The Provision of Public Goods Under Alternative Electoral Incentives

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September 4, 2002

ABSTRACT

Politicians who care about the spoils of office may underprovide a public good because its benefits cannot be targeted to voters as easily as pork barrel spending. We compare a winner-take-all system, where all the spoils go to the winner, to a proportional system, where the spoils of office are split among candidates proportionally to their share of the vote. In a winner-take-all system the public good is provided less often than in a proportional system when the public good is particularly desirable. We then consider the electoral college system and show that it is particularly subject to this inefficiency.

(JEL D82, L15)
Democratic societies delegate to elected representatives the power to tax and to provide public goods. A common complaint made by citizens and by the press is that a large fraction of public spending is not devoted to genuinely useful public projects, but rather to redistribution and pork-barrel projects. At least part of this redistribution does not seem to be from the rich to the poor, rather it is thought to be a wasteful result of jockeying by candidates for political advantage. Thus, it would be desirable to reduce the money tied up in redistribution, and increase the fraction devoted to public goods.¹

The economic literature has long recognized that the political process may lead to an inefficient provision of public goods. Conventional wisdom - due to Howard R. Bowen (1943), and presented in textbooks such as Joseph E. Stiglitz (1988) - is that the inefficiency comes from the difference between the Samuelsonian optimum and the policy preferred by the median voter. In this model the inefficiency can manifest itself either as underprovision or as overprovision of the public good, depending on the relative position of the median and average voters in the population. According to this view, the source of inefficiency is voter heterogeneity; democratic provision of public goods would be efficient in a society with identical voters. This model assumes exogenous restrictions on the policy space - such as linear taxes - and, as a consequence, ignores a fundamental distortion in majoritarian decision making: politicians have an incentive to tax minorities to target benefits to a majority.

Candidates face a trade-off. When public money are funneled to pork-barrel projects, fewer are available to finance public goods. The benefits from the public good may be higher on average, but they cannot be targeted to groups of voters as easily as the benefits from pork
barrel projects or pure transfers. We call this a trade-off between efficiency and targetability.\textsuperscript{2} The goal of our analysis is to model this trade-off and to explore how alternative political systems affect the provision of public goods and the distribution of resources across voters.\textsuperscript{3}

To highlight the basic idea, we focus on a model that is extremely simple. Two candidates compete for election, by making a (binding) electoral promise to each voter. Candidates only care about the outcome of the election. Voters are homogeneous: each voter will vote for the candidate promising him the most utility. Candidates are faced with only two possible choices: providing the public good or redistributing money. The public good gives every voter the same utility, and is the efficient choice: social welfare is higher when the public good is provided. Money (local public goods, pork-barrel projects), however, can be targeted to subsets of voters. Despite this simplicity, the solution is quite rich and a number of interesting issues emerge.

We show that targetability commands a premium, and therefore there is inefficiently low provision of public goods even in a world with identical voters. We then study how different electoral incentives affect the provision of public goods and the distribution of the resources. Our goal is to provide some insight in the difference between political systems.

Political systems differ in (at least) two important dimensions: in the way that votes are translated into seats in an assembly, and in the way assembly seats translate into influence on policymaking. In some countries, important decisions are made by bargaining in an assembly; in other countries, they are made by a premier elected by plurality voting. Proportional systems are usually associated with many parties having an influence on policymaking, through the process of post-election bargaining; examples are countries such as Italy.
and Belgium. Majoritarian systems are thought to favor the party with the highest share of the vote, in the sense that more power of policy setting is conferred to that party; this seems to be the case in countries such as Britain. We wish to collapse this complexity into one characteristic, namely the rewards that accrue to vote shares. Vote shares are rewarded differently in different political systems, thus leading to different incentives for parties in the election stage.

We focus on two extreme ways of rewarding vote shares. The first system is a winner-take-all system, where all the “spoils of office” go to the winner. The second is a proportional system: the spoils of office are split among the candidates proportionally to their share of the vote. The spoils of office represent the benefits for a party of being able to implement its policy, and the rents from power. Since we only deal with two candidates, we do not capture the richness of the contrast between plurality rule and proportional representation that comes from the process of coalition formation. However, in our view an important feature of proportional systems is the opportunity to translate vote shares into commensurate benefits (political and monetary) for the politician. We show that this feature alone, even in the absence of coalitional issues, generates important differences between the two systems at the electoral stage.

It seems obvious that a proportional system should generate different policy outcomes than a winner-take-all system. However, in a standard Downsian model with linear taxes the equilibrium would be the same under either system: both candidates would choose the ideal point of the median voter. Assar Lindbeck and Jurgen Weibull (1987) also show that the two systems lead to the same (efficient) outcome in a world where candidates engage in
pure redistribution. Similarly, absent the public good, our model yields the same prediction under both systems.

The reason the two systems differ in our model reflects a fundamental difference in the incentives they provide to candidates seeking public office. Let us consider the case where one candidate promises to provide the public good. Now, suppose that the opponent tries to defeat him by redistributing money to some majority of voters. The margin of victory of the redistributive plan against the public good declines as the public good becomes more valuable. This is because of the reduction in the number of voters to whom a candidate can promise benefits that are worth more than the benefits provided by the public good. When candidates maximize the share of the vote, the margin of victory is important for it determines candidates’ payoffs. Hence, in the proportional system the option of redistributing becomes less attractive as the public good becomes more valuable. In contrast, in a winner-take-all system all that matters about a particular redistributive plan is whether it beats the public good: the margin of victory is irrelevant. Thus, in a winner-take-all system the attractiveness of a redistributive plan does not depend on how valuable the public good is, as long as redistribution defeats the public good. We find that the proportional system is more efficient when the public good is very valuable, and the winner-take-all system is more efficient when the public good is not very valuable. We also find that the inequality in the redistribution of resources increases with the value of the public good under both systems.

