Chapter 2

1 A Model of Minimum Wage Effects on Labor Market Careers

Minimum wages can have complex effects on the labor market experiences of individuals through decisions to participate in the market, human capital investment activity, the duration of and frequency of unemployment spells, wage offers, and a myriad of other phenomena. The model we develop will abstract most of these issues, but in the conclusion and at selective points in the empirical analysis we will comment on how some of our theoretical and empirical analysis would change if some of these phenomena were introduced, and how policy implications could be skewed by their omission.

The model we will use to theoretically and empirically investigate minimum wage effects on labor market experiences views individuals and firms operating in stochastic environments that are governed by probabilistic laws that do not change over time. This is quite an abstraction from reality, but for purposes of examining minimum wage effects it may not be too egregious for a number of reasons. Because the majority of individuals paid the minimum wage are young, the major impact of minimum wage laws on labor market outcomes is likely to be concentrated in the first few years of labor market activity. Over such a relatively short period, the probabilistic structure of the labor market is not likely to change dramatically. Furthermore, pragmatically speaking, the data to which we have access (drawn from the Current Population Survey for the most part) are essentially static. To estimate a dynamic model using such data requires us to assume that the labor market environment does not change over time. In particular, under the assumption of stationarity (i.e., constancy of the probabilistic laws governing the economy) the rational behavior posited for all labor market participants and firms can be characterized in terms of decision rules that are time-invariant. If we were to allow the model to be nonstationarity, we would be forced to specify all of the conditions that individuals and firms will face over the infinite past and future in order to describe their decisions at any point in time. Clearly we don’t have access to the data which would allow us to undertake such a modeling effort, nor would such a complex theoretical analysis be particularly
enlightening.

Why attempt to build such a model in the first place? The point which we will make repeatedly in this book is that it is necessary to have a relatively complete, equilibrium model of employment relations if we are to comprehensively summarize the impact of minimum wages on labor market outcomes and the welfare of labor market participants and firms. To make welfare valuations, it is necessary to endow each set of agents in the economy with their own set of objectives. Subject to technological and budget constraints, and given the optimizing decisions of other agents in the market, each individual or firm acts so as to maximize the value of their objective function. We will view minimum wages as constraining the actions of all labor market participants, both individuals and firms, though as we shall see the minimum wage “constraint” may, under certain circumstances, increase the welfare of labor market participants, firms, or even both. In many cases, only one side of the market will experience a welfare increase as a result of a given minimum wage increase (and yes, it could be employers), while minimum wage increases at extremely high levels negatively impact both sides of the market, at least in an ex ante sense.

The objective function with which we endow labor market participants is one of expected wealth maximization. That is, individuals are assumed to care only about their (discounted) average earnings over their labor market careers; in particular, the variance of income flows does not favorably or unfavorably affect welfare. This is a strong assumption, and is made primarily for reasons of tractability. However, it is easy to show that when consumption decisions can be “decoupled” from earnings, as is the case when there exist perfect capital markets for borrowing and lending, expected wealth maximization behavior in the labor market is consistent with aversion towards consumption risk on the part of individuals. While young labor market participants may not have access to perfect capital markets, transfers between parents and children may serve the same role. In many cases, there is no strong reason to expect that young labor market participants will behave in ways other than those consistent with expected wealth maximization.

Perhaps less controversially, firms will be assumed to behave so as to maximize expected profits. The model employed throughout is search theoretic. At some points in the welfare analysis we will add general equilibrium elements by assuming that the rate of contacts between searching individuals and searching employers is determined using a Pissarides (2000) “matching function” setup. In his framework firms make
decisions regarding the number of vacancies to create and the contact rate is an increasing function of the number of searchers and the number of vacancies. There will be no other “general equilibrium” links between searchers and firms in our model, for example through the mechanism of public ownership. The general equilibrium analyses posit an expected value of zero for all new vacancies, though firms with filled jobs do earn positive profits, and the profit level will be a function of the minimum wage rate. In the partial equilibrium analysis, all firms earn nonnegative profits in equilibrium. We have thus deliberately kept our model of firm behavior and the link between firms and searchers as simple as possible subject to the restriction that it possess empirical implications broadly consistent with the CPS data to which we have access.

As is clear from the description of the model so far, many phenomena that could have significant impacts on the the effect of the minimum wage on labor market outcomes have been omitted. Perhaps the first thing that comes to mind is capital goods. If firms have access to production technologies that allow them to substitute capital for labor, then substantial minimum wage increases that significantly affect the price of labor, at least at the low end of the skill distribution, may lead employers to substitute machinery for manpower. Since we have no measures of capital utilization at the firms employing individuals in our the CPS data, we have no way of directly using this information to characterize the potential degree of substitutability between capital and labor at this part of the skill distribution.

Though it is common to assume the absence of capital in search-theoretic models of the labor market, clearly it would be preferable to allow firms to make these decisions. Over the past several decades we have seen the demand for lower-skilled individuals decrease precipitously in the United States, which has given rise to the marked increase in inequality in labor market earnings. While we cannot explicitly include in the model and the empirical work a number of important factors affecting the demand for labor, our Nash bargaining formulation of the wage determination process does allow us to represent the cumulative impact of these factors in an indirect way - through the bargaining power parameter. Recently, Cahuc et al (2005) have explicitly estimated bargaining power parameters for segments of the French work force, and find that higher-skilled workers tend to have more bargaining power than less skilled workers, even after accounting for other differences in the labor market environments these groups face. Of course, one problem with allowing this one
characteristic represent the plethora of omitted factors is the necessity we face of assuming that it is a “primitive” parameter, that is, that it remains the same when labor market policy (i.e., the minimum wage in our case) is even radically altered. Since many of our policy implications flow from the estimated value of the bargaining power parameter and our assumption that it is fixed, all of our findings and welfare analyses must be interpreted cautiously.

1.1 Characterization of the Labor Market Career

We will model labor market events as occurring continuously in time. By this we mean that there are no natural times, say weeks or months, at which labor market events always take place. Technically speaking, we view the labor market as a continuous-time point process, which means that at any point in time an unemployed individual can receive a job offer. Furthermore, at any point during an employment relationship the contract can be exogenously terminated. In some versions of the model, we allow for on the job search as well. In this case employed individuals at one firm may make contact with another that offers them an employment opportunity. In this model, jobs with a given employer may end due to some exogenous separation (such as the plant closing down or the individual changing locations) or for “endogenous” reasons - the employee finds a job at which she is more productive and terminates her position at her current employer. We will mainly focus on the case of unemployed search only because it is more straightforward to analyze and because such a model can be estimated with CPS data, the largest and most representative labor market data set available to us, and one that has more or less consistently collected the same information for four decades.

In order to fix ideas, consider the case in which there is no on-the-job search, so that individuals begin their labor market careers as unemployed searchers, eventually locating a job paying some “acceptable” wage (we will define below what we mean by acceptable). After some that job will end, and she will once again enter the state of unemployment, where the job search process will be repeated in exactly the same way as it was the first time around. This “cycling” continues unabated until the individual eventually leaves the market - which we will suppose is for an exogenous reason. In the language of stochastic process theory, the labor market career is an alternating, marked renewal process. It is alternating, because time in the labor mar-
ket is spent alternatively in unemployment and employment spells. It is a renewal process, because we will be assuming that the individual’s past labor market history plays no role in determining how long she will spend in a current spell of unemployment or employment. Finally, it is a “marked” process because when she is employed, we will not only be concerned with how long the employment spell lasts, but also at what wage she is employed. The wage rate is a marker, or subsidiary characteristic, of interest to us over and above the timing of events.

Let’s fix ideas with an example. Say that an individual begins her labor market career at time 0, an inconsequential normalization. Assuming that she will continue to participate in the market as an unemployed searcher or worker over her entire life, her labor market career can be completely characterized by the time at which she meets prospective employers and the value of the match associated with each contact, as well as the time at which employment matches she has accepted were (exogenously) terminated. For example, since she begins her labor market career in the unemployed search state at time 0, the first fifteen “events” in her labor market career might be given by the values in the following table.

<table>
<thead>
<tr>
<th>Event Number</th>
<th>State</th>
<th>Time of Event</th>
<th>Match Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>n</td>
<td>.891</td>
<td>6.243</td>
</tr>
<tr>
<td>2</td>
<td>n</td>
<td>3.168</td>
<td>4.329</td>
</tr>
<tr>
<td>3</td>
<td>n</td>
<td>15.554</td>
<td>3.871</td>
</tr>
<tr>
<td>4</td>
<td>n</td>
<td>15.558</td>
<td>10.918</td>
</tr>
<tr>
<td>5</td>
<td>e</td>
<td>38.921</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>n</td>
<td>44.236</td>
<td>7.891</td>
</tr>
<tr>
<td>7</td>
<td>n</td>
<td>56.793</td>
<td>12.119</td>
</tr>
<tr>
<td>8</td>
<td>e</td>
<td>157.421</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>n</td>
<td>164.772</td>
<td>10.145</td>
</tr>
<tr>
<td>10</td>
<td>e</td>
<td>322.510</td>
<td></td>
</tr>
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<td>...</td>
<td>...</td>
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</tr>
</tbody>
</table>
The interpretation of the figures in Table 2.1 is as follows. The individual initiated search at time 0, at which point she occupied the nonemployed state \( (n) \). At time .891 she encountered her first potential employer. [Time units are arbitrary here, but it may help to think of the unit of time as the week.] When she met her first employer, the productivity of the potential match was revealed to be 6.243; this is to be thought of as the “flow” value of productivity that will occur if she actually works at this employer. The value of this first potential match was insufficient to result in an employment contract so that the individual remained in the nonemployed state. The second potential employer was encountered at time 3.168 and the potential value of that match was 4.329, which also was deemed unacceptable. Only the fourth potential match resulted in an employment contract. This employment contract began at time 15.558 and had a flow value associated with it of 10.918. This match surplus is divided between the worker and the firm according to an idealized bargaining process which will be described in detail below. Whatever the division, it was sufficient to induce both parties to begin the employment contract, which was terminated (exogenously) at time 38.291. This caused the individual to reenter the nonemployment state, and she again began to search for a new acceptable employment contract immediately. After one unsuccessful encounter at time 44.236, she found another acceptable match with a value of 12.119 at time 56.793. This match was eventually terminated at time 157.421, and the nonemployed search process was repeated.

There are a number of things to note about this idealization of the labor market career. First is the implicit assumption that the only way an employment match can end is through an exogenous termination. This is clearly counterfactual, and can fairly readily be dispensed with if we allow for on-the-job search. If we posit that employed individuals encounter potential firms and can possibly be “bid away” by them if the match value at the new employer exceeds the match value at the current one,\(^1\) we will have a model in which jobs can end either by exogenous termination (e.g., the firm goes out of business due to a drop in demand for its product, or the individual experiences a negative health shock that doesn’t allow her to continue performing a particular job task) or by a “voluntary” separation in which the worker accepts employment at a firm making her a better offer. In this book we don’t pursue such an extension for two reasons. First, and most importantly, the CPS data really

\(^1\)For examples of such models see, for example, Dey and Flinn (2005), Postel-Vinay and Robin (2002), or Cahuc et al. (2005).
don’t offer us the possibility of examining job-to-job transitions in sufficient detail to make the estimation of such a model feasible. Secondly, allowing for on-the-job search will add another layer of complexity to our model and to some extent make the role of minimum wages in determining labor market outcomes a bit less transparent. For these reasons we ignore on-the-job search in what follows for the moment.

Another important point to note is that the events described in Table 2.1 are not strictly exogenous. Some events are conditionally exogenous; for example, given the decision to begin the labor market career at time 0 the arrival of the first possible job match at time .891 and the total flow value of that match, 6.243, are determined strictly randomly. However, the decision to reject that possible match is a behavioral one made by the searcher and the first firm encountered. This led the individual to continue in the nonemployed state and to continue on to meet her second potential match at a time which was, once again, determined randomly. Thus the labor market career is a sequence of exogenous events followed by decisions which lead to further exogenous events [conditional on the previous exogenous events and past decisions], and so on. The method of dynamic programming (DP) allows us to formalize this process.

