Interpreting Minimum Wage Effects on Wage Distributions: A Cautionary Tale

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Abstract

It is often tempting to attempt to infer the welfare effects of minimum wage changes from empirical observations on pre- and post-change employment and unemployment levels and wage or earnings distributions. Using a simple model of search, matching, and bargaining, we characterize the relationship between minimum wage levels, labor market outcomes, and the welfare of labor market participants. Using observations on wage distributions before and after changes in the nominal minimum wage, we determine what can and cannot be learned about welfare impacts from changes in various features of these distributions. Using Current Population Survey data from the U.S. data, we attempt to determine whether the minimum wage increases of 1996 and 1997 improved the welfare of young labor market participants. Our results point to a possible welfare increase as the result of the 1997 minimum wage change from $4.75 to $5.15, though there is no evidence for an increase (or decrease) in welfare associated with the 1996 increase from $4.25 to $4.75. While a number of the empirical results are not strictly consistent with the implications of our model, at a minimum our analysis illustrates the fact that well-specified behavioral models are required to evaluate the impact of changes in institutional constraints on the welfare of labor market participants.
1 Introduction

It is often tempting to attempt to infer the welfare effects of actual minimum wage changes from empirical observations on pre- and post-change employment and unemployment levels and wage or earnings distributions. For example, minimum wage increases are often explicitly or implicitly taken to be beneficial for the impacted population if changes in group employment rates are found to be positive or negligibly negative. As another example, although a very small percentage of U.S. workers are paid the minimum wage, larger impacts on welfare on the general population are often asserted to result from a sort of “ripple” effect from the bottom up, which is commonly referred to as spillover. Empirical evidence that demonstrates a change in the shape of the wage distribution above the (new higher) minimum wage is claimed to result from these spillover effects, and by their very nature these are assumed to be beneficial for individuals on the supply side of the market.

One of the goals of this paper is to rigorously define, characterize, and explain the phenomenon of spillover. To accomplish this task, we will utilize a simple model of search and bargaining in a stationary environment. While the model is admittedly highly stylized, in a companion paper (Flinn 2001) we show that it can be estimated using Current Population Survey (CPS) data and fits observed wage and unemployment duration distributions reasonably well. Thus the model at a minimum provides a parsimonious and readily interpretable view of the labor market as it is reflected in CPS data, and for this reason can be given some credibility. The standard stationary search-matching framework is readily adapted to allow for the existence of binding minimum wages rates. The qualitative features of the equilibrium which results from the imposition of a minimum wage [or an increase in value of an already binding minimum wage] in the model are roughly in accordance with empirical work on this subject that utilizes disaggregated data. Some of the implications of the model are: (1) the existence of a probability mass at the minimum wage \( m \) with an absolutely continuous distribution of the wage \( w \) to the right of \( m \); (2) decreases in employment rates with increases in the minimum wage; and (3) the possible existence of spillover effects in response to a change in the minimum wage rate. Using the model we will rigorously examine some particular welfare measures as well as the phenomenon of spillover.\(^1\) We show that welfare can increase even though employment rates decrease and with or without “spillover” in the wage distribution. Conversely, spillover effects of minimum wage changes do not indicate that the change was necessarily welfare-enhancing. The main point to which we want to draw attention is that the welfare effects of minimum wage changes can only be inferred by using empirical evidence on employment rates, wage distributions, and a formal model within which welfare can be rigorously defined and evaluated.

We will not devote much attention to the impacts of minimum wage levels on unemployment or employment levels. Instead, we will use our model to attempt to understand the impact of minimum wage levels on accepted wage offer distributions, both in terms of truncation and “shape-changing.” Several researchers have attempted to adapt econometric models for truncated and limited dependent variables to incorporate minimum wages into standard wage function estimation schemes. Some of the research efforts in this area include Meyer and Wise (1983a, 1983b), Dinardo et al (1996), and Dickens et al (1997). Meyer and Wise estimated a model of minimum wage effects using individual-level data which allowed them to infer what the wage distribution and employment

\(^1\)The definition of spillover we will utilize turns out to be identical to how the notion is defined in Assumption 1 of Dinardo et al (1996). While the authors of that paper utilize this assumption in performing a statistical decomposition of probability density functions, we will be interested in using the definition to formulate a nonparametric test for the existence of spillover.
level would have been in the absence of a minimum wage. The basic idea behind the econometric specification is to assume the form of the wage distribution in the absence of a minimum wage, and then to allow the minimum wage to alter this distribution by essentially aggregating probability mass around the minimum wage to that exact value. This results in a wage distribution which has a continuous and discrete component to it. While their model specification has been criticized by a number of researchers [e.g., Card and Krueger (pp.232-236) and Dickens et al. (1997)], primarily for relying on functional form assumptions for identification and for choosing a parameterization which rules out the possibility of employment increases in response to a minimum wage increase, it remains one of the better econometric attempts to identify minimum wage effects using individual-level data.2 From our perspective, the main weakness of their model is the arbitrary specification of the manner in which a minimum wage “distorts” the preexisting distribution wage distribution. In the model developed here, and in our companion paper (Flinn 2001), optimizing behavior by searchers and firms determines the nature of this “distortion,” and it is roughly consistent both with the Meyer and Wise econometric specification and with the empirical evidence cited in Card and Krueger.

Dickens et al (1997) attempt to compare wage distributions before and after changes in the nominal minimum wage in order to infer employment effects. To estimate employment impacts, they replace the Meyer and Wise identification condition that the minimum wage has no impact on the distribution of wages immediately above it with the condition that it has no impact on the wage distribution above some value $x \geq m$. While this assumption seems reasonable on the face of it, it will not in general be valid within the simple equilibrium framework developed here. In addition, as we have argued above, the ultimate goal of empirical wage research should probably be directed to determining welfare rather than employment impacts.

The focus of the Dinardo et al (1996) paper is on assessing the impacts of minimum wages and other institutional features of the labor market on wage and employment distributions. To perform their semiparametric statistical analysis, they are forced to make a number of rather controversial assumptions regarding the manner in which minimum wage changes impact both the wage distribution and employment rates. The wage distributions they fit also have the unfortunate characteristic of being everywhere continuous, which is inconsistent with the spike at the minimum wage rate clearly observable in U.S. data (particularly for young workers). While the goal of the analysis is to determine the impact of various institutional features of the labor market [one of which is the minimum wage] on the increasing levels of wage inequality observed over the past two decades in the U.S., the paper does not take a clear position on whether or not such increasing inequality is good or bad.

There are a number of ways to include minimum wages in equilibrium models of the labor market. In terms of our work, some of the more relevant theoretical, econometric, and empirical contributions to this literature include Eckstein and Wolpin (1990), Mortensen (1990), Burdett and Mortensen (1998), van den Berg and Ridder (1998), and Bontemps, Robin, and van den Berg (1999). In these models, which are not based on matching and bargaining but instead utilize assumptions of wage-posting by homogenous or heterogeneous firms, minimum wage restrictions do not typically result in mass points in the wage distribution at the minimum, and in a number of cases the wage distributions implied by the theory are very much at odds with empirical observation.3 However,  

\footnote{For another good example, see Heckman and Sedlacek (1981).}  \footnote{A notable exception to this statement can be found in Bontemps, Robin, and van den Berg (2000), which provides a perfect fit to the data at the expense of uniqueness of equilibrium and analytical complexity.}
this class of models contains examples of situations in which the imposition of minimum wages can be Pareto welfare-improving and allows for a number of labor market phenomena that cannot be captured within our framework. All of the various equilibrium labor market models based on search-theoretic foundations have their advantages and disadvantages, but our feeling was that a model based explicitly on bilateral bargaining between a given worker and firm may provide a slightly preferable framework in which to analyze disaggregated labor market data.

The plan of the paper is as follows. In Section 2 we develop a bargaining model in a continuous-time search environment both in the absence and in the presence of a binding minimum wage. Section 3 contains a discussion of various welfare measures and makes a case for focusing on the one featured in this paper, the unemployed value of search. In Section 4 we provide results and tests to determine whether a minimum wage change resulted in an increase or decrease in welfare. These tests are based on comparisons of various pre- and post-change wage distributions of the kind that are commonly available from the Current Population Survey. Section 5 provides details concerning the statistical properties of the tests that were conducted and practical details of the manner in which they were executed. Section 6 contains a discussion of the results from an empirical exercise we conducted in order to investigate whether the nominal minimum wage changes in 1996 and 1997 resulted in increases in the welfare of labor market participants. Section 7 concludes with a discussion of some general methodological issues associated with the evaluation of policy interventions such as a change in the nominal minimum wage.

2 Labor Market Search with Bargaining

In this section we describe the behavioral model of labor market search with matching and bargaining. The model is formulated in continuous time and assumes stationarity of the labor market environment. In the first subsection we derive the decision rules for terminating search and for dividing the match value between the worker and the firm in the absence of minimum wages. These results are relatively well-known, and are mainly presented to set ideas and for purposes of comparison with the case in which a binding minimum wage is present, which is analyzed in the following subsection.

Throughout we assume that there exists an invariant, technologically-determined distribution of worker-firm productivity levels that is given by $G(\theta)$. When a searcher and a firm meet, which happens at rate $\lambda$, the productive value of the match ($\theta$) is immediately observed by both the applicant and the firm. At this point a division of the match value is proposed using a Nash bargaining framework. Both the searcher and the firm share a common instantaneous discount rate of $\rho > 0$. The rate of (exogenous) termination of employment contracts is $\eta \geq 0$. While unemployed individuals search their instantaneous utility is given by $b$, which can assume positive or negative values. For simplicity, we assume that employed individuals do not receive alternative offers of employment, i.e., there is no on-the-job search. It is straightforward to adapt the current framework to that case.

2.1 Labor Market Decisions without Minimum Wages

We assume that the only factor of production is labor, and that total output of the firm is simply the sum of the productivity levels of all of its employees. Firms create vacancies costlessly and face no fixed costs of employing workers. For this reason, when the firm encounters a particular
potential employee its “disagreement” outcome is 0 [it earns no revenue but makes no wage payment]. The applicant’s disagreement value is the value of continued search, which we denote by $V_n$. Under expected wealth maximization it is well-known that for any given value of $V_n$ there exists a corresponding critical “match” value $\theta^* = \rho V_n$ which has the property that all matches with values at least as great as $\theta^*$ will result in employment while all those matches of a lower value will not (we will characterize the determination of this value below). For any $\theta \geq \theta^*$, the wage offer is given by

$$w(\theta, V_n) = \operatorname{arg\ max}_w \left[ V_e(w) - V_n \right]^\alpha \left[ \frac{\theta - w}{\rho + \eta} \right]^{1-\alpha},$$

(1)

where $\alpha \in (0, 1)$ is the bargaining power of the searcher and $V_e(w)$ denotes the value of being employed at wage $w$ to the searcher.