Another dimension of the political process that we consider involves two different ways of aggregating district level votes to obtain a winner in nation-wide elections. The first is to elect the candidate who obtains a majority of votes in a nation-wide electoral district. The
second way is to elect the candidate who gets the majority of the votes in the majority of districts. The second system will be called the *electoral college* since it is used in presidential elections in the United States. We show that the electoral college generates a more unequal distribution of resources and less efficient provision of global public goods. The reason for the difference is the following. If the electorate is made up of a single district, candidates only need to worry about “fending off” attacks by the other candidates on the majority of voters. In the electoral college system candidates need to worry about the other candidate going after 1/4 of the voters: 1/2 of the voters in 1/2 of the districts. This creates the need to concentrate promises on smaller groups of voters and means that the relative inflexibility in the targeting of the benefits from public goods must now lead to even worse inefficiency.

I. Related literature

A comprehensive survey of the literature is in Torsten Persson and Guido Tabellini (1998). Most of the literature on the inefficiency of democracy focuses on decision making in legislatures. For instance, David Baron (1991) models the legislative process via a sequential bargaining model. These models analyze specific, very structured legislative processes; they do not address candidates competing in large elections. In V. V. Chari et al. (1997), policy is determined through bargaining between a president and local representatives. This gives rise to a common pool problem whereby an excessive number of local public projects are financed from general taxation (see also Barry Weingast et al. (1981)). Similarly, Torsten Persson et al. (1997) compare congressional and parliamentary systems. In their model, the politician choosing the level of public good provision has the option of foregoing the public
good and appropriate the money for his own district; once again, a common pool problem arises. Our setup differs from these models since we focus on national candidates who do not represent any specific district: when proposing to increase transfers to a district, our candidates take into account the loss of another district. Thus, nationwide candidates mitigate the common pool problem by internalizing, in part, the costs of providing pork-barrel projects.

The idea of the tradeoff between policies with diffuse benefits and pork-barrel projects has long been present in the mainstream political science literature. In one variant (see David Mayhew (1974), pp. 52-61), legislators go for projects on which they can claim credit with the electorate. Public goods are bad for this because voters do not know whether the candidate was really instrumental in securing the provision of the public good, whereas it is easy for candidates to claim credit for a local pork-barrel project. This idea relies on incomplete information on the part of voters; one virtue of our model is that incomplete information is not central to our arguments.

There is work comparing the effects of alternative electoral systems on the distribution of resources, in the absence of a public good. Roger Myerson (1993) studies redistributive politics under alternative electoral systems when there is competition among more than two candidates. In Steven J. Brams and Michael D. Davis (1974) and James Snyder (1989), candidates redistribute resources across electoral districts. When the environment is asymmetric, the two systems - winner-take-all and proportional representation - yield different outcomes. All these models are purely about redistribution, hence they do not yield welfare implications.
David Austen-Smith and Jeffrey Banks (1988) analyze proportional representation in a three-candidate model of spatial competition that integrates the electoral and legislative processes. They obtain a sharp contrast between plurality rule with two candidates and proportional representation with three candidates. Under plurality rule, both candidates adopt the policy preferred by the median voter. Under proportional representation the equilibrium electoral platforms are symmetrically distributed around the median. The government is formed between the party that adopts the median position (which receives the fewest votes) and one of the other parties. The policy outcome that emerges from the legislative process is a compromise between the platforms of these two parties and is different from the median voter’s preferred policy.

Additional work on the comparison between majoritarian and proportional systems has been done by Torsten Persson and Guido Tabellini (1999). They construct a model of redistributive politics a’ la Lindbeck and Weibull in which a majoritarian system generates less public good provision than a proportional system. Persson and Tabellini also confront this prediction with cross-country data from around 1990; they find weak support for the prediction that majoritarian elections are associated with less public goods.

II. The model

Our setup does not restrict the set of feasible transfers in any way except to require that they satisfy a budget constraint. Myerson (1993) introduced this model (without the public good) to study redistributive politics.
A. Economy and Agents

There are two candidates, 1 and 2. There is a continuum of consumers/voters; the set of voters is denoted by $V$ which can be taken as the interval $[0, 1]$.

There are two goods, money and a public good. The public good can only be produced by using all the money in the economy.

Each voter has an endowment of one unit of money. The public good yields a utility of $G$ to each voter. Voters have no a priori preference for either candidate, and have linear utility over goods.

Candidates make binding promises to each voter. A candidate can offer to provide the public good (to all voters); alternatively, he can offer different taxes and transfers to different voters. Because a candidate’s promise is only relevant if he gets elected, each voter’s optimal behavior is to vote for the candidate who promises him the greatest utility.

B. Electoral Incentives

In our view, the opportunity to translate vote shares into commensurate benefits for the politician is an important feature of proportional systems relative to winner-take-all systems. We want to understand the consequences of this phenomenon on electoral promises. In our model, candidates are motivated to run by the prospect of spoils of office. We discuss two alternative systems, characterized by how these spoils are divided between candidates.

A proportional system, where the spoils are divided proportionally to the candidates’ share of the vote. Thus, candidates maximize the share of the vote.

A winner-take-all system, where all the spoils go to the winner. Thus, a candidate’s
payoff in the winner-take-all is 0 if his share of the vote is less than 1/2, 1 if it is greater than 1/2, and 1/2 if the share of the vote is exactly 1/2.

C. Game

A pure strategy for a candidate specifies whether he chooses to offer the public good or pure transfers (he cannot offer both, but see Alessandro Lizzeri and Nicola Persico (1998) for a model where this is possible). In the event he chooses transfers, a pure strategy specifies a promise of a transfer to each voter. Formally, a pure strategy is a function $\Phi : V \rightarrow [-1, +\infty)$ that satisfies the following condition: either $\Phi(v) = G - 1$ for all $v \in V$, or $\int_V \Phi(v) dv = 0$. This is a balanced budget condition. $\Phi(v) + 1$ represents the utility enjoyed by voter $v$.

There are two stages of the game:

**Stage 1** Candidates choose offers to voters simultaneously and independently.

**Stage 2** Each voter $v$ gets offers $\Phi_1(v), \Phi_2(v)$ from candidates 1 and 2. After observing the offers, voter $v$ votes for candidate $i$ if $\Phi_i(v) > \Phi_h(v)$. If the voter gets the same offer from both candidates, he randomizes with equal probability.