Finally, note that the only decision explicitly made by the searcher in this simple conceptualization of the labor market career is whether or not to accept a particular employment contract; though the wage associated with a given match value is assumed to be the outcome of a bargaining procedure, this procedure is largely “black-boxed,” and hence not strictly behavioral. Under the assumption that the environment is constant, the decision of whether or not to begin a particular employment “match” will be made by comparing the value of the match, denoted by $\theta$, with a constant, $\theta^*$. An employment contract will be initiated whenever a match value $\theta \geq \theta^*$. The parameter $\theta^*$ will be a function of all of the parameters that characterize the economic environment. From the partial labor market history contained in Table 2.1 it is clear that $7.891 < \theta^* \leq 10.145$.

Another decision, which is particularly relevant for young [and old] individuals, is whether to participate (i.e., undertake search) in the labor market. For the moment, we will ignore this decision and assume that all individuals in the population we are

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2It is possible to rationalize the Nash bargaining axiomatic solution as the outcome of a strategic bargaining game, as was famously done in Rubenstein (1982). We won’t explicitly attempt to model the interactions between negotiating firms and workers. For one attempt to do so in a bargaining model similar to ours, see Cahuc et al (2005).
considering are labor market participants. We shall return to a consideration of the participation decision at the end of this chapter.

1.2 The Stationary Labor Market Environment

The formal characterization of the environment within which firms and individuals interact begins with the notion of a counting process. Say that an individual begins a spell of search at time 0, as in the example above. If she were to search from time 0 until time \( t, t > 0 \), we can define the number of potential employers she encounters by the variable \( N(t) \). Looking again at the example in Table 2.1, consider the first search spell. The timing of the first four encounters with potential employers which occurred during this spell are given by the first four entries in the third column of the table. The first search spell was ended by the individual after the fourth contact because the match value was sufficiently high. If this search spell was not voluntarily “truncated” by the individual at this point, we could have imagined it continuing indefinitely, with one chance meeting with a potential employer followed by another.

Figure 2.1 plots the number of firms encountered as of time \( t, N(t) \), for this case, where we have followed the individual through her first 10 encounters in such a spell [which include the first four recorded in Table 2.1]. Note that that the value of \( N(t) \) is 0 until the first encounter, which occurs at time .891. At this point it jumps to 1, since there has been 1 contact up through \( t = .891 \). Then the value of \( N \) jumps to 2 at the time of the second contact, or \( N(3.167) = 1 \) but \( N(3.168) = 2 \). Thus \( N \) takes values in the set of nonnegative integers, is monotonically increasing, and, whenever changed, is incremented by 1.

The Poisson process is a particular type of counting process. There are a number of different yet equivalent ways to characterize it; for the purpose of defining the dynamic programming problem, the most useful one is probably the following.

Definition 1 \( \{N(t), t \geq 0\} \) is a Poisson process with parameter \( \lambda \ (> 0) \) if

1. \( N(0) = 0 \) (Initial condition)

2. \( \{N(t), t \geq 0\} \) has stationary, independent increments

\(^3\)A stochastic process \( X \) has independent increments if the change in the process over the interval \([t_1, t_2]\) is independent of the change in the process over the interval \([t_3, t_4]\) for any \( 0 < t_1 < t_2 < t_3 < t_4 \). A stochastic process has stationary independent increments if in addition the probability
3. \( P(N(t + \varepsilon) - N(t) \geq 2) = o(\varepsilon) \) for all \( t \geq 0 \)

4. \( P(N(t + \varepsilon) - N(t) = 1) = \lambda \varepsilon + o(\varepsilon), \) for all \( t \geq 0 \)

The notation \( o(\varepsilon) \) which appears in items (3) and (4) of the definition has the following interpretation. A function \( f(\varepsilon) \) has the property of being \( o(\varepsilon) \) if

\[
\lim_{\varepsilon \to 0} \frac{f(\varepsilon)}{\varepsilon} = 0.
\]

The practical import of this is the following. Consider the search interval of \([0, \varepsilon]\). When \( \varepsilon \) is “large,” say equal to 1 time unit, in principle a countably infinite number of contacts with potential employers can be made, although the probability of making a very large number of contacts will in general (depending on \( \lambda \)) be quite small. As we “shrink” the time interval by reducing the value of \( \varepsilon \), the probability of encountering more than 1 potential employer becomes negligibly small. Of course, when \( \varepsilon = 0 \), the probability of making any contact is 0. Thus items (3) and (4) mean that for \( \varepsilon > 0 \) but arbitrarily small, the probability of two or more events is arbitrarily close to 0 and the probability of one contact is approximately proportional to the length of the arbitrarily small interval. Below we shall refer to \( \lambda \varepsilon \) as the probability of experiencing a contact in the small interval \( \varepsilon \), and the reader should bear in mind that this interpretation is only “approximately” valid.

The parameter \( \lambda \) is referred to as the rate of the Poisson process. It has this interpretation since

\[
EN(t) = \lambda t,
\]

\[
\Rightarrow E(N(t)/t) = \lambda,
\]

so that the average number of contacts over any period of length \( t \) is \( \lambda \). For example, if \( \lambda = .2 \), the average number of contacts per period is .2. Conversely, the average wait between contacts is the reciprocal of .2, or 5 time periods in this case.

Note that under the assumption that the searcher-employer contact process is Poisson, every time a new nonemployed search process is begun it has the same probabilistic properties as the first search spell. In this sense, each search spell can without loss of generality be considered to “restart” at time 0. As mentioned above, distribution of \( N(t_2 + \varepsilon) - N(t_1 + \varepsilon) \) is the same as the probability distribution of \( N(t_2) - N(t_1) \) for all \( 0 < t_1 < t_2 \) and \( \varepsilon > t_2 - t_1 \).
a stochastic process with this property is referred to as a renewal process, since whenever a particular state, in this case nonemployment, is revisited, the “clock” is reset.

The labor market model we develop actually is built upon two separate and independent counting processes, the first being the nonemployed searcher-potential employer contact process and the second being the job dissolution process. Recall that we will be assuming that employment matches dissolve exogenously. This dissolution process will also be Poisson, with a rate parameter given by $\eta$, $\eta > 0$. Let an employment contract between a searcher and a firm begin at time 0 (an inessential normalization). Let $D(t)$ be the number of times the process would have exogenously dissolved as of time $t$; the times at which the contract would have been terminated may correspond to the times at which adverse demand shocks would have put the firm out of business. This termination process has the same properties as the searcher-firm contact process except the parameter characterizing the process may have a different value. Clearly, we have defined the two processes so as to be conceptually and probabilistically independent of one another.

The final component of the stationary labor market environment is the matching productivity distribution, $G(\theta)$. To ensure stationarity of the environment, we make the assumption that this matching distribution does not change over calendar time or as a function of the elapsed time searching during the spell. All contacts with potential employers are assumed to be independent and identically distributed (i.i.d.) draws from this distribution.

In summary, the labor market environment is described physically by the contact rate between firms and searchers, $\lambda$, the dissolution rate of on-going employment contracts, $\eta$, and the time-invariant matching distribution $G$. In order to completely characterize labor market equilibrium, we shall need a few other parameters but they are best described in the course of examining the decision-theoretic aspects of the model.

1.3 The Decision-Theoretic Model

The individuals in our model are posited to be taking actions so as to maximize their expected lifetime wealth. They can only maximize expected wealth due to the presence of “search frictions.” In this case, search frictions refer to the fact that
individuals do not know the location of the firms with employment vacancies, and even more importantly, do not know the identity of the firm with which they could achieve the highest productivity level. Prior to actually contacting a given firm and learning their productivity level there, all potential employers look alike to the individual. In an expectational sense, then, the individual is initially indifferent with respect to the identity of the firm which is contacted. Firms are only differentiated after contact.

It is important to be clear regarding our assumption as to the manner in which an individual’s productivity, $\theta$, is determined at a randomly selected firm. We assume that this productivity value is a draw from some fixed, known distribution $G(\theta)$. For the moment, we do not distinguish individuals in terms of observable characteristics. One might suppose that schooling, for example, tends to make one more productive in the labor market. Within our modeling framework, this kind of effect would be incorporated by conditioning the match distribution, and possibly other labor market parameters as well, on schooling. In general, let $x$ denote a characteristic or set of characteristic upon which population members are distinguished. Then the conditional matching function is $G(\theta|x)$. If $x$ was years of schooling completed, for example, we might assume that $G(\theta_0|x') \leq G(\theta_0|x)$ for all $\theta_0 > 0$ when $x' > x$. This is a first order stochastic dominance relationship, so that the likelihood that any match draw is less or equal to $\theta_0$ for a highly-educated individual is no greater than it is for a less-educated individual. Of course, a logical consequence of this is that the average match draws of the more highly-educated individual is at least as large as that of the less-educated person. Ceteris paribus, more highly-educated individuals will receive higher wages, but at any given firm their productivity can be lower than their less-educated colleague. Given an individual’s characteristics summarized by $x$, match draws across alternative employers will be independently and identically distributed according to $G(\theta|x)$.

There are many other alternative formulations one could make regarding the manner in which worker-firm productivity is determined. One leading approach is to assume that individual $i$ on the supply side of the market have a time-invariant productivity-determining characteristic $a_i$, while firm $j$ on the demand side of the market has a time-invariant productivity-determining characteristic $b_j$, with the pro-

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4 Under the informational assumptions of this model, jobs, or more properly potential jobs, are pure search goods. Once a potential employer is contacted, both the individual and the firm will learn the total value of the match as well as the share of that value accruing to each.
ductivity of individual $i$ at firm $j$ being given by $\theta_{ij} = a_i b_j$. This is the setup utilized by Postel-Vinay and Robin (2002) and by Cahuc et al. (2005) in their analysis of labor market dynamics using matched French employer-employee data. They skillfully exploit these data and are able to recover estimates of the employee skill distribution $F(a)$ without making parametric assumptions regarding the form of the distribution. Given a distribution of firm characteristics, $Z(b)$, each individual faces their own matching distribution $Z(b) \times a$. In a number of respects, this is more general than the matching process described in the previous paragraph in that it does not require to specify what factors, such as schooling, should be used to differentiate workers. On the other hand, it does place a number of restrictions on the distribution of match values across workers and firms that are probably overly strong.\(^5\)

At each moment in time, the individual makes decisions so as to maximize expected wealth given her current labor market state and her knowledge of the parameters that characterize the labor market environment. The method used to characterize her decision is dynamic programming (DP). To illustrate the DP approach to a continuous time model like ours, we will begin by assuming that workers receive the entire value of the match. In this case, the matching distribution $G$ is synonymous with a wage offer distribution $G(w)$ [since $w = \theta$ for any $\theta$ draw].

Say that the individual is current unemployed at some particular moment in time. The value of being in the state consists of the sum of a “flow” value which is the “current period return” [a period should be thought of as an instant, in this case] and the discounted value of next period’s problem given the current state and action taken. If the current state is denoted by $s$, the current choice or action by $a$, and next period’s state is given by $s'$, then

$$V(s) = \max_a R(s, a) + \beta E[V(s')|s, a], \quad (5)$$

where $R(s, a)$ denotes the current period return to the action $a$ taken when the state is $s$, $\beta$ is some positive scalar, $s'$ denotes next period’s state, and $E[V(s')|s, a]$ is the conditional expectation of the value of next period’s decision problem given current state $s$ and action $a$. In computing this expectation, the conditioning on $s$ and $a$ reflects the fact that in general the probability distribution of $s$ is not independent of

\(^5\)For example, this matching structure implies that any worker that is more productive than another worker at a given firm $b$ will be more productive at any other firm $b'$.\)
the current state and the action taken \( a \).

Consider first the value of being in state \( n \), that is, nonemployed and searching. Assume that the flow value of searching is given by the constant scalar \( b \). This flow value of nonemployment can reflect unemployment benefits, direct costs of search activity, and the value of other activities undertaken while the individual searches. Our strong stationarity assumptions require that \( b \) must be time invariant and that time spent in search does not change the parameters characterizing the labor market environment of the individual. These are both clearly counterfactual, especially for young labor market participants. In the case of unemployment benefits, for example, there are limitations on the length of time they can be received. Individuals typically face an unemployment benefit function which has the property that some amount \( B \) is received for a period \( T \) after which nothing is received.\(^6\) In such a case, clearly \( b \) is not time-invariant. Also note that young labor market searchers are often simultaneously investing in human capital. This investment in human capital would be expected to impact several parameters characterizing the labor market environment, such as the productivity distribution \( G \) and the rates of match arrivals and dissolutions of employment contracts. Introducing human capital accumulation into the model adds another state variable, the current level of the human capital stock, considerably complicating the analysis. The fact that human capital is not directly observable makes it necessary to rely on ad hoc assumptions regarding the nature of the human capital production function and the dependence of search parameters on the human capital stock and investment activity. For these reasons we have chosen to ignore it in the present analysis.