The value of employment at a wage of $w$ is easily determined. Consider an infinitesimally small period of time $\varepsilon$. Over this “period,” either the individual will continue to be employed at wage $w$ or will lose their job, which occurs at rate $\eta$. Then

$$V_e(w) = (1 + \rho \varepsilon)^{-1} \{ w \varepsilon + \eta \varepsilon V_n + (1 - \eta \varepsilon) V_e(w) + o(\varepsilon) \},$$

(2)

where the term $(1 + \rho \varepsilon)^{-1}$ is an “infinitesimal” discount factor associated with the small interval $\varepsilon$, $\eta \varepsilon$ is the approximate probability of being terminated from one’s current employment by the end of $\varepsilon$, and $o(\varepsilon)$ is a term which has the property that $\lim_{\varepsilon \to 0} o(\varepsilon)/\varepsilon = 0$. Note that the first term on the right hand side of [2] is the value of the wage payment over the interval, which is the total payment $w \varepsilon$ multiplied by the “instantaneous” discount factor [think of the payment as being received at the end of the interval $\varepsilon$]. After collecting terms and taking the limit of [2] as $\varepsilon$ goes to 0, we have

$$V_e(w) = \frac{w + \eta V_n}{\rho + \eta}.$$  

(3)

We now substitute [3] into [1] so as to simplify the problem as follows:

$$V_e(w) - V_n = \frac{w + \eta V_n}{\rho + \eta} - V_n$$

$$= \frac{w - \rho V_n}{\rho + \eta},$$

so that we get the well-known expression

$$w(\theta, V_n) = \operatorname{arg\ max}_w \left[ w - \rho V_n \right]^\alpha \left[ \theta - w \right]^{1-\alpha}$$

$$= \alpha \theta + (1 - \alpha) \rho V_n.$$

We can now move onto computing the value of nonemployment. Using the same setup as above for defining the value of employment, we begin with the $\varepsilon-$period formulation which is

$$V_n = (1 + \rho \varepsilon)^{-1} \{ b \varepsilon + \lambda \varepsilon \int \max[V_n, V_e(w(\theta, V_n))] dG(\theta)$$

$$+ (1 - \lambda \varepsilon) V_n + o(\varepsilon) \},$$

where $\lambda \varepsilon$ is the approximate probability of encountering one potential employer over the interval. Rearranging and taking limits, we have

$$\rho V_n = b + \lambda \int_{\rho V_n} \left[ V_e(w(\theta, V_n)) - V_n \right] dG(\theta).$$
Since
\[ V_e(w(\theta, V_n)) = \frac{\alpha \theta + (1 - \alpha) \rho V_n + \eta V_n}{\rho + \eta} \]
we have
\[ V_e(w(\theta, V_n)) - V_n = \frac{\alpha \theta - \alpha \rho V_n}{\rho + \eta}. \]

Then the final (implicit) expression for the value of search is
\[ \rho V_n = b + \frac{\alpha \lambda}{\rho + \eta} \int_{\rho V_n} [\theta - \rho V_n] dG(\theta). \]  

(4)

Note that this expression is identical to the expression for the reservation value in a model with no bargaining when \( \theta \) is the payment to the individual except for the presence of the factor \( \alpha \) [see, e.g., Flinn and Heckman (1982)]. This is not unexpected, since when \( \alpha = 1 \) the entire match value is transferred to the worker and search over \( \theta \) is the same as search over \( w \).

We can now summarize the important properties of the model. The critical “match” value \( \theta^* \) is equal to \( \rho V_n \), which is defined by [4]. Since at this match value the wage payment is equal to \( w^* = w(\theta^*, V_n) = \alpha \theta^* + (1 - \alpha) \theta^* = \theta^* \), the reservation wage is identical to the reservation match value. The probability that a random encounter generates an acceptable match is given by \( \tilde{G}(\theta^*) \), where \( \tilde{G} \) denotes the survivor function, \( 1 - G \). The rate of leaving unemployment is \( \lambda \tilde{G}(\theta^*) \). As we can see from [4], since \( \theta^* \) is an increasing function of \( \alpha \), the rate of unemployment is higher when searchers have more bargaining power.

The observed wage density is a simple mapping from the matching density. Since
\[ w(\theta, V_n) = \frac{\alpha \theta + (1 - \alpha) \theta^*}{\alpha}, \]
then the density function of observed wages is given by
\[ f(w) = \begin{cases} \frac{\alpha - g(\tilde{\theta}(w, V_n))}{\tilde{G}(\theta^*)} & w \geq \theta^* \\ 0 & w < \theta^* \end{cases}. \]

To illustrate the structure of the model both with and without binding minimum wages it will be useful to provide an example. We set the rate of arrival of offers (\( \lambda \)) to the value .5 (so that job contacts occur every 2 “periods” on average), the rate of job dissolutions (\( \eta \)) is set to .02 (so that the average length of a job is 50 periods), \( \rho \) is set to .01, and the instantaneous return from search (\( b \)) is set to -.1. The firm-searcher matching distribution is assumed to be uniform with support \([0, 10]\). We will compute the equilibrium wage distribution for \( \alpha = .3 \) and \( \alpha = .6 \) to illustrate the impact of bargaining power on the equilibrium wage functions both with and without minimum wages.

Figure 1.a plots the uniform p.d.f., which represents \( g(\theta) \) in this example. Figure 1.b plots the mapping from draws of \( \tilde{\theta} \) into wage offers under the two alternative values of \( \alpha \), that is \( w_\alpha(\theta, V_n(\alpha)) = \alpha \theta + (1 - \alpha) \rho V_n(\alpha) \). Note that \( \alpha \) affects the equilibrium mapping both directly
through the slope and indirectly through the disagreement point $\rho V_n(\alpha)$. Figures 1.c and 1.d plot the equilibrium wage p.d.f.s for the two $\alpha$ values. Increasing $\alpha$ in the uniform case simply results in increases in the lower and upper bound of the support of the equilibrium wage distribution, which is itself uniform. Since increases in $\alpha$ result in increases in the value of search, it is interesting to note that in this case the mean and variance of the wage distribution are both positively associated with the value of search.

2.2 Labor Market Decisions in the Presence of Minimum Wages

Now consider the case in which the interactions between applicants and firms are constrained by the presence of a minimum wage. The minimum wage, $m$, is set by the government and is assumed to apply to all potential matches. We assume that the only compensation provided by the firm is the wage. Thus there are no other forms of compensation that the firm can adjust so as to “undo” the minimum wage constraint.

We impose the minimum wage in the framework developed in the previous subsection. As should be clear, any $m \leq \hat{\theta}^*$ has no effect on the behavior of applicants or firms and thus would be meaningless. Thus we consider only the effects of an imposition of an $m > \hat{\theta}^*$.

Recall that the expected value of the match from the point of view of the firm is proportional to $(\hat{\theta} - w)$. Firms cannot earn positive profits on matches which have a value less than $m$. Since $m > \hat{\theta}^*$, an immediate implication of the imposition of the minimum wage is that fewer contacts will result in jobs - the standard employment effect.4

In terms of wage payments, the minimum wage acts solely as a side constraint on the Nash bargaining problem. Formally, the revised problem is given by

$$w(\theta, V_n) = \max_{w \geq m} \left[ V_n(w) - V_n \right]^{\alpha} \left[ \frac{\theta - w}{\rho + \eta} \right]^{1-\alpha},$$

where the only difference from [1] is the restriction $w \geq m$. The effect on the solution is relatively intuitive. Under the “constrained” Nash bargaining problem, there will exist a value of search which we denote $V_n(m)$ [Note that this value is not equal to the $V_n$ which we defined in the unconstrained problem - it will be defined below]. If we ignore the minimum wage constraint in determining the wage payment given a match value of $\theta$ and the search value $V_n(m)$, we get

$$w(\theta, V_n(m)) = \alpha \theta + (1 - \alpha) \rho V_n(m). \quad (5)$$

Under this division of the match value, the worker would receive a wage of $m$ when $\theta = \hat{\theta}$, where

$$\hat{\theta}(m, V_n(m)) = \frac{m - (1 - \alpha) \rho V_n(m)}{\alpha}.$$

4When moving from no minimum wage to a binding minimum wage of $m > \theta^*$ the change in the steady state employment rate is given by

$$\frac{\lambda \hat{G}(m)}{\eta + \lambda \hat{G}(m)} - \frac{\lambda \hat{G}(\theta^*)}{\eta + \lambda \hat{G}(\theta^*)} = \frac{\eta \lambda [\hat{G}(m) - \hat{G}(\theta^*)]}{[\eta + \lambda \hat{G}(m)][\eta + \lambda \hat{G}(\theta^*)]} < 0.$$

The situation is the same when considering the impact of moving from one minimum wage $m$ to another $m' > m$. 6
Then if $\hat{\theta} \leq m$, all “feasible” matches would generate wage offers at least as large as $m$. When $\hat{\theta} > m$, this is not the case. When $\theta$ belongs to the set $[m, \hat{\theta})$, the offer according to [5] is less than $m$. However, when confronted with the choice of giving some of its surplus to the worker versus a return of 0, the firm pays the wage of $m$ for all $\theta \in [m, \hat{\theta})$. Wages for acceptable $\theta$ outside of this set are determined according to [5].

We can now consider the individual’s search problem given this wage offer function. Using the $\varepsilon$–interval formulation,

\[
V_n(m) = \left(1 + \rho \varepsilon\right)^{-1} \left\{ b + \lambda \varepsilon \left[ \int_{\hat{\theta}(m,V_n(m))}^{\hat{\theta}(m,V_n(m))} \left[ m + \eta V_n(\theta) \right] d\theta \right] + \int_{\hat{\theta}(m,V_n(m))}^{\hat{\theta}(m,V_n(m))} \left[ \alpha \theta + (1 - \alpha) \rho V_n(\theta) + \eta V_n(\theta) \right] d\theta \right\}
\]

Taking limits after collecting terms, we have

\[
\rho V_n(m) = b + \frac{\lambda}{\rho + \eta} \left\{ \int_{\hat{\theta}(m,V_n(m))}^{\hat{\theta}(m,V_n(m))} [m - \rho V_n(\theta)] d\theta \right\}
\]

It makes some sense to refer to the value $\rho V_n(m)$ as the “implicit” reservation wage. Unlike the situation in which a binding minimum wage does not exist, this value is not the minimal acceptable wage and match value. The acceptable wage/match value is rather the imposed minimum value $m$. Nonetheless, the value $\rho V_n(m)$ is of critical importance in determining equilibrium wages and, we will argue below, the welfare effects of minimum wage changes.

Conditional on the value of a binding minimum wage $m$, the equilibrium wage cumulative distribution function is given by

\[
F_m(w) = \begin{cases} 
\frac{\alpha^{-1} g(\theta(w,V_n))}{G(m)} & w > m \\
\frac{\tilde{G}(m) - \alpha \tilde{G}(\theta(m,V_n))}{G(m)} & w = m \\
0 & w < m 
\end{cases}
\]

The minimum wage side constraint produces an equilibrium wage distribution which has a mass point at $m$ and has wages being continuously distributed on the interval $(m, \infty)$.

Let us reconsider our uniform example after a minimum wage of 7 has been imposed; since the distribution now has a mass point, it is necessary to plot the c.d.f. instead of the p.d.f. Figure 2.a plots the c.d.f. of the matching distribution. Figure 2.b contains the equilibrium wage offer mapping from $\theta$ to $w$ when $m = 7$. Note that for the case of $\alpha = .3$, the equilibrium wage function

\footnote{This statement is predicated on $\theta$ being a continuously distributed random variable with unbounded support.}
maps all values of $\theta \geq 7$ into an equilibrium wage payment of $w = 7$. For cases in which the distribution $G$ has bounded support, this illustrates that the imposition of a minimum wage can result in a degenerate wage offer distribution at the minimum (Figure 2.c). In the case of $\alpha = .6$, the equilibrium wage distribution has a substantial mass point at 7, with a relatively narrow range of wages above it.

3 Minimum Wage Effects on Welfare

There are a number of welfare criteria one could consider using to analyze the impacts of minimum wages even within the simple model of the labor market developed above. Here we provide a brief discussion of some of the more obvious candidates, which will help us to consider the relative advantages and disadvantages of the one that is the focus of the econometric and empirical analysis that follows. In discussing the welfare impact of minimum wage changes, we only allow policy makers (or population members acting as voters) to alter the minimum wage; all other primitive parameters (i.e., $\Psi \equiv (\rho \lambda \mu \alpha \eta)'$) are assumed to be invariant with respect to changes in policy. We also restrict attention to the welfare of individuals on the supply side of the market. We will offer a few thoughts on the welfare of firms at the conclusion of this section.

3.1 Value of Unemployed Search (WC1)

All individuals begin their labor market careers in the state of unemployment (the labor market participation decision is not explicitly considered). Therefore, the *ex ante* value of the labor market career is the value of unemployed search, or $V_n(m)$. This measure of welfare is well-motivated if the conceptual experiment is to select a fixed minimum wage at the time that a reference cohort enters the labor market. For purposes of theoretical and empirical analysis it also has the advantage of being a scalar measure.