A mixed strategy in this game could in principle be a very complicated object since the space of pure strategies is so large. However, it turns out that we only need to look at simple distributions. In this paper we discuss the case where the offers of transfers made by candidate $i$ to voters are realizations of the same random variable with c.d.f. $F_i : \mathbb{R} \rightarrow [0, 1]$. Of course, the fact that offers are realizations of the same random variable does not mean that each voter gets the same offer. Note also that at stage 2 each voter observes his realized promises, not random variables.

Because there are infinitely many voters, $F_i$ will be the empirical distributions of offers
in the electorate; $F_i(x)$ is the fraction of voters who receive promises below $x$ from candidate $i$. By manipulating $F_i$, candidate $i$ is able to target transfers to sections of the populations.

In the following sections we therefore reduce the representation of a mixed strategy to the probability of offering the public good (denote this by $\alpha_i$) and the distribution of transfers $F_i(\cdot)$ under candidate $i$’s strategy if he chooses to offer transfers. The budget constraint then is $\int_{-\infty}^{\infty} x dF_i(x) = 0$. The lower support of integration is explained by the fact that voters cannot be taxed more than their endowment.

Let $S(F_i, F_h)$ denote the share of the vote of candidate $h$ if he promises to transfer according to $F_h$ and candidate $i$ promises to transfer according to distribution $F_i$. The share of the vote of candidate $h$ is equal to the probability that any random voter receives an offer from $h$ which is higher than the offer he receives from $i$. Thus,

$$S(F_i, F_h) = \int_{-1}^{\infty} F_i(x) dF_h(x)$$

The game among candidates is constant sum and symmetric under either assumption on candidates’ objectives. Hence, in equilibrium both candidates must get 50 percent of the votes in expectation.

D. Discussion of the Model

A few features of our model are worth discussing. First, the nature of the redistribution. In this model private benefits can be targeted to arbitrarily small subsets of the population. Thus, the policy space is not exogenously restricted by grouping voters and then requiring different voters in the same group to receive the same electoral promise. Of course, for the logic of our results it is not necessary that candidates can literally target transfers to
each individual voter. We only need that candidates can discriminate among a relatively large number of groups of voters. This is certainly possible in reality since transfers can be different for voters in different geographical location, age group, profession, ....

Second, in our model each voter votes for the candidate who offers him the most. This is in contrast with models a’ la Lindbeck and Weibull (1987) or models of probabilistic voting, where increasing the electoral promise to any group delivers a continuous change in share of the vote from that group. The potential discontinuity of the share of the vote in our model, as one candidate’s promise to a voter equals and then exceeds his competitor’s, explains the role for mixed strategies. Mixed strategies provide the smoothness in candidates’ payoffs that allows us to obtain an equilibrium. This smoothness is exogenously assumed in models of probabilistic voting.

The assumption of identical voters is useful for highlighting the differences from the literature since we show that heterogeneity is not a driving force of our results. This assumption has the additional benefit of considerably simplifying the analysis. In Lizzeri and Persico (1998) we allow for a limited measure of voter heterogeneity and show that the substance of our results is unchanged.

Finally, the budget constraint. We require that transfers satisfy a budget constraint which is determined by the amount of public resources that a candidate would be able to allocate in case of winning the election. This is in keeping with the assumption of fully rational and perfectly informed voters. In connection with the budget constraint note that, with a finite population of $N$ voters, electoral promises could not be independent across voters, because the promises would have to sum up to the budget constraint. However, as
the number \( N \) goes to infinity, independent promises can be constructed so that the average deviation from the budget constraint converges to 0 with probability one. This is why we assume an infinite number of voters in our model.\(^9\)

III. **Winner-Take-All vs. Proportional System**

If \( G > 2 \), then under either assumption on candidates’ objectives the unique equilibrium involves both candidates promising to provide the public good. To see this observe that if candidate \( i \) offers the public good, he ties in the event candidate \( h \) also offers the public good, and wins if candidate \( h \) offers transfers since, because of the budget constraint, candidate \( h \) cannot offer more than \( G - 1 > 1 \) to more than 50 percent of the voters. Conversely, if candidate \( i \) chooses to offer money he gets less than 50 percent of the votes in the event that the other candidate offers the public good.\(^{10}\)

Section A. describes the benchmark case where \( G < 1 \), hence provision of the public good is dominated by a strategy of offering the same (zero) transfer to every voter.

Section B. and C. analyze the interesting parameter range, when \( 1 < G < 2 \). To develop an intuition for the results, it is useful to consider a simple example. Suppose \( G = 3/2 \) and assume that candidate 1 offers to provide the public good. Then candidate 2 can obtain almost 2/3 of the votes by offering a transfer of a little more than 1/2 (hence a utility of more than \( G \)) to a little fewer than 2/3 of the voters. These transfers are financed by taxing the entire endowment of the remaining voters. Suppose now that the value of the public good is 7/4. Now candidate 2 can offer more than the value of the public good to fewer than 4/7 of the voters. The number of voters to whom candidate 2 can offer more than the value
of the public good declines with $G$ although it is still sufficient to win as long as $G < 2$. This illustrates two fundamental aspects of the problem: First, provision of the public good is going to be inefficient since its benefits are not as targetable as transfers are. Second, in the proportional system the incentive to redistribute declines as the value of $G$ increases, while in the winner-take-all system the incentive to redistribute remains constant.

For values of $G$ between 1 and 2 there is no equilibrium in pure strategies. To see this, suppose candidate 1’s strategy was $\Phi_1$. If $\Phi_1(v) = G - 1$ for all $v$, i.e. candidate 1 promises each voter the public good, then candidate 2 can choose to promise more than $G - 1$ to more than 50 percent of the voters and obtain more than 50 percent of the votes. This is impossible in equilibrium. Suppose then that candidate 1 chooses to offer money. Now candidate 2 could take a set of voters $V_1$ with small positive measure such that $\Phi_1(v) > -1$ for $v \in V_1$, tax all their endowment and use the money to finance offers of $\Phi_1(v) + \epsilon$ to all other voters. The set $V_1$ and the $\epsilon$ can be chosen so that candidate 2 wins with a share of the vote arbitrarily close to one hundred percent. Thus, at equilibrium both candidates will be employing mixed strategies. We want to stress that there is a natural interpretation for these mixed strategies: choosing $F$ should be thought of as choosing the Lorenz curve, i.e. the empirical distribution of transfers, in the population.