Our model is set in continuous time, and therefore has no natural “periods” that can be defined which distinguish the “current” from the “future.” Our strategy will be to assume that there does exist a “decision period” of duration \( \varepsilon \) over which new actions are precluded. That is, the individual will take an action given the state \( s \) and will reap the reward from this action over the period \( \varepsilon \). At the conclusion of this decision period, she will take a new action given the state to which the system has then evolved. The decision period \( \varepsilon \) is in the end just an artifice, for we shall define behavior and the value of the problem in the limiting case as \( \varepsilon \to 0 \).

\(^6\)See van den Berg (1990) for a nice theoretical and econometric analysis of a single-spell search model with this type of declining unemployment benefit. Since we are looking at young searchers, many of whom have not worked enough to qualify for unemployment benefits, perhaps ignoring \( UI \) in this application is not too problematic.
Define the value of the unemployed search problem as

\[
V_n = (1 + \rho \varepsilon)^{-1} \left\{ b \varepsilon + \lambda \varepsilon \int \max[V_n, V_e(w)] \, dG(w) \right. \\
+ \left. (1 - \lambda \varepsilon) V_n + o(\varepsilon) \right\}.
\] (6)

As currently written, (6) includes no explicit action, since we assume that the individual has already decided to participate in the labor market. We will discuss how this decision can be made endogenous below.

In terms of the correspondence between [5] and [6], note that the term in [6] that corresponds to the \( \beta \) in [5] is \( \frac{b \varepsilon}{1 + \rho \varepsilon} \equiv \beta \varepsilon \), where \( \rho \) is the discount rate. The interpretation of the term \( \frac{b \varepsilon}{1 + \rho \varepsilon} \) is as follows. Over the short period \( \varepsilon \) the individual receives \( b \) per instant. Thus the total amount received at the end of this period is \( b \varepsilon \). However, since this amount is paid at the end of the period \( \varepsilon \), its beginning of period value must be suitably discounted. Applying the discount factor, the current period return of the action is \( \beta \varepsilon b \varepsilon \).

Now consider the next term. Assume that the searcher obtains exactly one job offer at wage \( w \). Her choice then will be either to accept employment at that wage or to continue searching. The value of accepting a wage offer of \( w \) is given by \( V_e(w) \) and will be discussed shortly. Given the receipt of the offer \( w \), the individual will choose the option associated with the highest value, so that the value of getting an offer of \( w \) is given by \( \max[V_n, V_e(w)] \). This choice is the only explicit behavior in the current setup. The expected value of getting an offer is then the expectation of \( \max[V_n, V_e(w)] \) taken with respect to the distribution of all possible wage offers, which is given by \( G(w) \) in this case. Given the receipt of an offer, the discounted expected value is \( \beta \varepsilon \int \max[V_n, V_e(w)] \, dG(w) \). The approximate probability of getting one offer is \( \lambda \varepsilon \) in the short interval \( \varepsilon \).

If no offer is received the individual will simply continue to search. This is true because in a stationarity environment, if a certain action was optimal at some arbitrary time when the individual faces choices in the set \( C \), then the same decision will be made at any other time when the individual occupies the same state and faces the same choices.\(^7\) Thus the value of not receiving an offer by the end of the period

\(^7\)We have already used this invariance property implicitly when we argued that given it was optimal to search at one point in time it will never be optimal to exit the labor market in the future. Obviously, if employment conditions, such as the wage offer distribution for example, were allowed to vary over time this would not be true in general.
is $\beta_\varepsilon V_n$, and the approximate likelihood of this event is $(1 - \lambda \varepsilon)$. In terms of the DP decomposition given in [5], the current period return is $\beta_\varepsilon b \varepsilon$, and the discounted expected value of future choices given search is

$$
\beta_\varepsilon \lambda \varepsilon \int \max[V_n, V_e(w)] dG(w) + \beta_\varepsilon (1 - \lambda \varepsilon) V_n + \beta_\varepsilon o(\varepsilon),
$$

where we recall that $\beta_\varepsilon o(\varepsilon)$ is the discounted value of all of the possible events that involve the arrival of 2 or more job offers in the period of length $\varepsilon$.

Before we can analyze the problem facing the nonemployed searcher further it is necessary to examine the situation of an employed individual being paid an instantaneous wage of $w$. Since we have precluded on-the-job search, and since it was initially optimal to accept a wage of $w$, an employed individual who has accepted a wage of $w$ will never quit and enter either the state of nonemployed search. Thus, she will simply remain at her job until such time as the employment match is exogenously terminated. Formally,

$$
V_e(w) = (1 + \rho \varepsilon)^{-1} \{w \varepsilon + \eta \varepsilon V_n + (1 - \eta \varepsilon) V_e(w) + o(\varepsilon)\},
$$

where the current period return is now given by $\beta_\varepsilon w \varepsilon$ and the expected future value of continuing to work at the job during this “period” is

$$
\beta_\varepsilon \eta \varepsilon V_n + \beta_\varepsilon (1 - \eta \varepsilon) V_e(w) + \beta_\varepsilon o(\varepsilon).
$$

This term is composed of the discounted value of being dismissed during the period and thus ending the period in the unemployment state multiplied by the probability of being dismissed $[\eta \varepsilon]$ plus the discounted value of ending the period in the same job multiplied by the probability of not being dismissed $[1 - \eta \varepsilon]$ plus the discounted value of the remainder term $o(\varepsilon)$, which reflects the value and probabilities of all other events which could occur in an interval of length $\varepsilon$.

We can determine the value of employment as follows. Multiply both sides of [7]
by $1 + \rho \varepsilon$ to get

\[
V_e(w)(1 + \rho \varepsilon) = w\varepsilon + \eta \varepsilon V_n + (1 - \eta \varepsilon)V_e(w) + o(\varepsilon)
\]

\[
\Rightarrow V_e(w)(\rho + \eta)\varepsilon = w\varepsilon + \eta \varepsilon V_n + o(\varepsilon)
\]

\[
\Rightarrow V_e(w) = \frac{w + \eta V_n}{\rho + \eta} + \frac{o(\varepsilon)}{\varepsilon},
\]

where the last line is obtained after dividing both sides of the second line by $\varepsilon$. Now taking limits, we have

\[
\lim_{\varepsilon \to 0} V_e(w) = \frac{w + \eta V_n}{\rho + \eta} + \lim_{\varepsilon \to 0} \frac{o(\varepsilon)}{\varepsilon}
\]

\[
= \frac{w + \eta V_n}{\rho + \eta}
\]

by the definition of the term $o(\varepsilon)$.

With this definition of $V_e(w)$, we can return to our consideration of $V_n$. First note that

\[
\max[V_n, V_e(w)] = \max[V_n, \frac{w + \eta V_n}{\rho + \eta}]
\]

\[
= \frac{1}{\rho + \eta} \max[V_n(\rho + \eta), w + \eta V_n]
\]

\[
= \frac{\eta V_n}{\rho + \eta} + \max[\rho V_n, w]
\]

\[
= \frac{\eta V_n}{\rho + \eta} + \frac{\rho V_n}{\rho + \eta} + \frac{\max[0, w - \rho V_n]}{\rho + \eta}
\]

\[
= V_n + \frac{\max[0, w - \rho V_n]}{\rho + \eta}. \tag{8}
\]

This is an important result, for it shows that for a given wage offer $w$ the option of accepting the employment match exceeds the value of the option of continuing to search when the wage offer $w$ exceeds the scalar value $\rho V_n$. This is an important enough result to warrant the following terminology.

**Definition 2** The reservation wage $w^*$ is equal to $\rho V_n$ and has the property that any wage offer $w \geq w^*$ will be accepted and any $w < w^*$ will be rejected.

The reservation wage $w^*$ completely summarizes the single decision rule utilized in this simple search model. Since the value of search $V_n$ will depend on all of the
parameters that characterize the labor market environment, so does the reservation wage. Below we shall discuss the manner in which we can solve for \( w^* \).

Using [8] we can rewrite [6] as

\[
V_n = (1 + \rho \varepsilon)^{-1} \left\{ b \varepsilon + \lambda \varepsilon \int [V_n + \frac{\max[0, w - \rho V_n]}{\rho + \eta}] dG(w) \right\} + (1 - \lambda \varepsilon) V_n + o(\varepsilon) \]

\[
= (1 + \rho \varepsilon)^{-1} (b \varepsilon + V_n + \lambda \varepsilon \int \frac{\max[0, w - \rho V_n]}{\rho + \eta} dG(w) + o(\varepsilon)) \}
\]

Then

\[
V_n(1 + \rho \varepsilon) = b \varepsilon + V_n + \frac{\lambda \varepsilon}{\rho + \eta} \int \left[ w - \rho V_n \right] dG(w) + o(\varepsilon)
\]

\[
\Rightarrow \rho V_n = b + \frac{\lambda}{\rho + \eta} \int \left[ w - \rho V_n \right] dG(w),
\]

where the second line is obtained after dividing the first line by \( \varepsilon \) and taking limits. Since \( w^* \equiv \rho V_n \), we can rewrite [9] as

\[
w^* = b + \frac{\lambda}{\rho + \eta} \int \left[ w - w^* \right] dG(w)
\]

Clearly [10] cannot in general be manipulated so as to yield a closed-form solution for \( w^* \). However, it is not difficult to establish that there exists a unique solution \( w^* \) to this equation which is relatively straightforward to compute. If we partially differentiate both sides of [10] with respect to \( w^* \), we see that the derivative of the left hand side (LHS) is simply 1, while the partial derivative of the RHS is

\[
\frac{\partial \text{RHS}(10)}{\partial w^*} = \frac{\lambda}{\rho + \eta} \left[ -(w^* - w^*) g(w^*) - \int_{w^*} dG(w) \right] = -\frac{\lambda}{\rho + \eta} \tilde{G}(w^*) < 0,
\]

where \( \tilde{G}(x) \), termed the survivor function, is defined as \( 1 - G(x) \). The distribution \( G \) has been assumed to be everywhere differentiable on its support, with an associated probability density function given by \( g(w) \). We will assume that the support\(^8 \) of the

---

\(^8\)Assuming that the distribution function \( G \) is everywhere differentiable, the support of the distribution is defined as the subset of the real line \( S \subseteq R \) such that \( g(s) > 0 \) for all \( s \in S \). In our discussion
distribution $G$ is the positive real line. Then the left hand side of (10) is a linear function of increasing function of $w^*$ that takes values on the interval $(-\infty, \infty)$, the right-hand side is a decreasing function of $w^*$ that takes values on the interval $(b, b + \lambda E(w)/(\rho + \eta))$. This last result follows from the fact that when the reservation wage is less than or equal to 0, the right hand side is simply $b + \lambda E(w)/(\rho + \eta)$, since all offers are excepted, while as $w^* \to \infty$, no offers are accepted and the value of the right hand side of (10) goes to $b$. Then there exists exactly one value of $w^*$ that solves (10), though that value of $w^*$ may be negative without further restrictions on the parameters.

We now illustrate the manner in which the reservation wage can be computed. We consider an example labor market characterized by the parameter values $b = -1$, $\lambda = .2$, $\rho = .005$, $\eta = .02$, and we assume that the wage offer distribution facing the individual is lognormal with parameters $\mu$ and $\sigma$ ($> 0$).\footnote{The log normal density is given by
\[
g(w; \mu, \sigma) = \frac{1}{w\sqrt{2\pi}\sigma} \exp\left\{-\frac{1}{2} \left(\frac{\ln(w) - \mu}{\sigma}\right)^2\right\}.
\]} For our example we have set $\mu = 1$ and $\sigma = 1$. Since the expected value of a log normally distributed random variable is given by $E(w; \mu, \sigma) = \exp(\mu + \frac{1}{2}\sigma^2)$, in this case we have $E(w) = 4.482$.

Figure 2.2 plots the LHS and RHS of [10] as a function of $w^*$. As we know, $LHS(w^*) = w^*$, while $RHS(w^*)$ is a monotone decreasing function. In the case of our example, the two lines intersect at the point $w^* = 7.439$, which is the value which completely characterizes all rational labor market behavior in this simple model. Note that $w^*$ is appreciably greater than the mean wage offer in this model. In particular, we might ask what is the probability that a wage offer will be accepted? This probability is given by the probability that a wage draw from $G$ exceeds the reservation wage, or $\tilde{G}(w^*; \mu, \sigma) = \tilde{G}(7.439; 1, 1) = 0.157$. Thus most offers in this case are rejected. The reason for the “choosiness” of the searcher we see in this example is due partially to the relatively low rate of discounting [with $\rho$ “small” the value of waiting for a good offer increases], the relatively low rate of exogenous terminations [when $\eta$ is small it is more worthwhile to wait for a better offer since it will be kept longer, on average], and the relatively high rate of offer arrivals, $\lambda$.

