prevalent, the government has less occasion to use its law
enforcement apparatus, and hence reduces its investment into it.

The model also provides insights into the role of pro-active law enforcement efforts, in
which the government attempts to monitor mafia activities and intervene independently,
rather than simply respond to complaints. We show that reliance solely on the latter,
purely passive, approach to law enforcement results in the government being displaced en-
tirely by the mafia. In an extension of the model, we show how the possibility of collusion
between the mafia and the government decreases taxation and increases the prevalence of
the mafia in the economy.
2 The Model

There are four players: two firms, a mafia, and a government. The sequence of play is as follows. At the beginning of the game, the firms have a contractual relationship, the total pre-tax value of which is normalized to 1. Nature chooses the division of the benefits of the contract, and determines which firm (called “firm 1” or F1) gets proportion $\alpha \in (0, \frac{1}{2})$ of those benefits, and which firm (“firm 2” or F2) gets $(1 - \alpha)$. Both the firms and the mafia observe Nature’s selections, but the government does not. Next, the firms choose the tax rate $\tau \in [0, 1]$. The tax rate is determined through a weighted average of the two firms preferred tax rates. After the firms choose the tax rate, the government chooses a proportion of the tax revenue collected to commit to law enforcement, $\lambda \in [0, 1]$. Neither the firms nor the mafia observe $\lambda$. Next, the mafia sets the fees ($\phi = (\phi_1, \phi_2)$) that each firm must pay for its services and makes a take-it-or-leave-it offer to each firm of the corresponding fee. Notice that we allow the mafia to price discriminate between the two firms. Following these offers, firms simultaneously choose whether to hire the mafia. When a firm hires the mafia, it pays the fee regardless of outcome.

Let $\mu_{Fi}$ be the probability that $Fi \in \{F1, F2\}$ hires the mafia. If one firm does not hire the mafia while the other does, the former firm appeals to the government, which then challenges the mafia. If both firms accept the mafia’s offers, the government official must choose whether to challenge the mafia or not. Denote the government’s choice of a probability with which to challenge the mafia, given that both firms hire the mafia, by $\gamma$. The idea underlying these assumptions is that, if just one firm hires the mafia, the government is obliged to attempt to provide that firm with protection from extortion. However, if both firms hire the mafia, then the government has the option of whether or not to engage in a fight against the mafia’s protection racket.

The probability that the government defeats the mafia when they are in conflict is function of the government’s level of investment in law enforcement $(\lambda \tau)$, $f : [0, 1] \rightarrow [0, 1]$. We assume that $f(\cdot)$ is increasing, concave, and satisfies $\lim_{x \rightarrow 0} f'(x) = \infty$.

If there is a conflict between the mafia and government, and the mafia loses, the mafia bears a cost $k$, which we interpret as the punishment imposed by the government. The winner of this conflict determines the division of post-tax benefits $(1 - \tau)$ between the firms. If the government wins, it imposes the outcome that corresponds to the actual realization of the contract, i.e., $(\alpha(1 - \tau), (1 - \alpha)(1 - \tau))$. If the mafia wins, it also imposes the actual realization if it was hired by both firms. However, if only one firm hired the mafia, and the mafia wins, it gives the entire surplus $(1 - \tau)$ to the firm that hired it. This assumption captures the idea that government enforcement is essentially fair, reflecting the agreed upon contract, but that firms hire the mafia to try to extort more than their rightful shares from

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As will be shown below, because firms’ equilibrium choices are identical, we can think of the present model as being one with any number of randomly matched firms with the equilibrium we solve for interpreted as the symmetric equilibrium of the corresponding game.

As will become clear, in equilibrium the firms will have identical preferences over the tax rate.

We assume that the mafia’s activities, including fighting the government, are financed by the fees paid by the firms, and that the mafia’s claims of its ability corresponding to the informational assumptions of the model are “credible” - guaranteed by some background repeated interaction that generates the mafia’s reputation.
Firms choose tax rate \((\tau)\) Nature determines contract division \((\alpha)\) Government chooses allocation of revenue to law enforcement \((\lambda)\) Mafia sets fees \((\phi)\) Firms choose whether to hire the Mafia \((\mu)\) If both firms hire the Mafia, Government chooses whether to challenge \((\gamma)\) Outcome of any conflict determined Firms choose whether to replace government official \((\rho)\)

Figure 1: Timeline of the game

the other firm. If both firms hire the mafia, this has an offsetting effect—neither firm can extort from the other.\(^9\) If only one firm hires the mafia, and this mafia is able to prevail over government law enforcement, then extortion occurs.

After the outcome of any conflict is determined and benefits are divided, firms choose whether or not to reelect the government, which is understood to compete against an otherwise identical challenger. The probability of reelection \(\rho\) can be represented by a finite-dimensional vector specifying the probability of reelection in each observationally distinct (for the firms) situation. The government receives a payoff, \(R\), if it is reelected and 0 if it is not reelected. The timeline of the game is summarized in Figure 1.

The mafia’s utility is its net revenue (fees collected minus costs imposed in defeat). The government’s utility is any tax revenue not spent \(((1 - \lambda)\tau)\) plus any electoral payoff. Each firm’s utility is the benefit it realizes from the contract (as enforced) net of taxes, minus the fee paid.

There are four observably distinct situations under which the firms must decide whether or not to reelect:

1. There is conflict between the mafia and government and the mafia wins.
2. There is conflict between the mafia and government and the government wins.
3. Both firms hire the mafia and there is no conflict.
4. Neither firm hires the mafia and there is no conflict.

Let \(M\) be the event that the mafia wins and \(G\) be the event that the government wins, given that there is conflict between the mafia and government. Let \(NG\) be the event that no conflict between the mafia and government occurs after neither firm has hired the mafia. Let \(NM\) be the event that no conflict between the mafia and government occurs after the firms have hired the mafia. The set \{\(M, G, NM, NG\)\} can be thought of as the range of the outcome function, whose arguments include \(\mu, \gamma, \lambda, \) and \(\tau\); to simplify notation, we suppress the functional representation below.

Let \(\rho_M, \rho_G, \rho_{NM}, \) and \(\rho_{NG}\) be the probabilities of reelection that correspond to each of the four outcomes enumerated above, respectively, where \(\rho = (\rho_M, \rho_G, \rho_{NM}, \rho_{NG}) \in [0, 1]^4\).

\(^9\)We justify this assumption in more detail later in the analysis.
Given that the firms are indifferent over $\rho$ at the time of the election, their optimal choice is induced by their preferences over the effects of that choice on other players’ behavior.

3 Equilibrium

Our solution concept is Perfect Bayesian Equilibrium, which requires that, after every possible history of play, the corresponding player choose an action that is optimal for her given her beliefs after that history. To avoid the proliferation of notation, we do not explicitly define the players’ beliefs about the unobserved action $\lambda$, but these beliefs are nonetheless a Bayesian inference from the equilibrium strategies whenever possible, as required by the Perfect Bayesian equilibrium concept.

Additionally, we require that, when a player is capable of credibly committing to any of a set of behavioral strategies, she play a behavioral strategy that maximizes her \textit{ex ante} expected utility at the earliest point in the history of the game at which such a credible commitment is possible. This requirement restricts our attention to those equilibria in which the firms induce, via their choice of a reelection rule ($\rho$), that behavior on the part of the mafia and government that the firms prefer. This is akin to assuming the firms vote retrospectively. Because firms’ reelection decisions do not affect their future utilities, they are, at the point of choice, indifferent between reelecting the government and not doing so. Thus, the firms’ choice of the re-election rule $\rho$ that maximizes their expected utility by inducing the “right” incentives of the government and mafia is credible. We focus our analysis on such re-election rules.

3.1 A Benchmark

An instructive question, within this context, is whether it is important that the government have the ability to challenge the mafia even when neither firm solicits the government’s help. What, for instance, would happen if the government could only engage in conflict with the mafia if appealed to by one of the firms? This would be equivalent to restricting $\gamma$ to be equal to 0. It turns out that if the government does not have the authority to challenge the mafia on its own, then regardless of tax policy or investment in law enforcement, both firms will hire the mafia. Moreover, the combination of the firms’ ability to appeal to the government for protection against the mafia and the arbitrarily stiff penalty that the government can impose on the mafia are never sufficient to induce the firms to finance the government if the government cannot also be expected to challenge the mafia when the firms do not appeal to the government directly.

\textbf{Proposition 1} \textit{If the government cannot challenge the mafia unless appealed to by one of the firms (that is, $\gamma$ is restricted to be 0), the game has a unique equilibrium in which the mafia completely replaces the government as the provider of enforcement services and the firms choose not to fund the government at all ($\tau^* = 0$).}

\textbf{Proof.} For ease of exposition, we defer the proof of this Proposition until section 3.3.

As we will see in the remainder of the paper, allowing the government to challenge the mafia, unsolicited, significantly alters players’ equilibrium play.
3.2 Hiring the Mafia

As noted earlier, the government makes two choices: the level of investment into law enforcement ($\lambda$) and the probability of challenging the mafia when both firms hire the mafia ($\gamma$). Somewhat surprisingly, the firms’ induced preferences over $\gamma$ at the time of the government’s action are such that, once both firms have paid the mafia, they are expectationally indifferent between the government challenging with certainty ($\gamma = 1$) and the government not challenging ($\gamma = 0$). This does not, of course, mean that firms are indifferent over the government’s strategy. Rather, firms care only to the extent that government behavior impacts the fees the mafia charges. This intuition is formalized in the following lemma which is instrumental in solving for equilibrium behavior.