A. The Game of Pure Redistribution

Before discussing how the provision of public goods differs in the two systems it is useful to show that the equilibrium in the game of pure redistribution is the same under the two systems.
Proposition 1 (Myerson (1993)). Suppose $G < 1$. Then the unique equilibrium under the winner-take-all and proportional systems involves both candidates drawing offers to all voters from a uniform distribution on $[-1, 1]$. Thus, for $i = 1, 2$

$$F_i(x) = \begin{cases} 
0 & \text{for } x \leq -1 \\
\frac{x+1}{2} & \text{for } -1 \leq x \leq 1 \\
1 & \text{for } 1 \leq x
\end{cases}$$

(1)

Proof. Let us show that this is an equilibrium. First, notice that the share of the vote from offering the public good is $F_1(G - 1) < F_1(0) = 1/2$, so offering the public good is a bad idea since it yields less than 50 percent of the votes. Suppose now $F_1$ satisfies equation 1, and candidate 2 offers money according to a distribution $F$. To satisfy the budget constraint it must be $\int_{-1}^{\infty} xdF(x) = 0$. We then have:

$$S(F_1, F) = \int_{-1}^{\infty} F_1(x)dF(x)$$

$$\leq \int_{-1}^{\infty} \frac{x+1}{2}dF(x) = \frac{1}{2} = S(F_1, F_2)$$

The second-to-last equality holds because of the budget constraint. Therefore deviation cannot increase the share of the vote for candidate 2. For a proof of uniqueness see Myerson (1993) and Alessandro Lizzeri (1997).

Notice that in this game candidates will choose to tax and redistribute even when no public good is provided at equilibrium. This is in contrast with the standard, median voter, models of democratic provision of public goods. This contrast is due to the fact that in our model the redistributive tools of candidates are not restricted to linear taxes.

When voters are risk-neutral, the outcome of the political process is efficient: since $G < 1$ citizens value money more than they value the public good, and they do not mind the risk
involved in the redistribution process.\textsuperscript{11}

B. A Winner-Take-All System

**Theorem 2** Suppose $1 < G < 2$. Under the winner-take-all system in the unique equilibrium both candidates offer the public good with probability $\alpha(G) = \frac{1}{2}$ for $G \in (1, 2)$. When candidates offer money they choose transfers according to the following distribution:

$$F^*(x) = \begin{cases} 
0 & \text{for } x \leq -1 \\
\frac{1}{2} \left( \frac{x+1}{2-G} \right) & \text{for } -1 \leq x \leq 1-G \\
\frac{1}{2} & \text{for } 1-G \leq x \leq G-1 \\
\frac{1}{2} \left( 1 + \frac{x+1-G}{2-G} \right) & \text{for } G-1 \leq x \leq 1 \\
1 & \text{for } 1 \leq x 
\end{cases}$$

(2)

When $G > 2$, the unique equilibrium is to offer the public good for sure: $\alpha(G) = 1$ for $G \in (2, \infty)$.

**Proof.** Case $1 < G < 2$ : We want to show that this is an equilibrium. First, the strategy $F^*$ is feasible since the average sum spent is $\int_{-1}^{\infty} x dF^*(x) = 0$. Suppose candidate 2 follows the equilibrium strategy: then the share of the vote of a candidate 1 who redistributes according to $F$, and meets candidate 2 who promises transfers is:

$$S(F^*, F) = \int_{-1}^{\infty} F^*(x) dF(x).$$

(3)

When $F$ is a best-response, it is never the case that the offers of candidate 1 fall outside candidate 2’s support: formally, $F(-1) = 0$ and $F(1) = 1$. The share of the vote accruing
to candidate 1 (expression (3)) is

\[
S(F^*, F) = \frac{1}{2} \left[ \int_{-1}^{1-G} \left( \frac{x + 1}{2 - G} \right) dF(x) + F(G - 1) - F(1 - G) \\
+ \int_{G-1}^{1} (1 + \frac{x + 1 - G}{2 - G}) dF(x) \right]
= \frac{1}{2} \left[ \frac{M_1 + M_2}{2 - G} + \frac{1 - G}{2 - G} (F(G - 1) - F(1 - G)) \\
+ (1 - \frac{G}{2 - G})(1 - F(G - 1)) + \frac{1}{2 - G} \right]
\]

where

\[
M_1 := \int_{-1}^{1-G} xdF(x)
\]

and

\[
M_2 := \int_{G-1}^{1} xdF(x)
\]

are the total transfers generated by promises in the interval \([-1, 1-G]\) and \([G-1, 1]\), respectively. It is obviously never a best response to promise anything in \((1-G, G-1)\) nor is it optimal to promise \(G-1\) to any positive measure of voters: indeed, any strategy that promises \(G-1\) to a mass \(m\) of voters is dominated by one that is identical, except that \(\varepsilon\) of the voters previously being offered \(G-1\) are now offered \(1-G\), and the remaining \(m-\varepsilon\) are offered \(G-1+\delta\). Thus, we can safely restrict to checking those deviations \(F\) for which

\[
F(G-1) - F(1-G) = 0.
\]

The above expression then reads

\[
S(F^*, F) = \frac{1}{2} \left[ \frac{M_1 + M_2}{2 - G} + (1 + \frac{G}{2 - G} - F(G - 1)) + \frac{1}{2 - G} \right]
\]

(4)

The problem of candidate 1 is to choose an \(F\) under the constraint that \(M_1 + M_2 \leq 0\) (budget constraint). First, clearly the candidate will choose to make \(M_1 + M_2 = 0\). Second, whenever \(F(G-1) < \frac{1}{2}\) we have \(S(F^*, F) < \frac{1}{2}\), so candidate 1 is sure to lose against
redistribution (but to win against the public good); whenever \( F'(G - 1) > \frac{1}{2} \), the candidate is sure to win against redistribution but is sure to lose against the public good. Finally, when \( F(G - 1) = \frac{1}{2} \) the candidate ties against the public good and against redistribution. Since \( \alpha = 1/2 \), the candidate is indifferent between any \( F \) such that \( M_1 + M_2 = 0 \). Similarly, given that candidate 2 plays the equilibrium strategy, candidate 1 is indifferent between offering the public good and offering money because the probability of victory is 1/2 in either case.