We now turn to a short consideration of how comparative statics exercises can be
conduted in this type of model.

Let us rewrite [10] in slightly different terms as

\[ w^* = Q(w^*; \omega), \]
\[ \Rightarrow 0 = w^* - Q(w^*; \omega) \]  
(11)

where \( Q \) is the RHS of [10] and \( \omega \) is a vector containing all of the parameters that characterize the labor market in this model. In general \( Q \) is a differentiable function of all of the elements of \( \omega \), which means that we can totally differentiate [11] as follows

\[ 0 = \left( 1 - \frac{\partial Q(w^*; \omega)}{\partial w^*} \right) dw^* - \frac{\partial Q(w^*; \omega)}{\partial \omega_i} d\omega_i \]
\[ \Rightarrow \frac{dw^*}{d\omega_i} = \frac{\frac{\partial Q(w^*; \omega)}{\partial \omega_i}}{1 - \frac{\partial Q(w^*; \omega)}{\partial w^*}}, \]  
(12)

where \( \omega_i \) denotes the \( i^{th} \) element of the parameter vector \( \omega \). Because \( \frac{\partial Q(x; \omega)}{\partial b} = 1 \) is negative, the denominator of [12] is always positive, so that

\[ \text{sgn} \left( \frac{dw^*}{d\omega_i} \right) = \text{sgn} \left( \frac{\partial Q(w^*; \omega)}{\partial \omega_i} \right), \]

where \( \text{sgn}(X) \) denotes the sign of the expression \( X \).

For example, consider the flow cost of job search, \( b \). Since \( \frac{\partial Q(x; \omega)}{\partial b} = 1 \), an increase in \( b \) results in an increase in the reservation wage, a result which is intuitive. Similarly, since

\[ \frac{\partial Q(x; \omega)}{\partial \lambda} = \frac{1}{\rho + \eta} \int_{w^*}^w [w - w^*] dG(w) > 0, \]

an increase in \( \lambda \) increases the reservation wage. From inspection of [10] it is obvious that \( \frac{\partial w^*}{\partial \rho} \) and \( \frac{\partial w^*}{\partial \eta} \) are both negative. It is not difficult to demonstrate that, under the lognormality assumption regarding \( G \), \( \frac{\partial w^*}{\partial \mu} \) and \( \frac{\partial w^*}{\partial \sigma} \) are both positive.

The search model we have described is simply a dynamic model of individual choice in a stationary environment. In this case the choices are limited to whether or not to accept an offered wage when one arrives. Such a model is inadequate for studying
minimum wage effects on labor market outcomes. Imagine that a minimum wage is imposed by the government, and that the imposition of this minimum wage, \( m \), has no effect on any other parameters of the model. Let \( w^*(\omega) \) be the original reservation wage optimally chosen in labor market environment \( \omega \). If the minimum wage is set at a value no greater than the reservation wage, i.e., \( m \leq w^*(\omega) \), then there is no effect on choices or outcomes. If \( m > w^*(\omega) \), then clearly the individual is worse off than before. The reason is that certain wage offers which were previously acceptable, those \( w \) in the interval \([w^*(\omega), m]\), are now precluded. Since the individual was free to choose a reservation wage equal to \( m \) previously but chose not to, she cannot be better off under this law. Thus in a “partial-partial” equilibrium search model the imposition of minimum wages cannot be beneficial for labor market participants.

For minimum wage laws to possibly have beneficial effects for searchers, the imposition of a minimum wage must change the search environment in some positive way for the individual. For minimum wages to alter the labor market environment requires, at a minimum, a partial equilibrium model of the interaction between the supply and demand sides of the market. The bargaining framework developed we describe below provides an acceptable context from this point of view.

1.4 Nash-Bargained Employment Contracts

As opposed to the model previously considered, we assume that jobs are not fundamentally differentiated by the wage they offer to a particular worker, but rather are distinguished by the productivity of the match between a particular worker and a particular firm. The flow revenue from such a match to the firm is given by \( \theta \) (normalize the product price to unity without loss of generality). The instantaneous profit to the firm is given by \( \theta - w \), where \( w \) is the instantaneous wage payment to the employee. By defining match-specific profits in this manner it is clear that we have assumed that the only factor of production is labor and that there are no fixed costs of employment. These assumptions are important in the derivation of the bargaining equilibrium we provide. We will consider a slightly more general model of firm behavior below.

When a potential worker meets a potential employer, the (flow) value of the match is assumed to be immediately observed. If the two parties enter into an employment contract, this contract will only specify a time-invariant instantaneous wage rate \( w \).
It is important to realize that the productivity value $\theta$ is *specific to the match* and not attributable to either the worker or the firm. In that sense, both have have a valid claim to it. How is it to be divided?

There are a variety of forms of "bargaining power" that we might consider in defining the surplus division problem. First is the notion that each individual should receive at least in compensation what he or she could earn from pursuing the next best option available to them. Under the assumptions which we have made about the search technology and the labor market environment, a searcher who does not receive an acceptable job offer will optimally continue to search. Thus the value of the next best option available to a potential employee bargaining over her share of $\theta$ is $V_n$. For the moment assume that the next best option available to a potential employer has a value of $V_e$, which denotes the value of holding onto a vacancy.

Bargaining results in a division of the flow value of the output produced. Given her wage payment $w$, the individual is indifferent concerning the actual value of the match. In other words, the value of an employment pair $\{w, \theta\}$ to the individual is given by $V_e(w, \theta) = V_e(w)$. Thus the value of the surplus of the employment contract which pays $w$ to the individual is $V_e(w) - V_n$. This difference is the rent which accrues to the worker from the employment contract.

The value of the employment match to the firm requires a little more discussion. In Pissarides’s general equilibrium model of search and bargaining in a stationary environment, which we will examine in more detail below, in order to search for an employee a firm must create a job vacancy which is costly to hold while the firm searches for an acceptable match. Since there is a large population of potential firm owners who could create a vacancy, a free entry condition applies in which the expected value of creating a vacancy is driven to zero. We will assume that such a condition holds so that $V_v = 0$. This is then the firm’s outside option in contract negotiations with the worker.

Let the firm’s value of an employment contract in which an employee has an instantaneous output level of $\theta$ and a instantaneous wage rate of $w$ be given by $Q(\theta, w)$. Then the solution to the generalized Nash bargaining problem is given by

$$w^*(\theta; \omega, \alpha) = \arg \max_w (V_e(w) - V_n)^\alpha (V_f(\theta, w) - 0)^{1-\alpha},$$

(13)

where $\alpha \in [0, 1]$ is termed the bargaining power parameter [in this case, it measures
the bargaining power of the individual while \( 1 - \alpha \) is the bargaining power of the firm, and where \( \omega \) contains all of the parameters describing the labor market environment with the exception of \( \alpha \). Note that \( V_v(w) - V_n \), measures the gain from participating in the employment contract paying a wage of \( w \) with respect to the next best alternative, which is to continue searching. As we argued in the last paragraph, the next best alternative to the firm with respect to engaging in this particular contract is equal to 0. Therefore the surplus value to the firm associated with a particular employment contract is equal to \( V_f(\theta, w) \). Note that \( V_f(\theta, w) > 0 \) in general, since if firms are induced to create costly vacancies they must be rewarded with some positive profits when the vacancy is filled for \( V_v \) to be equal to zero.

To determine \( V_f(\theta, w) \), we will assume that firms have the same discount rate as individuals, \( \rho \). If an employment contract lasts for duration \( t \), the ex post value of the contract is

\[
\int_0^t (\theta - w) \exp(-\rho u) du = \frac{(\theta - w)}{\rho} (1 - \exp(-\rho t))
\]

Since the probability density function of completed employment contracts is given by \( \eta \exp(-\eta t) \), the expected value of an employment contract \( \{\theta, w\} \) is given by

\[
V_f(\theta, w) = \frac{E_t(\theta - w)}{\rho} (1 - \exp(-\rho t))
\]

\[
= \frac{(\theta - w)}{\rho} \int (1 - \exp(-\rho t)) \eta \exp(-\eta t) dt
\]

\[
= \frac{(\theta - w)}{\rho} [1 - \eta \int \exp(- (\rho + \eta) t) dt]
\]

\[
= \frac{(\theta - w)}{\rho} [1 - \frac{\eta}{\rho + \eta}]
\]

\[
= \frac{\theta - w}{\rho + \eta}.
\]

We will use this expression for \( V_f(\theta, w) \) in explicitly solving the bargaining problem.

In concluding this section, it is appropriate to say a few words about \( \alpha \), a parameter which will figure prominently in the theoretical, econometric, and empirical work which follows. As we stated previously, there are essentially two aspects of bargaining advantage. One is the value of the next best option available to each of the two bargainers. The "threat point" of the firm, \( V_v \), has been fixed at 0, while the one of the individual has been set at \( V_n \) and is an endogenously determined value. Clearly,
the larger is the value of $V_n$ the higher the wage payment required for the individual to enter into any given employment contract.

The second aspect of bargaining advantage is the parameter $\alpha$. When $\alpha$ is equal to 1, the individual is assumed to have all of the bargaining “power” and extracts all of the surplus from the match. The case $\alpha = 1$ corresponds exactly to the simple search problem that we considered in the previous section, for in this case the matching distribution $G$ and the wage offer distribution are identical. At the other extreme, when $\alpha = 0$, firms possess all of the bargaining power. In this case, the wage payment is independent of the match value $\theta$, so that all employees are paid the same wage. If all employees are paid the same wage, then there is no motivation for further search, and all offers will be accepted. If the value of nonparticipation is fixed at 0, for example, and assuming that the instantaneous return associated with the search state $b < 0$, then it is not difficult to show that the common wage paid all workers will be $\hat{w} = -\frac{(q+\rho)}{\lambda G(\hat{w})}b$. In this case, firms will earn instantaneous profit of $\theta - \hat{w}$ on all matches $\theta \geq \hat{w}$.

In general, the bargaining power parameter $\alpha$ is not equal to 0 or 1. It then indicates the relative “strength” of the two parties in bargaining, conditional on their threat points. This parameter is admittedly difficult to interpret. We think of it as constituting a type of summary statistic of the labor market “position” of a particular group. For example, the match value distribution for low-skilled workers may be stochastically dominated by the match value distribution for high-skilled workers, but in low-skilled workers may be at a further disadvantage due to their having little bargaining power. Their low bargaining power may derive from there being many substitutes for them in the production process, for example. In this sense, the parameter cannot be really thought of as “primitive” since significant policy changes - such as a doubling of the minimum wage - may result in participation effects or substitution responses by firms which change the labor market “position” of the group, and hence change the bargaining power parameter. Thus comparative statics exercises and policy experiments performed with the estimates obtained from this model will only be valid locally, that is, for small changes in policy variables. This limitation is not too disturbing, since it probably applies to all empirical research.

\[10\] The same argument could be made for many of the other “primitive” parameters it must be admitted. When discussing estimates of the equilibrium model we shall provide some evidence that the rate of contacts between searchers and firms, $\lambda$, is not invariant with respect to changes in the minimum wage.
that has been conducted in this area.

1.5 The Search-Bargaining Model without Minimum Wages

We are now ready to combine the search and bargaining aspects of the model. Since \( V_e(w) = \frac{w + \eta V_n}{\rho + \eta} \), the individual’s surplus with respect to the alternative of continued search is

\[
V_e(w) - V_n = \frac{w + \eta V_n}{\rho + \eta} - V_n
= \frac{w - \rho V_n}{\rho + \eta},
\]

so that the solution to the bargaining problem is given by

\[
w(\theta, V_n) = \arg \max_w \left( \frac{w - \rho V_n}{\rho + \eta} \right)^\alpha \left( \frac{\theta - w}{\rho + \eta} \right)^{1-\alpha} = \alpha \theta + (1 - \alpha) \rho V_n.
\]

so that the wage is a weighted average of the match value and the reservation match value, \( \theta^* \).