**Lemma 1** The firms’ preferences over government action $\gamma$ are completely induced by the effects of that action on the mafia’s choices.

**Proof.** See the appendix. 

Because the firms are indifferent over all $\gamma \in [0, 1]$ at the time of the government’s action, the firms can credibly commit to any electoral response to that action at an earlier point in the game, and thus they can use electoral incentives to elicit that behavior on the part of the government that has the most desirable effect on the behavior of the mafia. Lemma 1 also implies that the firms’ decisions to hire the mafia and even the mafia’s choice of fees, conditional on its offering its services, are independent of $\gamma$.

Consider now the firms’ (possibly mixed) choice of whether to hire the mafia ($\mu$). We use two facts in deriving the firms’ expected utilities over their choice of whether to hire the mafia. First, as demonstrated in Lemma 1, if both firms hire the mafia, their expected utility from the contract is the same whether or not the government challenges. Second, if both firms hire the mafia they get the same contract division that the government would enforce.\(^{10}\)

Each firm will pay the fee named if and only if that fee is smaller than the expected loss associated with not paying it, which depends in turn on the probability that the other firm hires the mafia, the probability that the mafia defeats the government, the tax rate, and the disparity between the firms. By comparing firm 2’s expected utility from hiring the mafia to its expected utility from not doing so, given the probability that firm 1 hires the mafia ($\mu_1$), we can find the upper bound on the fee the mafia can charge ($\phi_2$) such that firm 2 prefers to hire:

\[
(1 - \tau)(1 - f(\lambda \tau))[\mu_1(1 - 2\alpha) + \alpha] \geq \phi_2. 
\]

The upper bound on $\phi_1$ is found analogously:

\[
(1 - \tau)(1 - f(\lambda \tau))[\mu_2(2\alpha - 1) + (1 - \alpha)] \geq \phi_1. 
\]

\(^{10}\)This second fact may seem like a strong symmetry assumption. To see why it is a reasonable reduced form, suppose that the mafia favored one firm or the other in its enforcement of the contract when hired by both firms. In this case, the disadvantaged firm would have an incentive to appeal to the government rather than hire the mafia. To counter-act this incentive, the mafia would have to lower the fee it charged the disadvantaged firm or lose that firm as a customer and be forced into conflict with the government. The mafia’s fee maximizing strategy, then, is to treat the two firms equally if hired by both.
These bounds on the fees the mafia can extract highlight how government law enforcement spending can mitigate the commitment problem even while the mafia remains ubiquitous in the economy. When the government invests in law enforcement, the risk to a firm of not hiring the mafia when the other firm does hire the mafia is lowered. This is because high levels of law enforcement spending make it more likely that the government will prevail in its law enforcement efforts and the firm that did not hire the mafia will, therefore, not be extorted. That is, law enforcement spending reduces the risk associated with the commitment problem. Consequently, the fees that the firms are willing to pay (and that the mafia can, therefore extract) are decreasing in the level of government spending on law enforcement. This intuition is formalized in the following result.

**Proposition 2** The maximal fee the mafia can extract from either firm is decreasing in the level of government spending on law enforcement.

**Proof.** The result follows from the fact that the left-hand sides of equations (1) and (2) are both decreasing in $\lambda \tau$. The derivatives with respect to $\lambda \tau$ are $- (1 - \tau) f'(\lambda \tau)[\mu_1(1 - 2\alpha) + \alpha] < 0$ and $- (1 - \tau) f''(\lambda \tau)[\mu_2(2\alpha - 1) + (1 - \alpha)] < 0$, respectively.

Figure 2 illustrates the actions taken by the two firms for all possible fees charged by the mafia ($\phi$). If the fees charged to each firm are sufficiently high, then neither firm hires the mafia (region $(0, 0)$). Similarly, if the fees are sufficiently low, both firms hire the mafia (region $(1, 1)$). The reason these two regions are not symmetric is that the firms obtain different shares of the contract ($\alpha \neq \frac{1}{2}$). Of course, since the mafia can price discriminate, it can also set the fees such that only one firm hires it, or it can choose moderate fees for both firms that induce a mixed strategy equilibrium ($\tilde{\mu}_1, \tilde{\mu}_2$) in this subgame.

Notice that the two firms have different motivations underlying their behavior. Recall that firm 1 is financially disadvantaged relative to firm 2 ($\alpha < \frac{1}{2}$). As a result, firm 1 has
more to gain by using the mafia to prey on firm 2, but less to lose if firm 2 should prey on it. As such, firm 1 is more willing to hire the mafia if firm 2 has not hired it and thus is susceptible to extortion. Firm 2, however, has greater incentive to protect what it already has (since it already has more), and less incentive to expropriate firm 1’s relatively more modest holdings. Firm 2, then, is tempted to hire the mafia not in order to extort firm 1’s resources but, rather, to provide itself with protection from extortion by firm 1. Hence, firm 2 is more willing to hire the mafia if firm 1 has hired it as well. This logic gives rise to the following proposition:

Proposition 3 Both firms have a dominant strategy of hiring the mafia if \( \phi_i < \alpha(1 - \tau)(1 - f(\lambda \tau)) \), and of rejecting the mafia’s offer if \( \phi_i > (1 - \alpha)(1 - \tau)(1 - f(\lambda \tau)) \). If \( \phi_i \in (\alpha(1 - \tau)(1 - f(\lambda \tau)), (1 - \alpha)(1 - \tau)(1 - f(\lambda \tau))) \), the financially disadvantaged firm (F1) is strictly predatory, seeking to extort the other firm by hiring the mafia only if the other firm does not. In contrast, the financially advantaged firm (F2) is strictly defensive, hiring the mafia only if the other firm does.

Proof. See the appendix. ■

3.3 The Mafia’s Fees

Consider next the mafia’s choice of what fee to charge each firm (\( \phi \)). If the mafia chooses \( \phi \) such that both firms hire it (\( \mu = (\hat{\mu}_1, \hat{\mu}_2) \)), then its expected utility is

\[
E[u_M(\phi, \mu = (\hat{\mu}_1, \hat{\mu}_2)); \lambda^*, \gamma^*, \tau)] = \hat{\mu}_1 \phi_1 + \hat{\mu}_2 \phi_2 - k f(\lambda \tau) [\hat{\mu}_1 (1 - \hat{\mu}_2) + \hat{\mu}_2 (1 - \hat{\mu}_1) + \gamma \hat{\mu}_1 \hat{\mu}_2],
\]

which is linear in \( \phi_1 \) and in \( \phi_2 \). This linearity implies that if the mafia chooses fees that will induce the mixed strategy it will only consider fees that induce one of the four corners of the mixed strategy region in Figure 2. Since these four corners correspond to the pure strategy equilibria, we can restrict attention to the optimal fee choices that induce each of the four pure strategy combinations. Moreover, all fee choices that induce only one firm to hire the mafia are dominated by the optimal fee choice that induces both firms to hire the mafia. The reason for this is two-fold. First, the mafia extracts higher total fees when it is hired by both firms. Second, the government and mafia are certain to be in conflict if only one firm hires the mafia, whereas if both firms hire the mafia, then the government and mafia are in conflict only if the government choose to challenge, which occurs with probability \( \gamma \). Consequently, we can restrict attention to the optimal fee that induces both firms to hire the mafia and fees that induce neither firm to hire the mafia.

If the mafia chooses \( \phi \) such that both firms pay, i.e., \( \mu = (1, 1) \), then it must prefer the highest possible fees such that they do. Hence, from (1) and (2), we get \( \phi_1 = \alpha(1 - \tau)(1 - f(\lambda^* \tau)) \) and \( \phi_2 = (1 - \alpha)(1 - \tau)(1 - f(\lambda^* \tau)) \), and so:

\[
E[u_M(\phi, \mu = (1, 1); \lambda^*, \gamma^*, \tau)] = (1 - \tau)(1 - f(\lambda^* \tau)) - k \gamma^* f(\lambda^* \tau)
\]

If the mafia chooses \( \phi \) such that neither firm hires it (\( \mu = (0, 0) \)), then

\[
E[u_M(\phi, \mu = (0, 0); \lambda^*, \gamma^*, \tau)] = 0.
\]
Comparing these expected utilities, the mafia prefers to be hired at its optimal fees if and only if its expected punishment is less than the total fees it collects.

\[ \gamma^* k f(\lambda^* \tau) < (1 - \tau)(1 - f(\lambda^* \tau)). \]  

The following lemma formally summarizes the above argument characterizing mafia’s behavior.

**Lemma 2** If \( \gamma^* k \leq (1 - \tau)\frac{(1 - f(\lambda^* \tau))}{f(\lambda^* \tau)} \), then the mafia offers its services for \( \phi_1 = \alpha(1 - \tau)(1 - f(\lambda^* \tau)) \) and \( \phi_2 = (1 - \alpha)(1 - \tau)(1 - f(\lambda^* \tau)) \), correctly anticipating that both firms will accept its offers. If \( \gamma^* k > (1 - \tau)\frac{(1 - f(\lambda^* \tau))}{f(\lambda^* \tau)} \), then the mafia offers its services for \( \phi_1 > (1 - \alpha)(1 - \tau)(1 - f(\lambda^* \tau)) \) and \( \phi_2 > \alpha(1 - \tau)(1 - f(\lambda^* \tau)) \), correctly anticipating that both firms will reject its offers.

