Shakeouts and Market Crashes*

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Abstract

Stock-market crashes tend to follow run-ups in prices. These episodes look like bubbles that gradually inflate and then suddenly burst. We show that such bubbles can form in the Zeira-Rob model in which demand size is uncertain. Two conditions are sufficient for this to happen: A declining hazard rate in the prior distribution over market size and a convex cost of investment. For the period 1971-2001 we fit the model for the Telecom sector, as well as for all the Nasdaq firms as one group.

1. Introduction

Stock-market crashes tend to follow run-ups in prices. The NYSE index rose in the late 1920s, and then crashed in October 1929, and the Nasdaq rose steadily through the 80’s and 90’s and crashed after March 2000. These episodes therefore look like bubbles that gradually inflate and then suddenly burst.

In a learning model of the Zeira-Rob type we study the possibility of bubble-like behavior of stock prices, but related purely to fundamentals. This model generates a crash when an irreversible creation of capacity overshoots demand. We add to Rob (1991) a convex adjustment cost for the growth of industry capacity and a declining hazard rate in the prior distribution over market size. Many of the ideas are also in Zeira (1988, 1999) and in Horvath, Schivardi and Woywode (2001).

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The first picture portrays the 30-year history of the Nasdaq index and its Telecom component. The first panel shows the financial side, the second also includes an index of the real activity. The two seem to be linked in that they both experience a sharp reduction in early 2000. Our aim is to understand what this link is – we wish to explain the slow rise in the stock market and the capital invested and then a sharp and sudden decline.

Our explanation of the run-up is of the “Peso Problem” type. Krasker (1980) introduced the term to describe the rational response of agents to a major event that is not realized within the sample period. Before the year 2000, this was how things looked to agents in the market. The “decreasing hazard” assumption is the technical feature of the model ensuring that, as the market grows, further growth looks ever more feasible, and the likelihood of a crash looks ever more remote. In such a situation it is rational to be more and more optimistic about the market’s growth potential as market size increases. If and when the crash does come, however, our model predicts that

- the more remote the possibility of a crash looks, and
- the more convex the adjustment cost of rapid creation of capacity,

the bigger the crash will be. We shall fit the model to the Telecom sector where a substantial crash decidedly did happen in early 2000.
2. Model

We begin with the simplest model, i.e., the rectangular-demand case of Rob (1991), to which we add convex adjustment costs for capacity. Capital has zero salvage value. In section 4 we generalize to downward-sloping demand and a positive salvage value for capital.

Demand.—The market demand function expresses willingness to pay as a function of the quantity, $Q$, supplied to the market,

$$P(Q, z) = \begin{cases} p & \text{if } Q < z \\ 0 & \text{if } Q \geq z \end{cases}$$

Demand is uncertain. Willingness to pay, $p$, is known. What is unknown is the extend of demand, the true realization of which is denoted by $z$, which we may think of as the number of new consumers, each demanding one unit of good per unit time. The parameter $z$ does not change over time. Its a random variable drawn at time $t = 0$ from a distribution $F(z)$ and never drawn again afterwards; 1 shows a family of demand curves indexed by various values of $z$, and highlights the demand curve that would occur if $z = z_2$. The distribution $F(z)$ is common knowledge among the potential entrants at $t = 0$. It will act as the prior distribution and it will be updated in a Bayesian fashion in light of experience. We allow
$F$ to have positive mass on $z = \infty$. A $z = \infty$ market will never crash.

**Production.**—Firms are infinitessimal and of indeterminate size. Production cost is zero, and the salvage value of production capacity is negligible. As long the price is positive, industry output is the same as the industry’s capacity to produce it. Let $k$ denote the industry’s capacity and let $n$ denote new capacity, i.e., aggregate investment. Capacity does not depreciate, and, as long industry price is positive, it is not scrapped. Therefore, capacity evolves as follows

$$k' = k + n.$$  \hspace{1cm} (1)

Initial capacity, $k_0 \geq 0$ is given.

**Investment.**—Adjustment costs of investment are rising at the industry level, but constant at the level of an individual firm. The unit cost, $c$, of adding capacity rises with aggregate investment

$$c = C(n),$$

where $C'(n) > 0$, to reflect a pecuniary or non-pecuniary congestion effect. Each firm perceives $c(n)$ to be independent of its of its own actions. An example is a scarce input into the construction of capacity the price of which rises with the amount of aggregate investment. Investment becomes productive capacity in the following period.

**Industry viability.**—To ensure that the industry can get off the ground in the first place, we assume that if no one else were to invest, it would be optimal to do so. The discounted proceeds that a unit of capacity would deliver if the market were never to crash would be $\beta p / (1 - \beta)$. The cost of creating a unit of capacity if no one else is creating it would be $C(0)$. Hence our viability condition is

$$C(0) < \frac{\beta p}{1 - \beta}.$$  \hspace{1cm} (2)

**Learning.**—All firms know the past history of prices and industry outputs. Based on this they revise their opinion about $z$. “Overshooting” happens when $k$ exceeds $z$ for the first time. Before overshooting the firms know only that $z \geq k$. After overshooting, we assume that firms learn $z$ exactly. Their prior over $z$ is the C.D.F. $F(z)$. To be clear, we re-state all this: The model has three distinct epochs, before and after overshooting

\[1\] In duration analysis, such a distribution would be called “defective.”
1. Before overshooting: Agents know only that $k \leq z$

2. At the overshooting date – “date $T$” – for the first time $k > z$, and firms learn $z$ exactly, perhaps because spare capacity $k_T - z$ becomes public information. This is illustrated in Figure 2.

3. Excess capacity amounting to $k_T - z$ is scrapped for a return of $s$ and equilibrium price remains at $s / (1 - \beta)$ thereafter. To simplify the algebra we assume that $s$ is negligible so that all firms get zero after the overshooting date.

*The unit value of capacity.*—Suppose that when information that firms have is only $k$ and the knowledge of whether $k$. The value of a unit of capacity in an industry with the state

$$V(k, z) = \begin{cases} v(k) & \text{if } k \leq z \\ 0 & \text{if } k > z. \end{cases}$$

We have, in other words, assumed that the value of all firms drops to zero if and when $k$ exceeds $z$ for the first time. The idea is that the salvage value of the capital
is low and that the capital does not depreciate. Therefore, once industry capacity outstrips demand, the product price crashes to zero, and with it the market value of the firm.\footnote{An example is the collapse in the price of on-line advertising at sites such as Yahoo and AOL, and the resulting fall in the prices of their shares – by a factor of three or more – and those of their competitors (Angwin 2002). Another example is the recent collapse of the Indian diamond industry after an over expansion. Section 5 elaborates on the events that take place after the crash.} At that date $z$ becomes publicly known, Section 5 discusses this in more detail.