This is the measure that is used in formulating the tests and the empirical work below. A major reason behind our choice is the direct link between this measure of welfare and the wage distribution. By looking at changes in the wage distribution before and after a minimum wage change we will in some cases be able to determine whether $V_n(m)$ changed as a result of the new minimum wage and in others we will even be able to determine the direction of the change. The tests we formulate are robust in the sense that they presume no prior knowledge of the values of the primitive parameters of the model, not even the functional form of the matching distribution. We have not been able to find analogous nonparametric and robust tests based on the alternative welfare measures discussed below. All of the welfare measures considered are to some extent interrelated, so that knowledge of the direction of change in the value of search will provide some information regarding the direction of change of other welfare measures, ones that admittedly may be more interesting in the context of a particular application.

We continue our example by considering the impact of minimum wages on some of the welfare measures discussed in this Section. Figures 3.a and 3.b contain plots of the value of search for each value of $m$ in the interval $[0,10]$ for the two cases of $\alpha = .3$ and $\alpha = .6$. For the case of $\alpha = .3$, to maximize the value of search a very high value of $m$ (8.02) is indicated. Conversely, for the relatively high bargaining power value of $\alpha = .6$ all binding minimum wages reduce the value of search. Therefore the optimal value of $m$ in this case is anything less than $\rho V_n(0)$. 

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3.2 Ex Post Welfare (WC2)

Another conceptual welfare experiment, in some sense more “realistic,” examines the effects of the imposition of a new minimum wage \( m' \) on the welfare of members of a cohort that has already entered the labor market and faces an environment \( \Psi \) and a current minimum wage of \( m \). We will assume throughout that the imposition of the new minimum wage is completely unanticipated by labor market participants.

Under any given minimum wage rate \( m \) and labor market environment \( \Psi \) we can assign a unique labor market value to each \( \theta, \theta \in R_+ \), and we will denote this value by \( S(\theta; m) \). For individuals with (hypothetical) draws of \( \theta < m \), the value of the state is that of continued search. For those with draws of \( \theta \) greater than or equal to \( m \) and less than \( \hat{\theta}(m) \) the value of the state is that of employment at a wage of \( m \), and for all values \( \theta \geq \hat{\theta}(m) \), the value of the state is that associated with being employed at a wage \( w(\theta, \rho V_n(m)) = \alpha \theta + (1 - \alpha) \rho V_n(m) \). To summarize, we have:

\[
S(\theta; m) = \begin{cases} 
V_n(m) & \text{if} \quad \theta < m \\
V_e(m; m) & \text{if} \quad m \leq \theta < \hat{\theta}(m) \\
V_e(w(\theta, \rho V_n(m)); m) & \text{if} \quad \hat{\theta}(m) \leq \theta 
\end{cases}
\]

The formal definition of Pareto improvement is then given by:

**Definition 1 (WC2)** Labor market participants in a market \( \Psi \) with a minimum wage \( m \) enjoy a welfare gain from the unanticipated imposition of a minimum wage \( m' \) if and only if \( S(\theta; m') > S(\theta; m) \) for all \( \theta \in R_+ \), and \( S(\theta; m') > S(\theta; m) \) for \( \theta \in \zeta \subseteq R_+ \), where \( \zeta \) is of positive measure.

When there is no binding minimum wage, so that \( m \leq \rho V_n(0) \), we can without loss of generality set \( m = 0 \). In this case, any match draw \( \theta < \rho V_n(0) \) results in a match not being formed, and therefore the value of such a draw is the value of (continued) search. When the match is at least as large as \( \rho V_n(0) \), then the value of the draw is given by the value of being employed at the wage generated by \( \theta \). Then

\[
S(\theta; 0) = \begin{cases} 
V_n(0) & \text{if} \quad \theta < \rho V_n(0) \\
V_e(w(\theta, \rho V_n(0)); 0) & \text{if} \quad \theta \geq \rho V_n(0) 
\end{cases}
\]

Definition WC2 clearly refers to Pareto-improving minimum wage changes, but when interpreting this criterion within the context of our model it is important to bear in mind that it does not refer to individuals, but rather to labor market states. The condition requires that for any labor market state any agent could occupy at any moment in time, the change to the binding minimum wage \( m' \) will either improve her welfare at this moment in time or will at least leave her indifferent between facing the binding minimum wage \( m' \) or none at all.

While the criterion refers only to improvements in the current values of participating in the labor market, due to the assumption that the population is homogeneous (in the sense that all individuals face the same labor market environment \( \Psi \) there is a strong dynamic consistency to this criterion as well. Since each individual will occupy all labor market states at some point in their (infinitely-long) labor market career, the condition states that the individual will also be at least as well off in every future labor market state they can occupy under the minimum wage \( m' \) as they would be in the absence of a binding minimum wage. It is perhaps surprising that such a seemingly strong condition can be satisfied for “reasonable” values of the labor market environment \( \Psi \) and choices of \( m' \).
When a binding minimum wage \( m \) exists, the necessary and sufficient condition for a Pareto improvement is given by:

**Proposition 2** The imposition of a new minimum wage \( m' \) in a labor market that currently has a binding minimum wage \( m \) is welfare-improving under WC2 if and only if

\[
V_n(m') \geq V_n(m) + \frac{\alpha(m' - \rho V_n(m))}{\rho + \eta}, \quad \hat{\theta}(m) \leq m' \tag{8}
\]

or

\[
V_n(m') \geq \frac{m + \eta V_n(m)}{\rho + \eta}, \quad m < m' < \hat{\theta}(m).
\]

For a proof see Flinn (2001). It immediately follows that when a binding minimum wage does not currently exist (say \( m = 0 \) without loss of generality) the following criterion is relevant.

**Corollary 3** The imposition of a minimum wage \( m' \) in a labor market that does not have one is welfare-improving under WC2 if and only if

\[
V_n(m') \geq V_n(0) + \frac{\alpha(m' - \rho V_n(0))}{\rho + \eta}. \tag{9}
\]

As can be seen from inspection of both of these cases, a necessary condition for \( m' \) to result in a Pareto improvement over \( m \) is that the value of the unemployed state must be greater under \( m' \) than under \( m \), or \( V_n(m') > V_n(m) \). Thus improvement with respect to WC2 implies improvement with respect to WC1, but the converse is not true. In other words, if we find that a minimum wage change was not welfare-improving using WC1, then it could not be the case that \( m' \) Pareto-dominates \( m \). If we find that \( m' \) is better than \( m \) using WC1, we cannot claim that \( m' \) Pareto-dominates \( m \).

The principal disadvantage in using WC2 as a welfare measure for purposes of conducting a “robust” empirical exercise is that the condition for Pareto-ranking two minimum wage rates depends on primitive parameters as well as the two scalars \( V_n(m) \) and \( V_n(m') \). Without knowledge of these model parameters, or at a minimum having access to estimates of them along with their corresponding sampling distributions, it is not possible to infer the welfare impact of minimum wage changes using WC2 using only pre- and post-change wage distributions and nonparametric testing methods. As a result, for this particular application WC1 is a more useful criterion from an implementation perspective than is WC2.

It is not possible to find a “Pareto optimal” minimum wage, i.e., one for which the value of each possible match value \( \theta \) is no less under minimum wage \( m \) than under any other possible minimum wage rate \( m' \) for all \( m' \) not equal to \( m \) which belong to \( R_+ \). Thus this criterion is most useful when comparing two minimum wage rates. In Figures 3.c and 3.d we plot the state values \( S(\theta; m) \) for \( m = 0 \) and \( m = 7 \) for both of the values of \( \alpha \) we are considering. When workers have little bargaining power (Figure 3.c), the binding minimum wage of 7 improves the value of any possible draw of \( \theta \), even for those that imply the “loss” of a job. The situation is very different for those with relatively high bargaining power. In this case, only match values in a small range beginning at 7 show an improvement under the binding minimum wage. It is easy to construct examples in which all \( \theta \) have lower state values under a higher minimum wage than under a lower one.
3.3 Average Welfare (WC3)

It is often the case that average welfare measures are employed in applications, where the averaging can be of a relatively general form. A characterization of this type of welfare comparison is as follows.

**Definition 4 (WC3)** A minimum wage $m'$ is at least as good as a minimum wage $m$ if and only if

$$\int S(\theta; m')d\mu(\theta|m') \geq \int S(\theta; m)d\mu(\theta|m),$$  \hspace{1cm} (10)

for some given weighting functions $\mu(\theta|m')$ and $\mu(\theta|m)$, where

$$\mu(\theta|x) \geq 0, \forall \theta \geq 0$$

$$\int d\mu(\theta|x) = 1$$

for $x = m, m'$.

There are at least two sensible interpretations that can be given to the weighting functions $\mu(\theta|x)$, though we shall argue that within the context of our particular application one is less compelling. One type of weighting function $\mu$ should be thought of as normative and could be taken to represent the preferences of a social planner, for example. The other type of $\mu$ could be said to be “positivistic” in the sense that it simply reflects some population distribution of match types under the minimum wage policy. Under this specification of $\mu$, the criterion given by [10] can be interpreted as a true average with respect to an objective population distribution rather than an “average” with respect to a social planner’s preferences.

3.3.1 Expected Steady State Welfare

The first interpretation we consider uses the steady state distribution of types in the population under policy $x$ as the weighting function $\mu(\theta|x)$. Under binding minimum wage $m$, we know that the probability that an individual will be in the unemployment state at any arbitrary point in time is given by $\eta/(\eta + \lambda G(m))$. Then the steady state c.d.f of $\theta$ conditional on unemployment is

$$\mu(\theta|u, m) = \frac{G(\theta)}{G(m)}, \theta < m,$$

while the steady state density of $\theta$ conditional on employment is

$$\mu(\theta|e, m) = \frac{G(\theta) - G(m)}{G(m)}, \theta \geq m.$$

Then the weighting function, unconditional on employment state, is

$$\mu(\theta|m) = \begin{cases} \mu(\theta|u, m)p(u|m) + \mu(\theta|e, m)p(e|m) & \text{if } \theta < m \\ \frac{\lambda G(\theta)}{\eta + \lambda G(m)} & \text{if } \theta \geq m. \end{cases}$$
Using this definition of the weighting function the welfare value of a minimum wage under criterion WC3 can be expressed as

$$\int S(\theta; m) d\mu(\theta|m) = V_n(m) \times \frac{\eta}{\eta + \lambda G(m)}$$

$$+ V_c(m; m) \times \frac{\eta + \lambda (\hat{G}(m) - \hat{G}(\hat{\theta}(m)))}{\eta + \lambda G(m)}$$

$$+ \int_{\theta(m)} V_c(\rho V_n(m); m) \frac{\lambda g(\theta)}{\eta + \lambda G(m)} d\theta.$$  \hspace{1cm} (11)

Under this criterion it is clear that the impact of the policy both on the state valuations and the steady state distribution determine the welfare value of the policy (these are simultaneously determined in any event). As a result, conditions under which \( m_0 \) will have an expected steady state welfare value greater than that associated with \( m \) are difficult to derive and are not particularly illuminating. Of most interest for our argument is the fact that \( V_n(m_0) > V_n(m) \) does not imply that \( \int S(\theta; m_0) d\mu(\theta|m) > \int S(\theta; m) d\mu(\theta|m) \). Therefore, no conclusions regarding changes in expected steady state welfare levels resulting from a minimum wage change can be drawn based on the tests conducted in the sequel (which are designed to detect changes in \( V_n(\cdot) \) resulting from a change in the minimum wage).

In Figures 3.e and 3.f we illustrate the relationship between the expected steady state welfare level and the minimum wage rate for our two values of \( \alpha \). While we are not able to formally characterize the relationship between \( V_n(m) \) and average steady state welfare, in our examples the welfare criteria behave in a similar manner with respect to changes in \( m \). Due to the absence of formal results, we cannot speculate as to whether this similarity could be expected to hold in other labor market environments \( \Psi \).

