Is Equality Stable?

Dilip Mookherjee

*Boston University*

Debraj Ray

*New York University*
Economic Inequality . . .

...is of interest, not only at some intrinsic level, but also for its possible connections to diverse variables.

Natural to study the evolution of inequality.

DOMINANT VIEW. Inequality as constant battle between convergence and “luck” (e.g., Becker and Tomes (1979), Loury (1981)).

In contrast, I want to emphasize a view of inequality as an inevitable consequence of the market mechanism.

BASIC ASSERTION. There are fundamental market forces responsible for the emergence of persistent inequality among ex ante identical agents, even in a world of perfect certainty.

Informal Description

- Economy populated by several dynasties.
- Each dynasty allocates resources to current consumption and bequests to descendants.
- Bequests may take the form of financial transfers, or educational expenditures for different professions.
- The returns to professions are endogenous.
- If several professional categories are necessary, wages *must* force separation in choices even if all individuals are ex-ante identical.
- This “broken symmetry” has no payoff implications for the generation alive today.
- But starting next generation, there *must* be inequality (not just in wages, but in payoffs).
- There must be individuals who are in low-paying professions, whose parents did not invest in them. And there must be others in high-paying professions whose parents did invest in them.
- Once such inequality sets in it will magnify and persist.
In our 2001 paper, we explore several aspects of this general setup:

- Under what conditions might every steady state of this model necessitate (utility) inequality?
- Under what conditions are there multiple steady states with varying levels of inequality?
- Are steady states efficient?
- Do competitive equilibria converge to some steady state?

Current presentation examines the starting point more carefully.

Specifically, two ingredients are used in basic argument:

- There must be missing (or least imperfect) credit markets. [No getting away from this one.]
- Individuals do not make financial bequests. If they could, such bequests could conceivably compensate for educational inequality.

This talk examines the interplay between financial and educational bequests, and its implications for the evolution of inequality.
Features of the Model

1. **Unit Continuum of Dynasties.**

Each dynasty composed of an infinite sequence of (one-person) generations.

2. **Each Individual “Chooses” a Profession for Her Child.**

$h = \text{profession or occupation}$. A collection of population weights $\lambda = \{\lambda(h)\}$ is an occupational distribution.

3. **Professionals are Inputs in Production of Single Final Good.**

That is, $\lambda$ is just a bundle of inputs, which produces output using a convex CRS technology.

4. **Professional Wages are Endogenous.**

Wage function $w = \{w(h)\}$, depends on $\lambda$.

**Assumption.** For each $\lambda$ there is a unique “supporting” wage function $w$. Conversely, every $w$ admits *some* scaling $kw$ such that $kw$ is a supporting wage function with unique profit-maximizing input choices.
Features, contd.

4. **Educational costs are paid upfront by the parent.**

Measured in units of final good, occupation $h$ costs $x(h)$ to acquire. We assume that there is $h$ with $x(h) = 0$ and $h'$ with $x(h') > 0$.

5. **Educational bequest supplementable by financial bequests.**

Assume bequest is at fixed rate of return $r$. Easy to modify.

6. **Set of professions may be finite or infinite.**

7. **Each individual obtains utility from own consumption and child’s wealth.**

Write as $u(c_t) + v(W_{t+1})$.

**Note.** Wealth includes wages $w$ and (interest-augmented) financial bequests $b$ made by parent. So $W = w + b(1 + r)$.

8. **No uncertainty.**
Equilibrium

Initially, each dynasty $i$ has occupation $h(i)$ and financial assets $m(i)$.

An *equilibrium* describes an entire sequence of such allocations.

At each date, the allocation determines an occupational distribution $\lambda_t$, as well as (supporting) wage function $w_t$.

Each individual must find her allocation to be optimal, given these prices and her budget.

More formally, dynasty $i$’s wealth at date $t$ is $W_t(i) = m_t(i) + w_t(h_t(i))$. Given this, $i$ chooses $(c_t, b_{t+1}, h_{t+1})$ to maximize

$$u(c_t) + v (b_t(1 + r) + w_{t+1}(h_{t+1}))$$  

subject to

$$c_t + x(h_{t+1}) + b_{t+1} = W_t.$$  

These choices must aggregate to the economy-wide distribution at every date.
Steady State

An equilibrium is a *steady state* if the joint distribution of financial wealth and occupations (and therefore the wage function) is unchanged over time.

A steady state can display some occupational mobility, but no wealth mobility (single-crossing).

**Simplifying Assumption.** All professions are occupied in steady state.
Benchmark: No Occupational Choice

Suppose constant wage $w$ at every date.

Only financial bequests permitted.

Then given resources $W$, individual chooses $b$ to maximize

$$u(W - b) + v(W'),$$

where $W' \equiv [1 + r]b + w$.

Define $\Psi(W, w)$ to be resulting choice of $W'$. $\Psi$ nondecreasing in $W$.

AN EXAMPLE.

$v = \delta u$ for some discount factor $\delta$ and $u(c) = c^{1-\sigma}/(1 - \sigma)$ for some $\sigma > 0$. Then

$$\Psi(W, w) = \frac{(1 + r)\rho}{1 + \rho + r} W + \frac{\rho}{1 + \rho + r} w$$

if $\rho W \geq w$, and equals zero otherwise, where

$$\rho \equiv [\delta(1 + r)]^{1/\sigma}.$$
Benchmark, contd.