We can now readily see why Proposition 1, which characterizes the equilibrium when \( \gamma \) is restricted to 0, holds. Recall that that modified game has a unique equilibrium in which the mafia completely replaces the government as the provider of enforcement services and the firms choose a tax rate of 0. To see why that is so, observe that the expected utilities in the modified game (i.e. when \( \gamma \) is restricted to 0) are identical to those in the unmodified game, so both the firms’ strategies with respect to hiring the mafia and, hence, the mafia’s expected utility, evaluated at \( \gamma = 0 \), are the same. It is clear that equation (3) evaluated at \( \gamma = 0 \) is always greater than equation (4), so the first part of Lemma 2 describes the optimal choice of \( \phi \). On the equilibrium path, \( \phi_1 > 0, \phi_2 > 0 \), and both firms hire the mafia. Because the government never fights the mafia in equilibrium, the government has no incentive to invest in law enforcement (\( \lambda^* = 0 \), for all \( \rho \)), which implies that the firms have no incentive to fund the government (\( \tau^* = 0 \)), which completes the proof of Proposition 1.

### 3.4 Investment in Law Enforcement

The government chooses the amount of tax revenues to allocate to law enforcement (\( \lambda \)) such that

\[ \lambda \in \arg \max E[u_G(\lambda, \tau, \rho^*(\tau, \cdot), \mu^*(\phi^*(\cdot, \cdot)), \gamma^*(\rho^*(\cdot, \cdot), \lambda, \cdot)]. \]  

The following result is useful in finding the government’s optimal allocation.

**Lemma 3** The government will never choose \( \lambda \) such that \( \gamma \in (0, 1) \).

**Proof.** See the appendix. ■

This lemma implies that we can restrict attention to cases where, if both firms hire the mafia, the government never challenges (\( \gamma = 0 \)) or challenges with certainty (\( \gamma = 1 \)). Moreover, if the government never challenges in equilibrium, then it has no incentive to invest in law enforcement since it is never called upon to fight. Likewise, if the mafia chooses fees such that neither firm hires it, the government has no incentive to invest.

In choosing how to allocate tax resources the government considers several factors. On the one hand, spending tax revenues on law enforcement is costly to the government because revenues directed toward law enforcement cannot be expropriated. On the other hand, investment in law enforcement may benefit the government by increasing the probability
that the government defeats the mafia when they are in conflict, an outcome that helps the government achieve reelection. And, of course, the government’s allocation decision also affects strategic play by other players (for instance, the mafia’s choice of fees). How these various considerations balance out depends on the level of tax revenues the firms provide to the government.

If the tax rate is sufficiently small, then the marginal increase in the government’s probability of winning a conflict with the mafia, and hence being reelected, from committing the last unit of tax revenue to law enforcement is greater than the marginal benefit of misappropriating the funds. Thus the government commits all of the revenue to law enforcement when taxes are sufficiently low.

For somewhat higher tax rates, the government allocates revenue to law enforcement only until the marginal expected electoral benefit from increasing the probability of victory over the mafia equals the marginal benefit of misappropriating the funds. This absolute amount of law enforcement \( \lambda \tau \) is constant as the tax rate increases, so the proportion of revenue committed to law enforcement decreases as revenue increases.

As long as the tax rate is not too high, the mafia will prefer to collect its fees and risk punishment from the government; however, for sufficiently high tax rates, the fees that the mafia can demand from the firms are too small to warrant such risks. In such circumstances, if the government chooses a high level of law enforcement, so that the mafia’s risk of punishment is high, the mafia will charge a fee that leads neither firm to hire it, effectively pricing itself out of the market. But if neither firm hires the mafia, then the government’s choice to commit resources to law enforcement is not optimal since it is never in conflict with the mafia. In this circumstance, the government should deviate to no investment in law enforcement. But, when it does so, the mafia faces little risk of punishment, and thus wants to charge fees that lead both firms to hire the mafia, which again makes the government’s resource allocation decision sub-optimal. Thus, for these moderately high tax rates, the government commits fewer resources to law enforcement and the mafia randomizes between charging fees that induce the firms to hire it and fees that induce the firms not to hire it. This strategy profile limits the mafia’s risk of punishment by the government. And, the government’s limited investment in law enforcement is rational because conflict with the mafia occurs relatively rarely (since the mafia sometimes prices itself out of the market) but not never.

For very high tax rates, the fees that the mafia can obtain from the firms are too small to warrant risking confrontation with even the weakest government, i.e. one that invests nothing in law enforcement. As such, the mafia prices itself out of the market and, since the firms never hire the mafia, the government invests no tax revenues in law enforcement.

The proportion of tax revenue that the government commits to law enforcement is formally derived in the appendix, and the absolute amount that the government spends on law enforcement, the fees charged by the mafia, and the likelihood of the firms hiring the mafia are each represented as a function of the tax rate in Figure 3.

3.4.1 Taxation, Government Corruption, and Law Enforcement

The firms fund the government through taxation in order to increase law enforcement and, thereby, weaken the mafia. The question arises, then, whether increasing government funding will actually lead to an increase in law enforcement, given the moral hazard problem that
the firms face vis-à-vis the government.

As already discussed, the government, in choosing how much to invest in law enforcement, balances two types of incentives. On the one hand, it is tempted to expropriate tax revenues. On the other hand, it has electoral incentives to invest in law enforcement. These electoral incentives come from the firms’ threat not to reelect the government should it fail to challenge and defeat the mafia. The government, then, will act in an increasingly uncorrupt manner as the electoral threat associated with losing to the mafia increases relative to the appeal of expropriating tax revenues.

The tax rate affects government incentives for expropriation. We will refer to the percentage of total tax revenues that the government expropriates as the level of government corruption. Although it is not illustrated directly, Figure 3 makes clear that, somewhat surprisingly, the level of government corruption is weakly increasing in the amount of taxation. That is, the more money the government collects in taxes, the higher the percentage of the money it expropriates. In region A, there is no corruption. The government invests all revenues in law enforcement because the electoral incentives loom large when the size of the government budget is small. At a somewhat higher level of taxation, the incentives for corruption become sufficiently strong that the government begins to expropriate tax resources. Within this range (region B), corruption is increasing in the level of taxation—the government keeps total law-enforcement spending constant, expropriating the surplus. Once taxes become high enough (region C), the level of corruption increases even faster as tax revenue increases. This is because, as taxes increase in this range, the firms hire the mafia less frequently. This weakens electoral incentives to invest in law enforcement because conflict between the government and mafia becomes less frequent. In this range, in fact, corruption is increasing so fast that total expenditures on law enforcement are actually decreasing in
the level of taxation—the more resources the government has, the fewer resources (in absolute, not percentage terms) it spends on law enforcement. Finally, when taxes are high enough (region D), the mafia cannot extract fees that make it worth being in business, so the government is never called on to challenge the mafia, and therefore it expropriates all tax revenues, spending nothing on law enforcement.

**Proposition 4** The level of corruption \((1 - \lambda)\) is weakly increasing in the tax rate.

When government corruption increases or decreases, it is not just the percentage of tax revenues that changes, but the absolute magnitude of resources invested in law enforcement \((\lambda \tau)\). Although corruption is monotonic in the tax rate, the absolute level of spending on law enforcement is not. In particular, as discussed above, in region C corruption increases so much as taxes increase that total spending on law enforcement actually decreases despite the fact that the size of the government’s budget has increased.

**Proposition 5** Government spending on law enforcement \((\lambda \tau)\) is not monotonic in the level of taxation. It is increasing in region A, positive and flat in region B, decreasing in region C, and 0 and flat in region D.

**Proof.** See the appendix.

### 3.4.2 Taxation and Mafia Viability

The dashed line in Figure 3 shows that the frequency with which the firms hire the mafia is weakly decreasing in the tax rate. When taxes are low (in regions A and B), the mafia dominates the economy in the sense that neither firm relies on the government for protection. However, government policy in these regions does have an effect on the mafia. In particular, government investment in law enforcement and the threat of punishment decrease the fees that the mafia charges the firms. Thus, in these regions the firms use tax and electoral policy to successfully limit the strength, if not the ubiquity, of the mafia. As taxes increase even further, into region C, both the mafia and government are active in enforcing contracts. Finally, if taxes become high enough (region D), the mafia is entirely eradicated. In order to achieve this outcome, the firms must turn over enough money to the state in the form of taxes such that the amount that the mafia is able to charge in fees is not sufficient to overcome the risk of punishment that the mafia faces even when the government invests nothing in law enforcement.

The level of taxation that drives the mafia out of business is \(\bar{\tau} = 1 - \frac{k f(0)}{1 - f(0)}\). Two comparative statics are evident. First, \(\bar{\tau}\) is decreasing in the government’s natural advantage relative to the mafia \((f(0))\). That is, in societies where existing government institutions make the government strong relative to mafias, it is relatively inexpensive to drive the mafia out of business. Second, \(\bar{\tau}\) is decreasing in \(k\). The larger the penalty the government is able to impose, the easier it is to eradicate the mafia.

**Proposition 6** The frequency with which the firms hire the mafia is weakly decreasing in the tax rate, with the mafia entirely eradicated if taxes are high enough. The level of taxation necessary to eradicate the mafia is decreasing in the government’s natural advantage relative to the mafia, \(f(0)\), and in the penalty the government is able to impose on the mafia, \(k\).

**Proof.** See the appendix.
3.5 Credible Electoral Incentives

Proposition 1 makes clear the importance of the firms’ inducing the government to challenge the mafia \((\gamma > 0)\) in order to keep the mafia in check, and Lemma 1 implies that the firms can credibly commit ex ante to any electoral response to achieve that goal. Lemma 1 also implies that the firms’ optimal choice of electoral behavior depends not only on the government’s response to the incentives created, but also on the incentives that that response creates for the mafia. From Proposition 2, if the mafia expects its offers of service to be accepted, then the fees the mafia offers are decreasing in the government’s investment in law enforcement \((\lambda \tau)\), and hence the firms wish to induce the government to invest in law enforcement as much as possible in order to mitigate the commitment problem. The firms also wish to induce the highest possible probability of the government challenging the mafia \((\gamma)\), because the government has greater incentive to invest in law enforcement if it anticipates confronting the mafia more often.