We can now compute the value of nonemployment. Instead of writing the value of employment as a function of the wage, we write it as a function of the “primitive parameters,” \( \theta \) and \( V_n \). Then rewriting \([9]\), we have

\[
\rho V_n = b + \lambda \int_{\rho V_n} [V_e(w(\theta, V_n)) - V_n] dG(\theta).
\]

Since

\[
V_e(w(\theta, V_n)) = \frac{\alpha \theta + (1 - \alpha) \rho V_n + \eta V_n}{\rho + \eta}
= \frac{\alpha \theta - \alpha \rho V_n}{\rho + \eta} + V_n,
\]

we have

\[
V_e(w(\theta, V_n)) - V_n = \frac{\alpha \theta - \alpha \rho V_n}{\rho + \eta}.
\]
Then the final (implicit) expression for the value of search is

$$\rho V_n = b + \frac{\lambda \alpha}{\rho + \eta} \int_{\rho V_n}^{\theta} [\theta - \rho V_n] dG(\theta). \tag{16}$$

We see that this expression is identical to the expression for the reservation value in a model with no bargaining when \( \theta \) is the payment to the individual, except for the presence of the factor \( \alpha \). This is not unexpected, since when \( \alpha = 1 \) the entire match value is transferred to the worker, and thus search over \( \theta \) is the same as search over \( w \).

Now we can summarize the important properties of the model. The critical "match" value \( \theta^* \) is equal to \( \rho V_n \), which is defined by (16). Since at this match value the wage payment is equal to \( w^* \equiv w(\theta^*, V_n) = \alpha \theta^* + (1 - \alpha)\theta^* \), we have that \( w^* = \theta^* \). Then the probability that a random encounter generates an acceptable match is given by \( \tilde{G}(\theta^*) \). The rate of leaving unemployment is \( \lambda \tilde{G}(\theta^*) \). As we can see from [16], since \( \theta^* \) is an increasing function of \( \alpha \), the likelihood of exiting a spell of unemployment is lower the larger is the worker’s bargaining power. This is the first indication we have that lower rates of leaving unemployment do not necessarily imply that the worker’s welfare level is low.

The observed wage density is a simple mapping from the matching density. Since

$$w(\theta, V_n) = \alpha \theta + (1 - \alpha)\theta^*$$

$$\Rightarrow \hat{\theta}(w, V_n) = \frac{w - (1 - \alpha)\theta^*}{\alpha},$$

then the probability density function of observed wages, \( h(w) \), is given by

$$h(w) = \begin{cases} \frac{\alpha^{-1} g(\hat{\theta}(w, V_n))}{\tilde{G}(\theta^*)} & w \geq \theta^* \\ 0 & w < \theta^* \end{cases} \tag{17}$$

An Example

In order to fix ideas, we present a detailed example of the computation of decision rules and the characterization of equilibrium for one particular labor market environment. We have chosen to work with very simple functional forms so as to make the computational steps as clear as possible. In the empirical work reported on latter,
functional forms which produce results more in line with the data will be employed.

We will assume that the matching distribution $G(\theta)$ is uniform, with the support of the distribution given by the interval $[0, 10]$. Thus

$$G(\theta) = \begin{cases} 
0 & \iff \theta < 0 \\
\theta/10 & \iff 0 \leq \theta \leq 10 \\
1 & \iff 10 < \theta 
\end{cases}$$

and

$$g(\theta) = \begin{cases} 
0 & \iff \theta < 0 \text{ or } 10 < \theta \\
1/10 & \iff 0 \leq \theta \leq 10 
\end{cases}.$$  

We have also assumed that $\lambda = .5$, $\eta = .02$, $\rho = .01$, and $b = -1$. These values imply that on average offers arrive to unemployed searchers every two time periods (recall that in continuous time a “period” is just a normalization that we use to express frequencies of events, and that events can occur at any time), jobs last for 50 periods on average, and searchers are “strongly” forward looking [i.e., the value of $\rho$ is close to 0]. We will characterize the equilibrium of the model for two values of $\alpha$, $\alpha = .3$ (low bargaining power of searchers) and $\alpha = .6$ (high bargaining power).

Under our distributional assumption regarding $G$, [16] becomes

$$\theta^* = b + \frac{\alpha \lambda}{\rho + \eta} \int_{\theta^*}^{10} (\theta - \theta^*) \frac{1}{10} d\theta. \quad (18)$$

$$\Rightarrow 0 = \frac{a}{20} (\theta^*)^2 + (1 - a)\theta^* + (b + 5a), \quad (19)$$

where

$$a = \frac{\alpha \lambda}{\rho + \eta}.$$  

This is a quadratic equation in $\theta^*$, and for any $\alpha \in (0, 1)$ there exists a unique solution $\theta^* \in (0, 10)$.

The labor market equilibrium in terms of equilibrium wage functions is represented in Figure 2.3. In 2.3.a we have graphed the population density of match draws, which is equal to .1 on the interval $[0, 10]$. Figure 2.3.b plots the wage functions corresponding to the two values of $\alpha$ we are considering. We note that the reservation match value $\theta^*$ is considerably higher when $\alpha = .6$. The reason is intuitive. Since the searcher gets to reap more of the surplus, it is worth "holding out" for a high draw. Since the exit rate from unemployment is $\lambda \tilde{G}(\theta^*)$, this implies that unemployment spells last longer
on average when $\alpha$ is high.

The wage density corresponding to the two values of $\alpha$ are graphed in Figures 2.3.c and 2.3.d. Because the parent distribution (of $\theta$) is Uniform and because the wage function is a linear mapping, the wage distributions are both Uniform as well. The support of the wage distribution associated with $\alpha = .3$ begins to the left of the other distribution and is more concentrated. Thus low $\alpha$ not only reduces the average accepted wage in this example but also reduces the variance in accepted wages.

\section*{1.6 Bargaining with a Minimum Wage Constraint}

The introduction of minimum wages into the search-bargaining framework is accomplished in a very straightforward manner. We assume that the labor market environment is exactly as described above, with the exception that a “side constraint” is introduced into the worker-firm bargaining problem \cite{13}. This constraint is that any employment contract must yield a wage payment of at least $m$ to the worker no matter what the value of $\theta$. The minimum wage is assumed to be set by the government and applies to all potential matches. This assumption represents the U.S. case relatively well, since the minimum wage applies to virtually all employment contracts in the labor force.\footnote{There are some provisions of federal and state minimum wage statutes which allow lower wage payments than $m$ to be paid to certain classes of workers. However, the class of workers to which these provisions apply is very small, and there is little empirical evidence that employers take advantage of them to any appreciable extent \cite[see, e.g., Card ...]} More controversially, we assume that the only compensation provided by the firm is the wage. Thus there are no other forms of compensation the firm can adjust so as to “undo” the minimum wage payment requirement.\footnote{A number of researchers have investigated the manner in which minimum wage laws could be “defeated” when compensation includes more than only wage payments. For example, Lazear \cite{...} investigates how the requirement of paying a high minimum wage may result in the firm providing less on-the-job training to young employees. Others \cite{...} have looked at the manner in which nonpecuniary compensation is reduced in response to minimum wage increases.}

The modified bargaining problem we develop is then represented by

$$w^*(\theta, m) = \arg \max_{w \geq m} \left( V_e(w) - V_n(m) \right)^\alpha \left( \frac{\theta - w}{\rho + \eta} \right)^{1-\alpha},$$

where $V_n(m)$ is the value of search given a minimum wage of $m$. If we define $\rho V_n(0) = \theta^*_0$, which is the reservation match value when there is no minimum wage, it is clear that any $m \leq \theta^*_0$ has no effect on the behavior of applicants or firms and thus would...
be a meaningless, or “slack,” constraint. Therefore we consider only the effects of an imposition of \( m > \theta^* \).

The first thing to note concerning the effect of the constraint on behavior is that, since the value of the employment contract to the firm is proportional to \( \theta - w \), no employment contract will be formed for which the match value \( \theta < m \), since in this case the firm would lose money. If there is no opportunity cost to the firm of forming a match, then any match for which it doesn’t lose money will be acceptable to it. Since \( \theta^* < m \), this implies that fewer encounters between searchers and firms will result in employment contracts, which will result in an increase in the unemployment rate. This loss of employment effect is consistent with that predicted by the simplest static competitive labor market models. We shall see latter the extent to which this prediction is robust with respect to alterations in modeling assumptions.

The effect of the addition of the minimum wage “side constraint” on the solution to the bargaining problem is relatively intuitive. Under the “constrained” Nash-bargaining problem, in which all employment contracts must pay at least a wage of \( m \), there will exist a value of search which we denote \( V_n(m) \). If we ignore the minimum wage constraint and solve [20] using \( V_n(m) \), we will get the wage offer function

\[
\tilde{w}(\theta, V_n(m)) = \alpha \theta + (1 - \alpha) \rho V_n(m).
\]  

(21)

Under this division of the match value, the worker would receive a wage of \( m \) when \( \theta = \hat{\theta} \), where

\[
\hat{\theta}(m, V_n(m)) = \frac{m - (1 - \alpha) \rho V_n(m)}{\alpha}.
\]

Then if \( \hat{\theta} \leq m \), all “feasible” matches would generate wage offers at least as large as \( m \). When \( \hat{\theta} > m \), this is not the case. When \( \theta \) belongs to the set \([m, \hat{\theta}]\), the offer according to [21] is less than \( m \). However, when confronted with the choice of giving some of its surplus to the worker versus a return of 0, the firm pays the wage of \( m \) for all \( \theta \in [m, \hat{\theta}] \). Wages for acceptable \( \theta \) outside of this set [i.e., when \( \theta \geq \hat{\theta} \)] are determined according to [21].

We can now consider the individual’s search problem given this wage offer function. Using the \( \varepsilon \) interval formulation, the value of search under a binding minimum wage
constraint is given by

\[ V_n(m) = (1 + \rho \varepsilon)^{-1} [b \varepsilon + \lambda \varepsilon \left\{ \int_{\hat{\theta}(m,V_n(m))}^{\hat{\theta}(m,V_n(m))} \frac{m + \eta V_n(m)}{\rho + \eta} \right\} dG(\theta) \\
+ \int_{\hat{\theta}(m,V_n(m))}^{\hat{\theta}(m,V_n(m))} \frac{\alpha \theta + (1 - \alpha) \rho V_n(m) + \eta V_n(m)}{\rho + \eta} dG(\theta) + G(m)V_n(m)] \\
+ (1 - \lambda \varepsilon)V_n(m) + o(\varepsilon) \]

\[ \Rightarrow (1 + \rho \varepsilon)V_n(m) = b \varepsilon + \lambda \varepsilon \left\{ \int_{\hat{\theta}(m,V_n(m))}^{\hat{\theta}(m,V_n(m))} \frac{m + \eta V_n(m)}{\rho + \eta} \right\} dG(\theta) \\
+ \int_{\hat{\theta}(m,V_n(m))}^{\hat{\theta}(m,V_n(m))} \frac{\alpha \theta + (1 - \alpha) \rho V_n(m) + \eta V_n(m)}{\rho + \eta} dG(\theta) + V_n(m)G(m)] \\
+ (1 - \lambda \varepsilon)V_n(m) + o(\varepsilon) \]

The integrand in the first integral on the right hand side of these expressions is the value of a job at a firm where the match productivity lies in the interval \([m, \hat{\theta})\), which we know pays a wage of \(m\) and has a total value of \((m + \eta V_n(m))/(\rho + \eta)\). The integrand of the second integral is the value of a job when the match value is greater than \(\hat{\theta}\), and \(V_n(m)\) is the value of encountering a firm where the match value is less than \(m\), which we know results in no job and continued search. The probability of this event is \(G(m)\).