**2.0.1. Equilibrium**

In this equilibrium strategies depend only on the industry state, $k$. All firms are of measure zero, so it does not matter whether new capacity is created by incumbents or by new entrants. The Bellman equation in the region $k < z$ is a function of the entry rule $n(k)$ which leads to next period capacity $k' = k + n(k)$, so that

$$v(k) = p + \beta v(k + n[k]) \frac{1 - F(k + n[k])}{1 - F(k)}$$  \hspace{1cm} (3)

It then takes a period to start producing, so the free entry condition in state $k$ is

$$C(n[k]) = \beta v(k + n[k]) \frac{1 - F(k + n[k])}{1 - F(k)}$$  \hspace{1cm} (4)

The industry-viability condition is (2)

**Definition 1.** Equilibrium is a pair of functions $v(k)$ and $n(k)$ defined for $k \in [0, z]$ that satisfy (3) and (4).

Substituting from (4) into (3), the relation is

$$v(k) = p + C(n[k])$$  \hspace{1cm} (5)

Let $c = c(n[k])$. Note that (5) gives us a linear relation between $v$ and $c$. Whether or not equilibrium is unique, (5) implies a unique positive relation between the two observables $n$ and $v$: $n = C^{-1}(v - p)$
With (4) this implies

\[ C(n) = \beta (p + C [n']) \frac{1 - F(k + n)}{1 - F(k)}. \]  \hspace{1cm} (6)

or, rearranged,

\[ C(n') = -p + C(n) \frac{1}{\beta} \frac{1 - F(k)}{1 - F(k + n)} \]

\[ = -p + C(n) \frac{1}{\beta} \exp \left\{ \int_k^{k+n} h(s) ds \right\} \]  \hspace{1cm} (7)

where \( h(z) = \frac{f(z)}{1-F(z)} \).

2.0.2. Evolution of \( k \) before the crash at date \( T \)

Assumptions on \( F \).—Suppose that the support of \( F \) is \([z_{\min}, \infty)\). Suppose furthermore that \( h(z) > 0 \) for all \( z > z_{\min} \), and that \( h'(z) < 0 \). That is, \( F \) has a strictly decreasing hazard. Assume, moreover, that\(^3\)

\[ \lim_{z \to \infty} h(z) = 0. \]

Now define the scalar \( n_{\infty} \) implicitly as the solution to the equation

\[ C(n_{\infty}) = \frac{\beta p}{1 - \beta}. \]  \hspace{1cm} (8)

Lemma 2. If \( n_{\infty} \equiv \lim_{t \to \infty} n_t \) exists, then it is unique and satisfies (8)

Proof. The RHS of (4) is at most equal to the RHS (8). Therefore

\[ n_t < n_{\infty}. \]  \hspace{1cm} (9)

Moreover, the probability of the market surviving for another period is

\[ \xi(k,n) \equiv \frac{1 - F(k + n)}{1 - F(k)} = \exp \left\{ -\int_k^{k+n} h(z) dz \right\} \]

\(^3\)The results seem to all go through even if \( F(z) \) is a defective distribution in which case \( \lim_{z \to \infty} F(z) < 1 \). This is possible when the hazard is declining so that \( \int_0^{\infty} h(z) dz < \infty \)
If \( h \) is decreasing, \( \xi \) is increasing in \( k \) and decreasing in \( n \). Since \( k_t \) is an increasing sequence, we then must have

\[
\xi (k_t, n_t) \geq \xi (k_0, n_\infty)
\]

Therefore for any \( t \), the return to a unit of incumbent capital is

\[
v_t = \sum_{j=t}^{\infty} \beta^{j-t} \prod_{\tau=t}^{j} \xi (k_\tau, n_\tau) p
\]

\[
\geq p \sum_{j=t}^{\infty} [\beta \xi (k_0, n_\infty)]^{j-t}
\]

\[
\geq \frac{p}{1 - \beta \xi (k_0, n_\infty)}
\]

Therefore, for any \( t \)

\[
n_t \geq n_{\text{min}} > 0,
\]

where \( n_{\text{min}} \) solves

\[
C (n_{\text{min}}) = \frac{\beta \xi (k_0, n_\infty) p}{1 - \beta \xi (k_0, n_\infty)}
\]

But then \( k_t \geq t n_{\text{min}} \), and so \( \lim_{t \to \infty} k_t = \infty \). Therefore the RHS of (7) converges to \( p + C (n) / \beta \). Rearranging, we get (8).

The following proposition concerns the full dynamics that pertain to those dates that precede the date of the crash. Now we can state the main result (which indeed does depend on there being a decreasing hazard):

**Proposition 3.** Before the crash (i.e., for \( t < T \)),

\[
n_{t+1} > n_t.
\]

**Proof.** We assume that \( C \) is a one-to one increasing map from \( R_+ \to R_+ \) and that its range is all of \( R_+ \) so that \( n (c) \equiv C^{-1} [c] \) is uniquely defined for all \( c \geq 0 \). Then write (7) as

\[
c' = -p + \frac{c}{\beta} \exp \left\{ \int_{k}^{k+n(c)} h (z) dz \right\}
\]

\[
\equiv \phi (c, k)
\]
Since $n(c)$ is strictly increasing, $\phi(c, k)$ is strictly increasing in $c$. Since $h(z)$ is a decreasing function, $\phi(c, k)$ is strictly decreasing in $k$.

Suppose, contrary to the claim, that $n_{t+1} \leq n_t$. Then $c_{t+1} \leq c_t$. Since $C(0) = 0$, we must have $n_{t+1} > 0$, or else (4) would be violated. But then $k_{t+1} > k_t$, and therefore
\[ c_{t+2} = \phi(c_{t+1}, k_{t+1}) < \phi(c_t, k_t) = c_{t+1}. \]
Iterating this argument leads to the conclusion that
\[ n_t \geq n_{t+1} \implies n_{t+1} > n_{t+2} > \ldots \geq n_{\min} \]
And since the initial value $n_t < n_{\infty}$, we conclude that $\lim_t n_t < n_{\infty}$. But $k_t \geq t n_{\min}$ and once again $\lim_{t \to \infty} k_t = \infty$, and therefore (8) must hold, and this is a contradiction. \[ \blacksquare \]

Since a bounded monotone sequence must have a limit, we conclude that $(n_t)$ indeed does have a limit and that this unique limit solves (8).

We assume that $C$ is a one-to one increasing map from $R_+ \to R_+$ and that its range is all of $R_+$ so that $n(c) \equiv C^{-1}[c]$ is uniquely defined for all $c \geq 0$.

**Lemma 4.** If $h$ is decreasing in $z$, $\phi$ is decreasing in $k$.

**Proof.** Differentiating,
\[
\frac{\partial \phi}{\partial k} = \left[ h(k + C^{-1}[c]) - h(k) \right] \frac{c}{\beta} \exp \left\{ \int_k^{k+C^{-1}(c)} h(z) \, dz \right\} < 0
\]
because $h$ is decreasing, whereas $C^{-1}[c] > 0$. \[ \blacksquare \]

Since $\phi$ decreases with $k$, the mode of convergence will therefore be as shown in Figure 3.