The government chooses its probability of challenging the mafia \((\gamma)\) to maximize the probability of reelection, given the level of investment in law enforcement \((\lambda \tau)\) and the reelection rule \((\rho)\). With this in mind, we can characterize the government’s best-response correspondence for its choice of \(\gamma\). The government’s expected utility from challenging with certainty is:

\[
E[u_G(\gamma = 1, \rho^*, \lambda | \mu = (1, 1))] = 1 - \tau + f(\lambda \tau)\rho_G R + (1 - f(\lambda \tau))\rho_M R.
\]

The government’s expected utility from never challenging is:

\[
E[u_G(\gamma = 0, \rho^*, \lambda | \mu = (1, 1))] = 1 - \tau + \rho_NM R.
\]

Comparing these, we find that the government’s best response correspondence is

\[
\gamma^*(\lambda, \rho): = \begin{cases} 
1 & \text{if } \rho_M(1 - f(\lambda \tau)) + \rho_G f(\lambda \tau) > \rho_NM \\
\gamma' \in [0, 1] & \text{if } \rho_M(1 - f(\lambda \tau)) + \rho_G f(\lambda \tau) = \rho_NM \\
0 & \text{if } \rho_M(1 - f(\lambda \tau)) + \rho_G f(\lambda \tau) < \rho_NM.
\end{cases}
\]  

(7)

In light of the best responses, what reelection rule will the firms adopt? Because any reelection rule is credible in equilibrium, we look for the reelection rule that maximizes the firms’ expected utilities given the paths of play described above. Clearly, the firms prefer the government to challenge with high probability (high \(\gamma\)) and high levels of investment in law enforcement (high \(\lambda\)) in order to reduce the fees that they pay to the mafia.

First, notice that the reelection probability conditional on both firms hiring the mafia and the government not challenging \((\rho_{NM})\) affects the government’s choice of whether to challenge, but does not directly enter into the government’s choice of investment in law enforcement. In particular, the government will challenge the mafia only if \(\rho_{NM} < \rho_M(1 - f(\lambda^* \tau)) + \rho_G f(\lambda^* \tau)\). Since the firms want the government to challenge, they will electorally punish the government for not challenging by choosing a \(\rho_{NM}\) that satisfies this constraint (e.g., 0). Further note that the reelection probability conditional on neither firm hiring the mafia \((\rho_{NG})\) has no effect on any decisions and so any \(\rho_{NG}\) is optimal.

Finally, consider the electoral responses when the government loses a conflict with the mafia \((\rho_M)\) or wins a conflict with the mafia \((\rho_G)\). Government investment in law enforcement
(\lambda^*) is weakly increasing in \(\rho_G\) and weakly decreasing in \(\rho_M\). That is, if the government is rewarded for winning and punished for losing, it has incentives to invest in law enforcement. Since the firms government law enforcement spending to be as large as possible, they want to provide electoral incentives for government victory. Thus, the firms want to make \(\rho_G - \rho_M\) as large as possible, which implies \(\rho_G = 1\) and \(\rho_M = 0\). Substantively, the firms reward the government for victory in conflicts with the mafia and punish the government for losses to the mafia, thereby inducing the government to invest in law enforcement. We summarize this argument in the following proposition:

**Proposition 7** The firms credibly commit to re-electing the government with certainty if the government challenges the mafia and wins, electing a new government if it challenges and loses, and re-electing the government with a probability of no more than \(\rho_M(1 - f(\lambda^*\tau)) + \rho_G f(\lambda^*\tau)\) if the government does not challenge the mafia.

### 3.6 Taxation and Commitment

The only remaining action to be determined is the firms’ choice of the tax rate (\(\tau\)). Because the mafia price discriminates among the firms, the firms are unanimous in their preferences over tax rate regardless of the contract division (\(\alpha\)).\(^{11}\)

The firms’ expected utility changes in each of the regions from Figure 3 because the government’s allocation of the tax resources is different in each region. In order to determine the optimal tax rate, the firms compare the locally optimal tax rate in each region and choose the one that maximizes their utility. Label the locally optimal tax rate in each region (including the boundaries) \(\tau^*_j, j \in \{A, B, C, D\}\). The following result will be useful in finding the optimal tax rate.

**Lemma 4** The optimal tax rate is never in regions B or D. It is always either \(\tau^*_A\) or \(\tau^*_C\).

**Proof.** See the appendix. ■

The intuition behind this lemma is that, because government investment in law enforcement is flat in regions B and D, the firms’ expected utilities are decreasing in the tax rate in those regions. Thus, the optimal tax rate can never be in the interior of B or D. It is feasible, however, for the optimal tax rate to be in the interiors of A or C or on their boundaries. In order to determine which it is, we must find the local optima and compare them. This intuition is illustrated in Figure 4.

In region A, if the local optimum is interior, it is given by the following first-order condition:

\[
(1 - \tau^*_A) \frac{f'}{f'} (\tau^*_A) = 1.
\]

At the interior optimum, the locally optimal tax rate in region A balances the marginal benefit of increased law enforcement that comes with increased government funding against

\(^{11}\)Another approach to modeling the firms’ choice of a tax rate would be to have the firms vote behind a “veil of ignorance”, prior to observing the realization of \(\alpha\). This alternative specification leads to the same equilibrium tax rate.
Figure 4: The firms’ expected utility as a function of the tax rate. The optimal tax rate can be in region A (left-hand figure) or region C (right-hand figure), but never regions B or D.

the marginal cost of increased taxation. If \((1 - \tau) \frac{f'}{f}(\tau) > 1\), for all \(\tau < (f')^{-1}\left(\frac{1}{R}\right)\), then there is a corner solution, denoted

\[
\bar{\tau}_A^* = (f')^{-1}\left(\frac{1}{R}\right). 
\]

Similarly, if the local optimum in region C is interior, it is given by the following first-order condition:

\[
(1 - \tau) \left( \frac{\partial \pi}{\partial \tau} \left( 1 - f(\hat{\lambda}\tau_C^*) \right) + (1 - \pi) \frac{\partial f(\hat{\lambda}\tau_C^*)}{\partial \lambda} \right) - \left( \pi \left( 1 - f(\hat{\lambda}\tau_C^*) \right) + f(\hat{\lambda}\tau_C^*) \right) = 0. 
\]

Increasing the tax rate in region C has three effects on the firms’ expected utilities. First, it diminishes the revenues associated with economic activity, which is a cost from the firms’ perspective \((- \left( \pi \left( 1 - f(\hat{\lambda}\tau_C^*) \right) + f(\hat{\lambda}\tau_C^*) \right) < 0)\). Second, it changes the probability that mafia charges fees that lead both firms to hire it \(\left( (1 - \tau) \frac{\partial \pi}{\partial \tau} \left( 1 - f(\hat{\lambda}\tau_C^*) \right) \right)\). As shown in the proof of Proposition 6, this effect is positive—increasing taxes decreases the probability that the firms hire the mafia, which makes the firms better off. In contrast, increasing taxes decreases total spending on law enforcement \(\left( (1 - \tau)(1 - \pi) \frac{\partial f(\hat{\lambda}\tau_C^*)}{\partial \lambda} < 0 \right)\), which makes the firms worse off. The optimal tax rate balances these marginal benefits and marginal costs.

If the first-order condition does not hold with equality for any tax rate in region C, then the locally optimal tax rate is either the lower corner or upper corner, respectively denoted

\[
\tau_C^* = 1 - k \frac{f(N\tau)}{1 - f(N\tau)} 
\]

and

\[
\bar{\tau}_C^* = 1 - k \frac{f(0)}{1 - f(0)}. 
\]

The globally optimal tax rate is found by comparing the expected utilities at these local optima (see Figure 4). This gives rise to the following result.
Of course, weakening the mafia comes at a price to the firms—taxation. The firms face a dilemma: if they must hire the mafia, they face a commitment problem. This is because the mafia can charge fees, which the firms must pay, even when they do not hire the mafia. Consequently, government law enforcement and taxation can sometimes diminish the fees the mafia can charge, thereby mitigating the commitment problem. However, this also means that the mafia is less attractive to the firms, simply because the firms do not have as much to lose. Thus the benefits of doing business are lower for the mafia. The mafia still faces a substantial cost, however, in the form of the risk of punishment. In equilibrium, these costs and benefits are exactly equal.

In region C, the firms pay higher taxes, only some of which are directed by the government toward law enforcement. Because taxes are high, the mafia cannot extract as much in fees from the firms, with relatively little threat of successful law enforcement by the modestly funded government. Thus, if the firms choose the lower tax rate, they reap the benefits of relatively low taxation and a non-corrupt government, but they bear the costs of a thriving mafia.

In region A, the firms pay relatively low taxes, all of which are directed by the government toward law enforcement. However, because taxes are low, the mafia can extract fairly high fees from the firms, with relatively little threat of successful law enforcement by the mafia. In equilibrium, these costs and benefits are exactly equal and, as a result, the mafia sometimes chooses to price itself out of the market.

What determines whether the firms prefer the lower or the higher tax rate?

