Economic Development Final, 1999

Your examination contains three questions, each of which carry the same weight. You have two hours to complete the exam. Remember that long answers by themselves do not have any value. Try to keep your answers brief and to the point.

Total Points: 80. Each question carries 25 points. I reserve 5 points for overall clarity in your exposition.

[1] (25 points) A laborer faces an infinite sequence of slack and peak seasons. He has an inter-seasonal discount factor of $\delta$. To begin with, assume that he has no access to capital markets — he cannot borrow or save. In the slack season he receives an income of $a$, while in the peak season he receives an income of $b$. Of course, $0 \leq a < b$. The laborer has a smooth, strictly concave utility function $u$ defined on consumption in any season, and he would like to smooth his consumption over time.

[i] (5 points) Let $v_s$ and $v_p$ denote the laborer’s normalized lifetime discounted utility viewed from the start of the slack and peak seasons respectively. Calculate these values.

[ii] (6 points) Now view matters from the start of a slack season. Suppose that a lender (with a linear utility function and the same discount factor of $\delta$) offers the laborer $L$ in the slack in return for $R$ in the peak. Assuming that there are no incentive constraints (as yet), characterize the set of all Pareto-optimal $(L, R)$-pairs (that dominate autarky for either agent).

[iii] (6 points) Suppose that a particular stationary contract is in force, but that the borrower could default on repayments in the peak season. If there is a default, assume that the borrower returns to autarky forever. Write down the borrower’s incentive constraint, and show that it implies the borrower’s participation constraint (part (i) may be useful here).

[iv] (8 points) Prove that if $u'(b) \geq \delta^2 u'(a)$, then it is impossible to design a stationary incentive-compatible contract that is better for both parties relative to autarky. On the other hand, if $u'(b) < \delta^2 u'(a)$, and if the lender chooses a stationary contract to maximize his return (subject to the appropriate constraints), show that such a solution always exists and describe it.

[2] (25 points) An individual’s capacity to work (measured in the number of efficiency units that he can supply) is given by the relation

$$\lambda(y) = \begin{cases} 0 & \text{for } 0 \leq y \leq a, \\ (y - a)^\alpha & \text{for } y > a, \end{cases}$$

where $a > 0$ and $\alpha \in (0, 1)$.

[i] (10 points) Suppose that this individual has a nonlabor income of $R$, and a reservation wage of zero. Solve out precisely for (a) the lowest piece rate at which this individual can supply effort, and (b) his total income at this minimum, both as functions of $R$. Take care to distinguish between interior and corner solutions. Show precisely how these quantities change with $R$ as $R$ varies all the way from zero to infinity.
For the rest of this question a purely graphical answer will (minimally) suffice, though you will have to use the features from part (i).

[ii] (7 points) Suppose that there is a total pool of nonlabor income \( Z \). A fraction \( \beta \) of the population have nonlabor income \( Z/\beta \); the rest have zero (it is easy to see that this adds up to \( Z \)). Draw the aggregate supply curve of efficiency units as a function of the market piece rate.

[iii] (8 points) Suppose that \( \beta \) goes up. Show carefully how the supply curve of efficiency units changes. Using this (and fixing some given demand curve for efficiency units), analyze how a change in \( \beta \) affects the equilibrium piece rate and efficiency units, and interpret your findings in the context of the inequality-output results studied in class.

[3] (25 points) An individual derives utility \( u(A, B) = A^\alpha B^\beta \) from the consumption of two goods \( A \) and \( B \). Assume that while \( \alpha \) and \( \beta \) are positive, \( \alpha + \beta \equiv \gamma < 1 \).

(i) (8 points) Normalize the price of \( A \) to 1 and let \( p \) be the (relative) price of good 2. If the individual has an aggregate consumption expenditure of \( c \) (defined in terms of units of good \( A \)), show that the “indirect utility function” \( v \) as a function of \( p \) and \( c \) is given by

\[
v(p, c) = K p^{-\beta} c^\gamma,
\]

where \( K \equiv (\alpha^\alpha \beta^\beta) / \gamma^\gamma \) is just a collection of multiplicative constants. [Feel free to use — without proof — standard properties regarding optimal budget shares of goods under Cobb-Douglas utility.]

Now suppose that goods \( A \) and \( B \) are produced by specific types of labor. One unit of unskilled labor produces one unit of \( A \), while one unit of skilled labor produces one unit of \( B \). To become skilled it costs \( x \) units payable in terms of good \( A \). Wages are paid competitively, so the wage is just the output price.

Each individual in each generation can be either skilled or unskilled (one unit), and their parents make this decision for them and pay for the cost of skill acquisition. Each individual maximizes dynastic utility \( \sum_t \delta^t u(A_t, B_t) \) for some common discount factor \( \delta \in (0, 1) \).

(ii) (17 points) Show that \( p \) is the equilibrium price of \( B \) in some steady state equilibrium if and only if

\[
p^\gamma - (p - x)^\gamma \leq \frac{\delta}{1 - \delta} [(p - x)^\gamma - 1] \leq 1 - (1 - x)^\gamma,
\]

and employing this inequality, prove that such steady states must exist and that all steady states involve \( p > 1 + x \), and that the set of steady state \( p \) lie in a connected interval, all of which display inequality in lifetime utility.

You may use results from the course without proof, but discuss the important conceptual steps that you need.