Answers to 2002 Midterm in Development Economics

[1] Was answered in “Answers to Problem Set 2”. But it needs some additional stuff for full credit. These are essentially statements identical to those found in Morris-Shin, to show that the strategy we have unearthed through the recursion is the unique equilibrium of the game.

[2] (a) Assume that each generation, starting with any $W$, leaves a starting wealth of precisely $W$ for the next generation. We will show that this policy is unimprovable and therefore optimal. Let $x(W)$ be the investment under this policy; then it is obvious that $x(W) = W/(1 + r)$ and consequently, lifetime consumption for any generation following this policy is $W - x(W) = rW/(1 + r)$. Therefore, assuming this policy to be in place in the future, a generation today solves, for each $W$,

$$\max_x u(W - x) + \frac{\delta}{1 - \delta} u \left( \frac{rW'}{1 + r} \right).$$

where $w$ is each generation’s wage income and where $W' = (1 + r)x$. Writing the FOC, we see that

$$u'(W - x) = \frac{\delta}{1 - \delta} ru'(rx).$$

[At this point, it would also be nice to argue that FOC must hold with equality.]

Because $r = (1 - \delta)/\delta$ by assumption, we may conclude that

$$u'(W - x) = u'(rx)$$
or $W - x = rx$. Solving, we are done.

[b] First notice that in any steady state, there has to be some unskilled labor as well as some skilled labor. For there to be net equality in consumption in steady state, therefore, investment in skilled labor has to earn the same rate of return $r$ as investment in financial capital.

This means that the wage differential $\bar{w} - w = (1 + r)S$. But $\bar{w} = \alpha \left( \frac{1 - \lambda}{\lambda} \right)^{1-\alpha}$ and $w = (1 - \alpha) \left( \frac{\lambda}{1 - \lambda} \right)^{\alpha}$, so this tells us that

$$\alpha \left( \frac{1 - \lambda}{\lambda} \right)^{1-\alpha} - (1 - \alpha) \left( \frac{\lambda}{1 - \lambda} \right)^{\alpha} = (1 + r)S.$$

It is easy to see that the LHS is monotone in $\lambda$ and therefore a unique solution $\lambda^*$ exists. The wages are therefore uniquely pinned down as $\bar{w}^* = \bar{w}(\lambda^*)$ and $w^* = w(\lambda^*)$.

Once we have this we can use part (a) as follows. Define a utility function $v$ by $v(c) = u(w^* + c)$. Then with $v$ in place of $u$, we have exactly the model of part (a). We don’t have to keep in mind $\bar{w}^*$ because the return in the labor market is exactly the same as in the financial market. By part (a), any initial $W$ will be maintained forever.

There is only one catch: $W$ has to be high enough so that people want to bequeath at least $S$, otherwise no one will not want to educate their children. In other words, the
minimum starting wealth for each generation (which includes the incremental returns to any skill endowment) must be at least $S(1 + r)$ (again using part (a)).

Notice that the financial wealth of skilled and unskilled generations will be different, but can differ by precisely the amount $(1 + r)S$. Skilled generations must have lower starting financial wealth.

[c] In terms of the model studied in class, a profession here is a pair (occupation, financial wealth). Because a skilled generation has starting financial wealth which is lower by precisely $S(1 + r)$ and because the extra return to skill is also the same amount, the distinctness condition is violated.

[3] (a) Notice that if an agent gets more than $a$, he will accept. So the principal will never offer more than $a$ in any equilibrium. Moreover, by offering a little more than $a$ to one agent and a little more than $b$ to the other he can always guarantee a payoff of $\pi(a) + \pi(b)$, so in no equilibrium can he get less than this. Indeed, there is one equilibrium in which he offers $(a, b)$ and both agents accept.

The other equilibria involve coordination failure among the agents. One accepts because he thinks the other will accept. In each of these equilibria, each of the agents could be strictly better off by saying no to the principal. For any $c$ less than $a$ but no less than $b$, it is (unfortunately for the agents) true that the strategy — “accept iff the offer is at least as good as $c$” — is a best response. So there are many equilibria of this kind, in which the principal offers $c \in [a, b]$ with the property that $2\pi(c) \geq \pi(a) + \pi(b)$ and the agents accept.

If coordination failure among the agents is removed at best the equilibrium in the first paragraph remains. [There are two of them by permutation across the agents.]

(b) I will accept any of the two following answers to this question:

**Case 1.** If an agent rejects, he cannot be made a second offer (he leaves the game). Under this assumption, I will give full credit to anyone who gives me the following argument. Suppose the principal makes an offer to one agent in period 1, who refuses. Then there is only one agent left. His outside option is now $a$, and this is what he must be given. If $\pi(a) > 0$, the principal will credibly make him an offer, which he will accept.

It follows that the first agent who refused will finally get a payoff of $b$. But this means that the principal can offer him $b$ to start with, and he will accept. Then the second agent will also get $b$.

Matters are different if $\pi(a) < 0$. Then if the first agent refuses, the second agent will never be hired by the principal. So refusal brings a credible payoff of $a$ for the first agent. So to get him to accept, the principal must offer $a$. Then the second agent can be offered $b$.

**Case 2.** Agents who reject in period 1 can be made fresh offers in period 2.

In period 2 we know that there are two equilibrium configurations: $(a, b)$ and $(b, a)$ (assuming both agents are available then). Let us suppose that in period 1 the principal tells agent 1 that if he refuses he will be offered $b$ tomorrow, either if he is alone, or via the contract pair $(b, a)$ if both are free. The first part (if agent 1 is alone) is credible because this is the only subgame equilibrium with a single agent. The second part (if both are free) is credible by part (a).
Given this, agent 1 will accept any offer which gives him at least $b$ in period 1. Knowing this, agent 2 might as well accept any offer that gives him at least $b$ as well. So the unique equilibrium payoff is $(b, b)$ to the agent and $1\pi(b)$ to the principal.