Introduction to Econometrics

Fall 2006

Assignment 7

Today’s Date: 11/13

Due Date: 11/20

Please show all of your work and clearly indicate your final response to each question.

1. We talked about estimating the parameter of the negative exponential distribution using the method of maximum likelihood. The negative exponential p.d.f. and c.d.f. are:

   \[ f(t) = \alpha \exp(-\alpha t) \]
   \[ F(t) = 1 - \exp(-\alpha t) \]

   where the parameter \( \alpha \) is positive.

Assume that \( t \) is the duration of unemployment of a labor market participant. You have access to data collected as follows. A large number of individuals are observed to become unemployed in a particular month, and then are followed for 12 months. Individuals who leave unemployment prior to the end of the 12 month observation period have their total length of time in unemployment recorded as \( t \). For those who are still unemployed at the end of the observation period, all we know is that their completed spell of unemployment will be greater than 12 months.

1. Let the subsample of individuals for whom \( t < 12 \) be denoted by \( C \). We showed that if we had access to a random sample of unemployment spells, the maximum likelihood estimator of \( \alpha \) was the inverse of the sample mean. Show that this estimator applied to group \( C \) only is inconsistent.

2. Derive a consistent estimator for \( \alpha \) that uses information from \( C \) and from the “censored” unemployment spells, i.e., those that last for over 12 months.

3. In a sample of 100 individuals, you know that 70 belong to group \( C \). Within this group, the average duration of a completed unemployment spell is 9 months. Is this enough information to find the value of your estimator in part 1.2 for this sample? If so, what is the estimate?
4. Say that instead of the exponential distribution assumption, the population distribution of unemployment durations is uniform on the interval \((0,a], a > 12\), with

\[
\begin{align*}
    f(t) &= a^{-1} \mathbb{1}_{t \in (0, a]}, \\
    F(t) &= \frac{t}{a} \mathbb{1}_{t \in (0, a]} + 1 \mathbb{1}_{t > a}.
\end{align*}
\]

Can you find a consistent estimator for \(a\) in this case? If so, what is it?

2. Text 10.2

3. Text 10.3

4. Text 10.6