Due Date: Monday, September 16

Be sure to show all of your work and clearly indicate your final response to each question.

1. Differentiate the following functions.
   (a) \( f(x) = x^{3.5} \)
   Ans: \( \frac{df(x)}{dx} = 3.5x^{2.5} \)

   (b) \( f(x) = \ln(6x); \quad x > 0 \)
   (c) \( f(x) = \exp(x^2) \)
   (d) \( f(x) = 6x^7 + 2x^{-1} \)
   (e) \( f(x) = \frac{\ln(x)}{x^4}; \quad x > 0 \)
   (f) \( f(x) = x^{-5}; \quad x \neq 0 \)
   (g) \( f(x) = \min[1, x]; \quad x \neq 1 \)

2. Find the partial derivatives \( \frac{\partial f(x, y)}{\partial x} \) and \( \frac{\partial f(x, y)}{\partial y} \) of the following functions:
   (a) \( f(x, y) = 8 + 4x^2 + 3y^5 \)
   Ans: \( \frac{\partial f(x, y)}{\partial x} = 8x \)
       \( \frac{\partial f(x, y)}{\partial y} = 15y^4 \)

   (b) \( f(x, y) = 4xy^3 \)
   (c) \( f(x, y) = \ln(2x^2) + \ln(3y^4); \quad x \neq 0, \quad y \neq 0 \)
   (d) \( f(x, y) = \exp(3x^4y^2) \)
   (e) \( f(x, y) = \ln(x) \exp(y); \quad x > 0 \)
   (f) \( f(x, y) = \frac{\ln(x)}{y^2}; \quad x > 0, \quad y \neq 0 \)
   (g) \( f(x, y) = \max[x, y]; \quad x \neq y \)
3. Totally differentiate the following functions:

(a) \( z = x^2 y \)

Ans: \( dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy \)
\[ \Rightarrow dz = (2x)dx + (1)dy \]

(b) \( z = x^4 + 2y^5 \)

(c) \( z = x + \ln(y); \ y > 0 \)

(d) \( z = \min\{2x, y\}; \ x \neq y \)

4. Find the value of \( x \) for which each of these functions attains its maximum.

(a) \( f(x) = x - .25x^3; \ x > 0 \)

Ans: First Order Condition:
\[ \frac{df(x^*)}{dx} = 0, \ x^* > 0 \]
\[ \Rightarrow 0 = 1 - .75(x^*)^2 \]
\[ \Rightarrow (x^*)^2 = \frac{1}{.75}, \ x^* > 0 \]
\[ \Rightarrow x^* = \sqrt{\frac{4}{3}} \]

Second Order Condition:
\[ \frac{d^2 f(x)}{dx^2} \bigg|_{x=\frac{\sqrt{4}}{3}} < 0 \ \text{(?)} \]
\[ -1.5(\sqrt{\frac{4}{3}}) < 0 \ \text{(?)} \]

Yes, so the solution yields a maximum

(b) \( f(x) = -x^2 \)

(c) \( f(x) = 4 + 5x - .1x^2; \ x > 0 \)

(d) \( f(x) = -|6 - x|, \ x > 0 \)