**Case \( G > 2 \):** straightforward, since redistributing resources cannot give utility greater than \( G \) to more than 50 percent of the voters.

**Uniqueness:** see Lizzeri and Persico (1998).

C. A Proportional System

We now discuss the system where candidates maximize the share of the vote.

**Theorem 3** Suppose \( 1 < G < 2 \). Under the proportional system in the unique equilibrium involves both candidates choosing to offer the public good with probability \( \beta(G) = G - 1 \) for \( G \in (1, 2) \). When candidates offer money they choose transfers according to the same distribution as in Theorem 2.

When \( G > 2 \), the unique equilibrium is to offer the public good for sure: \( \beta(G) = 1 \) for \( G \in (2, \infty) \).

**Proof. Case \( 1 < G < 2 \):**

Suppose that candidate 2 plays according to the equilibrium. As in the proof of Theorem 2 if candidate 1 offers money according to a distribution \( F \) and candidate 2 offer money according to distribution \( F^* \), we can write candidate 1’s share of the vote according to
equation (4). For the same reason as in the proof of Theorem 2, candidate 1 will choose
\( M_1 + M_2 = 0 \). Thus, we can rewrite equation (4) as follows:

\[
S(F^*, F) = \frac{1}{2(2 - G)} [2(G - 1)F(G - 1) + 3 - 2G]
\]

(5)

If candidate 2 chooses the public good instead, candidate 1’s share of the vote if he offers money according to \( F \) is \( 1 - F(G - 1) \). Thus, given that candidate 2 chooses the public good with probability \( G - 1 \) and money with probability \( 2 - G \), candidate 1’s expected share of the vote when he offers money is:

\[
(G - 1)(1 - F(G - 1)) + (2 - G) \frac{1}{2(2 - G)} [2(G - 1)F(G - 1) + 3 - 2G] = \frac{1}{2}
\]

Thus, given that candidate 2 plays according to the equilibrium strategy candidate 1’s share of the vote is \( 1/2 \) for any distribution \( F \). Furthermore, it is clear that given that candidate 2 plays according to \( F^* \), candidate 1’s share of the vote from choosing the public good is also \( 1/2 \). Thus, candidate 1 is happy to play according to the equilibrium strategy.

**Case** \( G > 2 \): identical to Theorem 2.

**Uniqueness**: see Lizzeri and Persico (1998). ■

In the above stylized model of proportional representation we implicitly assume that the implemented policy is that of the winning candidate. In reality, minority parties may have an influence on policy and the final outcome might depend in a complicated way on the outcome of some post-election bargaining game. We now show that the conclusions of this section are unchanged when we allow for more general ways of implementing policies after an election.
Consider a function $\pi (s) \to [0, 1]$. This function represents the probability that the implemented policy will be the platform of candidate 1 when candidate 1 has a share of the vote of $s$. We assume that $\pi$ is non-decreasing in $s$, and that $\pi (1/2) = 1/2$. Thus, the function $\pi$ may be thought of as a reduced form of a bargaining game between the two candidates.\textsuperscript{12} Given any such $\pi$, optimal behavior for voters still involves voting for the candidate who promises them the higher utility: this increases the chance that the policy of the favorite candidate is implemented.\textsuperscript{13} This implies that candidates’ equilibrium strategies are unchanged relative to our previous analysis, since candidates only care about shares of the vote, and not about policy. Thus, at equilibrium the public good is offered with probability $\beta (G)$ and, no matter what a candidate offers, he obtains a share of the vote of exactly 1/2. Since $\pi (1/2) = 1/2$, in equilibrium the probability that the public good is provided does not change from the outcome of our simple model.

D. Discussion and Comparison

<INSERT FIGURE 1 ABOUT HERE>

The distribution of money across voters is the same in both systems and is illustrated in the Figure. As the value of the public good $G$ increases, this distribution becomes more concentrated on the extremes. The probability of provision of the public good in the winner-take-all system is independent of $G$ for $1 < G < 2$. When candidates maximize the share of the vote the equilibrium probability of providing the public good $\beta (G)$ is not constant in
$G : \beta$ goes from 0 to 1 as $G$ increases from 1 to 2. We discuss these features below.

The presence of the public good makes redistribution more extreme than in Theorem 1. This is because, in order for transfers to be a profitable strategy against the public good, a majority of the voters must receive transfers that exceed the value of the public good; the resource constraint then dictates that the remaining voters receive correspondingly lower transfers. It is also easy to understand why no candidate offers transfers of value close to, but less than, the value of the public good. Indeed, a candidate offering transfers of $G - \varepsilon$ can win these votes only if the opponent redistributes. But then it is profitable to increase these transfers by $2\varepsilon$; this costs marginally more, and wins over the votes also in the event that the opponent promises the public good.

Another interesting feature of the equilibrium is that the redistribution of money is the same in both electoral systems. To understand this fact, observe that if candidate 1 offers transfers and candidate 2 offers the public good, the only relevant aspect of candidate 1’s redistributitional strategy is the fraction of voters who receive transfers that are worth less than the public good, i.e. $F(G - 1)$. Therefore, the consideration that pins down candidate 1’s redistributitional strategy $F(x)$ is that it must be a best response against a candidate 2 who redistributes, i.e. it must not be possible to change $F$ and increase the share of the vote of $F$ against $F^*$ without changing $F(G - 1)$. If this were possible, there would be an incentive to deviate from $F^*$ in both systems. Thus, the condition that pins down $F$ is the same under both systems.

