The Problem with Majority Rule

Shepsle and Bonchek Chapter 4
Majority Rule is problematic

1. Who’s the majority?
2. Sometimes there is no decisive winner
   • Condorcet’s paradox: A group composed of individuals with individually transitive preferences do not necessarily have transitive preferences as a collectivity
3. When the group’s preferences are intransitive there is either no stable outcome or the outcome is determined by the rules of the game.
   • Typically, the rule designating an agenda setter is decisive
Today, we're going to explore the implications of these problems by asking:

1. "How general" a problem is "cyclical" majorities?

2. What's so special about majority rule anyway?

3. What can be done?
Are "intransitive group preferences" a common problem?

• Sure, Andrew, Bonnie, and Chuck ran into trouble deciding, but…. 
……they had other issues too (Red Sox? You gotta be kiddin’ me!)

Are groups of normal, canoli-eating, Yankee game watchin’ people likely to have the same problem?
It depends….

Probability of group intransitivity = \( f(m,n) \)

where

- \( m \) is the number of alternative and
- \( n \) is the number of voters
Specifically….

\[
p(\text{intransitivity}) \propto \frac{\# \text{of} \ " \text{problem}" \ \text{preference configurations}}{(m!)^n}
\]

\[
p(\text{intransitivity}) \propto \frac{\# \text{of} \ " \text{problem}" \ \text{preference configurations}}{(m \times (m - 1) \times (m - 2) \times \ldots \times 2 \times 1)^n}
\]
Probability of a cyclical majority, $f(m,n)$

<table>
<thead>
<tr>
<th>$(m)$</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
<th>11</th>
<th>limit</th>
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<td>3</td>
<td>.056</td>
<td>.069</td>
<td>.075</td>
<td>.078</td>
<td>.080</td>
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<tr>
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<td>.150</td>
<td>.156</td>
<td>.160</td>
<td>.176</td>
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<td>.20</td>
<td>.215</td>
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<td>.251</td>
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<tr>
<td>6</td>
<td>.202</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.315</td>
</tr>
<tr>
<td>limit</td>
<td>?</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
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Example: Divide the Dollars

• Suppose there are three regions in a town and they've just been given $1000 dollars to divide - if they can agree how to divide it.
Divide the Dollars - details

- let $s(E)$, $s(C)$, and $s(W)$ be the shares going to East, Central, and West, respectively.

- a sharing scheme (strategy combination) $(s(E); s(C); s(W))$ is feasible if each component is non-negative, and the components sum to something less than $1000$.

- A sharing scheme is efficient if the values sum to $1000$ (nothing is wasted).

- Representatives make alternating offers until they settle on a division of the pie that defeats all additional proposals.
What happens?

• Well, we can say that the outcome will be efficient, but we can't say much more than that. Why?
Divide the Dollar: Proposal 1 “To share is fair”
Divide the Dollar: Proposal 2 “Go @#$%! West Man”
Divide the Dollar: Proposal 3 “West says to East: “I’m easy, I don’t want alot”
Divide the Dollare: Proposal 4 “Can’t we find a “fair” solution?”
Majority Cycle in “Divide the Dollar Game”

2. \((500,500,0) P_{EC} (333 \frac{1}{3}; 333 \frac{1}{3}; 333 \frac{1}{3})\)

3. \((700,0,300) P_{EW} (500,500,0)\)

4. \((333 \frac{1}{3}; 333 \frac{1}{3}; 333 \frac{1}{3}) P_{CW} (700,0,300)\)

5. \((500,500,0) P_{EC} (333 \frac{1}{3}; 333 \frac{1}{3}; 333 \frac{1}{3})\)

6. \((700,0,300) P_{EW} (500,500,0)\)