According to Proposition 8, the optimal tax rate can be in either region A or region C. The same Lemma implies that the expected utility is decreasing from the local optimum in region A until the boundary between region B and C. Further, if $\tau^*_A$ is the local optimum in region C, then the expected utility is decreasing for all tax rates greater than the local optimum in region A. Thus $\tau^*_C$ can never be the global optimum. The rest of the proposition follows from the argument in the text.

**Proposition 8.** The optimal tax rate is characterized by

$$\tau^* = \begin{cases} \tau^*_A & \text{if } E[u_F(\tau^*_A)] \geq \max\{E[u_F(\tau^*_C)], E[u_F(\tau^*_C)]; E[u_F(\tau^*_A)]\} \text{ and } \tau^*_A \leq (f')^{-1}\left(\frac{1}{R}\right) \\ \tau^*_A & \text{if } E[u_F(\tau^*_A)] \geq \max\{E[u_F(\tau^*_C)], E[u_F(\tau^*_C)]; E[u_F(\tau^*_A)]\} \text{ and } \tau^*_A > (f')^{-1}\left(\frac{1}{R}\right) \\ \tau^*_C & \text{if } E[u_F(\tau^*_C)] > \max\{E[u_F(\tau^*_A)], E[u_F(\tau^*_A)]; E[u_F(\tau^*_C)]\} \text{ and } \tau^*_C \leq 1 - k \frac{f(0)}{1 - f(0)} \\ \tau^*_C & \text{if } E[u_F(\tau^*_C)] > \max\{E[u_F(\tau^*_A)], E[u_F(\tau^*_A)]; E[u_F(\tau^*_C)]\} \text{ and } \tau^*_C > 1 - k \frac{f(0)}{1 - f(0)} \end{cases}$$

**Proof.** Lemma 4 implies that the optimum must be in region A or C. The same Lemma implies that the expected utility is decreasing from the local optimum in region A until the boundary between region B and C. Further, if $\tau^*_C$ is the local optimum in region C, then the expected utility is decreasing for all tax rates greater than the local optimum in region A. Thus $\tau^*_C$ can never be the global optimum. The rest of the proposition follows from the argument in the text.

Economic agents in this model face a commitment problem. For any given level of taxation, it is Pareto inefficient for both firms to hire the mafia. This is because the enforced contract is the same whether they both hire the mafia or both rely on the government, but when they both hire the mafia they also pay fees. The problem, of course, is that they do not trust each other not to individually hire the mafia in an attempt to extort the entire value of the contract.

The firms fund the government in order to solve this commitment problem. The threat of government challenge, should both firms hire the mafia, makes it relatively less attractive to the mafia to be hired. Consequently, government law enforcement and taxation can sometimes diminish the fees the mafia can charge, thereby mitigating the commitment problem. Of course, weakening the mafia comes at a price to the firms—taxation. The firms face a
trade-off. They would like to create a situation where neither hires the mafia. However, funding the government sufficiently to achieve this goal is costly. Consequently, even though appealing to the mafia is ex post inefficient, the firms will allow the mafia to persist. That is, the firms could drive the mafia out of business by funding the government sufficiently, but they choose not to do so because the increased tax burden would be more costly than the inefficiency of hiring the mafia. This intuition is summarized in the following corollary of Proposition 8.

**Corollary 1** For any tax rate the firms prefer jointly not to hire the mafia. Although the firms can always choose a tax rate, \( \tau = 1 - k \frac{f(0)}{1-f(0)} \), that would lead them not to hire the mafia, there are conditions under which they choose a lower tax rate that leads both of them to hire the mafia.

**Proof.** Given \( \tau \), and letting \( x \) represent firm i’s share of the contract, \( E[u_i(\mu = (1, 1))] = x(1 - \tau) - \phi_i \) and \( E[u_i(\mu = (0, 0))] = x(1 - \tau) \). Thus for all \( \phi_i > 0 \), firm i strictly prefers \( \mu = (1, 1) \) to \( \mu = (0, 0) \). From equation (5), if \( \tau > 1 - k \frac{f(0)}{1-f(0)} \), the mafia charges fees such that neither firm hires it. Proposition 8 establishes the conditions under which they choose \( \tau < 1 - k \frac{f(0)}{1-f(0)} \) and Proposition 6 establishes that, at those tax rates, they both hire the mafia with positive probability.

## 4 An Extension: The Possibility of Collusion

A common theme in the literature on state/mafia relations that we have not yet touched on involves collusion between the government and the mafia. In this section, we consider two simple extensions of our model that allow us to explore the implications of introducing the possibility of such collusion. In the first, the mafia can bribe the government not to confront the mafia in the absence of a firm’s request for intervention. In the second, the mafia can bribe the government not to intervene even when a firm has petitioned for assistance.

### 4.1 Passivity in the Absence of Complaints

Consider an extension in which, at the time when the government chooses whether or not to challenge the mafia, the mafia and the government also have the choice to enter into a credible agreement whereby the government does not challenge the mafia unless appealed to directly by one of the firms, in exchange for a payment from the mafia, \( \beta \). Thus, the mafia and the government can collude to prevent the government from breaking up the mafia’s protection racket. How does this possibility affect equilibrium play?

**Proposition 9** If \( R > k \), then equilibrium play in the extended model with the possibility of collusion is identical to equilibrium play in the model without the possibility of collusion.

**Proof.** See the appendix. ■

If electoral benefits are large relative to the punishment the government is capable of meeting out, then collusion is not in the interest of the mafia and so, will not happen. This is because the mafia’s bribe must be sufficiently large to compensate the government for the
forgone benefits of reelection. When the threat of government punishment is small, and the costs of compensating the government are large, the mafia is unwilling to pay the necessary bribe.

However, if \( R < k \), then the mafia has an incentive to bribe the government. How does this affect future play? As is clear from the earlier analysis, the firms’ willingness to hire the mafia is not a function of the government’s decision over whether or not to challenge the mafia, thus the firms’ hiring strategies are the same as in the game without collusion. However, the point at which the mafia is willing to charge fees that induce the firms to hire it does change because, with the ability to bribe the government, the costs to the mafia of being in business are lower.

The final question that must be answered is how much will be spent on law enforcement when collusion is possible. Even though the government will never challenge the mafia, it may have an incentive to invest money in law enforcement to increase the bribe it can extract from the mafia. The bribe the government can extract is given by \( \beta^* = f(\lambda^* \tau) R \), precisely the expected payoff the government associated with the possibility of reelection in the earlier model. Thus, with the exception of the change in when the mafia is hired expressed in equation (17), the government’s maximization problem is identical to the original model.

**Proposition 10** If \( R < k \), the tax rate in the game with collusion is weakly higher than the tax rate in the game without collusion. Moreover, the firms are weakly more likely to hire the mafia in the game with collusion than the game without collusion.

**Proof.** See the appendix. ■

The comparison of this result to Proposition 1 is instructive. Although the mafia can and does bribe the government to choose \( \gamma = 0 \) when both firms hire the mafia, the resulting outcome is not that which would result from a situation in which the government were incapable of challenging the mafia. When the government must be bribed not to challenge, the tax rate is positive, investment in law enforcement is positive, and the fees that the mafia charges the firms are lower. These differences are all attributable to the fact that the size of the bribe that the government receives is increasing in the absolute amount of resources that the government invests in law enforcement, giving the government an incentive to make such an investment even though it will not challenge the mafia. Thus the firms have higher utility under a government that is corrupt in this way than they would have under a government that was constrained not to challenge.

### 4.2 Inaction in the Presence of Complaints

One could also imagine a second, more dramatic, form of collusion between the mafia and the government, whereby the mafia can bribe the government not to intervene even when directly appealed to by one of the firms. In order to think about collusion of this sort, we need to consider what the electoral consequences are if the government challenges or fails to challenge when one firm hires the mafia and the other firm appeals to the government. In this scenario, the firms have different preferences over government action: the firm that hired the mafia wants the government not to challenge while the firm that did not hire the mafia wants the government to challenge. As such, we assume that no matter what it does in this scenario, the government wins the election with probability \( 1/2 \).
Given this, how large must the payment from the mafia be in order to dissuade the
government from challenging if appealed to by one of the firms? If the mafia offers a bribe
of $\beta$, then the government’s expected utility from not challenging is $E[u_G(\text{no challenge})] = (1-\lambda)\tau + \frac{1}{2}R + \beta$ and its expected utility from challenging is $E[u_G(\text{challenge})] = (1-\lambda)\tau + \frac{1}{2}R$. Clearly, then, the government can be persuaded not to challenge with any arbitrarily small, positive bribe. Hence, this form of collusion implies that the government will not intervene
to aid a firm which is being extorted by a mafia hired by another firm.

Collusion of this form makes the commitment problem between the firms significantly
worse because the risk of extortion is heightened. This allows the mafia to extract higher fees from the firms. The fact that the mafia can extract higher fees from the firms does not affect the rest of the analysis in any qualitative way. It will still always be the case the either both or neither firm hires the mafia and, therefore, the type of collusion we will see on the equilibrium path is the sort discussed in the pervious sub-section. The only effect of adding this type of collusion, then, is that the mafia finds it more profitable to be in business because it can extract higher fees. As a result, the cut-point at which the mafia prices itself out of the market shifts rightward. Thus, under this type of collusion, the mafia will dominate the economy more frequently and will extract greater payments in return for its “protection” services.

5 Conclusion

Mafias play a critical role as alternative providers of contract enforcement and revenue protec-
tion in transition economies. However, their presence also creates the possibility of extortion. This threat of extortion, we have argued, creates a commitment problem between firms.