We now discuss the intuition for why public good provision differs between winner-take-all and proportional systems. When a candidate evaluates a deviation in the way he distributes
transfers, offering more than $G - 1$ to more than 50 percent of the voters is profitable when the opponent offers to provide the public good, but not when he redistributes. When an opponent redistributes the best deviation is to downgrade the offers of $G - 1$ down to offers of $1 - G$, since no voter is receiving offers in the interval $[1 - G, G - 1]$. The money saved through this reallocation can be used to buy share of the vote. The vote share gained by this deviation increases with $G$, since the higher $G$ the greater the savings. Thus, in the proportional system the appeal of this deviation increases with $G$. To discourage this deviation, at equilibrium the probability $\beta$ that the opponent offers the public good must be high enough, and increasing with $G$. By contrast, in the winner-take-all system the value of this deviation is independent of $G$ because the margin of victory is irrelevant: as soon as a deviation guarantees more than 50 percent of the votes, it yields a payoff of 1. In fact, the value of the opposite deviation, offering more than $G - 1$ to more than 50 percent of the voters, is also independent of $G$ for the same reason. In the winner-take-all system, to discourage both kind of deviations it must be equally likely that the opponent offers the public good and redistribution, hence $\alpha = 1/2$ independent of $G$.

This discussion suggests that what matters for our results is that, when confronted with an opponent who promises the public good, the attractiveness of the option of redistributing resources is different in the two systems.

E. Efficiency and Constitutional Design

We are now in a position to compare electoral systems. When $G < 1$ and $G > 2$ equilibrium is the same under the two systems, so we can concentrate on the case where $1 < G < 2$. It is important to distinguish between ex ante and ex post Pareto efficiency. This is because

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candidates’ strategies involve randomization, and thus there are two notions of allocation, one before and one after the uncertainty is realized. Before electoral promises are made, allocations are lotteries over consumption. Ex ante Pareto efficiency ranks such allocations, taking the point of view of a voter who considers the expected utility of the outcome of the election before receiving an electoral promise. In contrast, after the winning policy is implemented, an allocation is a consumption vector that specifies how much each voter consumes. Ex post Pareto efficiency ranks such allocations, from the viewpoint of a voter who has already received this consumption.

In the Proposition below we show that, under both electoral systems, while the equilibrium is ex post efficient, it is ex ante inefficient. It is also possible to rank electoral systems in terms of ex ante efficiency.

Since each candidate is equally likely to get elected, the probability of provision of the public good is 1/2 in the winner-take-all system and $G - 1$ in the proportional system: when $G < 3/2$ the probability of provision is higher in the winner-take-all system, when $G > 3/2$ it is higher in the proportional system. Thus, risk neutral voters prefer (ex ante) the proportional system if and only if $G > 3/2$.

If voters are risk averse, they care not only about the probability of public good provision, but also about the distribution of transfers at equilibrium. However, notice that while the probability of providing the public good may differ in the two systems, the distribution of transfers is the same; furthermore, whenever the outcome of the two systems differ, the bet associated to the public good has higher mean, and lower spread, than the bet associated with transfers. Hence, risk-averse voters are ex ante unanimous in their preference for the
system that yields higher provision of the public good, just as in the case of risk-neutrality.

**Proposition 4** (i) *The equilibrium outcome is ex post efficient in both electoral systems for any value of G.*

(ii) *For* $1 < G < 2$, *the equilibrium outcome is ex ante inefficient in both electoral systems.*

(iii) *Suppose voters are risk averse or risk neutral. Then when* $G > (<) 3/2$ *the proportional (winner-take-all) system yields higher ex ante utility for voters.*

**Proof. Part i.** It is obvious that if $G < 1$ or $G > 2$ the equilibrium is *ex post* efficient. Suppose therefore that $1 < G < 2$. If ex post it turns out that the public good was provided, this is an *ex post* efficient allocation because there is no distribution of transfers that gives more utility to all voters. Suppose then that the public good was not provided. This implies that the empirical distribution of money across the population is as specified by equation (2). Clearly, this allocation cannot be dominated *ex post* by any other distribution of money. Moreover, providing the public good requires that all the money be taken away from all voters. Thus, the 50 percent of the voters who in equilibrium get more than $G$ would be worse off *ex post* if the public good is provided.

**Part ii.** Consider the winner-take-all system. To see that the equilibrium outcome is *ex ante* inefficient, observe that for each voter, from the *ex ante* perspective, the equilibrium outcome is a lottery that puts probability $\alpha(G)$ on the provision of the public good and probability $1 - \alpha(G)$ on the distribution in equation (2). *Ex ante*, all voters prefer the allocation that puts probability one on the provision of the public good. The same reasoning takes care of the proportional system.
**Part iii.** Denote with $H_P$ and $H_W$ the equilibrium distribution of utilities in the proportional and winner-take-all systems, respectively. It is easy to verify that the function

$$D(y) = \int_{-\infty}^{y} (H_P(z) - H_W(z)) \, dz$$

is greater than zero for all $y$ if and only if $G < 3/2$. Thus, $H_W$ dominates $H_P$ in the sense of second order stochastic dominance if and only if $G < 3/2$.

Ex ante efficiency appears to be a more natural concept when evaluating the efficiency of political systems. This is because the choice of a political system precedes the electoral stage. At the stage of constitutional design, it is reasonable to believe that citizens evaluate political systems knowing the equilibrium distribution of resources, but before receiving a specific electoral promise.

**IV. The Inefficiency of the Electoral College**

In some electoral systems, candidates to national level offices, like president, are selected on the basis of the majority of votes in a nationwide district. This is the system that was discussed in sections B. and C.. In other systems, like the United States for presidential elections, the selection is made on the basis of a majority of votes in a majority of districts. This is the electoral college system. Similarly, in Britain, to form a majority government it is only necessary to win a majority of the districts, and only a majority of the votes in each of those districts is required. Thus, the electoral college model is also a model of the British (Westminster) system. Here we argue that the electoral college system is even more inefficient than the systems described in the previous sections in generating incentives for candidates to effectively provide public goods.
The electoral college system is characterized by a winner-take-all rule both at the district level and at the national level. To win in our two-candidate model, a candidate must obtain more than 50 percent of the votes in more than 50 percent of the districts. A district will be denoted by \( d \). We assume a continuum of measure 1 of districts.\(^{16}\) All districts are identical both in size and in the benefits they receive from the public good. A strategy must now specify, in the case that redistribution is chosen, the aggregate transfer to each district as well as the distribution inside the district.