After subtracting \(V_n(m)\) from both sides, we get

\[ \rho \varepsilon V_n(m) = b \varepsilon + \lambda \varepsilon \left\{ \int_{\hat{\theta}(m,V_n(m))}^{\hat{\theta}(m,V_n(m))} \frac{m + \eta V_n(m)}{\rho + \eta} - V_n(m) \right\} dG(\theta) + G(m)(V_n(m) - V_n(m)) + o(\varepsilon) \]

\[ \Rightarrow \rho \varepsilon V_n(m) = b \varepsilon + \lambda \varepsilon \left\{ \int_{\hat{\theta}(m,V_n(m))}^{\hat{\theta}(m,V_n(m))} \frac{m - \rho V_n(m)}{\rho + \eta} \right\} dG(\theta) \]

\[ + \int_{\hat{\theta}(m,V_n(m))}^{\hat{\theta}(m,V_n(m))} \frac{\alpha \theta + (1 - \alpha) \rho V_n(m) - \rho V_n(m)}{\rho + \eta} dG(\theta) + o(\varepsilon) \]
After dividing both sides by $\varepsilon$ and taking limits as $\varepsilon \to 0$, we arrive at

$$
\rho V_n(m) = b + \frac{\lambda}{\rho + \eta} \left\{ [m - \rho V_n(m)] [G(\hat{\theta}(m, V_n(m))) - G(m)] + \alpha \int_{\theta(m, V_n(m))} [\theta - \rho V_n(m)] dG(\theta) \right\}.
$$

It is important to note a fundamental difference between the value $\rho V_n(m)$, which we might want to refer to as the “implicit” reservation wage in the presence of a binding minimum wage constraint, and $\rho V_n(0)$, the corresponding “explicit” reservation wage [and match] value when no minimum wage constraint is binding. The value of $\rho V_n(0)$ is an acceptance value, that is, it completely characterizes the decision of whether or not an employment contract is struck. When a binding minimum wage is present, employment contacts are formed whenever $\theta \geq m$. The value of $\rho V_n(m)$ is only instrumental in determining the equilibrium wage contract and wage distribution. Put another way, when there is no binding minimum wage constraint, the smallest possible observed wage will be equal to $\rho V_n(0)$, and the distribution of wages will be continuous as long as $G$ itself is. When there is a binding minimum wage, the smallest observed wage will be given by $m$, and we know that $\rho V_n(m) < m$. The observed wage distribution will consist of a mass point at the minimum wage, the size of which is given by $G(\hat{\theta}(m, V_n(m)) - G(m))$, while the distribution of wages immediately above $m$ will be continuous [once again, as long as $G$ is]. In the presence of a binding minimum wage, the observed wage distribution is given by

$$
p(w) = \begin{cases} 
\frac{\alpha^{-1}g(\hat{\theta}(w, V_n(m)))}{G(m)} & w > m \\
\frac{G(\hat{\theta}(m, V_n(m)) - G(m))}{G(m)} & w = m \\
0 & w < m
\end{cases}
$$

An Example (Continued)

We consider the same search environment as above and determine the impact of the imposition of a minimum wage of 7 on the equilibrium wage distribution. Recall that in the previous example we computed the wage distribution under a low and high value of $\alpha$, .3 and .6. We noted that without a minimum wage the acceptance match value was an increasing function of $\alpha$. In our previous example the critical match value when $\alpha = .6$ was equal to 6.204, so that a minimum wage of 7 is binding.
for both values of $\alpha$.

As noted above, when the underlying population match distribution is continuous, the imposition of a binding minimum wage results in a mixed discrete-continuous equilibrium wage distribution in which there exists a mass point at the value $m$ with a continuous distribution of wages beginning immediately “to the right” of $m$. To represent a random variable $w$ that does not have a density everywhere on its support we must plot the cumulative distribution function (c.d.f.) instead of the density function (as was done in Figure 2.3). Figure 2.4.a exhibits the c.d.f. of the match distribution. The equilibrium wage function with the binding minimum wage of 7 is shown in Figure 2.4.b. In this case, since the minimum wage is binding for both values of $\alpha$, the lower bound of the support of both distributions is the same. However, we see that for $\alpha = .3$ all positive wage offers are equal to the minimum. This is due to the bargaining power of searchers being low and the minimum wage being high relative to the upper bound on the distribution of $\theta$. Instead, when $\alpha = .6$, there is significant clustering of wages at $m$ though most wages observed will be greater than $m$. The critical value we have defined as $\hat{\theta}_\alpha(m)$, which is the highest value of $\theta$ that will yield a wage payment of $m$, is equal to 7.557 for $\alpha = .6$ (as opposed to 10 when $\alpha = .3$).

Figures 2.4.c and 2.4.d display the wage distributions for the two values of $\alpha$. The wage is a degenerate random variable in the case of $\alpha = .3$, that is, it assumes the constant value $m = 7$ for all employees. In the case of $\alpha = .6$, while there is a considerable mass point at $w = 7$, about 82 percent of all wage observations are greater than 7. The conditional distribution of wages greater than 7 is Uniform.

This completes our discussion of the case of search and bargaining with a minimum wage constraint when given contact rate $\lambda$. The strengths of the model are its simplicity and the fact that it delivers some empirical implications broadly in accord with stylized facts. Clearly, it is highly stylized and partial equilibrium in nature. We now turn to two important extensions of this framework. In the first, we maintain the assumption of no on-the-job search, but allow contact rates to be determined in equilibrium following the matching function framework developed by Pissarides (2000). In this way we introduce some general equilibrium features into the model, albeit in a very simple way.

The second extension introduces on-the-job search into the model. This makes the model a considerably more realistic description of the labor market in which young
participants find themselves. While characterization of the decision rules used by agents is more complex than is the case without OTJ search, it is not difficult to describe the qualitative features of the model and the welfare effects of the minimum wage in this richer framework.

1.7 Endogeneity of the Rate of Contacts

As in Pissarides (2000), we adopt a highly stylized model of the decision to participate in the labor market. While we continue to assume that agents are identical when entering the market, in the ex ante sense, we assume that individuals in the population are heterogeneous in the value of remaining outside of the market. In this case, outside options include schooling, leisure, etc., and since a large proportion of the 16-24 year old population that is the focus of our analysis does not participate in the market, this is an important consideration. Let the (normalized) value of not participating in the labor market for an individual be given by \( \rho V_o \), and assume that in the population this random variable has a distribution \( Q(\rho V_o) \). If an individual enters the labor market they enter as an unemployed searcher, the normalized value of which is \( \rho V_n(m) \) given the prevailing minimum wage \( m \). Thus all agents in the population with a value of \( V_o \) less than or equal to \( V_n(m) \) will enter the market under \( m \). In the steady state the participation rate is given by \( Q(\rho V_n(m)) \).

We adopt the standard set up (in the macroeconomics literature) for modeling firms’ decisions to create vacancies. At any moment in time, any firm can create a vacancy, which is a precondition for adding a worker to its staff. Think of it as setting up the workplace in advance. One rationale for why this activity has to be accomplished in advance is that a potential employee must be evaluated in this setting to determine her match value \( \theta \) at that particular job. As is standard in this literature, we assume that there exists a constant returns to scale matching technology,

\[
M(\tilde{u}, v) = vq(k),
\]

\[13\] This is only strictly true in the steady state. For example, if the minimum wage is changed and as a result the value of unemployed search declines, those individuals currently working at a match at which the value of the employment contract exceeds the value of the outside option will remain in the market even if the value of search is now less than the value of their outside option. Once they lose their job however, which occurs with probability one in finite time, they will no longer continue to search and will instead leave the labor market.
where \( k \equiv \tilde{u}/v \), \( \tilde{u} \) is the size of the set of unemployed searchers, and \( v \) is the size of the set of vacancies. The contact rates differ depending on which side of the market the agent is on. On the supply side of the market, the contact rate is given by

\[
\lambda = \frac{M(\tilde{u}, v)}{\tilde{u}} = \frac{vq(k)}{\tilde{u}} = \frac{q(k)}{k}.
\]

From the vacancy holders point of view, the contact rate is

\[
\frac{M(\tilde{u}, v)}{v} = \frac{vq(k)}{v} = q(k).
\]

Assuming that there exists a population of potential (firm) entrants with an outside option value of 0, firms create vacancies until the point that expected profits are zero. Let the cost of creating a vacancy be given by \( \psi > 0 \). Then the expected value of creating a vacancy is given by

\[
\rho V_v = -\psi + q(k)\tilde{G}(r)(J - V_v),
\]

where \( r \) denotes the acceptance match value (which is equal to the maximum of \( \rho V_n(m) \) and \( m \)), \( q(k)\tilde{G}(r) \) is the rate at which a firm fills a vacancy (\( q(k) \) is the rate at which it meets job applicants and \( \tilde{G}(r) \) is the probability that the match value drawn is greater than the lowest acceptable value, \( r \)), and \( J \) is the expected value of a filled vacancy (where the expectation is taken with respect to the distribution of acceptable matches \( \theta \geq r \)). By setting \( r \) to the maximum of \( \rho V_n(m) \) and \( m \), we allow for the possibility that the minimum wage \( m \) is not binding. To close the model, we assume that firms keep creating vacancies up to the point at which the expected value of a vacancy is 0 - this is referred to as the free entry condition (FEC). Imposing \( V_v = 0 \), we have

\[
0 = -\psi + q(k)\tilde{G}(r)J. \tag{29}
\]

We can solve for the equilibrium number of vacancies given the expected value of a filled vacancy and the size of the set of unemployed searchers using (29) and other
pieces of the model. As we shall see in the following chapter, in the steady state the probability that a labor market participant is unemployed is given by

$$u = \frac{\eta}{\eta + G(r)q(k)/k}.$$  

If we denote the size of the set of labor market participants by $l$, then the size of the set of unemployed searchers (relative to the entire population) is

$$\tilde{u} = lu = \frac{\eta l}{\eta + G(r)q(\tilde{u}/v)/(\tilde{u}/v)}.$$  

With endogenous contact rates, a labor market equilibrium in the presence of a minimum wage (that may or may not be binding) is characterized by the quadruplet $(l, u, v, \rho V_n(m))$, which is a solely a function of the parameters $(\rho, b, \eta, \alpha, G, Q, q, \psi)$. An equilibrium, if one exists, can be constructed by first fixing a value of $\lambda$. Let $x \equiv \rho V_n(m)$. Then given $\lambda$, (??) determines $x(\lambda)$. The participation rate is then determined as $l(\lambda) = Q(x(\lambda))$. From (30) we have $\tilde{u}(\lambda) = l(\lambda)\eta/(\eta + \lambda G(\max\{x(\lambda), m\}))$. Finally, $\tilde{u}(\lambda)$ and $J(\lambda)$ are used with (29) to determine $v(\lambda)$. We define $T(\lambda) = q(l(\lambda)\omega(\lambda)); \omega)/(l(\lambda)\omega(\lambda))$. There exists a unique equilibrium if and only if there exists a unique value $\lambda^*$ such that $\lambda^* = T(\lambda^*)$. In general, without further restrictions on the parameter space and the functional forms of $q$, $G$, and $Q$, there may exist no or multiple equilibria. For example, if we restrict the outside option value, $\rho V_o$, to be positive for all individuals, and if we fix all other parameters aside from $b$ at some given values for which the model is well-defined\(^\text{14}\), then there will sufficiently negative values of $b$ for which no one participates in the market.

Given existence, multiple equilibria can arise depending on specific properties of the distribution functions $Q$ and $G$. In performing the empirical exercises and policy simulations reported below, we assumed particular functional forms for $q$, $Q$, and $G$. When performing the policy simulations, in which $m$ is varied over some range of values, we have found a number of cases of nonexistence. However, when an equilibrium existed we found it to be unique in the sense that $\lim_{n \to \infty} T^n(\lambda^0) = \lim_{n \to \infty} T^n(\lambda^0) = \lambda^* \in [\lambda^0, \lambda^0]$, where $\lambda^0$ and $\lambda^0$ are small and large starting values

\(^{14}\)By well-defined, I mean that the distribution of match values has finite expectation, the discount rate is strictly greater than 0, etc.
of $\lambda$ in the iterative successive approximation process.

To illustrate some of the properties of a labor market equilibrium with and without a binding minimum wage, we present the following example.