### 3.3.2 Normative Interpretation

Instead of thinking of the weighting function \( \mu \) as representing an objective distribution of types, we can instead think of it as representing the preferences of some fictitious agent. If \( \mu \) represents a set of preferences, then following usual practice we will insist that they be invariant with respect to policy choices, so that \( \mu(\theta|x) = \mu(\theta) \) for all \( x \). As is conventional we normalize these policy invariant preferences so as to make \( \mu(\theta) \geq 0 \) for all \( \theta \geq 0 \) and \( \int \mu(\theta) = 1 \), and we denote the set of preference functions that satisfy these conditions by \( \Theta_\mu \). In this case the welfare criterion WC3 is greatly simplified, since

$$\int S(\theta; m') d\mu(\theta|m') \geq \int S(\theta; m) d\mu(\theta|m)$$

$$\Rightarrow \int S(\theta; m') d\mu(\theta) \geq \int S(\theta; m) d\mu(\theta)$$

$$\Rightarrow \int (S(\theta; m') - S(\theta; m)) d\mu(\theta) \geq 0.$$  \hspace{1cm} (12)

The following results follows immediately.

**Proposition 5** Average welfare associated with \( m' \) exceeds that associated with \( m \) for all \( \mu \in \Theta_\mu \) if and only if \( m' \) dominates \( m \) under WC2.
Proof. Sufficiency is easy to establish, since if \( S(\theta; m') \geq S(\theta; m) \) for all \( \theta \), then because \( \mu(\theta) \geq 0 \) for all \( \theta \) and all \( \mu \in \Theta_\mu \), \( \int (S(\theta; m') - S(\theta; m))d\mu(\theta) \geq 0 \) for any choice of \( \mu \in \Theta_\mu \). Necessity is obvious as well, since if \( S(\theta; m') < S(\theta; m) \) for some set of values \( \theta \in \xi \subseteq R_+ \), average welfare is lower under \( m' \) for all weighting functions \( \mu(x) = 1, x \in \xi \). □

Proposition 6 \( V_n(m') > V_n(m) \) is a necessary condition for \( m' \) to be preferred to \( m \) for all \( \mu \in \Theta_\mu \).

Proof. If \( m' \) dominates \( m \) under WC2 then \( V_n(m') > V_n(m) \). Since WC2 is necessary and sufficient for average welfare to be greater under \( m' \), \( V_n(m') > V_n(m) \) is a necessary but not sufficient condition for average welfare to be greater under \( m' \). □

The normative interpretation of the weighting function can be used to supply another motivation for the use of WC1.\(^6\) Since \( S(\theta; m) \) is nondecreasing in \( \theta \) for all \( m \), the lowest welfare value in the population is associated with \( S(0; m) \). Using a Rawlsian criterion we would then assign \( \mu(0|m) = 1 \) and \( \mu(x|m) = 0, x > 0 \). But since \( S(0;m) = V_n(m) \), this is simply WC1.

We would argue that neither of the two “average” welfare criteria is particularly compelling in this application. In the stationary environment that provides the setting of our analysis, all individuals will occupy all states (i.e., values of \( \theta \)) at some point in their infinitely-long lives. Thus it seems particularly unlikely for a social planner to have strong preferences over the values of \( \theta \) associated with particular individuals at any arbitrary point in time. The average steady state welfare criterion may be less objectionable on the face of it, but since agents don’t behave so as to maximize this objective it is somewhat difficult to justify a social planner’s interest in it. As was the case with the measure WC2, in order to determine whether a minimum wage \( m' \) dominates the minimum wage in an average sense requires knowledge of primitive parameters in addition to the scalars \( V_n(m') \) and \( V_n(m) \). For purposes of conducting a robust empirical exercise this is a serious disadvantage.

3.4 The Welfare of Firms

To this point we have not explicitly mentioned the welfare of the other important set of agents in the model, firms. This omission has been in part intentional, since the data to which we have access only contains information on outcomes for agents on the supply side of the labor market. For the sake of completeness, we do offer a few observations regarding this issue.

Due to search frictions, matching heterogeneity, and the costless creation of job vacancies, firms earn positive profits in the short- and long-run. Let the number of firms in the market be denoted \( K \), and let the number of individuals participating be given by \( N \); both \( K \) and \( N \) are assumed to be large enough that laws of large number results are applicable.\(^7\) In the steady state the number of agents employed is

\[
N_e(m) = \frac{\lambda G(m)}{\eta + \lambda G(m)} N,
\]

and the average number of workers per firm is given by \( AE(m) = N_e(m)/K \). The average instantaneous profit per worker at the firm under minimum wage \( m \) is

\[
AP(m) = \int_m (\theta - w(\theta; m, \rho V_n(m)))dG(\theta)/\tilde{G}(m).
\]

\(^6\)I am grateful to Juan Dubra and an anonymous referee for bringing this point to my attention.

\(^7\)The partial equilibrium model we consider is silent as to the determination of the number of firms in the market and the number of individuals participating on the supply side. Such an extension would clearly be of interest and could yield welfare results of a very different nature.
Then the steady state instantaneous profit of any firm is proportional to \( \Pi(m) = AE(m)AP(m) \). Given the stationarity of the environment it seems natural to equate the value \( \Pi(m) \) with the welfare of the representative firm.

Assuming existence of the matching density everywhere on the real line and the differentiability of the value of search with respect to \( m \), we have

\[
\frac{\partial \Pi(m)}{\partial m} = \frac{\partial AE(m)}{\partial m}AP(m) + AE(m)\frac{\partial AP(m)}{\partial m}.
\]

In terms of the employment effect, from our discussion above we know that it is unambiguously negative, with

\[
\frac{\partial AE(m)}{\partial m} = -\frac{N}{K(\eta + \lambda G(m))^2} < 0.
\]

The impact on average profit per employee is given by

\[
\frac{\partial AP(m)}{\partial m} = \frac{g(m)}{G(m)}AP(m) - p(w = m) - (1 - \alpha)p\frac{\partial V_n(m)}{\partial m}p(w > m), \tag{14}
\]

where \( p(w = m) \) is the probability that an employed agent is paid the minimum wage and \( p(w > m) \) is the probability that an employed worker is paid more than \( m \). Note that under the model the first term on the right hand side of [14] must be positive; this reflects purely the selection effect of improving match quality by imposing a higher “standard.” The second term is always negative and reflects the fact that all those receiving the minimum wage must now be paid more. The third term is of ambiguous sign. When the minimum wage change increases the value of search, and hence the welfare of individuals under welfare criterion \( WC1 \), then this term must be negative. Since a minimum wage change may decrease the value of search, the firm may benefit from the minimum wage change among those workers paid more than the minimum wage due to the reduction in the value of their threat point. While there is a clear interpretation of the various impacts of a minimum wage change on the representative firm’s welfare, there is no way to sign the change without access to the primitive parameters that describe the labor market.

### 4 The Impact of Minimum Wage Changes on Wage Distributions

In this section we link the impact of minimum wage changes on the value of unemployed search, \( V_n(m) \), to changes in steady state wage distributions. In this way the steady state wage distributions, which we assume are observable, can be used to indirectly infer the impact of an observed minimum wage change on (one measure of) the welfare of labor market participants.

#### 4.1 Results using Unconditional Wage Distributions

Within this model the effects of imposing a minimum wage on the accepted wage distribution are complex. The minimum observed wage will always increase in response to the imposition of a binding minimum wage or when a binding minimum wage is increased. While intuition might lead one to expect that comparing wage distributions associated with the same labor market environment
\(\Psi\) and different minimum wage levels in terms of first order stochastic dominance (FOSD) criteria might be a reliable guide to underlying welfare levels, this is not typically the case. Using the welfare criterion we have made our focus of interest, the fact that the wage distribution under the new [higher] minimum wage does not first order stochastically dominate the old one is informative about welfare, but the converse is not the case. We now provide the demonstration of this claim.

**Definition 7** Distribution \(A\) first order stochastically dominates distribution \(B\) if \(B(x) \geq A(x)\) for all \(x\) and \(B(x) > A(x)\) for some \(x\).

Under our model we have the following necessary and sufficient condition for first order stochastic dominance.

**Proposition 8** Let the c.d.f. of wages under the minimum wage \(x\) be given by \(F_x\). Then \(F\) first order stochastically dominates \(F\) iff

\[
\frac{\tilde{G}(m)}{G(m')} \geq \frac{\tilde{G}(z-(1-\alpha)\rho V(m'))}{G(z-(1-\alpha)\rho V(m'))} \quad \text{for all } z \geq m'.
\]

**Proof.** Clearly \(F(w) > F(w')\) for all \(w < m\) since \(F(w) = 0\) for all \(w < m\). For any \(z \geq m'\),

\[
F_m(z) = 1 - \frac{\tilde{G}(z-(1-\alpha)\rho V(m'))}{G(m')}
\]

and

\[
F_m(z) = 1 - \frac{\tilde{G}(z-(1-\alpha)\rho V(m))}{G(m)}
\]

so that

\[
\begin{align*}
\Leftrightarrow 1 - \frac{\tilde{G}(z-(1-\alpha)\rho V(m))}{G(m)} & \geq \frac{\tilde{G}(z-(1-\alpha)\rho V(m'))}{G(m')} \\
\Leftrightarrow \frac{\tilde{G}(z-(1-\alpha)\rho V(m'))}{G(m')} & \geq \frac{\tilde{G}(z-(1-\alpha)\rho V(m))}{G(m)} \\
\Leftrightarrow \frac{\tilde{G}(m)}{G(m')} & \geq \frac{\tilde{G}(z-(1-\alpha)\rho V(m'))}{G(z-(1-\alpha)\rho V(m'))}.
\end{align*}
\]

**Corollary 9** If \(V(m') \geq V(m)\), \(m' > m\), then \(F\) first order stochastically dominates \(F\).

**Proof.** The left-hand side of \([15]\) is by construction greater than or equal to 1. Then the inequality is satisfied since

\[
\begin{align*}
\Rightarrow \frac{V(m')}{\alpha} & \geq \frac{V(m)}{\alpha} \\
\Rightarrow \frac{\tilde{G}(z-(1-\alpha)\rho V(m')}{G(z-(1-\alpha)\rho V(m'))} & \leq 1.
\end{align*}
\]
Unfortunately, this result is not of much practical significance since \( V_n(m') \geq V_n(m) \) is only a sufficient condition for stochastic dominance of \( F_{m'} \) with respect to \( F_m \) and is not a necessary one. A more useful result from the viewpoint of testing is the following.

**Corollary 10** If \( F_{m'} \) does not first order stochastically dominate \( F_m \), \( m' > m \), then \( V_n(m') < V_n(m) \).

These results suggest that observed wage distributions before and after minimum wage changes can reveal whether the minimum wage increase worsened welfare, but cannot be used to infer whether or not welfare increased. In particular, the finding that the new wage distribution first order stochastically dominates the old one is consistent with \( V_n(m') \leq V_n(m) \). The finding that \( F_{m'} \) does not first order stochastically dominate \( F_m \) implies that \( V_n(m) > V_n(m') \).

Obviously \( F_{m'} \) may not first order stochastically dominate \( F_m \) due to a variety of features of the two distribution functions. Our model specification places restrictions on the way in which FOSD can fail. In particular, if \( F_{m'} \) does not FOSD \( F_m \) there must exist some \( x^* \) such that \( F_{m'} \leq F_m \) for all \( x \leq x^* \) and \( F_{m'} > F_m \) for all \( x > x^* \). That is, the c.d.f.s should intersect either never (in which case \( F_{m'} \) first order stochastically dominates \( F_m \) or once and only once (in the case of failure of FOSD). Multiple crossings of the c.d.f.s could be produced by sampling variability or model misspecification.

As these results make clear, it is difficult to assess welfare impacts from changes in wage distributions. We now turn our attention to another characteristic of the relationship between the pre- and post change wage distributions that may have some information value and to which it is possible to give a reasonably intuitive and yet precise definition of the notion of spillover.