In our model, firms pay taxes and use electoral pressure to persuade the government to
invest tax revenues in law enforcement. Firms are willing to bear the cost of taxation and
the risk of government expropriation because law enforcement mitigates the commitment
problem, lowering the fees extracted by the mafia by diminishing the risk of expropriation
should a firm choose not to hire the mafia. For some equilibrium levels of taxation, in fact,
law enforcement spending not only diminishes the fees extracted by the mafia but decreases
the mafia’s presence in the economy. However, this decreased mafia presence is purchased by
the firms at the cost of increased taxation and greater government corruption. The extent
to which the mafia dominates the economy depends on how the firms evaluate the relative
costs of taxation and government corruption, on the one hand, and paying the mafia’s fees,
on the other.

We further showed that the stronger government institutions for fighting and punishing
the mafia, the more likely the firms are to be willing to bear the costs of a tax rate that
eliminates the mafia from the economy. Conversely, the possibility of collusion between the
government and the mafia makes it more likely that the mafia will dominate the economy
and increases the fees the mafia can extract from firms.

This modeling approach suggests a variety of interesting avenues for future work. The
mafia in our model was relatively benign. While it could serve as both a gun-for-hire ex-
tortionist (if hired by one firm) or as the overseer of a protection racket (if hired by both
firms), the mafia made only take-it-or-leave-it offers. An alternative model would allow the
mafia to use its coercive capacity to make “take-it-or-take-it” offers to the firms. That is, the mafia might often serve as an extortionist on its own behalf, demanding fees from the firms in exchange for “protection.” Allowing for a more aggressive mafia of this sort would alter incentives in a variety of ways, presumably, for example, increasing the firms’ interest in funding government law enforcement.

Another interesting extension would consider other forms of corruption by the government. For instance, governments might have an incentive to “hold up” firms in exchange for law enforcement protection from an extortionist mafia. Such incentives would be further exacerbated if the costs governments bear for taxing different firms vary across firms or industries (Gehlbach 2003). If such rent extraction has efficiency implications, this might provide incentives for firms to sell themselves to the government, providing government officials with an equity stake in the firm in order to mitigate the hold up problem.

Thus, within the context of the relationship between states, mafias, and firms seeking contract enforcement and revenue protection there are a rich variety of theoretical models to be explored. We leave them for future research.

A Appendix

A.1 Proof of Lemma 1

We compare the firms’ expected utilities associated with $\gamma = 1$ and $\gamma = 0$, respectively.

$$E[u_{1,2}(\gamma = 1, \mu = (1, 1), \cdot)] = (1 - f(\lambda \tau)) \frac{1 - \tau}{2} + f(\lambda \tau) \frac{1 - \tau}{2} = \frac{1 - \tau}{2} = E[u_{1,2}(\gamma = 0, \mu = (1, 1), \cdot)]$$

$\blacksquare$

A.2 Proof of Proposition 3

We can write F1’s expected utility from hiring the mafia with certainty as:

$$E[u_{1}(\mu_1 = 1, \mu_2)] = \mu_2 \alpha (1 - \tau) + (1 - \mu_2)(1 - f(\lambda \tau))(1 - \tau) + f(\lambda \tau) \alpha (1 - \tau) - \phi_1.$$

F1’s expected utility from appealing to the government with certainty is:

$$E[u_{1}(\mu_1 = 0, \mu_2)] = \mu_2[(1 - f(\lambda \tau)) \times 0 + f(\lambda \tau) \alpha (1 - \tau)] + (1 - \mu_2) \alpha (1 - \tau).$$

Similarly, we can write for F2:

$$E[u_{2}(\mu_1, \mu_2 = 1)] = \mu_1 (1 - \alpha)(1 - \tau) + (1 - \mu_1)(1 - f(\lambda \tau))(1 - \tau) + f(\lambda \tau)(1 - \alpha)(1 - \tau) - \phi_2,$$

and

$$E[u_{2}(\mu_1, \mu_2 = 0)] = \mu_1[(1 - f(\lambda \tau)) \times 0 + f(\lambda \tau)(1 - \alpha)(1 - \tau)] + (1 - \mu_1)(1 - \alpha)(1 - \tau).$$
Comparing expected utilities, we obtain that \( \mu_2 = 1 \) is the best response for F2 if (1) holds, and \( \mu_1 = 1 \) is the best response for F1 if (2). Rearranging terms shows that F2 is indifferent if

\[
\mu_1 = \frac{\phi_2}{(1 - 2\alpha)(1 - \tau)(1 - f(\lambda \tau))} - \frac{\alpha}{(1 - 2\alpha)} \equiv \tilde{\mu}_1,
\]

and that F1 is indifferent if

\[
\mu_2 = -\frac{\phi_1}{(1 - 2\alpha)(1 - \tau)(1 - f(\lambda \tau))} + \frac{(1 - \alpha)}{(1 - 2\alpha)} \equiv \tilde{\mu}_2.
\]

\[\blacksquare\]

### A.3 Proof of Lemma 3

\[E[u_G(\lambda, \cdot)] = (1 - \lambda)\tau + [(1 - \mu_1^*(\cdot))(1 - \mu_2^*(\cdot))\rho_{NG}^*(\cdot)
\]
\[+ (\mu_1^*(\cdot)(1 - \mu_2^*(\cdot)) + \mu_2^*(\cdot)(1 - \mu_1^*(\cdot)))(1 - f(\lambda^*\tau))\rho_{NG}^*(\cdot) + f(\lambda^*\tau)\rho_{G}^*(\cdot))
\]
\[+ \mu_1^*(\cdot)\mu_2^*(\cdot)((\gamma^*(\cdot)(1 - f(\lambda^*\tau))\rho_{M}^*(\cdot) + f(\lambda^*\tau)\rho_{G}^*(\cdot)) + (1 - \gamma^*(\cdot))\rho_{NM}^*(\cdot))]R. \quad (8)
\]

In order to determine the possible equilibrium paths of play in all subgames beginning with the government’s choice of how much to invest in law enforcement \( \lambda \), recall that, in equilibrium, either both firms hire the mafia \( (\mu = (1, 1)) \) or neither does \( (\mu = (0, 0)) \). Thus, we can restrict attention to the equilibrium behavioral strategy profiles in which \( \mu = (0, 0) \) and \( \mu = (1, 1) \). From equation (7), the government’s choice of whether or not to challenge the mafia \( (\gamma) \) is a function of the government’s resource allocation decision \( (\lambda) \). There are two cases to consider:

1. \( \rho_{NM} < (1 - f(0))\rho_M + f(0)\rho_G \)
2. \( \rho_{NM} \geq (1 - f(0))\rho_M + f(0)\rho_G. \)

Since \( f(\lambda \tau) \) is increasing in \( \lambda \), we know from equation (7) that in case 1, \( \gamma = 1 \), regardless of \( \lambda^1 \).

Now consider case 2. From equation (7) we know that if \( \lambda^1 = 0 \), then \( \gamma = 0 \). This yields the following expected utility for the government:

\[E[u_G(\lambda^1 = 0, \gamma = 0)] = \tau + \rho_{NM}\delta\tau
\]

If, however, \( \lambda^1 > 0 \) and \( \gamma \in (0, 1) \), then the government’s expected utility is:

\[E[u_G(\lambda^1 > 0, \gamma \in (0, 1))] = (1 - \lambda^1)\tau + \gamma((1 - f(0))\rho_M + f(0)\rho_G)\delta\tau.
\]

Note from equation (7) that if \( \gamma \in (0, 1) \), then \( \rho_{NM} = (1 - f(0))\rho_M + f(0) \). Consider, then, the deviation from \( (\lambda^1 > 0, \gamma \in (0, 1)) \) to \( (\lambda^1 = 0, \gamma = 0) \). We have that:

\[E[u_G(\lambda^1 = 0, \gamma = 0)] = \tau + \rho_{NM}\delta\tau = \tau + ((1 - f(0))\rho_M + f(0))\delta\tau,
\]

which is clearly larger than \( E[u_G(\lambda^1 > 0, \gamma \in (0, 1))] \). Hence, if \( \lambda^1 > 0 \), then \( \gamma \not\in (0, 1) \). Moreover, if \( \gamma = 0 \), then \( \lambda^1 \) must be 0. Thus, if \( \lambda > 0 \), then \( \gamma = 1 \). \[\blacksquare\]
A.4 Derivation of $\lambda^*$

From Lemma 3, $\lambda > 0$ implies $\gamma = 1$, which, from equation (7) implies that $\rho_{NM} < (1 - f(\lambda \tau))\rho_M + f(\lambda^* \tau)\rho_G$. Recall from equation (5) that the mafia charges fees that lead both firms to hire the mafia only if $k < \frac{(1-\tau) (1-f(\lambda^* \tau))}{\gamma f(\lambda^* \tau)}$. If the mafia chooses fees such that $\mu = (0, 0)$, then $\lambda = 0$. Hence, if $\lambda > 0$ we can further restrict attention to cases where equation (5) is satisfied. Given that $\gamma = 1$ if $\lambda > 0$, the government’s expected utility is given by:

$$E[u_G|\lambda, \mu = (1, 1), \gamma] = (1 - \lambda)\tau + [(1 - f(\lambda^* \tau))\rho_M + f(\lambda^* \tau)\rho_G] R.$$ 

At an interior solution, the optimal level of investment in law enforcement, labeled $\lambda'$, satisfies the following first-order condition:

$$f'(\lambda' \tau) = \frac{1}{(\rho_G - \rho_M) R},$$

which implies that, if it is interior, $\lambda' = (f')^{-1}\left(\frac{1}{\rho_G - \rho_M} R\right)$. Notice, further, that if $\lambda'$ is interior, then

$$\lambda' \tau = (f')^{-1}\left(\frac{1}{\rho_G - \rho_M} R\right) \equiv \lambda'^*$$

is invariant to the tax rate. We must also consider corner solutions. The assumption that $\lim_{x \to 0} f'(x) = \infty$ rules out $\lambda' = 0$. However, if $f'(\lambda \tau) > \frac{1}{(\rho_G - \rho_M) R}$, for all $\lambda \leq 1$, then there is a corner solution at $\lambda' = 1$.