**Imperfect Persistence Assumption.** Increase in next period’s assets following a unit change in assets today is bounded below unity.


Under imperfect persistence, for any given $w$, $\Psi(., w)$ intersects the $45^0$ line once and only once.

In constant-elasticity example, imperfect persistence satisfied iff

$$\rho \equiv [\delta(1 + r)]^{1/\sigma} < 1 + \frac{1}{r}. \quad (3)$$

Under imperfect persistence, policy function precipitates unique limit wealth $\Omega(w)$. In example with (3) satisfied,

$$\Omega(w) = \begin{cases} w & \text{if } \rho \leq 1, \\ \frac{\rho}{1 - r(\rho - 1)}w & \text{otherwise}. \end{cases}$$
Steady State Inequality

Define special wage function $w^*$ by

$$w^*(h) - w^*(h') = (1 + r)[x(h) - x(h')]$$

for every pair of occupations $h$ and $h'$, and scaled suitably to serve as a supporting wage function.

By CRS, there is a unique wage function of this form. Let $\underline{w}^*$ be the lowest wage in $w^*$, and $\overline{w}^*$ be the highest wage.

**Note.** Easy to compute $w^*$ from parameters of system.

**Proposition 1** As long as

$$\overline{w}^* > \Omega(\overline{w}^*),$$

(4) every steady state must involve (utility) inequality. If (4) fails, a steady state with perfect equality exists, and must display the wage function $w^*$.

Intuition: Argue by contradiction.
Configurations that Favor $\bar{w}^* > \Omega(w^*)$.

Very broadly: the bequest motive cannot be so strong as to overwhelm all earning differences.

**Discounting.** For instance, if $\delta(1 + r) \leq 1$ in constant-elasticity example, $\Omega(w)$ must equal $w$, and so the inequality condition must hold.

**Occupational Variety.** Inequality condition more likely to apply when occupational structure exhibits large differences in training costs. In example, condition reduces to

$$\frac{M}{w^*} > \frac{\rho - 1}{1 + r(1 - \rho)},$$

$M$ is the cost of the most expensive occupation.

**Poverty.** Inequality condition more likely to hold in poor economies. Limit wealth $\Omega(w)$ moves more than one-to-one with $w$.

**Growth.** Inequality condition more likely to hold in growing economies. Attenuates bequest motive. In constant-elasticity example, suppose wages start at base $w$ and grow at rate $g$. Then steady state wealth also grows at $g$, but with “level coefficient”

$$\hat{\Omega}(w) = \{\rho/[1 - r(\rho - 1) + g(1 + \rho + r)]\}w.$$

[Smaller than old value.]
What do Steady States Look Like?

Consider the special case of a “rich” set of occupations. Explain why important.

Assume a continuum of occupations, with $x \in [0, M]$.

![Figure 1: A Steady State Wage Function.](image)
What do Steady States Look Like?, contd.

**Wealth Clustering at Bottom.** Across occupations with wages between $w$ and $\Omega(w)$, *wages must be linear in costs with slope* $(1+r)$. All individuals in occupations in this zone will have identical wealth.

**Wealth Inequality Elsewhere.** To encourage the settlement of “high-cost” occupations, the rate of return on occupational choice must depart from the financial rate. And this departure *must* create inequality.

**Inequality May “Accelerate” with Wealth.** Typically, will need higher and higher rates of return to support the more expensive occupations. True in the constant-elasticity example.

**Financial Bequests...** are made at the bottom of the occupational ladder. But this result to be qualified for two reasons: (a) possible gaps in occupational structure; (b) interpretation of an occupational category.

**Unique Steady State.** True with the continuum structure; not true with a “small” set of occupations. To see why, look at figure.
Equilibrium Dynamics

A distribution of wealth prevails at any date; this will map into a distribution of wealth for the next generation.

Several cases possible. We focus on one. Assume that initial wealth is perfectly equal at $W_0$, and suppose that

$$\bar{w}^* > \Omega(\bar{w}^*).$$

**Observation 1.** Let tomorrow’s wage function be $w_1$, with lowest wage $w_1$. Let $W_1 \equiv \Psi(W_0, w_1)$. Then for every $x$ with $w_1(x) \leq W_1$,

$$w_1(x) = w_1 + (1 + r)x.$$

**Observation 2.** For any $N$, there exists a threshold such that starting from any equal initial wealth above this threshold, there is perfect equality for at least $N$ generations.

**Note.** This “declining wealth” story can be replaced by an argument based on economy-wide growth + endogenous training costs.
But Equality Can’t Last Forever ...
Summary

1. We outline an approach to studying the evolution of economic inequality.

2. A wide diversity of occupations and endogenous variations in their market returns are crucial to our argument.

3. Main point: when bequest motive is low “relative to” occupational cost diversity, inequality is *inevitable*, even starting from ex-ante identical individuals and in a world of perfect certainty.

4. Above condition more likely to hold when:
   - there is low discounting.
   - the range of training costs is large.
   - the economy is poor; or when
   - the economy grows quickly