Answers to Development Midterm, 2000


[2]
(a) [5 points] Suppose that the number of people in each sector is constant over time; say at \( s \) for the traditional sector and \( 1-s \) for the market. Give the necessary and sufficient condition for person \( i \) to stay in the traditional sector without exclusion. Use this to argue that if person \( i \) is in the traditional sector, then so is \( j \) for \( j < i \).

**Answer.** Let \( \delta(i) \) denote the discount factor of person \( i \). Conditional on some trade occurring today, \( i \) can deviate. If he does so, his return is

\[
D + \frac{\delta(i)}{1 - \delta(i)} (1 - s)M,
\]

because he is in the market from tomorrow on where the expected return at each date is just \( (1 - s)M \).

If he does not do so, the return is

\[
T + \frac{\delta(i)}{1 - \delta(i)} sT,
\]

where the first term is \( T \) and not \( sT \) because all this is conditional on a trade occurring today. The second expression must weakly exceed the first for there not to be a deviation; i.e.,

\[
\frac{\delta(i)}{1 - \delta(i)} [sT - (1 - s)M] \geq D - T.
\]

Now we show that if \( j < i \), then the same inequality must hold as well. Two steps are involved. One is to note that \( \frac{\delta}{1 - \delta} \) goes up when \( \delta \) goes up; the second is to note that \( [sT - (1 - s)M] \) is positive (otherwise the above inequality would not hold to start with). Combining, we see that the left-hand side goes up when \( \delta \) goes up.

(b) [5 points] Use part (a) to construct a mapping which has \( s \) — the expected size of the traditional sector — on the horizontal axis, and the fraction of the population that can credibly want to be (without exclusion) in the traditional sector as a “best response” to \( s \). Argue that this mapping represents a complementarity. Describe this complementarity precisely and explain the various sources of the complementarity (in words).

**Answer.** The important part of this question is that I wanted you to recognize that there must be a flat part to the mapping before it starts to climb. In fact, if you did this carefully the next question would have dropped out as a bonus.

Let \( s \) be the expected number of people in the \( T \)-sector. We want to construct the map \( f(s) \) which describes the measure of people who credibly want to be in that sector. Notice first that

\[
f(s) = \text{ for all } s \text{ such that } sT \leq (1 - s)M.
\]
for $s$ beyond this threshold, $f(s)$ is given by the condition that

$$\frac{\delta(f(s))}{1 - \delta(f(s))} [sT - (1 - s)M] = D - T.$$ 

It is easy to see that this uniquely determines $f(s)$ for each $s$. To show that $f$ is strictly increasing in this range, raise $s$. Then $[sT - (1 - s)M]$ must rise. To maintain equality above, \(\frac{\delta(f(s))}{1 - \delta(f(s))}\) must fall. Because $\delta$ is decreasing in $i$, this means that $f(s)$ must rise.

Verbally, the fact that $s$ increases has two effects: it raises the expected value of trades in the traditional sector and lowers it in the market sector. So now the traditional sector is more attractive. This by itself is not sufficient, Now one has to argue that because of this higher attraction, some slightly more impatient people can credibly stay in the traditional sector.

(c) [5 points] Prove that it cannot be an equilibrium for the market to shut down entirely. Prove that it can be an equilibrium for the traditional sector to shut down entirely. Explain the asymmetry in words.

If the market shuts down entirely, then even the most impatient people are in the traditional sector. Notice that the no-deviation constraint now reduces to

$$\frac{\delta}{1 - \delta} T \geq D - T.$$ 

But now we have a contradiction, because for $\delta$ small enough this constraint cannot be satisfied. Therefore the market cannot shut down entirely. At the same time, the traditional sector can shut down entirely. For then the expected return to being in the traditional sector is $0$, while in the market it is $M > 0$. There is no paradox here because by assumption, contract-breaking is not possible in the market. This is the asymmetry which allows for one corner solution but not the other.

(d) [5 points] Provide (and interpret) conditions under which the traditional sector is (partially) active in at least one equilibrium.

**Answer.** For the traditional sector to be partially active we need the existence of some $s > 0$ such that

$$\frac{\delta(s)}{1 - \delta(s)} [sT - (1 - s)M] = D - T.$$ 

Rearranging, this is equivalent to the condition that

$$\delta(s) = \frac{D - T}{D - (1 - s)(T + M)}.$$ 

One way to guarantee this is to have the discount factor going down very slowly as $i$ goes up, with all the drop coming near the end. For then, while $s$ is close to 1, the right-hand side is certainly less than one. But we can keep the left-hand side above 1 by having $\delta$ hover near one until the very end of the distribution. Verbally, this says that the condition for having a partially active traditional sector is implied by having lots of patient people and only a small fraction of impatient people.
[3] (a) [8 points] Suppose for the moment that $s$ is forever fixed. Then show that if $s > 1$, then $B$ is a dominant choice, while if $s < -1$, $A$ is a dominant choice. Show that between these two values there are all always multiple equilibria, depending on expectations (about the choices of other individuals, in the same and in future generations).

**Answer.** If $s > 1$, then being in sector $B$ guarantees you a payoff of at least

$$\alpha s + (1 - \alpha)s = s > 1,$$

which is strictly higher than being in Sector $A$. So being in $B$ is dominant. Similar trivial argument applies for the case in which $s < -1$.

(b) [12 points] Now go back to the case where $s$ unfolds as a stochastic process. Show that the thresholds in part (a) now change from 1 and -1 to $\alpha$ and $-\alpha$.

**Answer.** Suppose that an individual today believes that tomorrow, people will go to Sector $B$ as long as $s > S$, where $S$ is some threshold. Then today, an individual will choose sector $B$ as long as

$$\alpha s + (1 - \alpha)[E(s'|s) + 2P(s'|s > S)] > 1.$$

To understand this, first note that we are only writing down a sufficient condition (not a necessary one). This is why I am pessimistic in the current period and assume that no one else is going to $B$. This explains the $\alpha s$ for the first period. In the second period, I certainly get the conditional expectation of $s'$ given $s$, plus if the state exceeds $S$ tomorrow, I will have a population of measure 2 coming in.

Now noting that $E(s'|s) = s$ (there is no drift), we may rewrite the above as

$$s + 2(1 - \alpha)P(s' > S|s) > 1.$$

This defines a mapping $g(S)$ from tomorrow’s anticipated threshold to a threshold today, given by

$$g(S) + 2(1 - \alpha)P(s' > S|g(S)) = 1.$$

The only consistent expectation is one which replicates itself: that is, $g(S) = S$. Calling this $s^*$, we see that

$$s^* + 2(1 - \alpha)\frac{1}{2} = 1,$$

or that $s^* = \alpha$. The other case is proved similarly.