### 2.0.3. Example: Pareto $F$

A decreasing hazard distribution is the Pareto:
\[
F(z) = 1 - \left( \frac{z}{z_{\min}} \right)^{-\rho}.
\]
Its hazard rate is
\[
\frac{f(z)}{1 - F(z)} = \left( \frac{z}{z_{\min}} \right)^{\rho} \left( \frac{z}{z_{\min}} \right)^{-\rho - 1}
\]
\[
= \rho \frac{z_{\min}}{z}.
\]
Figure 3: Convergence of the solution, $c_t$, to the difference equation (13).

Now (6) reads

$$C(n) = \beta \left( p + C[n'] \right) \left( \frac{k}{k'} \right)^\rho$$

Then, since $k' = k + n$, this equation reads

$$C(n) = \beta \left( p + C[n'] \right) \left( \frac{1}{1 + \frac{n}{k}} \right)^\rho$$

which we can rearrange this into the difference equation

$$C(n') = -p + C(n) \frac{1}{\beta} \left( 1 + \frac{n}{k} \right)^\rho \quad (15)$$

This is the specific form depicted in Figure (3).

Assume now that

$$C(n) = Cn \quad \text{and} \quad \rho = 1.$$ 

Then (15) reads

$$n' = -\frac{p}{C} + \frac{1}{\beta} n \left( 1 + \frac{n}{k} \right) \quad (16)$$
The second difference equation is

\[ k' = k + n \]  

(17)

**2.0.4. Saddle path analysis of the transformed system**

The pair of difference equations [(16), (17)] in \((n, k)\) has no finite steady state for \(k\). In order to be able to linearize around a steady state, we change variables from \(k\), the level of capacity, to its rate of growth

\[ x = \frac{n}{k}. \]

We shall now analyze the evolution of the pair \((n, x)\). The change of variables transforms the pair (16) and (17) into the following pair of difference equations

\[ n' = -\frac{p}{C} + n \frac{1}{\beta} (1 + x)^\rho, \]

(18)

\[ x' = \frac{x}{(1 + x)} \left[ -\frac{p}{Cn} + \frac{1}{\beta} (1 + x)^\rho \right]. \]

(19)

These equations have the unique steady state. Now the steady state of the system is

\[ \begin{pmatrix} n \\ x \end{pmatrix} = \begin{pmatrix} \frac{\beta p}{(1 - \beta)c} \\ 0 \end{pmatrix} \]

So let us linearize around it. The Jacobian evaluated at the steady state is

\[ \begin{bmatrix} \frac{1}{\beta} & \frac{p \beta}{(1 - \beta)c} \\ 0 & 1 \end{bmatrix} \]

(20)

The characteristic roots are \(\left(\frac{1}{\beta}, 1\right)\). As is standard we set \(n' = n\) and \(x' = x\) to find two curves crossing in the steady state

\[ n' = n \implies n = \frac{p}{c} \left( \frac{\beta}{[1 + x]^\rho - \beta} \right) \equiv \Phi(x) \]

\[ x' = x \implies n = \frac{p \beta}{c} \left( \frac{1}{[1 + x]^\rho - \beta (1 + x)} \right) \equiv \Psi(x) \]
These are the two demarcation curves in the phase diagram, and they cross at the steady state. Both are downward sloping (at least if $\rho = 1$) the ratio

$$\frac{\Psi(x)}{\Phi(x)} = \frac{[1 + x]^\rho - \beta}{[1 + x]^\rho - \beta - \beta x} = \begin{cases} > 1 & \text{if } x > 0 \\ = 1 & \text{if } x = 0 \\ < 1 & \text{if } x < 0 \end{cases}.$$ 

So, $\Psi(x)$ is steeper than $\Phi(x)$, and the two curves cross at the steady state, as shown in Figure 4. The area where either $n < 0$ or $x < 0$ is not relevant for the pre-overshooting stage of the game, hence it is shaded.

To be able to draw the typical arrows on the phase diagram rewrite the system as

$$n' = -\frac{p}{C} + n \frac{1}{\beta} (1 + x)^\rho \equiv A(x,n),$$

$$x' = \frac{x}{(1 + x)} \left[ -\frac{p}{Cn} + \frac{1}{\beta} (1 + x)^\rho \right] \equiv B(x,n).$$

Then,

$$A(x, \Phi[x]) = n \quad (21)$$
and

\[ B(x, \Psi[x]) = x \tag{22} \]

The vertical arrows.—First we show that if \( n > \Phi(x) \), we move even higher. And the opposite if \( n < \Phi(x) \). That is,

**Claim 5.**

\[ n \geq \Phi(x) \implies A(x, n) \geq n. \]

**Proof.** The relevant portion of the phase diagram is that for \( x \geq 0 \). For all such \( x \),

\[ -\frac{\partial A(x, n)}{\partial n} \geq \frac{1}{\beta} > 1 \]

Together with (21) the claim follows. ■

The horizontal arrows.—Next we show that if \( n > \Psi(x) \), we move to the right. And if \( n < \Psi(x) \), we move to the left. That is,

**Claim 6.**

\[ n \geq \Psi(x) \implies B(x, n) \geq x. \]

**Proof.** We have

\[ \frac{\partial B(x, n)}{\partial n} = \frac{x}{(1 + x)} \frac{p}{Cn^2} > 0 \]

if \( x > 0 \). Together with (22) the claim follows. ■

These two claims pin down the arrows that are displayed in Figure 4 along with the saddle path. The evolution of \( n \) and \( x \) is valid only before the overshooting date. The shaded area is not admissible because \( x \) cannot be negative. In Figure 5 we plot the time path for \( n \). Mathematica was used to draw the stable manifold. This figure shows in greater detail what would happen to the trajectory if it did not start on the saddle path.

### 3. Empirical part I: testing some predictions

We might think of running a couple of regressions at this point to have a crude reality check. In trying to bring the “right” data to the model we concentrate
Figure 5: Equilibrium time path for \( n_t \) as it approaches \( n_\infty \) on the Telecom data-set (representing more closely an “industry”). We would estimate the two structural equations:

\[
\begin{align*}
\nu_t &= p + C(n_t) \\
\tau_{t+1} &= -\frac{p}{C} + \frac{1}{\beta}n_t \left(1 + \frac{n_t}{k_t}\right)^\rho
\end{align*}
\]

There might be different ways of specifying the cost function and the Pareto distribution. Next we propose 4 specifications.