### 4.2 Results using Conditional Wage Distributions

Consider a wage rate \( w \) such that \( w > m' > m \). Then under either value of the minimum wage the density of accepted wages at \( w \) exists.\(^9\) Consider the ratio of the density at \( w \) under \( m' \) and \( m \), which is in essence a likelihood ratio. Then we define

\[
L(w; m, m') = \frac{\alpha^{-1}g(\theta(w, V_n(m')))}{\alpha^{-1}g(\theta(w, V_n(m)))} \frac{G(m)}{G(m')} = \frac{\tilde{G}(m) \times g(\theta(w, V_n(m')))}{\tilde{G}(m') \times g(\theta(w, V_n(m)))}.
\]

We might refer to the ratio \( \tilde{G}(m)/\tilde{G}(m') \) in \( L(w; m, m') \) as the truncation effect of the minimum wage change. Since \( \tilde{G}(m) > \tilde{G}(m') \), this effect is always greater than 1 and is independent of the value of \( w, w > m' \); we will write it as \( T(m, m') \). We view this effect on the ratio of wage densities at \( w \) as mechanical and uninteresting from a welfare perspective. Instead, what we will refer to as the spillover effect is the term

\[
S(w; m, m') = \frac{g(\theta(w, V_n(m')))}{g(\theta(w, V_n(m)))}.
\]

\( ^8 \)Note that \( F_m \) can never FOSD \( F_m \circ \) for the simple reason that \( F_m(w) > 0 \) and \( F_m(w) = 0 \) for all \( w \in [m, m') \).

\( ^9 \)For purposes of this discussion we assume that the matching distribution has unbounded support, which implies that the wage distribution will share this characteristic as well whenever \( \alpha > 0 \).
We have a decomposition of the likelihood ratio of the wage density at \( w \) before and after the wage change that is given by

\[
L(w; m, m') = T(m, m') S(w; m, m').
\]

It will be convenient to work with an additive decomposition of the log likelihood ratio, or

\[
\ln L(w; m, m') = \ln T(m, m') + \ln S(w; m, m').
\]

Using the logarithmic decomposition, it is clear that the truncation effect shifts \( \ln L \) by the uniform amount \( \ln T(m, m') \). Furthermore we know that \( \ln T(m, m') > 0 \) for any two binding minimum wages \( m' > m \). Our main interest is in the manner in which the \textit{shape} of the wage density above \( m' \) changes with a change in the minimum wage. We will assess this by looking at the manner in which \( \ln L(w; m, m') \) varies in \( w \). That is, we are interested in

\[
\frac{\partial \ln L(w; m, m')}{\partial w} = \frac{\partial \ln S(w; m, m')}{\partial w}.
\]

We work with the logarithm of the likelihood ratio so that the truncation effect can be ignored.

\textbf{Definition 11} The quantity \( \partial \ln S(w; m, m')/\partial w \) is the \textbf{shape perturbation} at \( w \) associated with the minimum wage increase from \( m \) to \( m' \). We denote this quantity by \( \text{SP}(w; m, m') \).

In general, minimum wage changes result in changes in the shape of the density above the new minimum wage. The following conditions are necessary and sufficient for there to be an absence of spillover as a result of a minimum wage change.

\textbf{Proposition 12} Assume that \( G(\theta) \) is continuously differentiable on its support \( Q \subseteq R_+ \), where \( Q \) is a connected set. Then there is no spillover when moving from minimum wage \( m \) to \( m' \) if and only if at least one of the following holds:

1. \( V_n(m') = V_n(m) \)
2. \( g(\theta) = \tau^{-1} \exp(\beta \theta) \) for all \( \theta \in Q \), where \( \tau = \tau(\beta, Q) = \int_Q \exp(\beta x) \, dx < \infty \).

\textbf{Proof.} For \( \text{SP}(w; m, m') \) to be 0 for all \( w \) requires that \( S(w; m, m') \) be independent of \( w \) for all \( w > m' \), or

\[
S(w; m, m') = \frac{g(\frac{w}{\alpha} - \frac{1-\alpha}{\alpha} \rho V_n(m'))}{g(\frac{w}{\alpha} - \frac{1-\alpha}{\alpha} \rho V_n(m))} = k \forall w > m'.
\]

If \( V_n(m') = V_n(m) \), then \( S(w; m, m') = 1 \) for all \( w > m' \), so condition 1 is obvious.

Rewrite \( S(w; m, m') \) as \( g(y + b)/g(y + a) \). This expression is independent of \( y \) for all values of \( y \) if and only if \( g(x + y) = \tau g(x)g(y) \) for all values of \( x \) and \( y \) such that \( g(x + y) \), \( g(x) \), and \( g(y) \) are well-defined and non-zero. After rewriting

\[
g(x) = \exp(r(x)),
\]

the condition \( g(x + y) = \tau g(x)g(y) \) implies

\[
r(x + y) = \ln \tau + r(x) + r(y),
\]

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which for continuous \( r \) is equivalent to
\[
  r(x) = -\ln \tau + \beta x
\]
for some \( \beta \). Then
\[
g(x) = \tau^{-1}\exp(\beta x),
\]
where \( \tau \) is a constant such that \( \tau = \int_0^\theta \exp(\beta x) \, dx. \)

When looking for “spillover effects” we will be focusing primarily on whether or not condition 2 in the above proposition is satisfied, since it is the only “global” one of the two. By this we mean that satisfaction of condition 2 does not depend on the particular values of \( m \) and \( m' \) chosen so long as they are both binding minimum wages.

The types of distributions which satisfy condition 2 are relatively familiar ones. When \( \beta = 0 \) we have \( g(x) = \tau^{-1} \) on \( Q \), which implies that \( Q \) is a finite interval \([\theta, \overline{\theta}]\), with \( 0 \leq \theta < \theta < \infty \), so that \( \tau = |\theta - \overline{\theta}| \). In this case \( G \) is a uniform distribution. When \( \beta < 0 \), we have the case of a negative exponential distribution. When \( Q = R_+ \), then \( \tau = |\beta|^{-1} \). When \( Q \) is a proper subset of \( R_+ \), then \( g \) is a truncated negative exponential density. Finally, when \( \beta > 0 \), for integrability of the density we require that \( Q \) be a proper subset of \( R_+ \). In other analytic respects this case closely resembles that of \( \beta < 0 \).

The power of Proposition 12 lies in its demonstration that these distributions are the only ones in the class of distributions considered that cannot exhibit ‘spillover’ for any values of \( m \) and \( m' \). Furthermore, the negative exponential distribution is the only one that cannot exhibit spillover if we restrict our attention to distributions with support \( R_+ \).

A result which follows immediately from this proposition that we shall make use of in the sequel is the following.

**Corollary 13** If condition 2 of Proposition 12 is satisfied, then\[ f_m(w) = \delta(m, m') f_{m'}(w), \forall w > m' > m, \]
where\[ \delta(m, m') = \frac{\mathcal{G}(m') g(-\frac{1-\alpha}{\alpha} \rho V_n(m))}{\mathcal{G}(m) g(-\frac{1-\alpha}{\alpha} \rho V_n(m'))}. \]

We now develop a test for what we have defined above as spillover effects that does not require us to assume any particular functional form for \( G \), and in fact does not make much use of the model structure. In this sense we can consider this to be a general test for whether or not a minimum wage change results in shape perturbations. It is important to keep in mind that the absence of shape perturbation effects does not imply that there are no general welfare effects associated with a minimum wage change.

We propose to test proportionality of the population wage densities after a minimum wage change over a subset of the support of the wage distribution. We employ the following result.

**Proposition 14** Let \( f(w; m) = \delta f(w; m') \) for all \( w \in S \subset \Omega \), where \( S \) is a connected set strictly contained in \( \Omega \), with \( S = (a, b) \). Then \( F_m(w | w \in S) = F_{m'}(w | w \in S) \).

---

I am very much indebted to Bernard Salanié for suggesting this method of proof.
Proof. The conditional c.d.f. \( F_x(w|w \in S) \) is given by
\[
F_x(w|w \in S) = \frac{\int_a^w f_x(z) \, dz}{\int_a^b f_x(z) \, dz}, \quad x = m, m'; \quad w \in S.
\]
Since \( f_m(w) = \delta_{m'}(w) \) for \( w \in S \),
\[
F_m(w|w \in S) = \frac{\int_a^w \delta_{m'}(z) \, dz}{\int_a^b \delta_{m'}(z) \, dz} = F_{m'}(w|w \in S), \quad \forall w \in S.
\]

To implement our test for shape perturbation requires that we restrict attention to the subset of wages from the two minimum wage regimes that are greater than the largest minimum wage. Let \( m' \) denote the greater of the two (binding) minimum wages and \( m \) the lesser of the two. The null hypothesis that we wish to test is
\[
H_0: F_m(w|w > m) = F_{m'}(w|w > m'), \quad \forall w > m'. \tag{16}
\]
Using consistent, nonparametric estimates of \( F_m(\cdot|w > m) \) and \( F_{m'}(\cdot|w > m') \) we can implement this test; the details are provided below. There are two possible outcomes of the test. If the null is not rejected, then there is no evidence of spillover and we conclude that the value of search did not increase unless the distribution \( G \) belongs to the family specified in condition 2 of Proposition 12. We will augment our testing procedure in the following way. If the support of \( G \) is \( R_+ \), then the only distribution that exhibits no spillover is the negative exponential. We will assume that the support of \( G \) is \( R_+ \), so that conditional on not finding evidence of spillover we will proceed to test whether the conditional distribution(s) of wages are negative exponential. We will do this by first pooling all observations from the two months before testing whether the distribution of the total sample of wage observations greater than \( m' \) was generated by i.i.d. draws from a negative exponential distribution.\(^{11}\) We perform an omnibus test that has power against all alternative distributions.

4.3 Results using Matched Data

By matched data we mean observations on the same individuals under (at least) two different minimum wage regimes. A number of researchers have examined the impact of minimum wage changes on labor market outcomes using panel data,\(^{12}\) though none have attempted to assess welfare effects per se from such data. Given the assumptions of the model, it is trivial to learn the welfare effect of a minimum wage given access to wage information for at least one individual who was paid more than the minimum wage in each year. Let there exist a set of individuals in the population \( I \) who remained at the same employer both before and after the minimum wage change and who have a wage rate greater than the (respective) minimum in both years. Let \( i \) be a member

\(^{11}\)Recall that pooling the data from the two months only occurs after not rejecting the null hypothesis that the conditional distributions are equal. While the test could be carried out using only one of the wage distributions, efficiency considerations point to using the pooled data.

\(^{12}\)See, for example, Egge et al (1970), Linneman (1982), Smith and Vavrichek (1992), and Currie and Fallick (1996).
of this set, and let their pre- and post-minimum wage change wage observations be denoted by \( w_i \) and \( w'_i \). Then

\[
\begin{align*}
w_i &= \alpha \theta_i + (1 - \alpha) \rho V_n(m), \quad \theta_i > \hat{\theta}(m, V_n(m)) \\
w'_i &= \alpha \theta_i + (1 - \alpha) \rho V_n(m'), \quad \theta_i > \hat{\theta}(m', V_n(m'))
\end{align*}
\]

so that

\[
w'_i - w_i = \rho (1 - \alpha) \{V_n(m') - V_n(m)\}.
\] (17)

The following result is immediate.

**Proposition 15** *The minimum wage change was welfare improving if and only if*

\[
E(w'_0 - w | w > m, w'_0 > m) > 0.
\]

**Proof.** According to [17] the wage difference for any \( i \in I \) is equal to \( \rho (1 - \alpha) \{V_n(m') - V_n(m)\} \), which is independent of \( i \), so that the expectation over the set \( I \), or any subset of \( I \), is equal to this constant. The constant is positive if and only if \( V_n(m') - V_n(m) > 0 \). \[ \blacksquare \]

Because the constant is the same for all members of \( I \), we note the following strong implication.

