1. You want to estimate the following bivariate linear regression

\[ \ln w = \beta_0 + \beta_1 s + \varepsilon, \]

where \( \ln w \) is the logarithm of the wage rate and \( s \) is a variable equal to 1 if the individual has completed more than high school and equals 0 if not. The random variable \( \varepsilon \) is independently and identically distributed in the population with mean 0 and variance \( \sigma^2 \). You have access to a sample of size 200 drawn randomly from the population.

Define

\[ X = \begin{bmatrix} 1 & s_1 \\ 1 & s_2 \\ \vdots & \vdots \\ 1 & s_{200} \end{bmatrix}, \]

and

\[ y = \begin{bmatrix} \ln w_1 \\ \ln w_2 \\ \vdots \\ \ln w_{200} \end{bmatrix}. \]

In the sample you find that

\[ X'X = \begin{bmatrix} 200 & 106 \\ 106 & 106 \end{bmatrix} \]

and

\[ X'y = \begin{bmatrix} 475.25933 \\ 263.60782 \end{bmatrix} \]

1. With the information given to you, can you compute unbiased estimates of \( \beta_0 \) and \( \beta_1 \)? If so, what are the estimates?
2. Do you have sufficient information to compute an unbiased estimate of $\sigma^2_\varepsilon$? If so, what is it?

3. Say that you reformulate the regression model so that

$$\ln w = \delta_1 (1 - s) + \delta_2 s + u.$$ 

Can you find unbiased estimates of $\delta_1$ and $\delta_2$ that are simple linear functions of your estimates of $\beta_0$ and $\beta_1$? If so, what are they?

4. We can define estimated residuals based on estimates of $\beta$ and $\delta$ in both specifications of the model, which we will denote by $r_1$ and $r_2$. Is $\sum_{i=1}^{200} r_{1,i} = 0$?

Is $\sum_{i=1}^{200} r_{2,i} = 0$?

2. W 3.1
3. W 3.6
4. W 3.7
5. W 3.10