$\lambda'$ is the optimal choice of investment in law enforcement, given that the government challenges. However, no investment in law enforcement ($\lambda = 0$) could be optimal if the government chooses not to challenge. In order to determine when $\lambda'$ is preferred to $\lambda = 0$, we need to consider two cases.

Case 1: $\rho_{NM} < (1 - f(0))\rho_M + f(0)\rho_G$.

In this case, if the government chooses to deviate from $\lambda = \lambda'$ to $\lambda = 0$, $\gamma$ nonetheless remains equal to 1. From the concavity of $f(\cdot)$ and the definition of an optimum, it follows that $\lambda = 0$ cannot be optimal in this case unless $\lambda'$ is itself equal to 0, which is never true.

Case 2: $\rho_{NM} \in ((1 - f(0))\rho_M + f(0)\rho_G, (1 - f(\lambda' \tau))\rho_M + f(\lambda' \tau)\rho_G)$.

In this case, if the government chooses to deviate from $\lambda = \lambda'$ to $\lambda = 0$, this will also lead it to switch from $\gamma = 1$ to $\gamma = 0$. Thus, comparing

$$E[u_G(\lambda', \gamma = 1)] = (1 - \lambda')\tau + ((1 - f(\lambda' \tau))\rho_M + f(\lambda' \tau)\rho_G) R$$

to

$$E[u_G(0, \gamma = 0)] = \tau + \rho_{NM} R,$$

we find that the government will choose $\lambda = \lambda'$ in this case only if

$$(((1 - f(\lambda' \tau))\rho_M + f(\lambda' \tau)\rho_G) - \rho_{NM}) R > \lambda' \tau.$$  \hspace{1cm} (11)
Otherwise it will choose \( \lambda = 0 \).

We have established the conditions under which \( \lambda' \) is preferred to \( \lambda = 0 \), conditioned on the firms hiring the mafia. Notice that, given equation (10), the condition in equation (11) is purely a function of parameters, so that the case where \( \lambda' \) is optimal and the case where \( \lambda = 0 \) is optimal are mutually exclusive.

Now it remains to consider the consistency of these conditions with the conditions under which the firms hire the mafia. There are three possibilities. From equation (5), if \( k > (1 - \tau)\frac{1-f(0)}{f(0)} \), then the firms never hire the mafia. If so, the government never challenges, and so \( \lambda = 0 \). If \( k < (1 - \tau)\frac{1-f(\lambda\tau)}{f(\lambda\tau)} \), then the firms always hire the mafia; the government challenges and chooses \( \lambda = \lambda' \) if equation (11) is satisfied and the government does not challenge and does not invest in law enforcement if it is not satisfied. Finally, we need to consider the case \( k \in \left( (1 - \tau)\frac{1-f(\lambda\tau)}{f(\lambda\tau)}, (1 - \tau)\frac{1-f(0)}{f(0)} \right) \). In this case, there is no pure strategy equilibrium.

Define the critical value \( \hat{\lambda} \) as the choice of \( \lambda \) such that the mafia is exactly indifferent between charging a fee that induces both firms to hire it and charging a fee that induces neither firm to hire it:

\[
k = (1 - \tau)\frac{1 - f(\hat{\lambda}\tau)}{f(\hat{\lambda})}.
\]

Let \( \pi \) be the probability that the mafia choose \( \phi \) such that neither firm hires it and \( 1 - \pi \) be the probability that the mafia chooses fees such that both firms hire it. Then, in equilibrium the mafia must choose this probability such that \( \hat{\lambda} \) is optimal for the government. The government’s expected utility is:

\[E[u_G(\lambda|\tau, \pi)] = (1 - \lambda)\tau + (\pi(1 - f(\lambda\tau))\rho_M + f(\lambda\tau)\rho_G)R.
\]

The mafia chooses \( \pi \) such that the following holds:

\[(1 - \pi)f'(\hat{\lambda}\tau) = \frac{1}{R(\rho_G - \rho_M)} \iff \pi = 1 - \frac{1}{Rf'(\hat{\lambda}\tau)(\rho_G - \rho_M)} \tag{13}\]

Combining all these cases, we can formally characterize the proportion of tax revenue invested in law enforcement in the following lemma:

**Lemma 5** If \(((1 - f(\lambda'))\rho_M + f(\lambda')\rho_G) - \rho_{NM}) R > \overline{\lambda}\tau$, then

\[
\lambda^* = \begin{cases} 
1 & \text{if } \tau < (f')^{-1}\left(\frac{1}{(\rho_G - \rho_M)R}\right) \\
\hat{\lambda} & \text{if } \tau \in \left( (f')^{-1}\left(\frac{1}{(\rho_G - \rho_M)R}\right), 1 - k\frac{f(\lambda\tau)}{1-f(\lambda\tau)} \right) \\
0 & \text{if } \tau > 1 - k\frac{f(0)}{1-f(0)}
\end{cases} \tag{14}
\]

where \( \hat{\lambda} \) is implicitly defined by (12). If \(((1 - f(\lambda'))\rho_M + f(\lambda')\rho_G) - \rho_{NM}) R < \overline{\lambda}\tau$, then when have that \( \lambda^* = 0 \) for all tax rates.
A.5 Proof of Proposition 4

We will make use of the following result.

Lemma 6 \( \hat{\lambda}\tau \) is decreasing in \( \tau \).

**Proof.** From equation (12), \( \frac{f'}{f'(1)} \left( \hat{\lambda}\tau \right) = \frac{1}{k} \). Since the right hand side is obviously decreasing in \( \tau \), the left-hand side must be as well. Define \( g(\cdot) = \frac{f'}{f'(1)}(\cdot) \). Then \( g'(\cdot) = \frac{f''}{f'(1)} + \frac{f'}{f'(1)^2} > 0 \). Thus, in order for the left-hand side to be decreasing in \( \tau \), \( \hat{\lambda}\tau \) must be decreasing in \( \tau \). ■

The optimal government resource investment is given by equation (14):

\[
\lambda^* = \begin{cases} 
1 & \text{if } \tau < (f')^{-1}(\frac{1}{R}) \\
\frac{(f')^{-1}(\frac{1}{R})}{\tau} & \text{if } \tau \in \left( (f')^{-1}(\frac{1}{R}), 1 - k \frac{f(N\tau)}{1 - f(N\tau)} \right) \\
\hat{\lambda} & \text{if } \tau \in \left( 1 - k \frac{f(N\tau)}{1 - f(N\tau)}, 1 - k \frac{f(0)}{1 - f(0)} \right) \\
0 & \text{if } \tau > 1 - k \frac{f(0)}{1 - f(0)}. 
\end{cases}
\]

The level of corruption is \( 1 - \lambda^* \). Thus, it suffices to show that \( \lambda^* \) is weakly decreasing in \( \tau \). For \( \tau \in [0, (f')^{-1}(\frac{1}{R})] \), \( \lambda^* = 1 \), which is constant and, therefore, weakly decreasing. For \( \tau \in \left( (f')^{-1}(\frac{1}{R}), 1 - k \frac{f(N\tau)}{1 - f(N\tau)} \right) \), \( \lambda^* = \frac{(f')^{-1}(\frac{1}{R})}{\tau} \), which is strictly decreasing in \( \tau \). For \( \tau \in \left( 1 - k \frac{f(N\tau)}{1 - f(N\tau)}, 1 - k \frac{f(0)}{1 - f(0)} \right) \), \( \lambda^* = \hat{\lambda} \). Lemma 6 demonstrates that \( \hat{\lambda}\tau \) is decreasing in \( \tau \), which implies that \( \hat{\lambda} \) is decreasing. If \( \tau \in \left( 1 - k \frac{f(0)}{1 - f(0)}, 1 \right) \), then \( \lambda^* = 0 \) which is constant and, therefore, weakly increasing in \( \tau \). ■

A.6 Proof of Proposition 5

The optimal government resource investment is given by equation (14):

\[
\lambda^* = \begin{cases} 
1 & \text{if } \tau < (f')^{-1}(\frac{1}{R}) \\
\frac{(f')^{-1}(\frac{1}{R})}{\tau} & \text{if } \tau \in \left( (f')^{-1}(\frac{1}{R}), 1 - k \frac{f(N\tau)}{1 - f(N\tau)} \right) \\
\hat{\lambda} & \text{if } \tau \in \left( 1 - k \frac{f(N\tau)}{1 - f(N\tau)}, 1 - k \frac{f(0)}{1 - f(0)} \right) \\
0 & \text{if } \tau > 1 - k \frac{f(0)}{1 - f(0)}. 
\end{cases}
\]

In the first region, \( \lambda^*\tau = \tau \), which is increasing in \( \tau \). In the second region, \( \lambda^*\tau = \frac{\lambda}{N\tau} \), which is constant in \( \tau \), by equations (14) and (10). In the third region, \( \lambda^*\tau = \hat{\lambda} \tau \), which is decreasing in \( \tau \) by Lemma 6. In the fourth region, \( \lambda^*\tau = 0 \), which is constant in \( \tau \). ■

A.7 Proof of Proposition 6

From equation (5), if \( \tau < 1 - k \frac{f(N\tau)}{1 - f(N\tau)} \), the mafia charges a fee such that both firms hire it. If \( \tau \in \left( 1 - k \frac{f(N\tau)}{1 - f(N\tau)}, 1 - k \frac{f(0)}{1 - f(0)} \right) \), the mafia charges a fee that induces the firms to hire
the mafia with probability \(1 - \pi\). From equation (13), \(f'' < 0\), and \(\frac{\partial \pi}{\partial \tau} < 0\), it follows that 
\[
\frac{\partial \pi}{\partial \tau} = \frac{f''(\lambda\tau)\frac{\partial \lambda}{\partial \tau}}{R(f(\lambda\tau))^2} > 0,
\]
hence \(1 - \pi\) is decreasing in \(\tau\). By equation (5) if \(\tau > 1 - k\frac{f(0)}{1 - f(0)}\), the mafia charges a fee such that neither firm hires it. 