**Model 1:** Assuming \( C(n) = Cn \) and \( \rho = 1 \)

\[
v_t = p + Cn_t
\]

and

\[
\tau_{t+1} = -\frac{p}{C} + \frac{1}{\beta}n_t \left(1 + \frac{n_t}{k_t}\right)
\]

leading to the regressions

\[
v_t = a_0 + a_1 n_t + \varepsilon_t
\]

and

\[
\tau_{t+1} = a_2 + a_3 n_t \left(1 + \frac{n_t}{k_t}\right) + \omega_t
\]

\[
\rightarrow a_2 + a_3 n_t + a_4 \frac{n_t}{k_t} + \omega_t
\]
obtaining parameter estimates with OLS for both equations.

**Model 2:** Assuming \( C(n) = Cn \) only

\[
v_t = p + Cn_t
\]

and

\[
n_{t+1} = -\frac{p}{C} + \frac{1}{\beta} n_t \left( 1 + \frac{n_t}{k_t} \right)^\rho
\]

leading to the regressions

\[
v_t = a_0 + a_1 n_t + \varepsilon_t
\]

and

\[
n_{t+1} = a_2 + a_3 n_t \left( 1 + \frac{n_t}{k_t} \right)^\rho + \omega_t
\]

obtaining parameter estimates with OLS for the first regression and using Non-linear least squares for the second.

**Model 3:** Assuming \( C(n) = Cn^\lambda \)

\[
v_t = p + C \left( \frac{n_t^\lambda - 1}{\lambda} \right) + \varepsilon_t
\]

and

\[
\frac{n_t^\lambda - 1}{\lambda} = -\frac{p}{C} + \frac{1}{\beta} \left( \frac{n_t^\lambda - 1}{\lambda} \right) \left( 1 + \frac{n_t}{k_t} \right)^\rho + \omega_t
\]

so we estimate

\[
v_t = a_0 + a_1 \left( \frac{n_t^\lambda - 1}{\lambda} \right) + \varepsilon_t
\]

with Max. Likelihood finding \( \hat{\lambda} \) and then in a second step we estimate

\[
\left( \frac{n_{t+1}^\lambda - 1}{\lambda} \right) = a_2 + a_2 \left( \frac{n_t^\lambda - 1}{\lambda} \right) \left( 1 + \frac{n_t}{k_t} \right)^\rho + \omega_t
\]

with NLLS.

**Model 4:** In the last specification we calibrate the parameter \( a_2 = 1.1 \) as it should be in according theory \( (= \frac{1}{\beta}) \). So the first eq. remains

\[
v_t = a_0 + a_1 \left( \frac{n_t^\lambda - 1}{\lambda} \right) + \varepsilon_t
\]
and the second becomes
\[
\left(\frac{n_t^{\lambda + 1} - 1}{\lambda}\right) = a_2 + 1.1 \left(\frac{n_t^{\lambda} - 1}{\lambda}\right) \left(1 + \frac{n_t}{k_t}\right)^{\rho} + \omega_t
\]

**Remarks:**

1. We are *jointly testing* the model basic structure and assumptions on cost functional forms and prior for the first equation and cost functional form for the second.

2. We should justify the error term in the regressions since our model implies they hold exactly. It cannot be heterogeneity given homogeneity of firms. It could be *measurement error* or *errors in optimization* or *approximation errors* (Rust HE99). An alternative way is modelling the error in a structural way including it as an *unobserved state variable* (or heterogeneity). Notice that if measurement error is invoked then it is problematic to estimate the second equation of the model since both dependent and independent variables suffer from measurement errors.

**Variables used:** Our TELECOM data contain the series:

- 1. *total_sales*, used to proxy \(k_t\), from which we can calculate \(n_t = k_{t+1} - k_t\) [the variable *property_plant_equipment* delivered similar results]
- 2. value-weighted stock-price index *NASDAQ_telecom*, used to proxy \(v_t\)

**TABLE 1:** Estimation results of the first structural eq. in the 4 models.

<table>
<thead>
<tr>
<th>COEFF</th>
<th>model 1</th>
<th>model 2</th>
<th>model 3</th>
<th>model 4</th>
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<tr>
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<td>(19.56)</td>
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<td>(0.0024)</td>
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<td>0.86</td>
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The comparison shows how, in general across the 4 models, the first structural equation (not embodying the Pareto assumption) performs very well.
TABLE 2: Estimation of the second structural eq. in the 4 models.

<table>
<thead>
<tr>
<th>COEFF</th>
<th>model 1</th>
<th>model 2</th>
<th>model 3</th>
<th>model 4</th>
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<td></td>
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<tr>
<td>$a_4$</td>
<td>1.05</td>
<td>constr=$a_3$</td>
<td>constr=$a_3$</td>
<td>constr=$a_3$</td>
</tr>
<tr>
<td>(st.dev.)</td>
<td>(0.42)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>fixed at 1</td>
<td>2.7</td>
<td>4.1</td>
<td>-7.4</td>
</tr>
<tr>
<td>(st.dev.)</td>
<td>(0.9)</td>
<td>(0.7)</td>
<td>(0.5)</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.64</td>
<td>0.67</td>
<td>0.87</td>
<td>0.63</td>
</tr>
<tr>
<td>adj-$R^2$</td>
<td>0.63</td>
<td>0.65</td>
<td>0.86</td>
<td>0.62</td>
</tr>
</tbody>
</table>

It seems that something is wrong with the Pareto model. We have that when we impose the coefficient $a_3 = \beta$ to be an acceptable value ($a_3 = 1.1$ in our exercise) model 4 produces unacceptable values for the parameter of the Pareto. At the same time if, like in model 3, $a_3$ is left unconstrained, acceptable values are obtained for $\rho$ but unacceptable for $a_3$. All this might suggests that Pareto beliefs are not a proper assumption. When we simulate the series from the fitted model and we compare them to the real ones, we have a pictorial view of how the Pareto model badly tracks them. Probably we need a non-monotone hazard to generate an acceptable $n_t$ series. In a following section we might answer the question of why model 3 has such a low estimate for $a_3$.

4. Empirical Part II: Adding shocks to the model

4.1. Theoretical Framework

The difference equations (16) and (17) are deterministic. But neither will fit the data exactly. We will add two shocks to the model: A shock to the willingness to pay, $p$, and a shock to the entry cost $c$. But since these are shocks that the agents experience themselves, their optimal decision rules change. We now explain this minor extension of the model.

The demand shock.—Assume that

$$Price = \begin{cases} 
    p + \varepsilon_t & \text{if } Q < z, \\
    0 & \text{if } Q \geq z.
\end{cases}$$
and that $\varepsilon_t$ is i.i.d., and independent of $z$; $P^\varepsilon(\varepsilon)$ is the distribution of $\varepsilon$, that $\int \varepsilon dP^\varepsilon(\varepsilon) = 0$, and $\varepsilon_t \geq -p$. Then, observing that $p + \varepsilon_t > 0$ still leads agents to infer only that $z > k_t$.