The electoral college system is more inefficient than a nationwide winner-take-all system. This increase in the inefficiency is due to the fact that a candidate who offers to provide the public good must now worry about his opponent going after the majority of voters in the majority of districts. If \( G < 4 \), a candidate can offer more than \( G - 1 \) to more than 50 percent of the voters in more than 50 percent of the states and zero to the rest. Such a strategy leads to sure victory against a candidate who chooses to offer the public good with probability one. Thus, if \( G < 4 \) there is no equilibrium where the public good is provided with probability 1.

**Theorem 5** Suppose elections are conducted under an electoral college system. Let \( \mu_i^d \) be the average transfer offered by candidate \( i \) to voters in district \( d \).

(i) Suppose \( G < 1 \). In equilibrium candidates never promise to provide the public good, and both candidates make offers to voters according to the following process: each candidate draws \( \mu_i^d \) from a uniform distribution on \([-1, 1]\). In district \( d \) candidate \( i \) makes offers to voters according do a uniform distribution on \([-1, 2\mu_i^d + 1]\).

(ii) Suppose \( 1 < G < 4 \). The equilibrium probability of providing the public good is less
than or equal to 1/2.

(iii) For $G > 4$, the public good is provided for sure at equilibrium.

Proof. Part (i) Take a district where candidate $i$ has dedicated $\mu^d_i$ resources and candidate $h$ has dedicated $\mu^d_h$ resources. If $\mu^d_i > \mu^d_h$ then, by following the equilibrium strategy, candidate $i$ gets strictly more than 50 percent of the votes in district $i$ and wins the district. This means that the candidate with more resources in the district gets all the electoral college votes in the district. If $\mu^d_i < \mu^d_h$, party $h$ wins the district, and if $\mu^d_i = \mu^d_h$ then the two candidates split the district equally. Moreover, given $\mu^d_i, \mu^d_h$ and given the fact that candidate $i$ is distributing the resources uniformly, candidate $h$ cannot do any better than choose a uniform. With any other distribution he would still lose if $\mu^d_i > \mu^d_h$, win if $\mu^d_i < \mu^d_h$ and tie if $\mu^d_i = \mu^d_h$. The fact that candidate $h$ does not know $\mu^d_i$ when choosing the optimal distribution of resources in district $d$ does not change the optimality of the uniform distribution.

Let us now consider how candidates distributed resource across districts. By preceding argument, districts behave exactly like voters: if a candidate dedicates more resources to the district he gets the district. Thus, choosing $\mu^d_i$ from a uniform distribution on $[-1, 1]$ is part of an equilibrium.

Part (ii) Suppose that candidate 1 was offering to provide the public good with probability greater than 1/2. Since $G < 4$, candidate 2 could offer more than $G$ to more than 50 percent of the voters in more than 50 percent of the districts: this leads to an expected payoff strictly greater than 1/2, which is impossible at equilibrium.

Part (iii) Since $G > 4$, when your opponent follows the equilibrium strategy there is no way to use transfers and offer more than $G$ to more than 1/4 of the voters, and that is the
minimum share of the vote needed to reach an expected payoff of 1/2. So, offering money is
dominated by offering the public good. ■

In the previous result, we have not proved that an equilibrium exists when \( 1 < G < 2 \).
In any finite game that approximates our game, an equilibrium will exist and will exhibit
the same qualitative features presented in Theorem 5.

The logic of Theorem 5 (i) is a hierarchical version of the logic of Theorem 1. The forces
that generate a uniform distribution in a nationwide election also push toward a uniform
distribution in each state. However, it cannot be an equilibrium that all districts get the same
resources. If this were the case a candidate could deviate by targeting higher average offers
to a majority of the districts. Thus, there must be an ex-post difference in the amount of
resources that each candidate offers to voters in different districts. It is then straightforward
to apply the same logic to the distribution of resources across districts. This yields a uniform
distribution on \([-1,1]\).

When \( G < 1 \), there is a big difference between the equilibrium in a nationwide district
election discussed in Proposition 1 and the outcome described in Theorem 5. This reflects
the incentives to “go after 25 percent of the voters” in the electoral college system as opposed
to going after the majority in a nationwide district. The aggregate distribution of resources
is much more unequal in the electoral college system. Recall that in Proposition 1 we saw
that in a nationwide district voters’ consumption is distributed uniformly on \([0,2]\). In the
electoral college some voters get to consume as much as 4 while a lot more voters consume
less than 1. More formally, the outcome of Theorem 5 is dominated by the nationwide
district outcome in the sense of second degree stochastic dominance;\(^{17}\) thus, whenever voters
are risk-averse they will prefer a system with a nationwide district.

The contrast between Theorems 2 and 5 is also interesting. When $G > 1$, the electoral college system delivers a probability of providing the public good which is not higher than in the nationwide district when $1 < G < 2$, and is strictly lower when $2 < G < 4$.

The result in Theorem 5 is related to the literature that discusses the inefficiency of pork barrel spending in legislatures where representatives represent different districts. Chari et al. (1997), for instance, show that voters in each district have an incentive to select a representative who is strongly in favor of pork-barrel spending. This arises because of a common pool problem where candidates in each district do not take into account the costs to other districts. Our setup differs from these models since we focus on national candidates who do not represent any specific district: when proposing to increase transfers to a district, our candidates take into account the loss of another district.

V. Conclusion

We have presented a political-economic model where the provision of a public good is determined by the electoral incentives of office-seeking candidates. When candidates have the option of redistributing resources, public goods will be underprovided relative to the efficient outcome. This happens because benefits from the public goods cannot be targeted to groups of voters as easily as the benefits from pork barrel projects or pure transfers.

In this setup, we have compared different electoral systems: the electoral college system is always less efficient than a system with a nationwide district. The winner-take-all system is less efficient than a proportional system when the public good is very desirable, and is
more efficient when the public good is not very desirable. In both the winner-take-all and in the proportional system, the redistributinal platforms that candidates propose become less egalitarian as the public good becomes more desirable.