**Corollary 16** *The wage differences \( (w'_i - w_i) = (w'_j - w_j) \), for all \( i, j \in I \).*

This result implies that the variance in wage changes over the set \( I \), or any subset of \( I \), is 0, which is clearly something we don’t expect to see in any actual data set. We shall discuss this issue further when we carry out the empirical exercise.

It is obvious that there are many reasons to expect increases in the average wage paid to job stayers [those paid more than the minimum wage in this case] that are unrelated to changes in the nominal minimum wage. However, it is the case that the model places an additional restriction on the magnitude of the increase in the average wage for our reference group that can result solely from a change in the statutory minimum wage. This restriction is developed in the following two results.

**Proposition 17** *Let \( m \) be a binding minimum wage. Then

\[
\frac{d \rho V_n(m; \Psi)}{dm} \leq 1.
\]

**Proof.** Let \( x \equiv \rho V_n(m) \). Then the implicit reservation wage under a binding minimum wage \( m \) is the solution to

\[
0 = x - b - k \{(G(\hat{\theta}) - G(m))(m - x) + \alpha \int_{\hat{\theta}}^{\theta} (\theta - x) dG(\theta)\},
\] (18)

where \( k \equiv \lambda / (\rho + \eta) \) and \( \hat{\theta} = (m - (1 - \alpha)x) / \alpha \). Then implicitly differentiating [18] delivers

\[
\frac{dx}{dm} = \frac{k(G(\hat{\theta}) - G(m)) - kg(m)(m - x)}{1 + k(G(\hat{\theta}) - G(m)) + k\alpha(1 - G(\hat{\theta}))}.
\]

Since \( \hat{\theta} > m \), all terms in the denominator are positive or nonnegative. Since \( m - x \leq 0 \), the numerator is less than or equal to \( k(G(\hat{\theta}) - G(m)) \). \[ \blacksquare \]

This result implies, among other things, that any nominal minimum wage greater than \( \rho V_n(0) \) must be binding.
Corollary 18 Let \( m \) and \( m' \) be two binding minimum wages in the labor market environment \( \Psi \), with \( m < m' < \infty \). For an individual at the same job under \( m \) and \( m' \), and for whom \( w > m \) and \( w' > m' \),
\[
w' - w \leq m' - m.
\]

Proof. An individual at the same job [i.e., the same \( \theta \)] is paid \( w' = \alpha \theta + (1 - \alpha)\rho V_n(m'; \Psi) \) under \( m' \) and \( w = \alpha \theta + (1 - \alpha)\rho V_n(m; \Psi) \) under \( m \), so that
\[
w' - w = (1 - \alpha)\rho (V_n(m'; \Psi) - V_n(m; \Psi)), \quad w > m, w' > m'.
\]
By Proposition 17 \( \rho (V_n(m'; \Psi) - V_n(m; \Psi)) \leq m' - m \), and the result follows since \( (1 - \alpha) \leq 1 \).

Since the data we utilize below spans a minimum wage increases of 50 cents (in 1996) and 40 cents (in 1997), by Corollary 18 we should not observe a change in the wages of job stayers (paid more than the minimum wage in both periods) of more than 50 cents in 1996 and 40 cents in 1997. Thus our model places strong restrictions on both the magnitude of the change in the average wage of job stayers and the variance of wage changes within this group of individuals.

5 Tests for Welfare Impacts

To summarize the main theoretical results derived above, we found that:

1. If \( F_{m'} \) first order stochastically dominates \( F_m \) for \( m' > m \) then we can draw no conclusion regarding the relationship between \( V_n(m') \) and \( V_n(m) \). If \( F_{m'} \) does not first order stochastically dominate \( F_m \) then \( V_n(m') < V_n(m) \).

2. If \( F_{m'}(w > m') = F_m(w > m') \) then there is no spillover. No spillover indicates no difference in \( V_n(m') \) and \( V_n(m) \) as long as the matching distribution \( G \) does not have an associated density of the form \( g(\theta) = \tau^{-1} \exp(\beta \theta) \). If we find no evidence of spillover, then it is possible to test that the conditional distribution is not of this form; if we find that this is the case, then we can conclude that there was no change in the value of search.

3. Using wage observations before and after the minimum wage change for individuals who worked at the same job at both points in time, \( E(w' - w|w' > m', w > m) > 0 \) if and only if \( V_n(m') > V_n(m) \).

In this section we discuss practical issues involved with implementing tests of these conditions.

The implications concerning the comparisons of entire conditional or unconditional distributions call for tests of equality and first order stochastic dominance. The general statement of the testing problem is as follows. Consider two distributions \( A(x) \) and \( B(x) \), which could either be conditional or unconditional c.d.f.s. We assume that we have two independent random samples drawn from each distribution, \( X^A \) and \( X^B \), with the number of draws from each given by \( n_A \) and \( n_B \) respectively. Let the empirical distribution of each be defined by
\[
\hat{A}_{n_A}(x) = n_A^{-1} \sum_{i=1}^{n_A} \chi[X_i^A \leq x], \quad \hat{B}_{n_B}(x) = n_B^{-1} \sum_{i=1}^{n_B} \chi[X_i^B \leq x],
\]
where $\chi$ denotes the indicator function. Using these empirical c.d.f.s, we will perform tests of the formal hypotheses:

$$A(x) = B(x), \ \forall x \in Q \subset R$$  \hspace{1cm} (19)

and

$$A(x) \leq B(x), \ \forall x \in Q \subset R,$$  \hspace{1cm} (20)

where the set $Q$ is defined relative to the particular testing situation.

The test of [19] is conducted using the familiar Kolmogorov-Smirnov (two sample) test statistic,

$$S_n^1 = \left( \frac{n_A n_B}{n} \right)^{1/2} \sup_{x \in Q \subset R} |\hat{A}_{n_A}(x) - \hat{B}_{n_B}(x)|,$$

where $n = n_A + n_B$. McFadden (1989) noted that the Kolmogorov-Smirnov test could be modified to test the hypothesis of first order stochastic dominance. The test statistic associated with the hypothesis that $B$ first order stochastically dominates $A$ is

$$S_n^2 = \left( \frac{n_A n_B}{n} \right)^{1/2} \sup_{x \in Q \subset R} (\hat{A}_{n_A}(x) - \hat{B}_{n_B}(x)).$$

Without relatively strong assumptions on the distributions $A$ and $B$, such as absolute continuity, the asymptotic distributions of the test statistics are intractable. As an alternative, a number of authors have worked on applying bootstrap and other resampling methods to nonparametric testing problems; two important papers in this literature are Romano (1988,1989). We follow Abadie (2001) in using a bootstrap procedure to compute the $p$-values of the test statistics [19] and [20] that circumvents the need for strong assumptions on $A$ and $B$. This is a significant advantage in our application, since, as Abadie notes, the presence of heaping at the minimum wage is not consistent with a continuity assumption. Moreover, self-reported (or proxy-reported) wage data such as those we use in our empirical application, display large numbers of observations heaped at reporting “focal points” (which are typically integers). For all of these reasons, the bootstrap methodology (or some other type of resampling procedure) has much to recommend it. The details of the bootstrap procedure that we use can be found in Abadie (2001), as well as a set of relatively weak regularity conditions sufficient to imply consistency.

When testing for exponentiality of the truncated wage distributions (where the truncation point corresponds to the greater of the two minimum wages), we proceed as follows. After pooling the observations from the two monthly samples, we first form the nonparametric estimator $\hat{F}_{m \vee m'}(w \mid w > m')$, where the subscript $m \vee m'$ indicates that the wage distribution corresponds to both the $m$ and $m'$ minimum wage regimes. Then, under the assumption that the distribution of wages is truncated negative exponential (with the truncation point equal to $m'$), we find the maximum likelihood estimator of the parameter of this distribution. Given that $G$ is negative exponential, we have

$$f_{m \vee m'}(w; \gamma, \rho V_n(m')) = \alpha^{-1} g(\frac{w - (1 - \alpha)\rho V_n(m')}{\alpha})$$

$$\quad = \frac{\gamma}{\alpha} \exp\left( -\frac{\gamma}{\alpha} [w - (1 - \alpha)\rho V_n(m')] \right),$$

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$$f_{m \vee m'}(w; \gamma, \rho V_n(m')) = \alpha^{-1} g(\frac{w - (1 - \alpha)\rho V_n(m')}{\alpha})$$

$$\quad = \frac{\gamma}{\alpha} \exp\left( -\frac{\gamma}{\alpha} [w - (1 - \alpha)\rho V_n(m')] \right),$$

\footnote{In the following wage equations the wage distribution is conditional on the value of $V_n(m')$, but could equivalently be written conditional on the value $V_n(m)$ both because (i) the test for exponentially is conducted conditional on $V_n(m') = V_n(m)$ and (ii) the truncated wage distribution is independent of the value of search under the null hypothesis that $G$ is negative exponential.}
and since
\[ \tilde{F}_{m \vee m'}(m'; \gamma, \rho V_n(m')) = \exp\left(-\frac{\gamma}{\alpha}[m' - (1 - \alpha)\rho V_n(m')]\right), \]

then
\[ f_{m \vee m'}(w|w > m'; \gamma, \rho V_n(m')) = \frac{\gamma}{\alpha} \exp\left(-\frac{\gamma}{\alpha}[w - m']\right). \]

Thus the parameter we are interested in estimating is the ratio \( \delta = \gamma / \alpha \), the maximum likelihood estimator of which is
\[ \hat{\delta} = \frac{n}{\sum_{i=1}^{n} (w_i - m')}, \]

where \( n \) is the number of observations in the pooled sample. Given \( \hat{\delta} \) we can define the Kolmogorov-Smirnov statistic by
\[ S_n^3 = \sup_{x \in [m', \infty)} \left| \tilde{F}_{m \vee m'}(x| x > m') - (1 - \exp(-\hat{\delta}[x - m'])\right|, \]

where \( \tilde{F}_{m \vee m'}(x| x > m') \) denotes the nonparametric maximum likelihood estimator of the c.d.f. of the conditional wage distribution using the pooled data. Since \( S_n^3 \) is a function of estimated parameters (\( \hat{\delta} \) in this case), the distribution of the \( K - S \) statistic under the null must be modified to incorporate sampling variability in \( \hat{\delta} \). The critical values for the test statistic are taken from a table in Lawless (1982, p. 448) which were adapted from Stephens (1974). Practically speaking, the problem of sampling variability in \( \hat{\delta} \) is not of major significance because of the extremely large sample sizes with which we work.

The requirements for testing the third set of implications, those that pertain to individuals working at the same job before and after the minimum wage change who are paid more than the prevailing minimum wage in each period, are of a different nature. The result that was obtained applied to each and every individual who belonged to this set, and therefore was in theory “testable” if we had information on the requisite wage information for only one individual. Strictly speaking, no testing procedure is necessary to examine this implication.

In practice, wages are measured with error and there are many determinants of wage rates that are neglected in our modeling framework. For example, the assumption of fixed wage rates at a given job match (in the absence of minimum wage changes) is clearly counterfactual. Particularly for young workers, such as those in our sample, empirical studies often find that there exists a substantial amount of wage growth for those remaining in the same job. Our strategy of dealing with these issues is admittedly somewhat ad hoc, but (we hope) reasonable. We compare the wage growth of those on the same job (to the extent to which we can perform this conditioning accurately) over years in which the minimum wage changes with the wage growth registered by job stayers over a year-long period that does not include a change in the nominal minimum wage. This amounts to the common difference-in-differences estimator often employed in empirical labor economics; the novel aspect of its use here arises from the fact that a partial equilibrium model has been employed to define which differences are the relevant ones to examine in assessing the welfare impact of a minimum wage change.