*The cost shock.*—We also assume a time-specific fixed-cost component $\eta_t$ of investment costs so that they become

$$\tilde{C}(n, \eta_t) = C(n) + \eta_t.$$  \hfill (25)

Like $\varepsilon$, we assume that $\eta_t$ is i.i.d., and independent of $z$; $P^\eta(\eta)$ is the distribution of $\eta$, that $\int \eta dP^\eta(\eta) = 0$.

The state of the industry is $(k, z, \varepsilon, \eta)$. But we expect that $\varepsilon$ will not affect the evolution of $k$, so we shall just assume it and then verify it in equilibrium. The value of a unit of capacity is positive only if $z > k$. That is,

$$V(k, z, \varepsilon, \eta) = \tilde{v}(k, \varepsilon, \eta) I_{\{k \leq z\}}$$

where

$$\tilde{v}(k, \varepsilon, \eta) = p + \varepsilon + \beta E_{z,\varepsilon} \{ V(k + n[k, \eta], z, \varepsilon) \mid z > k \}$$  \hfill (26)

The marginal investment condition is

$$C(n[k, \eta]) + \eta = \beta \left( \frac{1 - F(k + n[k, \eta])}{1 - F(k)} \right) \int \tilde{v}(k + n[k], \varepsilon', \eta') dP^\varepsilon(\varepsilon') dP^\eta(\eta')$$  \hfill (27)

Substituting into (27) implies

$$\tilde{v}(k, \varepsilon, \eta) = p + C(n[k, \eta']) + (\varepsilon' + \eta').$$  \hfill (28)

To get to the second estimating equation, substitute from (29) into (28) to get

$$C(n_t) + \eta_t = \beta \left( \frac{1 - F(k_t + n_t)}{1 - F(k_t)} \right) \int \tilde{v}(k_t + n_t, \varepsilon_{t+1}, \eta_{t+1}) dP^\varepsilon(\varepsilon_{t+1}) dP^\eta(\eta_{t+1})$$

$$= \beta \left( \frac{1 - F(k_t + n_t)}{1 - F(k_t)} \right) \left[ p + C(n_{t+1}) + \varepsilon_{t+1} + \eta_{t+1} \right] dP^\varepsilon(\varepsilon_{t+1}) dP^\eta(\eta_{t+1})$$
But \( \varepsilon' \) and \( \eta' \) are both zero mean, and so we end up with

\[
C(n_t) + \eta_t = \beta \left( \frac{1 - F(k_t + n_t)}{1 - F(k_t)} \right) [p + C(n_{t+1})]
\]

Rearranging,

\[
C(n_{t+1}) = -p + \frac{1}{\beta} C(n_t) \left( \frac{1 - F(k_t)}{1 - F(k_t + n_t)} \right) + \omega_t, \tag{30}
\]

where

\[
\omega_t = \frac{1}{\beta} \left( \frac{1 - F(k_t)}{1 - F(k_t + n_t)} \right) \eta_t
\]

Unfortunately this error term is correlated with the regressor, \( n_t \), for 2 reasons: (i) \( \eta_t \) is negatively related to \( n_t \) because it is an index of the fixed cost of entry at \( t \), and (ii), there is some heteroskedasticity. This explains why we are estimating such a low coefficient on \( n_t \). The conditional expectation of the least-squares coefficient of \( n \) will be biased downwards because

\[
\frac{d}{dn_t} E(\omega_t | n_t) = \frac{1}{\beta} \left( \frac{1 - F(k_t)}{1 - F(k_t + n_t)} \right) \frac{d}{dn_t} E(\eta_t | n_t) < 0
\]

where the other terms in the derivative drop out, at least approximately, because \( E(\eta_t) \) is on average zero.

The industry-viability condition now is

\[
C(0) + \eta_{\text{max}} < \frac{\beta(p - \varepsilon_{\text{min}})}{1 - \beta}
\]

4.2. Estimation

[TO BE INCLUDED: 1.instrumental variables 2.autocorrelation controls]

5. Estimating the Demand Hazard

To summarize, the demand hazard is a very important piece in our model: we showed that if it decreasing the industry looks safer and safer over time and firms are more and more willing to enter the market or to invest more units of capital in it. This is the essential mechanism that drives up the value per unit of capital until the crash. With the use of a different dataset derived from Gort and Klepper [4] (from now on GK and GK dataset) we would like to estimate the demand hazard and check if our assumption of decreasing monotonicity is cross-validated.
5.1. Description of the data

Gort and Klepper collected data on 46 different industries representing "new products". Only for 16 of those data on sales and prices time series are available. The following table reports the products whose time series were used in our estimations.

<table>
<thead>
<tr>
<th>product_index</th>
<th>product name</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>computers</td>
</tr>
<tr>
<td>7</td>
<td>crystal piezo</td>
</tr>
<tr>
<td>8</td>
<td>ddt</td>
</tr>
<tr>
<td>9</td>
<td>electrocardiographs</td>
</tr>
<tr>
<td>10</td>
<td>electric blankets</td>
</tr>
<tr>
<td>15</td>
<td>freezers home and farm</td>
</tr>
<tr>
<td>18</td>
<td>lasers</td>
</tr>
<tr>
<td>21</td>
<td>missiles guided</td>
</tr>
<tr>
<td>24</td>
<td>nylon</td>
</tr>
<tr>
<td>26</td>
<td>penicillin</td>
</tr>
<tr>
<td>27</td>
<td>pens ball point</td>
</tr>
<tr>
<td>40</td>
<td>styrene</td>
</tr>
<tr>
<td>43</td>
<td>tape recording</td>
</tr>
<tr>
<td>45</td>
<td>television receivers monochrome</td>
</tr>
<tr>
<td>48</td>
<td>transistors</td>
</tr>
<tr>
<td>50</td>
<td>tubes cathode ray</td>
</tr>
</tbody>
</table>

For details on how the data were collected we refer the reader to [4]. In what follows the time series are named as:

1. product abbreviation code: product_index
2. units sold: units_sold
3. price series: price
4. period of the real business cycle in which the observation is measured: nber_rbc_code
5. dummy for war periods: war

In the appendix we report several figures depicting our time series.
5.2. Construction of the Measure of Demand Hazard

We assumed that that \( h(z) > 0 \) for all \( z > z_{\min} \), and that \( h'(z) < 0 \). That is, \( F \) has a strictly decreasing hazard and also that \( \lim_{z \to \infty} h(z) = 0 \).

The procedure we chose to check the above in our cross-industry data is the following:

1. We want to interpret a "sharp" decline in the units sold time series (or its flattening out) after a continuous consecutive rise as a "failure" that is a realization of \( z \). Consequently we search for such a point in the units sold timeseries using a discriminant analysis similar to the one carried out in [4]. Industries for which such a point exist record a value of \( z=1 \) at such level of units_sold. Pictures in the appendix report the level in units_sold timeseries that discriminant analysis suggest being \( z \) for each industry. Of course it might be the case that such point does not exist for some industries: in this case our \( z \) dummy for such industries will assume the value 0 everywhere (meaning industry is a censored observation).