This model addresses an important feature of political competition, the trade-off between efficiency and targetability. Policies whose benefits are uniform across the population are penalized in a political equilibrium, relative to policies whose benefits are (or can be made) more concentrated.\textsuperscript{18}

Of the many simplifications that make this model easy to analyze, three merit particular emphasis. First, our model is one of office seeking candidates and ignores the possibility that politicians can be driven by ideological considerations. This is a very stylized representation, but one that, we believe, captures an important incentive in electoral competition. It would be desirable to extend this model to allow for politicians that are motivated both by ideological considerations and the benefits of office.\textsuperscript{19} Such an extension would allow a comparison with citizen-candidate models where politicians care about the policy, and cannot commit to electoral platforms.\textsuperscript{20}

The assumption that electoral promises are binding is an extreme; in reality a politician cannot perfectly commit to honor his promises. It would be desirable to understand how the analysis would change if commitment were only imperfect, but this requires a dynamic analysis that is beyond the scope of the present paper.\textsuperscript{21}

Finally, an unrealistic feature of our analysis is that with positive probability some voters are taxed all their endowment, they are fully expropriated. This is a consequence of the assumption that candidates know voters’ endowments and taxes are non distortionary. If
either of these assumptions is relaxed, all voters would retain control of a positive amount of resources.

We feel that the most interesting extensions of our model involve further explorations of the political process. It would be interesting to remove the restriction to two candidates. This would be particularly important for a comparison of proportional representation with plurality rule, since these systems seem to be associated with different numbers of candidates. What we show in this paper is that there are differences between the two political systems even when they have the same number of candidates (two). We expect the main features of our analysis to carry over to the environment with more than two candidates, so long as the proportional system allocates benefits to politicians in a less extreme fashion than plurality rule.

Such an extension would permit an analysis of issues that were mostly ignored in this paper, namely the process of legislative bargaining and coalition formation, and its influence on the provision of public goods. Austen-Smith and Banks (1988) provide a benchmark analysis of this issue in a unidimensional spatial model with strategic voters. It would be desirable to build on their analysis to extend our model in this direction. This is a matter for future research.


Footnotes

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1. Here we identify pork-barrel projects (or local public goods) with redistribution, not public goods. This is because the benefits from pork-barrel projects are concentrated on few voters. Indeed, pork-barrel projects are widely understood as a means of (non-monetary) redistribution disguised as public goods. What characterizes public goods in our model is that their benefits cannot be easily targeted to subsets of the population.

2. See Section E. for a discussion of the notion of efficiency used in the paper.

3. The informal idea that distortions will arise out of majoritarian decision making if policies with concentrated benefits coexist with policies whose benefits are diffuse, can be attributed to James Buchanan and Gordon Tollock (1962).

4. In the conclusion we discuss these issues further.

5. An exception is Stephen Coate and Stephen Morris (1995), who focus on the fact that politicians have private information on the effects of government policy.

6. This is a convenient assumption because it will allow us to invoke the law of large numbers on a number of occasions. This is meant to be an approximation for a game with a large (but finite) number of voters.
7. Alessandro Lizzeri and Nicola Persico (1998) relax this assumption and show that the analysis is robust.

8. This assumption is made solely to simplify the notation and presentation of the results; all the results hold for any increasing utility function. Thus, we will feel free to interpret the results for the case in which voters are risk averse.

9. See Myerson (1993), footnote 1, for further discussion.

10. Consider an election with \( n \) candidates. Let all candidates offer the public good, and assume that at this strategy combination each candidate receives a share of the vote of \( 1/n \). Then, it is only when \( G \) is greater than \( n \) that this is an equilibrium. When \( 1 < G < n \) the equilibrium is going to be inefficient. This suggests that the inefficiency is increasing in the number of candidates. However, a full analysis of the case of \( n \) candidates is not an easy extension of the two candidate model.

11. For a discussion of the appropriate notion of efficiency see Section E. With risk neutrality and \( G < 1 \), the equilibrium outcome is efficient according to all definitions discussed in that section.

12. Because of our assumption of discrete investment in the public good, it is appropriate to model the compromise between two parties as a probability rather than, say, a coefficient in the convex combination between the two policies. A policy outcome that is a convex combination between public good and redistribution would be inconsistent with that assumption.

13. More precisely, this behavior is the limit of optimal behavior in finite elections.
14. Timothy Besley and Stephen Coate (1998) discuss a model where it is important to distinguish between alternative notions of efficiency in a dynamic setting.

15. Another possible notion of efficiency is surplus maximization. Notice that in our model voters are treated equally *ex ante* by a candidate’s strategy. For such strategies, surplus maximization coincides with *ex ante* Pareto efficiency.

16. Just as in the previous analysis this assumption is made for convenience, to avoid the complications of meeting the budget constraint with a stochastic strategy. Things would be approximately the same when there are a large number (e.g., 50) of districts.

17. A random variable $A$ dominates a random variable $B$ is the sense of second order stochastic dominance iff

$$B \sim A + \varepsilon, \text{ with } E(\varepsilon|A) = 0.$$ 

That is, $B$ is equal, in distribution, to $A$ plus a noise term. This definition is equivalent to the one presented in the proof of Proposition 4.

18. The analysis can be extended to allow for voters who have heterogeneous valuations for the public good (see Lizzeri and Persico (1998)). In this case the benefits from the public good are less uniform across the population, and this makes the public good more likely to be provided than when voters are homogeneous. This is in contrast with the conclusions from the textbook median-voter model, where public goods provision becomes more efficient as the population becomes more homogeneous.

19. For an empirical discussion of the importance of ideology in legislatures see for instance Keith Poole and Howard Rosenthal (1996).
20. See Besley and Coate (1997) and (1998), and Martin Osborne and Al Slivinski (1996).

21. See Persson et al. (1997) for a recent dynamic analysis of policy making that does not assume commitment on the part of politicians.

22. David Baron and Daniel Diermeier (1998) provide a more recent analysis of similar issues. They focus on the dynamics of parliamentary systems.
TITLE FOR FIGURE 1: “Redistribution of resources at equilibrium”