7. \((333 \frac{1}{3}; 333 \frac{1}{3}; 333 \frac{1}{3}) P_{CW} (700,0,300)\)….etc
Shift the tax burden: Proposal 1 “share the love”
Shift the tax burden: Proposal 2 “Family values: Protect inheritance, and protect our nation’s farms”
Shift the tax burden: Proposal 3 “Soak the rich!”
Shift the tax burden: Proposal 4 “Save our cities!”
Shift the tax burden: Proposal 5 “Family values: Protect inheritance, and protect our nation’s farms”
Cycling majorities shifting the tax burden

<table>
<thead>
<tr>
<th>Proposal</th>
<th>Proposer</th>
<th>Cut taxes on</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Rich</td>
<td>wealth,</td>
<td>land</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Wage-earners</td>
<td>income,</td>
<td>land</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Rich</td>
<td>income,</td>
<td>wealth</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Farmer</td>
<td>wealth,</td>
<td>land</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Wage-earners</td>
<td>?</td>
<td>?</td>
<td>?</td>
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Conclusion

• Majority rule seems to be deeply flawed in handling the “most political” of political problems
What's so special about majority rule?
May showed that

Majority rule $\iff A, N, M$

So arguing for against majority rule means arguing for or against A, N, or M
Condition A (Anonymity)

Social preferences depend only on the collection of individual preferences, not on who has which preference.
Condition N (Neutrality)

changing rank of $j$ and $k$ in each group members preferences changes rank of $j$ and $k$ in group preferences (i.e. naming the alternatives is arbitrary)
Condition M (Monotonicity)

- if \( j \) is at least as good a \( k \) from the group’s standpoint, and \( j \) becomes more desirable to one of the members, then \( j \) is now strictly better than \( k \) from the standpoint of the group.
When does majority rule make sense?

ex. Should grades be determined by majority rule?

ex. Should what I have for breakfast be decided by majority rule?

ex. Should amendments to the constitution be decided by majority rule?

ex. Should students at a public high school be allowed to vote on whether or not to have organized prayer at football games?
We already saw that

- Majority rule creates practical problems in some situations
- May not be normatively appealing in all situations

So why don’t we ditch it?
Arrows theorem – Majority rule is not special

- The pathologies of majority rule apply to “any” group decision procedure that meets some minimal standards
These minimal standards can be thought of as generalizations of May’s conditions for majority rule.

<table>
<thead>
<tr>
<th>May Condition</th>
<th>Arrow Condition</th>
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</thead>
<tbody>
<tr>
<td>Anonymity</td>
<td>Dictatorship</td>
</tr>
<tr>
<td>Neutrality</td>
<td>Independence</td>
</tr>
<tr>
<td>Monotonicity</td>
<td>Pareto Optimality</td>
</tr>
</tbody>
</table>
A (Anonymity) is a special case of what Arrow called "Non-Dictatorship" (D)

There is no distinguished individual \( i^* \) \( \in \mathcal{G} \) whose preferences dictate the group preference, independent of other members.
N (Neutrality) is a special case of what Arrow called "Independence from Irrelevant Alternatives" (I)

if j and k stand in a particular relationship to each other for each member of the group, and this relationship does not change, then neither should the group preference between j and k
M (Monotonicity) is a special case of what Arrow called "Unanimity" (P) or Pareto Optimality.

If every member of G prefers j to k (or is indifferent between them), then the group preference must reflect a preference for j over k (or an indifference between them).
Arrow argued that any reasonable procedure for making group choices should involve D, I, and P, and two other criteria:

**Condition U (Universal admissibility)** (each $i \in G$ may adopt any strong or weak complete and transitive preference ordering over the alternatives in $A$)

**Rationality assumption** $R_G$ is complete and transitive.
Arrow’s theorem

There exists no mechanism for translating the preferences of rational individuals into a coherent group preference that simultaneously satisfies conditions U, P, I, and D.
Conclusion

Arrow showed that if you accept U,P,I as "untouchable" (May shows us that advocating majority rule amounts to making U,P,I untouchable) you have accept either

1. Dictatorship
2. The potential for intransitivity