2. At this point our dataset is ready: using survival analysis tools to estimate the empirical hazard implied by the \{sales, z\} series where sales replaces time in ordinary survival analysis while \( z \) is the ordinary failure dummy.

5.3. Results

The following picture summarizes our main findings:

\(^4\)Units sold increases until \( z \) for that particular industry is reached: after that units sold should be constant or decreasing according to our model.

\(^5\)REMARK: "analysis time" is units_sold according to our procedure.
Empirical hazard estimate from GK dataset.

The picture reports kaplan-meyer empirical hazard estimate\(^6\) on the entire sample of 16 industries. As it is clear from the picture we cannot accept the assumption of a decreasing hazard across the board. A strong non-monotonicity is apparent. Good news is that a consistent part of the hazard is initially decreasing: if the crash happens in this portion of the demand the predictions of the model could be supported. Sometimes it is difficult to interpret variation in the hazard: the cumulated hazard is an alternative smoothed version of it. Its being concave

\[^6\]The Nelson-Aalen estimator of the cumulative hazard is defined up to the largest observed time as:

\[ \hat{H}(t) = \sum_{j|t_j \leq t} \frac{d_j}{n_j} \]

where \( n_t \) is population alive at \( t \) and \( d_t \) is number of failures up to time \( t \). The empirical hazard is:

\[ h(t_j) \equiv H(t_j) - H(t_{j-1}) = \frac{d_{t_j}}{n_{t_j}} \]
suggest the hazard being decreasing. I report below Nelson-Aalen cumulative hazard estimate:

It display a very "rough concavity", overall.

Interestingly enough, using an alternative dataset by Jorgenson we find the following hazard estimate on 35 SIC-double digit industry definition (hence very different from GK dataset):
Empirical Hazard Estimate from Jorgenson dataset. The pattern from this dataset is quite similar. The integrated Nelson-Aalen hazard is reported below.
It appears to be quite concave avvalorating the assumption of an overall decreasing hazard.

5.4. Actual and simulated series for Telecom sales

To show how well the model fits the data. In Figure ?? we preview things by plotting the time-path of $k$ and the actual Telecom sales, which crash at age 30
6. General Demand and positive salvage value

In this section we introduce downward sloping demand and a positive salvage value \( s \) for each unit of capital withdrawn from the industry.

*Positive salvage value for exiting capacity.*—Assume that

\[
s < C(0)
\]

which makes it unprofitable for a firm to create a capacity in the industry at hand and then salvage that capital for other purposes.

*Demand.*—Let \( p = \tilde{D}(q, z) \) be the demand curve for the output \( q \) of an industry. We take the following special form:

\[
\tilde{D}(q, z) = \begin{cases} 
D(q) & \text{if } q \leq z, \\
0 & \text{if } q > z.
\end{cases}
\]

It is illustrated in Figure 6. The figure shows three possible demand curves indexed by realizations of the random variable \( z \), and highlights the demand curve that would occur if \( z = z_2 \).

*Evolution of capacity.*—Let \( k_t \) be industry capacity. Since \( D(q) \) is already known, a positive the equilibrium price reveals only that \( z \) exceeds \( k \) and nothing else. Let \( n_t \) denote net entry or exit of capacity at date \( t \). Since firms are all alike,
gross and net entry and exit are the same. Capacity does not depreciate so that its law of motion is

\[ k_{t+1} = k_t + n_t. \]

*Saturation level of capacity.*—The analog of (2) is

\[ C(0) < \frac{\beta D(0)}{1 - \beta}. \]

But if demand slopes down, then even in the absence of a crash we will generally reach a level of capacity, \( k^* \), at which price will be too low to sustain new investment. This level \( k^* \) satisfies the equation

\[ C(0) = \frac{\beta D(k^*)}{1 - \beta}. \]

Evidently, if \( z > k^* \) this industry will never see a crash.

6.1. Analysis of the general case

We analyze the model backwards, starting with period \( T \).
Period $T$ and beyond.—This is when $k$ exceeds $z$ for the first time. The situation is described in Figure 7. Excess capacity then is $k_T - z$, and all the producers realize it right away, before production has taken place. Exactly $k_T - z$ of the capacity is then permanently withdrawn. If less were withdrawn, price would be zero. If more were withdrawn, price would remain above the flow value, $(1 - \beta) s$ of using capital elsewhere. Therefore price falls to

$$p_T = (1 - \beta) s$$

and remains there for ever. The value of a unit of capacity falls to $s$, and stays there. Therefore $s$ is the salvage value of capital. Figure 7 illustrates this.

The value of capital in place before $T$.—Since no one knows $z$ before date $T$, the state of the industry is just $k$. Prior beliefs over $z$ are $F(z)$. Incumbents take as given the entry rule

$$n = g(k).$$

Therefore they estimate that the probability of excess capacity being created by
the following period is
\[
\frac{F(k + g[k]) - F(k)}{1 - F(k)} \equiv \delta(k) .
\] (35)

Let \( v(k) \) be the value of a unit of capital in place at the start of a period. Then the analog of (3) is
\[
v(k) = D(k) + \beta [\delta(k) s + [1 - \delta(k)] v(k + g[k])] .
\] (36)

**Entry of new capacity.**—Given that \( k \) units of capacity are already in place, (??) tells us that the cost of new entry is \( c(k) \) – it is the unit cost of all the capital created during the period. Positive entry \((n > 0)\) implies that entrants expect that the discounted return will cover their entry cost. Therefore if \( g(k) > 0 \), then the analog of (4) is
\[
C[g(k)] = \beta [\delta(k) s + [1 - \delta(k)] v(k + g[k])]
\] (37)

6.1.1. **The value of capital in equilibrium**

Before the crash, (36) and (37) apply. Combining them, we have
\[
v(k) = D(k) + C(k) .
\] (38)

If a crash does occur, the value of capital falls to \( s \).

6.1.2. **If \( z \geq k^* \) then \( k_t \) approaches \( k^* \)**

This subsection shows that in equilibrium, the probability of a crash conditional on information available at the time is always positive. Moreover if \( z \) happens to exceed \( k^* \), capacity will gradually approach \( k^* \). Markets for which \( z < k^* \) will experience a crash, whereas markets for which \( z > k^* \) will not. First we show that \( k \) must strictly increase until the crash.

**Proposition 7.** \( k < k^* \implies g(k) > 0 \).

**Proof.** Suppose, on the contrary, that \( n \equiv g(k) = 0 \). But then from (35), \( \delta(k) = 0 \), and \( k \) becomes constant. But then by (32) and the fact that \( D() \) is strictly decreasing,
\[
C(0) < \frac{\beta D(k)}{1 - \beta}.
\]
which is incompatible with free entry. ■

Next, we show that the capital stock never reaches $k^*$.