A.8 Proof of Lemma 4

A firm that will receive a share \(x\) of the contract has an expected utility given by:

\[
E[u_F(\tau, \cdot)] = \begin{cases} 
  x(1 - \tau) f(\tau) & \text{if } \tau < (f')^{-1}(\frac{1}{R}) \\
  x(1 - \tau) f(\lambda\tau) & \text{if } \tau \in [(f')^{-1}(\frac{1}{R}), 1 - k\frac{f(\lambda\tau)}{1 - f(\lambda\tau)}] \\
  x(1 - \tau) \left(\pi + (1 - \pi)f(\lambda\tau)\right) & \text{if } \tau \in (1 - k\frac{f(\lambda\tau)}{1 - f(\lambda\tau)}, 1 - k\frac{f(0)}{1 - f(0)}) \\
  x(1 - \tau) & \text{if } \tau > 1 - k\frac{f(0)}{1 - f(0)}. 
\end{cases} 
\] (15)

If \(\tau \in [(f')^{-1}(\frac{1}{R}), 1 - k\frac{f(\lambda\tau)}{1 - f(\lambda\tau)}]\), the expected utility is \(x(1 - \tau)f(\lambda\tau)\). Equation (14) shows that \(\lambda^*\tau = \frac{1}{R}\), which from (10) is constant in \(\tau\). Thus, it is clear that the expected utility is decreasing in \(\tau\), so \(E[u_F(\tau_B^*)] \leq E[u_F(\tau_A^*)]\).

If \(\tau > 1 - k\frac{f(0)}{1 - f(0)}\), the expected utility is \(x(1 - \tau)\), which is decreasing in \(\tau\), so \(E[u_F(\tau_B^*)] \leq E[u_F(\tau_C^*)]\). ■

A.9 Proof of Proposition 9

At the point where this decision is made, the government’s expected utility from not colluding with the mafia is precisely as before: \(E[u_G(\gamma = 1)] = (1 - \lambda^*)\tau + f(\lambda^*\tau)R\). If the government does collude, it is sure not to be reelected, so its expected utility is \(E[u_G(\gamma = 0)] = (1 - \lambda^*)\tau + \beta\). The government, then, will collude if and only if

\[
\beta \geq f(\lambda^*\tau)R \equiv \beta^*. 
\] (16)

If the mafia chooses to collude with the government, it will pay the lowest effective bribe, \(\beta^*\). The mafia’s payoff from colluding with the government is \(E[u_M(\beta^*)] = \phi_1 + \phi_2 - \beta^*\). The mafia’s expected utility from not colluding with the government is as before: \(E[u_M(\beta = 0)] = \phi_1 + \phi_2 - f(\lambda^*\tau)k\). The mafia, then, is willing to collude with the government as long as \(\beta^* < f(\lambda^*\tau)k\), which, from (16), implies \(R < k\). ■

A.10 Proof of Proposition 10

The mafia’s utility from being hired by the firms, given that there will be collusion, is

\[
E[u_M(\mu = (1, 1))] = \phi_1^* + \phi_2^* - (1 - \tau)(1 - f(\lambda^*\tau)) - f(\lambda^*\tau)R.
\]

The expected utility from pricing itself out of the market is 0. Thus, the mafia will charge fees that induce the firms to hire it if

\[
R < (1 - \tau)\frac{1 - f(\lambda^*\tau)}{f(\lambda^*\tau)}.
\] (17)
Notice that, since $R < k$, for a fixed $\lambda^*\tau$, this condition is easier to satisfy than the condition in equation (5).

For $R < k$, the optimal investment decision is

$$
\lambda^C = \begin{cases} 
1 & \text{if } \tau < (f')^{-1}(\frac{1}{R}) \\
(f')^{-1}(\frac{1}{R}) & \text{if } \tau \in \left( (f')^{-1}(\frac{1}{R}), 1 - R \frac{f(\lambda^*\tau)}{1 - f(\lambda^*\tau)} \right) \\
\hat{\lambda} & \text{if } \tau \in \left( 1 - R \frac{f(\lambda^*\tau)}{1 - f(\lambda^*\tau)}, 1 - R \frac{f(0)}{1 - f(0)} \right) \\
0 & \text{if } \tau > 1 - R \frac{f(0)}{1 - f(0)}. 
\end{cases}
$$

(18)

The fact that the government’s optimal investment is the same (but for the cut-points) as it was in the previous model implies that the firm’s choice of optimal tax rate will also be similar. In particular, the firms gain the same benefit from government investment in law enforcement as they did in the model without collusion. In the model without collusion, government investment in law enforcement made it more costly for the mafia to operate, which mitigated the commitment problem. In the model with collusion, government investment in law enforcement increases the cost to the mafia of bribing the government, which makes it more costly for the mafia to operate, which also mitigates the commitment problem. Thus, the firms expected utility in the game with collusion, given $R < k$, is

$$
E[u_F(\tau, \text{collusion})] = \begin{cases} 
x(1 - \tau)f(\tau) & \text{if } \tau < (f')^{-1}(\frac{1}{R}) \\
x(1 - \tau)f(\lambda^*) & \text{if } \tau \in \left( (f')^{-1}(\frac{1}{R}), 1 - R \frac{f(\lambda^*\tau)}{1 - f(\lambda^*\tau)} \right) \\
x(1 - \tau) \left( \pi + (1 - \pi)f(\lambda^*) \right) & \text{if } \tau \in \left( 1 - R \frac{f(\lambda^*\tau)}{1 - f(\lambda^*\tau)}, 1 - R \frac{f(0)}{1 - f(0)} \right) \\
x(1 - \tau) & \text{if } \tau > 1 - R \frac{f(0)}{1 - f(0)}. 
\end{cases}
$$

The only difference between the firms’ optimization problem in the game with collusion and the game without collusion is the cut-points. Thus the optimal tax rate in the collusion extension is analogous to the tax rate characterized by Proposition 8:

$$
\tau^C = \begin{cases} 
\tau^*_A & \text{if } E[u_F(\tau^*_A)] \geq \max\{E[u_F(\tau^*_A)], E[u_F(\lambda^*_C)], E[u_F(\tau^*_C)]\} \text{ and } \tau^*_A \leq (f')^{-1}(\frac{1}{R}) \\
\tau^*_A & \text{if } E[u_F(\tau^*_A)] \geq \max\{E[u_F(\tau^*_A)], E[u_F(\lambda^*_C)], E[u_F(\tau^*_C)]\} \text{ and } \tau^*_A > (f')^{-1}(\frac{1}{R}) \\
\tau^*_C & \text{if } E[u_F(\tau^*_C)] > \max\{E[u_F(\tau^*_A)], E[u_F(\lambda^*_C)], E[u_F(\tau^*_C)]\} \text{ and } \tau^*_C \leq 1 - R \frac{f(0)}{1 - f(0)} \\
\tau^*_C & \text{if } E[u_F(\tau^*_C)] > \max\{E[u_F(\tau^*_A)], E[u_F(\lambda^*_C)], E[u_F(\tau^*_C)]\} \text{ and } \tau^*_C > 1 - R \frac{f(0)}{1 - f(0)}. 
\end{cases}
$$

Thus the tax rate in the game with collusion differs from that of the original game only if $E[u_F(\tau^*_C)] > \max\{E[u_F(\tau^*_A)], E[u_F(\tau^*_C)]\}$ and $\tau^*_C \in \left( 1 - k \frac{f(0)}{1 - f(0)}, 1 - R \frac{f(0)}{1 - f(0)} \right)$, in which case the tax rate is higher with collusion.

From equation (17), the firms hire the mafia in the game with collusion if $R < (1 - \tau) \frac{1 - f(\lambda^*\tau)}{f(\lambda^*\tau)}$. From equation (5), in the game without collusion the firms hire the mafia if $k < (1 - \tau) \frac{1 - f(\lambda^*\tau)}{f(\lambda^*\tau)}$. By assumption, $R < k$. If the optimal tax rate is in region A, both firms hire the mafia with certainty. Moreover, the size of region A is the same in both games. If the tax rate is in the interior of Region C the firms hire the mafia with probability $1 - \pi = \frac{1}{R f(\lambda^*)}$. 

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which is the same in both games. If, the optimal tax rate in the game without collusion is on the upper border of region C and the optimal tax rate in the game with collusion is in \((1 - k \frac{f(0)}{1-f(0)}, 1 - R \frac{f(0)}{1-f(0)})\), then the firms hire the mafia with 0 probability in the game without collusion and with positive probability \(1 - \pi\), in the game with collusion. Finally, if the optimal tax rate in both games is on the upper border of region C, then the firms hire the mafia with probability 0 in both games. ■
Works Cited


