Your examination contains four questions, each of which carry the same weight. You have two hours to complete the exam. Remember that long answers by themselves do not have any value. Try to keep your answers brief and to the point.

**Total Points: 80. Each question carries the same points.**

**[1] (20 points)** A group of $n$ agents are engaged in a joint venture. The shares in the output of the joint venture are given by the numbers $\lambda_1, \lambda_2, \ldots, \lambda_n$, where these are nonnegative and sum to one. The output of the joint venture is equal precisely to the sum of the efforts contributed by the $n$ players. If $e_i$ is the effort contributed by person $i$, his cost is given by $(1/2)e_i^2$.

[i] (5 points) Precisely set up and solve for the efficient effort contributions and estimate the total potential surplus available from the joint venture.

[ii] (5 points) Now set up the problem as a game of voluntary contributions and solve for the Nash equilibrium. Derive an expression for the total surplus under Nash equilibrium.

[iii] (5 points) Find the allocation of shares that maximizes total surplus under the Nash equilibrium. What happens to this maximum surplus — in per-capita terms — as the number of players gets larger? Explain your answer intuitively.

[iv] (5 points) Discuss the reasons for any difference between the results you obtain here and the corresponding results discussed in class for the case in which effort costs are linear.

**[2] (20 points)** Consider the following model of income distribution. There is a continuum of individuals, indexed on $[0,1]$. We start with a cumulative distribution of wealth (= inheritance) on $[0,1]$. Individuals must decide to become entrepreneurs or workers.

If the former, they must make an upfront investment of $I$ from their own resources, whereupon they can open a firm. There are no capital markets by assumption, so this is possible only if their inheritance exceeds $I$. Output is a strictly concave function $f(L)$ of labor input $L$. The net income of entrepreneurs is the (variable) profit of their firm, plus leftover inheritance, if any, after their investment. Of this, a given fraction is consumed, and the rest bequeathed to a single successor.

If the latter, no investment need be made. Workers enter the labor market. Each worker supplies inelastically one unit of labor, and receives a wage. They consume the same given fraction of their wage plus inheritance (if any), and bequeath the rest.

There is no uncertainty.

[i] (7 points) Set up this model formally. Be sure to include the following details: (a) describe in each period the equilibrium choices, given some wage, (c) show how labor supply and demand are matched to obtain the equilibrium in each period, and (e) along the equilibrium, describe the evolution of the inheritance distribution over time (assume that the intertemporal rate of return on inheritances is zero).

[ii] (7 points) Describe what the steady state distribution of inheritances looks like, and explain why there may be multiple steady states. Use your model to show the interplay between
distributional effects and aggregate outcomes (such as wages, labor, aggregate income), by moving over different steady state inheritance distributions.

[iii] (6 points) Discuss the main parallels and differences from the Galor-Zeira model studied in class.

[3] (20 points) Suppose that a village couple’s fertility decisions are influenced by the number of children per-capita born to the previous generation in that village. Specifically, assume that if the per-capita number of children in the previous generation is \( n_{t-1} \), then the couple’s psychological return from having \( n \) children today is given by \( a(n_{t-1})n - c(n) \), where \( a \) is an increasing function describing this couple’s benefits from having children and \( c \) is a smooth convex increasing function describing the costs of children.

Assuming that all couples in all generations are identical, find an implicit equation that links fertility choices across generations. By using graphical analysis to describe the steady states of this equation, show that a temporary policy designed at reducing fertility may have persistent and long run effects.

[4] (20 points) Consider the version of the Acemoglu-Zilibotti model of financial markets studied in class, with the following modifications (a) each agent in a continuum \([0, 1]\) of identical individuals has one unit of wealth in the safe asset and a total of \( w \) units of wealth in the risky sectors, and this division between “safe” and “risky” cannot be changed, (b) there are only three securities (and three states of nature) in the risky sector, labelled 1, 2 and 3, with minimum scale requirements given by the values 0, \( a \), and \( 2a \) respectively (for some \( a > 0 \)), (c) the riskfree return \( r = 1 \) and the risky return \( R = 2 \). [In all other respects, retain the static allocation model studied in class, with uniform uncertainty and logarithmic preferences.]

[i] (6 points) For each value of \( w \), describe the possible equilibria in the financial sector (which sectors are open, which allocations prevail, the indirect utility received by each agent, etc).

[ii] (7 points) Without (necessarily) attempting to solve this completely, show how you would go about formulating the planner’s problem and the conditions that need to be satisfied for this solution to be different from the equilibrium. Distinguish between the cases \( w \in [3a, 4a) \), \( w \in [4a, 6a) \) and \( w \geq 6a \), and explain why these distinctions are important.

[iii] (7 points) Prove that there are values of \( w \) for which only two sectors are open, it is feasible to open three sectors (that is, \( w > 3a \)), and yet the planner’s solution coincides with that of society. Describe how this observation differs from the Acemoglu-Zilibotti model, and explain the reason for the difference.