**Proposition 8.**

$$k < k^* \implies k + n < k^*$$

**Proof.** If, contrary to the claim, $k + n \geq k^*$, then it follows that $n > 0$. Then $D(k + n) < D(k^*)$, and (37) then implies

$$c(n) = \beta \left[ \delta (k + n) s + [1 - \delta (k + n)] v (k + n) \right] \leq \beta \left[ \delta (k + n) s + [1 - \delta (k + n)] \frac{D(k + n)}{1 - \beta} \right].$$

But since $k + n > k^*$,

$$\frac{\beta D(k + n)}{1 - \beta} < C(0) \implies \frac{\beta D(k + n)}{1 - \beta} < C(n),$$

which means that the entrants lose money, i.e., a contradiction to (37). ■

**Corollary 9.** No crash can occur if $z \geq k^*$.

Not every market has a crash. Only those for which $z < k^*$. The situation is described in Figure 8. It is assumed in the figure that $z < k^*$ so that the crash will indeed occur. This is the subject of the next proposition.

**Proposition 10.** If $z < k^*$, the crash occurs with probability 1.

**Proof.** Suppose on the contrary that the capital stock does not reach $k^*$ so that for some values of $z$ sufficiently close to $k^*$ the crash does not take place. That is, suppose that

$$\lim_{t \to \infty} k_t = \tilde{k} < k^*.$$
Realized demand curve

\[ kT - 1 \quad z \quad k^* \quad q \]

\( D(q) \)

(1 - \( \beta \)) price

Capacity just before the crash

Maximal price drop

Actual price drop

Figure 8: Fall in product price at \( T \)

But then the survival probability is bounded away from 1: Since \( D'(\cdot) \leq 0 \),

\[
\sigma(k + n) \leq \frac{1}{D(\tilde{k}) + c - s} \left( \frac{c}{\beta} - s \right)
\]

\[ = \frac{1}{D(\tilde{k}) - D(k^*) + D(k^*) + c - s} \left( \frac{c}{\beta} - s \right) \]

\[ \leq \frac{1}{D(\tilde{k}) - D(k^*) + \left( \frac{1 - \beta}{\beta} c + c - s \right)} \left( \frac{c}{\beta} - s \right) \]

(using (32))

\[ = \frac{1}{D(\tilde{k}) - D(k^*) + \left( \frac{c}{\beta} - s \right)} \left( \frac{c}{\beta} - s \right) = 1 - \eta, \]

where \( \eta > 0 \). Now the probability of surviving for \( T \) periods can be decomposed into a sequence of conditional probabilities, and so this probability is

\[
\prod_{t=0}^{T-1} \sigma(k_{t+1}) \leq (1 - \eta)^T \to 0 \quad \text{as} \quad T \to \infty.
\]
6.1.3. Size of shakeout

The fraction of capital that exits during the shakeout is, approximately,

\[ E(k_T - z \mid k_{T-1} \leq z < k_T) \approx \frac{1}{2} \frac{k_T - k_{T-1}}{k_{T-1}}. \]

This equality is the exact when \( F \) is uniform, and it should be a fairly good approximation for other \( F \)'s that have continuous densities. See Figure 9. Essentially, the size of the shakeout depends, at least to a first approximation, on the rate of growth of \( k \) at \( T \).

Now, this is a rate of growth per unit of time, and in this case the time unit matters. As the time interval becomes short, mistakes in the creation of capacity disappear. Therefore, if capacity can be created quickly, and if information about demand comes in right away, we would not expect ever to see much excess capacity, or a large shakeout. Does the information sector fit that description?
6.1.4. Other coincident shakeouts and crashes

Examples of shakeouts and crashes that occurred together.

1. The radio market expanded rapidly in the 1920’s and then crashed spectacularly in 1929 with the most notable stock being that of RCA. It remains to be checked whether there was a shakeout in telecommunications in the 1930’s.

2. The tire industry.—Figure 1 of Jovanovic and MacDonald (1994) shows that this is exactly what happened to the tire industry 80 years ago: Stock prices crashed in 1919 and a massive shakeout occurred between 1922 and 1929. Of the 275 tire producers that were around in 1922, roughly half were gone by 1929. Yet during this period automobile sales nearly doubled. Therefore capacity did not unexpectedly outstrip demand. Moreover, Agarwal and Gort (1996) find that entry falls, but remains positive during the shakeout phase.

3. Market share instability and excess volatility.—Mazzucato and Semmler (1999) find that in the automobile industry stock prices were the most volatile during the period 1918-1928 when market shares were the most unstable. This is not inconsistent with the model, if we interpret exiting capital as a symptom of market-share loss and, presumably, a market-share gain for the capital that remains in the industry after the shakeout.

7. Appendix A: General Information on NASDAQ

The NASDAQ Stock Market, like the other major national securities markets, has formal company listing requirements, electronic surveillance of trading, real-time dissemination of quotations and trade reports. “Over-the-counter” (OTC) securities are those securities which are not listed on NASDAQ or any of the national U.S. securities exchanges. NASDAQ began as an efficient and modernized system to store and display quotations on OTC securities. It has evolved into an organized securities market as described above and today is distinctly separate from the OTC market. References to The NASDAQ Stock Market as an “OTC market” are anachronistic.

Prior to the creation of NASDAQ in 1971 and its development of an electronic quote dissemination system, dealer quotations and indications of dealer interest
in OTC securities were disseminated in paper copy on a daily basis. These copies were printed on pink-colored paper, hence the designation of OTC securities as “pink sheet” stocks. The “Pink Sheets” are still published by the National Quotation Bureau (212-868-7100).

There are more than 4000 of those but the total number keeps on changing. The following picture keeps track of the number of firms in NASDAQ for the recent years; total numbers are reported also for the other two major electronic markets: Amex and NYSE.

NASDAQ staff produces several indexes to account for the general performance of NASDAQ:

- NASDAQ Composite Index (contains all the companies on NASDAQ)
- NASDAQ National Market Composite Index (All US based firms)
- NASDAQ-100 Index
- NASDAQ Financial-100 Index
- NASDAQ Industrial Index
- NASDAQ National Market Industrial Index
- NASDAQ Bank Index
- NASDAQ Biotechnology Index
The following figure represents the time series evolution of 3 of them:

- CCMP is NASDAQ composite index
- CUTL is NASDAQ telecommunications index
- IXK is NASDAQ computer index

8. Appendix B: NASDAQ timeline

The following is a brief timeline of NASDAQ:
1961- Congress authorizes the Securities and Exchange Commission (SEC) to conduct a study of fragmentation in the over-the-counter market. The SEC proposes automation as a possible solution and charges the National Association of Securities Dealers, Inc. (NASD) with its implementation.