**Example 3** Assume that $\rho V_o$ is normally distributed with mean 5 and variance 4 in the population, and that $\ln \theta$ is normally distributed with mean 2 and variance .25. The discount rate, in “monthly” units, is .05/12, the exogenous dismissal rate $\eta$ is .038, the utility flow in unemployment $b$ is -20, and the flow cost of a vacancy $\psi$ is 120. The matching production function $M$ is given by

$$M(\tilde{u},v) = \tilde{u}^{\omega} v^{1-\omega}, \quad (31)$$

and the Cobb-Douglas parameter $\omega$ is set at .4. The equilibrium of this model, with and without a binding minimum wage of $m = 6$, is as follows:

<table>
<thead>
<tr>
<th>Outcome</th>
<th>$m = 0$</th>
<th>$m = 6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>0.098</td>
<td>0.121</td>
</tr>
<tr>
<td>$l$</td>
<td>0.583</td>
<td>0.620</td>
</tr>
<tr>
<td>$v$</td>
<td>0.017</td>
<td>0.018</td>
</tr>
<tr>
<td>$\rho V_n(m)$</td>
<td>5.416</td>
<td>5.610</td>
</tr>
<tr>
<td>$\tilde{u}$</td>
<td>0.057</td>
<td>0.075</td>
</tr>
<tr>
<td>$e$</td>
<td>0.529</td>
<td>0.545</td>
</tr>
</tbody>
</table>

The outcome $e$ in the last row of the table is the proportion of the population employed in equilibrium. We note that the imposition of a binding minimum wage of 6 in this labor market environment has some notable impacts. First, the unemployment rate among labor force participants increases by about one-third. The percentage of population members in the labor market increases from 0.583 to 0.620; this increase is large enough to counterbalance the increased unemployment rate among labor market participants so that the proportion of the total population that is employed actually increases from 0.529 to 0.545. Another striking feature of the new equilibrium is the increase in the value of unemployed search. Thus an imposition of this particular binding minimum wage in the particular labor market resulted in increased levels of unemployment among labor market participants, increased levels of employment in
the population, and an increased value of unemployed search. In the next section we turn to the consideration of constructing welfare measures to quantify the value of minimum wage changes to agents on the supply and demand side of the labor market.

1.8 The Model with On-the-Job Search

We now consider the impacts of minimum wages in a labor market with a less restrictive search technology than has been the case so far.\textsuperscript{15} As above, we will begin by considering the situation in which contact rates are fixed, but are state dependent. If an individual is unemployed, offers arrive at rate $\lambda_n$, and while employed offers arrive at rate $\lambda_e$ (previously we set $\lambda_e = 0$). All other parameters describing the labor market environment are identical.

As was true above, we can describe an implicit reservation wage/match value by $\theta^*(m)$, which emphasizes the dependence of this parameter on the minimum wage $m$. In order to fix ideas, we will first describe the how the model works in the absence of a binding minimum wage so that we can assume that $m = 0$ and will just denote the critical match value by $\theta^*$. When an acceptable match is found, which is one for which $\theta > \theta^*$, then the wage of the worker is set using the Nash-bargained wage function $w(\theta, \theta^*)$. The mechanics of this process are identical to the case in which $\lambda_e = 0$.

The wage setting mechanism is considerably more complicated when a currently employed individual meets a new potential employer. We denote the worker’s current state, that is their employment relationship, by the pair $(\theta, w)$, and let the characteristics of the other employment option be denoted $(\theta', w')$. Let there be a currently employed individual with wage $w$ and match value $\theta > \theta^*$, who meets a new potential employer with match value $\theta'$. We assume that the potential match value will only be reported to the current employer if the employee has an incentive to do so. One situation in which this will be the case is when $\theta' > \theta$. When this occurs, we assume that a bargaining process for the individual’s services begins between the current and potential employers and stops when one of the firms’ surplus reaches zero. This will clearly be the current employer when $\theta' > \theta$. Let the maximal value of the match $\theta$ to the worker be given by $Q(\theta)$. Then the objective function for the Nash bargaining problem when $\theta' > \theta$ is:

\textsuperscript{15} A full treatment of this problem is contained in Flinn and Mabli (2005).
\[ S(\theta', w', \theta) = \{ V_e(\theta', w') - Q(\theta) \}^\alpha \times \{ V_f(\theta', w') \}^{1-\alpha} \]

where \( V_f(\theta', w') \) denotes the new firm’s value of the match.

The firm’s value of the current employment contract is defined as follows. Over the small period of time \( \varepsilon \), the firm earns a profit of \((\theta - w) \varepsilon \). With “probability” \( \eta \varepsilon \), the match is exogenously terminated and the firm earns no profit from the match thereafter. With probability \( \lambda \varepsilon \), the worker receives a job offer from an alternative firm. If he reports this offer to his current firm, his wage might be renegotiated. With probability \((1 - \lambda \varepsilon - \eta \varepsilon)\), the worker does not receive another job offer and he is not exogenously dismissed over the period \( \varepsilon \). In this case the status quo is maintained.

Then the value of the firm’s problem is:

\[
V_f(\theta, w) = \left( \frac{(\theta - w)\varepsilon}{1 + \rho \varepsilon} \right) + \left( \frac{\eta \varepsilon}{1 + \rho \varepsilon} \times 0 \right) + \left( \frac{\lambda \varepsilon \varepsilon G(\tilde{\theta}(w))}{1 + \rho \varepsilon} \times V_f(\theta, w) \right) + \left( \frac{1 - \lambda \varepsilon \varepsilon - \eta \varepsilon}{1 + \rho \varepsilon} \times V_f(\theta, w) \right) + \left( \frac{o(\varepsilon)}{1 + \rho \varepsilon} \right),
\]

where \( V_f(\theta, w(\theta, \tilde{\theta})) \) represents the equilibrium value to a firm of the productive match \( \theta \) when the worker’s next best option has a match \( \tilde{\theta} \). The function \( \tilde{\theta}(w) \) is defined as the maximum value of \( \theta \) for which the contract \( (w) \) would leave the firm with no profit. This value is implicitly defined by \( V_f(\tilde{\theta}(w), w) = 0 \). Any encounter with a potential firm in which the match value is less than \( \tilde{\theta}(w) \) will not be reported by the employee. In this case, the firm’s value remains \( V_f(\theta, w) \). Any new contract with a match value greater than \( \tilde{\theta}(w) \) will be reported to the current firm and will result in either a renegotiation of the current contract or a separation. A separation occurs if the new match value \( \theta' > \theta \), in which case the worker quits his current firm to work at the new firm with match value \( \theta' \). After rearranging terms and taking limits as \( \varepsilon \to 0 \), we have

\[
V_f(\theta, w) = \left( \rho + \eta + \lambda \varepsilon \tilde{G}(\tilde{\theta}) \right)^{-1} \times \{ \theta - w + \lambda \varepsilon \int_{\tilde{\theta}(w)}^\theta V_f(\theta, w(\theta, \tilde{\theta}))dG(\tilde{\theta}) \}.
\]
The worker’s value of being employed is defined as follows. With probability $\eta\varepsilon$, the match is exogenously terminated and the worker returns to unemployment, the value of which is $V_n(m)$. With probability $\lambda\varepsilon$, the worker receives a job offer from an alternative firm. Reporting the offer to his current firm will result either in the renegotiation of his wage at the current firm or in his separation from the current firm to work at the alternative firm. With probability $(1 - \lambda\varepsilon - \eta\varepsilon)$, the worker does not receive another job offer and he is not exogenously dismissed over the period $\varepsilon$. In this case the status quo is maintained. Thus, the worker’s value of employment at a current match value $\theta$ and wage $w$ is given by

$$V_e(\theta, w) = \left( \frac{w\varepsilon}{1 + \rho\varepsilon} \right) + \left( \frac{\eta\varepsilon}{1 + \rho\varepsilon} \times V_n(m) \right) + \left( \frac{\lambda\varepsilon}{1 + \rho\varepsilon} \int_{\theta(w)}^{\theta} V_e(\theta, w(\theta, \bar{\theta}))dG(\bar{\theta}) \right) +$$

$$+ \left( \frac{\lambda\varepsilon}{1 + \rho\varepsilon} \int_\theta^{\theta(\bar{\theta})} V_e(\theta, w(\bar{\theta}, \theta))dG(\bar{\theta}) \right) + \left( \frac{\lambda\varepsilon G(\theta(w))}{1 + \rho\varepsilon} \times V_e(\theta, w) \right) +$$

$$+ \left( \frac{(1 - \lambda\varepsilon - \eta\varepsilon)}{1 + \rho\varepsilon} \times V_e(\theta, w) \right) + \left( \frac{\sigma(\varepsilon)}{1 + \rho\varepsilon} \right),$$

where $V_e(\theta, w(\theta, \bar{\theta}))$ is the equilibrium value of employment to a worker with match value $\theta$ when his next best option has a match value of $\bar{\theta}$. Note that when an employee encounters a firm with a new match value $\bar{\theta}$ which is lower than his current match value but is capable of being used to increase the value of his current employment contract (i.e., $\theta > \bar{\theta} > \theta(w)$), his new value of employment at the current firm becomes $V_e(\theta, w(\theta, \bar{\theta}))$. Instead, when the match value at the newly-contacted firm exceeds that of the current firm, the employee changes employers. The value of employment at the new firm is given by $V_e(\theta, w(\bar{\theta}, \theta))$. Thus, the match value at the current firm becomes the determinant of the threat point faced by the new firm and plays a role in the determination of the new wage. Finally, when the match value at the new firm is less than $\theta(w)$, the contact is not reported to the current firm since it would not result in any new improvement in the current contract. Because of this selective reporting, the value of employment contracts must be monotonically increasing both within and across consecutive job spells. Declines can only be observed following a transition into the unemployed state.
After rearranging terms and taking limits, we have

\[ V_e(\theta, w) = \left( \rho + \eta + \lambda \tilde{G}(\bar{\theta}(w)) \right)^{-1} \times \{ w + \eta V_n(m) + \lambda \int_{\bar{\theta}(w)}^{\theta} V_e(\theta, w(\bar{\theta}, \bar{\theta}))dG(\bar{\theta}) \} - 1 \times \{ w + \eta V_n(m) + \lambda \int_{\bar{\theta}(w)}^{\theta} V_e(\theta, w(\bar{\theta}, \bar{\theta}))dG(\bar{\theta}) \} \times \{ \rho + \eta + \lambda \tilde{G}(\bar{\theta}(w)) \}^{-1} \times \{ w + \eta V_n(m) + \lambda \int_{\bar{\theta}(w)}^{\theta} V_e(\theta, w(\bar{\theta}, \bar{\theta}))dG(\bar{\theta}) \} \}

With a new match value of \( \theta' > \theta \), the surplus attained by the individual at the new match value with respect to the value she could attain at the old match value after extracting all the surplus associated with it is

\[ V_e(\theta', w(\theta', \theta)) - Q(\theta) \]

where \( Q(\theta) = V_e(\theta, w^*(\theta)) \) is the value of employment to the employee if he receives the total surplus of the match \( \theta \). In this case, the equilibrium wage function is \( w^*(\theta) = w(\theta, \theta) \). Then,

\[ Q(\theta) = \left( \rho + \eta + \lambda \tilde{G}(\bar{\theta}(w)) \right)^{-1} \times \{ w^*(\theta) + \eta V_n + \lambda \int_{\bar{\theta}(w)}^{\theta} V_e(\theta, w(\bar{\theta}, \bar{\theta}))dG(\bar{\theta}) \}, \]

where we have used the fact that \( \tilde{\theta}(w^*(\theta)) = \theta \).

The model is closed after specifying the value of nonemployment \( V_n \). Up until this point, we have assumed that \( \theta^* \) is the value of \( \theta \) required for an unemployed searcher and a firm to initiate an employment contract. In the absence of a minimum wage (i.e. with \( m = 0 \)), \( \theta^*(m) \) is the reservation match value and the minimal acceptable wage. All match values greater than this value will result in employment. With a minimum wage, however, this is the "implicit" reservation match value. The minimal acceptable wage and match value is, instead, the imposed minimum value \( m \).

Similar to the no-on-the-job search model, we only consider the case in which \( m \geq \theta^*(m) \). When \( m < \theta^*(m) \), the minimum wage constraint is nonbinding since all matches greater than \( \theta^*(m) \) result in employment at wages greater than the minimum. In addition, all acceptable matches to a firm must be greater than or equal to \( m \). Any match values less than \( m \) would result in negative profits. To summarize, we have \( \theta > m > \theta^*(m) \), for all \( \theta \) among employed individuals.

With a binding minimum wage, the searcher’s steady state value of being unemployed is
\[ V_n(m) = \left( \rho + \lambda_n \tilde{G}(m) \right)^{-1} \times \left\{ b + \lambda_n \int_m V_e(\tilde{\theta}, w(\tilde{\theta}, \theta^* (m)))dG(\tilde{\theta}) \right\}. \]

When an employed agent meets a new potential employer, the solution to the Nash bargaining problem is given by

\[ w(\theta', \theta) = \arg \max_{w \geq m} S(\theta', w, \theta) \]

where \( S(\theta, w, \theta^* (m)) = \{V_e(\theta, w) - Q(\theta)\}^\alpha \times V_f(\theta, w)^{1-\alpha} \). When an unemployed agent meets a new potential employer, the solution to the Nash bargaining problem is given by

\[ w(\theta, \theta^* (m)) = \arg \max_{w \geq m} S_n(\theta, w, \theta^* (m)) \]

where \( S_n(\theta, w, \theta^* (m)) = \{V_e(\theta, w) - V_n\}^\alpha \times \{V_f(\theta, w) - 0\}^{1-\alpha} \). Regardless of whether the searcher is employed, the minimum wage acts as a side constraint in the Nash bargaining problem. Unlike the model in which on-the-job search was absent, our current model does not produce a tractible form for the wage function. In order to solve the model, we must resort to numerical approximations of the value functions.