We conclude this section with a brief discussion of the general problem of nonstationarity to which we have just alluded. In comparing wage distributions at two points in time, our tests rely critically on the assumption of constancy of the economic environment. When two time points are separated by a substantial amount of time this assumption becomes untenable. There are essentially
two responses available to the analyst. The first is to modify the model so as to allow for limited
forms of nonconstancy in the environment. For example, one could posit exogenous shifts in the
matching distribution as a function of calendar time or the labor market age of participants. The
principle disadvantage of such an approach is that, generally speaking, it will no longer be possible
to form nonparametric tests for the welfare impacts of minimum wage changes. Tests for welfare
impacts of minimum wage changes will then only be consistent given the validity of the particular
form of nonconstancy introduced into the model and the availability of consistent estimators of any
parameters required to characterize the evolution of the environment.

Given our explicit interest in developing robust, nonparametric tests for welfare impacts, we
have chosen a second route to minimize the problem of nonconstancy of the environment. Because
we use Current Population Survey data in conducting the empirical exercises below, we have access
to monthly information on wage distributions. As a result, we have the opportunity to look at the
impact of the minimum wage change immediately before and immediately after the change, with
the observations on the wage distributions being separated only by (approximately) one month.
We believe that it is unlikely that there were any substantial changes in the economic environment
over such a brief period of time either at the aggregate or individual level.

6 Data and Empirical Results

The data used in the empirical work are drawn from recent Current Population Surveys. The first
of the two minimum wage changes we examine occurred on October 1, 1996 (when there was an
increase from $4.25 to $4.75 per hour) and the second took place on September 1, 1997 (when
there was an increase from $4.75 to $5.15 per hour). Using the Outgoing Rotation Group (ORG)
subsamples we are able to compare the wage distributions for the month immediately preceding
the minimum wage change with the month immediately following it. To assess the welfare impact
of the earlier minimum wage change, we compare the wage distribution of the September 1996
sample with that of the October 1996 sample (by the sampling design of the CPS these samples
are independent in the sense that they do not contain wage observations on the same individuals).
In order to assess the welfare impact of the most recent minimum wage increase we compare the
August 1997 and September 1997 wage distributions.

As a methodological exercise, we also have conducted the same tests comparing wage distribu-
tions centered on the date of the change in the mandated minimum wage but separated by a
longer time interval (approximately five months on either “side” of the date of the minimum wage
change). To examine the impact of the 1996 minimum wage change we compare the monthly wage
distributions from May 1996 and February 1997. We compare the monthly wage distributions from
April 1997 and January 1998 to examine the impact of the 1997 minimum wage change.

To obtain further evidence on the validity of the tests we conduct, we also applied them to data
from a period that spanned no change in the nominal minimum wage. The date upon which we
centered the comparisons was October 1, 1998, so that the “one month” comparison involves the
wage distributions for September and October 1998, while the “nine month” comparison involves
the wage distributions for May 1998 and February 1999. Since there was no minimum wage change
in this period, the model implies that no significant shifts in the wage distributions should be
observed.

To conduct the tests that involve comparisons of the wages earned by the same individual at
two points in time we utilize information from the March CPS surveys of 1996-1998. In particular,
we compare the wages of individuals who satisfy the test restrictions (i.e., are employed and paid more than the prevailing minimum wage in each period) in March of year $t$ and March of year $t + 1$ and who work in the same industry-occupation category in both years). In this manner we form three matched samples: March 1996-7, March 1997-8, and March 1998-9. In the first two matched panels, the year long period between observations spans a nominal minimum wage change. The last matched panel does not span a minimum wage change and is essentially used for benchmark purposes as was explained in the previous section.

We focus our attention on labor market participants between the ages of 16 and 24, inclusive. This age group has by far the largest proportion of employed members paid exactly at or within a few cents of the minimum wage. We have reasoned that if the minimum wage is to have a substantial impact on the labor market outcomes and welfare of any particular group in the population it is most likely to be this one.

The CPS is a household survey of addresses which has the structure of a rotating panel. Dwelling units are selected to be in the survey for 4 consecutive months, then are out of the sample for eight months, and then return for four consecutive months. Detailed information concerning each household member’s current job, if they are employed at the time of the monthly survey, are only asked to individuals in their 4th and 8th month of participation in the sample (who comprise the Outgoing Rotation Group subsample). Thus we have selected individuals for inclusion in the sample who: (1) were between 16-24 years of age; (2) were in the 4th or 8th month of survey participation; and (3) reported themselves to be currently working. We have not excluded individuals who report being enrolled in school full-time since this group accounts for a substantial proportion of employees paid the minimum wage.

We should mention a few characteristics of the CPS data and U.S. minimum wage laws which complicate any empirical analysis of this issue. First, the minimum wage is set in terms of hourly compensation rates, though many employees are not paid on an hourly basis. Employed individuals in the Outgoing Rotation Group are asked whether or not they are paid on an hourly basis. If they respond that they are paid on that basis, they are asked to report their hourly rate. All employed individuals in the ORG are also asked their gross weekly earnings and their usual weekly hours of work. For individuals paid on an hourly basis, we use their hourly wage report as a measure of their wage rate. For employed individuals who do not report an hourly wage, we attempt to infer one by using the standard procedure of dividing the gross weekly wage by their reported usual hours of work.$^{14}$ Since individuals whose wages are inferred from their report of gross weekly wages and usual hours are likely to have a noisier measure of their “true” rate of hourly compensation, we are less likely to observe them clustered tightly around or exactly at the prevailing minimum wage, even when that is their true “target” hourly compensation rate. This problem provides another rationale for focusing attention on young labor market participants, since they are much more likely to be paid on an hourly basis than are older workers.

The second issue is that of proxy respondents. When CPS interviewers contact a household, one individual in the household provides all of the information for each person living in it. This person is often the head of the household or the spouse of the head. Since many minimum wage workers live as a dependent in someone else’s household, often their parents’, the measurement problems we referred to in the previous paragraph are likely to be exacerbated. While these measurement problems significantly reduce the appeal of the CPS, it remains the best large-scale

\[14\] This procedure fails when usual weekly hours are not reported, which they are not when individuals report that they have no set weekly work schedule. Few cases are excluded for this reason, however.
and representative survey of the U.S. population that can be used to study minimum wage effects on labor market outcomes.

The final issue concerns problems with creating short panels of repeated observations on the same individual. Since the CPS does not provide unique individual-level identifiers that would allow researchers to match individuals across years in a straightforward manner, a more circuitous procedure must be utilized. We begin by matching household identification numbers across March surveys in years $t$ and $t+1$; the household identifier is unique so that errors in matching introduced at this stage should be virtually non-existent. We perform the person matching as follows. When we find a household-level match between the two successive years, for each individual in the household in $t$ we determine whether there is anyone in the same household in $t+1$ who is (1) of the same sex and (2) zero to two years of age older. If any individual in the household in year $t+1$ satisfies those two conditions we say that they are the same person. Of course, the presence of same sex twins, for example, will cause problems, but it is likely that the success rate of this matching procedure is quite high.

6.1 Cross-Sectional Wage Distribution Results

The empirical results from the cross-sectional comparisons are reported in Table 1. For the comparisons of the unconditional wage distributions, we conducted tests of equality of the distributions and first order stochastic dominance (i.e., that the later month’s distributions dominates that of the earlier month). When comparing the conditional distributions, those that involve the wage observations in each month that were larger than the greatest of the two monthly minimum wages, we compute the same two test statistics as well as the one associated with the test for the negative exponentiality of the pooled distribution of wages. The $p$-values associated with each test statistic are reported as well.\(^\text{15}\)

The first panel of the table contains information relevant to the evaluation of the minimum wage change that occurred in 1996. From the results reported in the first line we see that the null hypothesis of equality of the distributions is strongly rejected. This is to be expected, since the distributions are very different (by construction) in their lower tails. In particular, there is no probability mass in the interval $[4.25,4.75)$ for the October 1996 distribution while there is a substantial proportion of wage observations in that interval in September 1996. We also find strong evidence that the October 1996 wage distribution first order stochastically dominates the September 1996 wage distribution. This implies that the value of search could have increased as a result of the minimum wage change, but does not establish that it did.

The second line of Table 1 reports test results based on the wage distributions for the same two months which are truncated from below at the larger minimum wage value, which in this case is 4.75 (all wages are strictly greater than 4.75). The test of equality indicates whether or not spillover is present. Given the $p$-value of 0.664, there is no evidence of spillover. Furthermore, the $p$-value associated with the test for negative exponentially of the matching distribution indicates that such a distributional assumption is untenable.\(^\text{16}\) Taken together, these findings suggest that based on

\(^{15}\)The $p$-values of the test statistics for equality and FOSD are computed by the bootstrap procedure defined above. The $p$-values for the test statistics associated with the null of negative exponentiality are taken from the tables in the sources cited above.

\(^{16}\)Some care is required in interpreting this test result. There is a substantial amount of “heaping” in wage reports, and this measurement problem is to a large extent responsible for the extremely large values of the Kolmogorov-Smirnov statistic that are calculated. To modify the test to allow for measurement error would require that we take
the cross-sectional wage distributions there is no evidence that the minimum wage change in 1996 had a significant impact on the value of search.

The distributions on which these test statistics are based are presented in Figure 4. In Figures 4.a and 4.b we see the large differences in the unconditional wage distributions before and after the minimum wage change that account for the emphatic rejection of the null hypothesis of equality. The lack of difference in the truncated wage distributions (Figures 4.c and 4.d) can be seen from the small differences in these distributions for virtually all \( w > 4.75 \). The “lumpiness” in the empirical wage distribution also accounts (to some extent) for the rejection of the null hypothesis that \( \theta \) follows a negative exponential distribution. Whether the observed heaping is “real” or simply reflects reporting error cannot be determined from these data.

The results in lines 3 and 4 of Table 1 repeat the above exercise, but use months further removed from the date of the minimum wage change (May 1996 and February 1997). Once again, in terms of the unconditional wage distributions the results are similar to what was observed when September and October 1996 were compared. There is strong evidence that the wage distributions are not equal, and there is strong evidence that the wage distribution in February 1997 first order stochastically dominates the wage distribution in May 1996. The fact that the \( p \)-value is even higher than in line 1 probably is attributable to exogenous price increases, productivity growth, etc. The results in line 4 mirror those in line 2. There is no evidence of spillover from the comparison of the truncated wage distributions, and the test of negative exponentiality of the matching distribution again indicates decisive rejection of the null hypothesis. Thus using these results our (tentative) conclusions are unchanged: there is no evidence that the minimum wage change of October 1996 significantly affected the value of search, one important welfare criterion.

The middle panel of Table 1 contains the results of our investigations regarding the minimum wage change that took place on September 1, 1997. Comparing the unconditional wage distributions of August and September of 1997 we see there is no evidence of equality and strong evidence for the null hypothesis that the September wage distribution stochastically dominates the one observed in August. The results that pertain to the truncated distributions for the two months (with lower bound 5.15) are presented in the next line. In this case there is some limited evidence for spillover as indicated by the \( p \)-value associated with the test for distributional equality, though it is not strong. There is evidence against the null hypothesis that the truncated wage distribution in September first order stochastically dominates the truncated wage distribution in August. As before, the null that the truncated wage distributions are negative exponential is rejected. Taken altogether, these results contain some qualified support for the notion that the minimum wage change of September 1997 did lead to a change in the value of unemployed search, and that change could have been positive.

The histograms for the unconditional and conditional wage distributions in August and September 1997 are presented in Figure 5. In terms of the unconditional wage distributions (Figures 5.a and 5.b), the patterns observed are similar to what we observed in Figures 4.a and 4.b. However, the relationship between the truncated wage distributions for the two months is very different than was the case for the previous regime shift. Figure 5.d clearly indicates the reason for the rejection of the null hypothesis of stochastic dominance of the September (truncated) wage distribution with respect to the one of August.

We then reexamined the impact of the September 1997 minimum wage change using the April 1997 and January 1998 wage distributions. The results are very similar to what was found using the
August and September 1997 wage distributions. In particular, there is a strong indication that the unconditional wage distributions are not equal and that the January 1998 distribution first order stochastically dominates the April 1997 distribution. In terms of the truncated wage distributions, there is evidence of spillover and no indication of stochastic dominance.