1971 - February 8, the National Association of Securities Dealers Automated Quotation (NASDAQ) begins trading, with median quotes for 2,500 over-the-counter securities.

1976 - NASDAQ purchased the assets of the automated quotation NASDAQ System from its builder and operator, Bunker Ramo Corporation.

1980 - NASDAQ releases inside quotations, which promptly narrows displayed and published spreads on more than 85 percent of its securities.

1982 - The National Market System (NMS) is created for NASDAQ’s 40 highest volume securities for up-to-the-minute reporting of trades.

1984 - Small Order Execution System (SOES) automatically executes small orders against the best quotations, making greater volume and efficiency in trading possible.

1993 - NASDAQ develops three news indexes that enable investors to track key growth industries: biotechnology, computer and telecommunications.

1996 - The NASDAQ Web site—www.NASDAQ.com—begins operating, quickly becoming one of the most visited investor sites on the Internet.

1997 - The SEC approves NASDAQ’s proposal to reduce the minimum quotation increment from 1/8 of a dollar to 1/16 of a dollar for stocks trading above $10. New SEC Order Handling Rules begin to phase-in, narrowing spreads and enhancing market information.

2000 - NASDAQ membership votes overwhelmingly to restructure and spin off NASDAQ into a shareholder-owned, for-profit company.

9. Appendix C: NASDAQ index formulas

The section illustrates how the NASDAQ index is computed. The formulas provided help in understanding what the preceding picture actually plots. Some definitions first:

- The market capitalisation is obtained by multiplying the number of shares in issue with the mid price.

- The mid price of a security is obtained by taking the average between the
best bid price and the best offer price available on the market during normal business hours.

- The **number of shares outstanding** are used to calculate the market capitalisation for each component of the index. These shares represent capital invested by the firm’s shareholders and owners, and may be all or only a portion of the number of shares authorised\(^7\).

- **Constituent** is any firm listed on NASDAQ.

The Nasdaq Composite Index is weighted arithmetically where the weights are the market capitalisations of its constituents. The index is the summation of the market values (or capitalisations) of all companies within the index and each constituent company is weighted by its market value (shares in issue multiplied by the mid price). The formula used for calculating the index is straightforward. However, determining the capitalisation of each constituent company and calculating the capitalisation adjustments to the index are more complex. The index value itself is simply a number which represents the total market value of all companies within the index at a particular point in time compared to a comparable calculation at the starting point. The daily index value is calculated by dividing the total market value of all constituent companies by a number called the divisor. The divisor is then adjusted when changes in capitalisation occur to the constituents of the index (see Revision of the Divisor) allowing the index value to remain comparable over time.

\[ I_t = I_0 \frac{\text{total market value}_t}{\text{divisor}_t} = I_0 \frac{\sum_{i=1}^{N_t} P_{it} S_{it}}{D_t} \]

where \(t\) is the date at which we want to calculate the index \(I\), \(t=0\) is a reference date or base date we start with (like February 1971 for the composite index which is set to 100) \(P_{it}\) is the price of a share of company \(i\) at date \(t\), \(S_{it}\) is the total number

\(^7\)Shares that have been issued and subsequently repurchased by the company for cancellation are called treasury shares, because they are held in the corporate treasury pending reissue or retirement. Treasury shares are legally issued but are not considered outstanding for purposes of voting, dividends, or earnings per share calculation. Shares authorised but not yet issued are called un-issued shares. Most companies show the amount of authorised, issued and outstanding, and treasury shares in the capital section of their annual reports. It is possible to back out the total number of outstanding shares of each company from the balance sheet. In COMPUSTAT it is possible to obtain market capitalization by using the following DATA items: (DATA6+DATA199*DATA25-DATA60)
of shares outstanding for company i at date t and D_t is a divisor, introduced to make the index comparable over time (basically keeps track of changing in the pool of firms or their share policies and allows the composite index only to track growth rates over periods) and defined below:

\[ D_t = \sum_{i=1}^{N_0} P_{i0} S_{i0} + \sum_{j=1}^{t} G_{j-1} \frac{I_0}{I_{j-1}} \]

where G_{j-1} is net new money raised at time j-1 through the issue of new companies, new shares, rights issues, capital reorganisations or even capital re-payments. This figure may be negative. If G is zero between periods the index boils down to:

\[ I_t = \frac{\text{total market value}_t}{\text{total market value}_0} I_0 = I_0 \sum_{i=1}^{N_t} \frac{P_{it} S_{it}}{\sum_{i=1}^{N_0} P_{i0} S_{i0}} \]

10. Appendix D: Derivation of (19)

In the law of motion for \( k \),

\[ k' = k + n, \]

divide by \( n' \) to obtain

\[ \frac{k'}{n'} = \frac{k}{n'} + \frac{n}{n'} \]

\[ \frac{k'}{n'} = \frac{k}{n'} + \frac{n}{n'} \]

\[ \frac{k'}{n'} = \left(1 + \frac{k}{n}\right) \frac{n}{n'}. \]

Inverting both sides,

\[ \frac{n'}{k'} = \frac{n'}{n} \frac{1}{(1 + \frac{k}{n})} \]

\[ = \frac{1}{(1 + \frac{k}{n})} \frac{n}{n} \left[ -\frac{p}{Cn} + \frac{1}{\beta} \left(1 + \frac{n}{k}\right)^\rho \right] \]

\[ = \frac{1}{(1 + \frac{k}{n})} \left[ -\frac{p}{Cn} + \frac{1}{\beta} \left(1 + \frac{n}{k}\right)^\rho \right] \]

\[ = \frac{x}{(1 + x)} \left[ -\frac{p}{Cn} + \frac{1}{\beta} (1 + x)^\rho \right] \]
Therefore (16) and (17) are equivalent to (18) and (19).

11. Appendix E: units_sold timeseries and the z-levels by industry

11.1. Censored Industries [z undisclosed]
11.2. Uncensored Industries [z disclosed]
12. Appendix F: Alternative Measures of Demand Hazard

Given our GK dataset it is possible to engineer alternative measures of demand hazard. We were unable due to time constraints to implement the following two:

1. Look for a point where a "sharp" decline in the price timeseries is observed and record the corresponding value of units_sold timeseries: it is interpreted as z in the model. This procedure is quite intuitive: when prices start falling sharply, we think that demand is saturated in that market or simply that demand is overshooted (z is hence discovered).

Unfortunately we do not have data on stocks for firms belonging to industries to whom our dummy assigns a failure(=overshooting of demand). We could cross check if at those dates we observe a decrease in the value of capital.
2. Use GK shakeout dummy as z. GK recorded the duration of time before shakeout for each industry. A problem with this measure is that shakeout is defined as a "sharp" decline in the number of firms which does not strictly correspond to z in our model\(^9\).

References


\(^9\)Our model bears no predictions with regards to the number of firms in an industry.