In the next section we show that individuals can be paid the minimum wage only at their first job in an employment spell and only until they receive an alternative offer with which they can renegotiate their current employment contract.

1.8.1 The On-the-Job Search Model with Endogenous Contact Rates

In using the matching function in general equilibrium search models usual, researchers typically assume that there is no on-the-job search. In a model with on-the-job search, searchers can be unemployed or employed. Most treatments of this problem assume that employed and unemployed searchers devote equal amounts of time to search, so that the total number of searchers equals the number of labor market participants (see, e.g., Pissarides (2000)). Estimates of contact rates of unemployed and employed searchers in partial equilibrium analyses consistently indicate that the contact rate of the unemployed is at least three times larger than that of the employed, making this assumption unattractive. We define the searching population as equal to \( U + vE \), where \( U \) is the size of the unemployed population, \( E \) is the size of the employed population, and \( v \) is a technological parameter partially characterizing the
search technology. Defining the searching population in this way, the contact rate per vacancy is given by \( \frac{M(S,V)}{V} = q(k) \) and the contact rate per effective searcher is \( \frac{M(S,V)}{S} = \frac{q(k)}{k} \). The contact rate per unemployed (employed) searcher is the probability that the searcher is unemployed (employed) multiplied by the contact rate per effective searcher:

\[
\lambda_n = \frac{U}{S} \cdot \frac{M(S,V)}{S}
\]

\[
\lambda_e = \frac{vE}{S} \cdot \frac{M(S,V)}{S}
\]

We will make a functional form assumption regarding \( q(k) \).

Assuming that there exists a population of potential firm entrants with an outside option value of 0, firms create vacancies until the point that expected profits are zero. Assuming the cost of creating a vacancy is given by \( \psi > 0 \), the expected value of creating a vacancy is given by

\[
\rho V_v = -\psi + \frac{M(S,V)}{V} (J(m, \theta^*(m)) - V_v)
\]

where \( V_v \) is the value of a vacancy, \( \frac{M(S,V)}{V} \) is the rate at which a firm fills a vacancy, and \( J(m, \theta^*(m)) \) is the expected value of a filled vacancy,

\[
J(m, \theta^*(m)) = \frac{\lambda_e}{\lambda_n + \lambda_e} \int_{m' \theta} V_f (\theta', \theta) g(\theta') g_{ss}(\theta)d\theta' + \frac{\lambda_n}{\lambda_n + \lambda_e} \int_{m \theta} V_f (\theta, \theta^*(m)) g(\theta) \tilde{G}(m)d\theta
\]

where \( g_{ss}(\theta) \) is the steady state distribution of match values and \( V_f (\theta', \theta) \) is the value to the firm of employing a worker whose productivity value is \( \theta' \) and whose threat point is \( \theta \).

Assuming a free entry condition (FEC), \( V_v = 0 \) and we have

\[
0 = -\psi + \frac{M(S,V)}{V} (J(m, \theta^*(m)))
\]

This equation can be used to solve for the flow cost of creating a vacancy, \( \psi \), given the equilibrium number of vacancies \( V \), the expected value of a filled vacancy \( J(m, \theta^*(m)) \), and the size of the set of searchers \( S \).

We note that with exogenous contact rates, the steady state unemployment rate
is defined by

$$U = \frac{\eta}{\eta + \lambda_n G(m)}$$

whereas with endogenous contact rates, the steady state unemployment rate is defined by

$$U = \frac{\eta}{\eta + \frac{U}{s M(SV)} G(m)}.$$

Unlike the no on-the-job search case, we do not allow for the decision whether to participate in the labor market. We assume that all individuals are either in the unemployed or employed states.

With endogenous contact rates, a labor market equilibrium in the presence of a minimum wage is characterized by the quadruplet $$(U, \nu, V, \theta^*(m))$$ which is solely a function of the primitive parameters $$(\rho, b, \eta, \alpha, \psi)$$ and the parameters of the match distribution $$G(\theta)$$

### 1.8.2 An Analysis of the Model with On-the-Job Search

We now determine when workers receive the minimum wage and analyze how the existence of a minimum wage affects labor market outcomes conditional on whether workers are allowed to search while employed.

In the case of no on-the-job search ($$\lambda_e = 0$$), the wage equation derived from the Nash bargaining framework is

$$w(\theta, \theta^*(m)) = \alpha \theta + (1 - \alpha) \theta^*(m)$$

where $$\theta$$ is the worker’s current match value, $$\theta^*(m)$$ denotes the match value associated with unemployment, and $$\alpha$$ is the worker’s bargaining power parameter. Examining the wage equation, we find that under this division of the match, the worker would receive a wage of $$m$$ when $$\theta = \hat{\theta}_m(m, \theta^*(m))$$, where

$$\hat{\theta}_m = \frac{m - (1 - \alpha) \theta^*(m)}{\alpha}$$

For notational convenience we will write $$\hat{\theta}_m(m, \theta^*(m))$$ as $$\hat{\theta}_m$$. It is assumed that $$\hat{\theta}_m > m$$ in order to avoid the uninteresting case in which all ”feasible” matches (i.e. those greater than $$m$$) generate wage offers at least as large as $$m$$. According to the wage equation, when $$\theta \in [m, \hat{\theta}_m)$$, the wage is less than $$m$$. However, when confronted
with the choice of giving some of its surplus to the worker versus a return of 0, the firm pays the wage of \( m \) for all \( \theta \in [m, \hat{\theta}_m) \). Wages for \( \theta \geq \hat{\theta}_m \) are determined according to the wage equation \( w(\theta, \theta^*(m)) \).

Figure 2.5 depicts the no OTJ search case. Here, the wage function maps a single productivity value \( \theta \) to a wage for each \( \theta > \theta^*(m) \). The line \( w = \theta \) depicts the wage at which the firm breaks even for each productivity value \( \theta \). The firm earns a profit of \( (\theta - w) \) for a worker with match value \( \theta \), depicted as the vertical distance between the line \( w = \theta \) and the wage function \( w(\theta) \). Once we introduce a minimum wage into the graph, we see that some \( \theta \) (particularly \( \theta \in [\theta^*(m), m) \)) are no longer available to the worker. This is the standard (negative) employment effect. Additionally, for all current match values \( \theta \in [m, \hat{\theta}_m) \), the firm wants to pay the worker the wage \( w(\theta) < m \), but cannot do so under the minimum wage law. Therefore, it pays the worker \( w = m \) which lowers its profits from \( (\theta - w(\theta)) \geq 0 \) to \( (\theta - m) \geq 0 \) for each \( \theta \in [m, \hat{\theta}_m] \). For all \( \theta > \hat{\theta}_m \), the firm pays the worker \( w(\theta) > m \) and earns a profit of \( (\theta - w(\theta)) > 0 \).

When we extend the model to allow workers to search on-the-job, the wage that is derived from the Nash bargaining framework is represented by the function \( w(\theta', \theta) \). Extending the original model to allow for on-the-job search produces two immediate implications regarding when workers receive the minimum wage. We prove both results below. We show that within any employment spell, only at the first job in the sequence can the employee be paid the minimum wage. Moreover, the minimum wage may only be paid to the employee at the beginning of their first job in the employment spell, since the arrival of alternative, viable employment opportunities with other firms will result in a renegotiated wage payment larger than the minimum. This implies that individuals will spend only a small amount of employment time at jobs paying the minimum wage if the contact rate of employees with other firms with vacancies is sufficiently large.

Since workers can be paid the minimum wage only at their first job, any minimum wage earner must use \( \theta^*(m) \) as their best outside option, which is unemployment. The definition of the match value \( \hat{\theta}_m \) when allowing for OTJ search is identical to its definition without OTJ search. That is, \( \hat{\theta}_m \) is the maximum match value with which the worker receives the minimum wage while using \( \theta^*(m) \) as a match value that represents their best outside option. We can re-write the value of nonemployment that we derived in the previous section without having to re-write the value of employment.
The value of nonemployment is
\[ V_n(m) = (\rho + \lambda_n \tilde{G}(\theta^*(m)))^{-1} \times \{ b + \lambda_n \int_{\theta^*(m)}^{\tilde{\theta}(m)} V_{\tilde{\theta}}(\tilde{\theta}, w(\tilde{\theta}, \theta^*(m))) = m \} dG(\tilde{\theta}) + \lambda_n \int_{\theta^*(m)}^{\tilde{\theta}(m)} V_{\tilde{\theta}}(\tilde{\theta}, w(\tilde{\theta}, \theta^*(m))) > m \} dG(\tilde{\theta}) \]

We now formally state and prove each proposition:

**Proposition 4** A worker can receive the minimum wage only at his first job.

**Proof.** Let \( \theta^*(m) \) be the reservation match value when there exists a minimum wage; let \( \theta^1 \) such that \( \theta^1 \in [m, \hat{\theta}_m] \) be the match value that the worker draws upon exiting unemployment; and let \( \theta^2 \) such that \( \theta^2 > \theta^1 \) be the first match value the worker draws while employed at his first job. Note: the match values \( \theta^1 \) and \( \theta^2 \) are not the first and second match values denoted by his current match value; and the second inequality follows from the fact that the wage function is monotonically increasing in its first argument. Thus, \( m < w(\theta^2, \theta^1) \) implies that the wage at his second job will always be greater than the minimum wage. □

**Proposition 5** A worker can receive the minimum wage at his first job only until the first alternative job offer arrives with which he can renegotiate his current contract.

**Proof.** The wage function \( w(\theta', \theta) \) is monotonically increasing in its first and second argument: \( w_1(\theta', \theta) > 0 \) and \( w_2(\theta', \theta) > 0 \) (where \( w_1 \) and \( w_2 \) denote the partial derivatives of the wage function with respect to its first and second arguments, respectively) by the nature of the Nash bargaining framework.

If an unemployed individual draws \( \theta^1 \in [m, \hat{\theta}_m] \), then he will accept the offer and receive a wage \( w(\theta^1, \theta^*(m)) = m \). If he receives a second draw \( \theta^2 \) such that \( \hat{\theta}_m \geq \theta^2 > \theta^1 \) then he will leave the current firm and receive a wage \( w(\theta^2, \theta^1) > m \).
at his second job by proposition 1. Alternatively, consider the situation in which
the individual draws a match value \( \theta^2 \), such that \( m < \theta^2 < \hat{\theta}_m \), coming out of
unemployment in which case his wage is \( w(\theta^2, \theta^*(m)) = m \). Here the superscript
2 refers to the second draw from the previous example, but is the first draw in this
example. If he subsequently draws a match value \( \theta^1 \) (the first match value from
the previous example) such that \( m < \theta^1 < \theta^2 \), he renegotiates his contract at his
current firm to \( w(\theta^2, \theta^1) \), which we have already seen is greater than the minimum
wage. Thus coming out of unemployment, the worker can receive the minimum wage.
When another job offer arrives, the worker renegotiates his contract and receives a
wage greater than the minimum wage. ■

While some of these arguments are on the technical side, the intuition is imme-
diate under the worker-firm bargaining structure we employ. For an individual in
an on-going employment spell, their welfare is nondecreasing over the course of the
spell. Since the wage is the only characteristic of the job that directly appears in the
employee’s utility function, wages must be nondecreasing over the employment spell.

While minimum wage constraints “distort” participation decisions and job accep-
tance decisions involving unemployed searchers, they have no effect on the mobility
decisions of employedsearchers. That is, all mobility decisions remain efficient, with
the worker always choosing to go to the employer at which her productivity is high-
est. When a new potential match arrives that does not dominate the current value,
it still may be welfare-improving to the individual in its role as an outside option.
While these dominated offers have no effect on aggregate productivity, they do bring
renegotiation and a shift in the division of rents in favor of employees. These dom-
inated offers bring wage increases, and only the starting wage in the first job after
employment can pay \( m \), all subsequent wages paid at that firm or other firms worked
at in the same employment spell must be larger than \( m \) (and are strictly increasing).