The last panel of the table presents test results from the comparison of wage distributions that span no nominal change in the minimum wage. The first line of the panel compares the wage distributions for September and October 1998. Since there is no minimum wage change, if the economic environment is constant the wage distributions should be equal. The p-value of 0.344 indicates that there is no strong evidence that the wage distributions are in fact different. The test for stochastic dominance provides some evidence that the wage distribution for October does stochastically dominate that of September, but we would argue that unless there is a strong indication that the distributions are unequal the results of such a test should be ignored. The conditional wage distributions, those that involve only wage observations greater than 5.15, indicate that there was no evidence of spillover, which should indeed be the case if the economic environment was constant over this brief period given no change in the minimum wage. The fact that there is evidence that the matching distribution is not negative exponential leads us to conclude that our test results indicate that there was no significant change in the value of search between the two months, as our model would predict. The graphical evidence presented in Figure 6 indicates the basis for these test results.

This is not the case when we consider the longer interval stretching from May 1998 through February 1999. In this case, the tests based on the unconditional distributions indicate that the distributions are not equal, and also that the February 1999 wage distribution first order stochastically dominates the May 1998 distribution. The tests based on the truncated wage distributions also provide some indication of spillover. We believe that the differences among the results reported in the first two lines and the last two lines of the third panel can be attributed to the length of the measurement period. Comparing two distributions separated by a substantial length of time, even nine months, leads to spurious findings of welfare changes. In order to measure the welfare impact of minimum wage changes in a robust manner it is necessary to utilize wage information from before and after the minimum wage change that is separated by a minimal amount of time.

6.2 Empirical Analysis using the Matched CPS Sample

In this section we utilize the matched CPS samples in an attempt to determine something about the welfare effects of the minimum wage changes of September 1996 and October 1997. In describing the CPS data we noted that it is not possible to be absolutely certain that we have accurately matched individuals from the March surveys of year \( t \) and year \( t + 1 \), though our error rate should be small. More problematic for the utilization of the results of the theoretical results which were derived in Section 4.3 is our inability to determine whether the individual is working at the same job in the two periods. This is primarily due to the fact that the CPS does not collect any information on the length of time individuals have been employed by their current employers. It is not possible to convincingly circumvent this problem, and our “solution” is admittedly problematic. We will think of individuals as being at the same job in the two periods if the industrial and occupational classification of their job is the same in March of years \( t \) and \( t + 1 \). In determining whether they are the same, we use a relatively crude classification system that distinguishes between about 15 occupational and 20 industrial categories. While we could have utilized three-digit occupation and industry codes, we felt that reporting error, especially given the problem of proxy respondents,
would have resulted in too few individuals being classified as “stayers.”

To utilize the analytic results from Section 4.3 we impose the following restrictions on the matched samples. For the matched sample of March 1996 and March 1997 records we only include individuals who were paid more than $4.25 in 1996 and $4.75 in 1997. To be included in the matched sample of 1997 and 1998 individuals must earn more than $4.75 in 1997 and more than $5.15 in 1998. Finally, for our “baseline” matched sample of 1998 and 1999 individuals were required to report wages greater than $5.15 in each March survey. After imposing the additional requirement that the individual be employed in the same industry-occupation category in both years, our final samples sizes were 115, 92, and 145 for the 1996-7, 1997-8, and 1998-9 matched panels, respectively. Selected moments of the wage distributions for each panel are presented in Table 2.

The first thing to note is that the average wage in each of the matched samples that span an increase in the nominal minimum wage is that the mean wage rate increased markedly from the first year of the panel to the second. From Proposition 15 this would imply that the value of search also increased, and under our model assumptions this increase would solely be attributed to the minimum wage change. There are a number of reasons for us to question this inference, however. First, we note that Corollary 18 implies that the mean wage should not have risen by more than $0.50 in the 1996-7 panel or more than $0.40 in the 1997-8 panel. Furthermore, for the 1998-9 panel in which there was no intervening minimum wage increase, there should have been no mean wage change instead of the large positive increase that was observed.

Second, and related to this last point, is the fact that the mean wage changes registered in the matched samples which spanned a minimum wage increase were actually smaller in absolute value than was the increase observed for the sample in which the wage observations did not span a minimum wage increase. In particular, the average wage changed by $1.18 in the panel with no minimum wage change as opposed to $0.94 and $1.02 for the panels in which the nominal minimum wage changed. If we consider the panel that spans no minimum wage change as measuring the baseline growth in wages recorded by individuals who stay at the sample employer, then there is no evidence that the minimum wage changes of the earlier years boosted the rate of wage growth through changes in bargaining power.

Third, our comparisons of cross-sectional wage distributions has illustrated the problems with assuming constancy of the economic environment over periods of time as long as 9 months. Therefore, aside from the problems of within job wage growth mentioned in the second point, there is real potential for the environmental parameters to change markedly over a period as long as a year. Given the structure of the CPS, there is no way to obtain a matched sample of individuals over a period shorter than one year. Other panel data sets could be used for this purpose, though by construction they will not be as representative of the U.S. population as is the CPS.

Putting these caveats aside for the moment, let us briefly consider whether other moments appearing in Table 2 are roughly consistent with the implications of the model. The strong, and clearly counterfactual, result in Corollary 16 implies that the variance of wage changes should be 0 in all of our matched samples. This would be the case if the correlation between wage observations for the same individual was unity. While the estimated correlation is quite high, at least in the 1996-7 and 1997-8 panels, it is clearly not 1. As expected, this model implication is strongly rejected by the data, though it must be mentioned that the correlation between wage observations could be reduced because some sample members actually are working in different jobs in the two different March periods (and thus have two different match values) as well as by the inevitable errors in the reporting of wage information.
In summary, we have raised a number of issues concerning the inferences it is possible to draw based on the matched CPS samples of individuals whose wages were strictly greater than the minimum wage in each year of the panel and who were employed in the same industry-occupation category. To obtain more valid measures under our model one would have to access to wage observations on these individuals considerably closer in time than one year. However, we can cautiously advance the interpretation that there is no panel evidence that points to significant welfare gains as a result of changes in the nominal minimum wage in 1996 and 1997.

7 Conclusion

While it is tempting to infer the welfare effects of minimum wage changes from empirical observations on pre- and post-change wage distributions, in this paper we have attempted to point out the hazards of doing so. We have focused on wage distributions in this paper, but this statement applies with equal force to the case in which the lack of change in employment levels following a minimum wage increase is taken to imply welfare increases. The particular welfare criterion utilized in this paper, which is motivated by a simple equilibrium matching and bargaining model, reflects both employment probability and wage distribution effects of minimum wage changes and hence is preferable to measures which take into account only employment or only wage information. We have discussed alternative welfare measures that arise naturally under our model specification. Our choice of the value of unemployed search as the measure of interest was based on the simplicity of looking at a scalar-valued characteristic, the fact that it is closely related to the alternative welfare measures discussed, and the fact that its relationship with the wage distribution is so immediate. This last advantage gives us the possibility of drawing accurate influences regarding changes in the value of search by examining certain properties of wage distributions before and after nominal minimum wage changes.

The empirical application we have presented usefully summarizes the general points we wish to make. The fact that the wage distribution following a minimum wage change stochastically dominates the wage distribution before the change does not necessarily imply that welfare has increased. We found evidence that this was the case for both the 1996 and 1997 minimum wage changes. Using the truncated wage distributions, we found no evidence that the 1996 minimum wage change resulted in spillover. As was established analytically, the absence of spillover was consistent with no changes in the value of search if the matching distribution belonged to a particular parametric family, the leading member of which is the negative exponential distribution. After conducting tests that led to a rejection of that possibility, we conclude that there is no evidence from the cross-sectional data that the 1996 minimum wage change resulted in a welfare change. This was not the case for the 1997 minimum wage change, in which case spillover was detected. That said, unless the form of the matching distribution is known the direction of the welfare change cannot be inferred. Thus all we can say that there is some evidence that welfare changed as a result of the 1997 minimum wage increase and that the effect could have been positive (based on the stochastic dominance results using the unconditional wage distributions).

We showed that the direction of wage changes for individuals who worked at the same job both before and after the minimum wage change and who were paid more than the relevant minimum wage in each period could be used to very directly infer welfare impacts in principle. The pitfalls involved with using such an approach include nonstationarity in the economic environment over a period as long as a year and on-the-job wage growth. While we found substantial growth in mean
wages over the sample periods spanning minimum wage increases, patterns of growth were not consistent with many implications of the model. Moreover, these average wage increases were lower than what was observed for a “control” period in which nominal minimum wages did not change. Our conclusion from the panel evidence is necessarily a tentative one, but there is no indication that the minimum wage changes of 1996 and 1997 significantly improved the welfare of young labor market participants.
Table 1
Tests of Distributional Equality, First Order Stochastic Dominance,
and a Negative Exponential Distribution of Match Values
(p-values)

<table>
<thead>
<tr>
<th>Change in $m$</th>
<th>Months</th>
<th>Samples</th>
<th>$n_0$</th>
<th>$n_1$</th>
<th>Test Statistics</th>
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<tr>
<td>4.25 to 4.75</td>
<td>9/96 and 10/96</td>
<td>${w_0 \geq 4.25, w_1 \geq 4.75}$</td>
<td>1575</td>
<td>1527</td>
<td>3.288</td>
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<td></td>
<td></td>
<td>${w_0 &gt; 4.75, w_1 &gt; 4.75}$</td>
<td>1345</td>
<td>1409</td>
<td>0.623</td>
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<tr>
<td>5/96 and 2/97</td>
<td></td>
<td>${w_0 \geq 4.25, w_1 \geq 4.75}$</td>
<td>1473</td>
<td>1514</td>
<td>4.044</td>
</tr>
<tr>
<td></td>
<td></td>
<td>${w_0 &gt; 4.75, w_1 &gt; 4.75}$</td>
<td>1223</td>
<td>1398</td>
<td>0.674</td>
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<tr>
<td>4.75 to 5.15</td>
<td>8/97 and 9/97</td>
<td>${w_0 \geq 4.75, w_1 \geq 5.15}$</td>
<td>1673</td>
<td>1409</td>
<td>5.224</td>
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</tr>
<tr>
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<td>1446</td>
<td>6.513</td>
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<td>1087</td>
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<td>1.334</td>
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<td>1643</td>
<td>0.836</td>
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<td>1552</td>
<td>0.755</td>
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<td>${w_0 \geq 5.15, w_1 \geq 5.15}$</td>
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<td>1324</td>
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<td>${w_0 &gt; 5.15, w_1 &gt; 5.15}$</td>
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<tr>
<td>$\Delta m$</td>
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<tr>
<td>$\bar{w}_1$</td>
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<td>$\bar{w}_2$</td>
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<td>$\hat{\sigma}_1$</td>
<td>3.371</td>
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<tr>
<td>$\hat{\sigma}_2$</td>
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<tr>
<td>$\hat{\sigma}_{1,2}$</td>
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<td>9.774</td>
<td>8.503</td>
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<td>$\hat{\rho}$</td>
<td>0.758</td>
<td>0.752</td>
<td>0.557</td>
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<td>$\hat{\sigma}_{w_2-w_1}$</td>
<td>2.596</td>
<td>2.617</td>
<td>3.749</td>
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<td>$N$</td>
<td>115</td>
<td>92</td>
<td>145</td>
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References


Figure 4.a
Empirical Unconditional C.D.F.s
September and October 1996 CPS

Figure 4.b
Difference in October and September
Unconditional C.D.F.s

Figure 4.c
Empirical Conditional C.D.F.s
September and October 1996 CPS

Figure 4.d
Difference in October and September
Conditional C.D.F.s
Figure 6.a  
Empirical Unconditional C.D.F.s  
September and October 1998 CPS

Figure 6.b  
Difference in October and September  
Unconditional C.D.F.s

Figure 6.c  
Empirical Conditional C.D.F.s  
September and October 1998 CPS

Figure 6.d  
Difference in October and September  
Conditional C.D.F.s