1 Time Inconsistency of Optimal Taxation

We study optimal taxation in this Chapter. By “optimal” we mean that the government raising taxes does this in the interest of the agents of the economy and not in the personal interest of the bureaucracies that constitute the government itself. This is a strong assumption, but we will show that, even in this case, we need to find ways to control the sovereign (Ed Prescott, who has been awarded the Nobel Prize in 2005 for his work with F. Kydland on Time Inconsistency).

1.1 Distortionary Taxation

Suppose a government has to raise an amount $g$ to provide the economy with a public good (e.g., military defence, police protection, communication infrastructures, education and health care,....). The public good consists of units of the single consumption good in the economy at time $t + 1$. The government raises taxes. We consider two different tax systems:

- taxes on savings, and
- lump-sum taxes.

We will show that lump-sum taxes are to be preferred in terms of Pareto efficiency. Taxes on savings are most commonly used though because they are easier to implement in real economies.

Consider a two-period economy with a representative agent. A production technology allows the agent to save any non-negative amount $S_t$ at $t$ and receive $(1 + r)S_t$ at $t + 1$. With lump-sum taxes $\tau$ the representative agent’s budget constraints are:

$$c_t + S_t = w_t, \quad c_{t+1} = S_t(1 + r) - \tau$$

$$\tau = g$$

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With taxes on savings, instead:

\[ c_t + S_t = w_t, \quad c_{t+1} = S_t (1 + r) (1 - \tau) \]

\[ S_t (1 + r) \tau = g \]

If the representative agent has period utility \( u(c) \), and discounting \( \beta \), the first order conditions of his/her maximization problem with taxes include:

\[ \frac{u'(c_{t+1})}{u'(c_t)} = \beta (1 + r) [\text{with lump-sum taxes}] \]

\[ \frac{u'(c_{t+1})}{u'(c_t)} = \beta (1 + r) (1 - \tau) [\text{with taxes on savings}] \]

We say that the saving choice of the agent is distorted with taxes on savings but not with lump-sum taxes. What we mean is that the first order condition which determines savings at the margin is changed (with respect to the benchmark with no taxes) with taxes on savings but not with lump-sum taxes. This is the source of the inefficiency of taxes on savings: A planner with a constraint that \( g \) must be raised faces aggregate endowment \( y_t \) and \( y_{t+1} - g \) at \( t+1 \) (and a saving technology with return \( 1 + r \)) and chooses the consumption allocation which results from lump-sum taxes. [Make sure you understood this point!]

1.2 Optimal Choice of Public Goods Provision Financed by Taxes on Savings

Consider the economy in the previous section. Assume that the government must now choose an amount \( g \) of public good provision. The public good has utility for the representative agent in the amount \( v(g) \). The government is aware that taxes on savings distort the margin (as we have shown in the previous section), but has no other financing mean available.

The government will have to choose public good provision \( g \) and tax rate \( \tau \) to maximize the representative agent’s utility while satisfying the constraint that the public good needs to be financed by taxes on capital: \( g = (1 + r) \tau S_t \). The government, on the other hand, cannot prescribe a saving choice to the representative agent. (This is what distinguishes a government from a planner: The government has power of raising taxes, but
agents make their own economic choices; the planner is instead a conceptual construct we use as a benchmark to define Pareto optimality, who chooses all of the agents’ allocations, including savings. Write down the planner’s problem and its first order conditions; recall that there is a representative agent and hence the objective function of the planner does not depend on parametric weights.

Recall that the public good is in units of the consumption good at time \( t+1 \). Consider the case in which at time \( t \) the government decides the amount of the public good to be provided and the tax rate to finance it. We solve this problem in two steps: first we solve for the amount of savings that the agent chooses given any \( g \) and hence any \( \tau \) (given \( g \), \( \tau \) is determined by the financing constraint \( g = (1 + r) \tau S_t \)); then we solve for the optimal level of \( g \) chosen by the government.

The first step, that is, the choice problem of the representative agent, is the following:

\[
\max_{c_t,c_{t+1},S_t} u(c_t) + \beta (u(c_{t+1}) + v(g))
\]

subject to:

\[
\begin{align*}
&c_t + S_t = w_t, \quad c_{t+1} = S_t (1 + r) (1 - \tau) \\
&(1 + r) \tau S_t = g
\end{align*}
\]

Assume \( u(c) = \frac{1}{1-\gamma} c^{1-\gamma} \), with \( \gamma < 1 \); and assume \( w_{t+1} = 0 \). Then the first order condition with respect to savings \( S_t \) (substitute (1), not (2), in the utility function of the agent) is (derive it; it takes a bit of algebraic work):

\[
\frac{S_t}{w_t - S_t} = (\beta (1 + r) (1 - \tau))^{\frac{1-\gamma}{\gamma}}
\]

For this equation it is relatively easy to see that \( \frac{\partial S_t}{\partial \tau} < 0 \). [To prove it either use the Implicit Function Theorem or reason as follows: the right-hand-side is increasing with \( S_t \); the left-hand-side is decreasing in \( \tau \) if \( \gamma < 1 \), as we assumed; therefore, if \( \tau \) increases, the left-hand-side decreases and hence the right-hand-side must also decrease, which can only be if \( S_t \) decreases.] We can also similarly establish, by substituting (2) into (3) and re-doing the argument, that \( \frac{\partial S_t}{\partial g} < 0 \) (do this; \( S_t \) appears in (2) and hence the argument needs be modified a little bit).
We conclude therefore that, for given public good provision \( g \) financed through taxes on savings, the representative agent saves an amount \( S(g) \), with \( S'(g) = \frac{\partial S}{\partial g} < 0 \).

We can now solve for the second step, the optimal level of public good provision \( g \) of the government. It is the solution of the following maximization problem (note that we have substituted the expression for consumption into the utility function, as well as the expression for savings, \( S(g) \), and for taxes, \( \frac{g}{(1+r)S(g)} \), from the financing constraint):

\[
\max_g \ u(w_t - S(g)) + \beta u \left( S(g) \left( 1 + r \right) \left( 1 - \frac{g}{(1+r)S(g)} \right) \right) + v(g)
\]

The first order condition can be written as follows:

\[
\beta v'(g) = \beta u'(c_{t+1}) - S'(g) (\beta u'(c_{t+1})(1 + r) - u'(c_t))
\]

Let’s analyze this condition carefully. Note the following:

- the left-hand-side is the marginal benefit of \( g \) and the right-hand-side is its marginal cost;
- the term \(-S'(g) (\beta u'(c_{t+1})(1 + r) - u'(c_t))\) is positive if \( \tau > 0 \):
  \( S'(g) < 0 \);
  \( \beta u'(c_{t+1})(1 + r) - u'(c_t) > 0 \) if \( \tau > 0 \), from the first order condition of the representative agent problem, which include \( \frac{u'(c_t)}{u'(c_{t+1})} = \beta(1 + r)(1 - \tau) \);
  it increases with the absolute value of \( S'(g) \), and with \( \beta u'(c_{t+1})(1 + r) - u'(c_t) \).

We can now compare the solution of the government problem with the solution of the planner’s problem (that I have asked you to write down at the beginning of this section). The first order conditions of the planning problem (or of the government problem with lump-sum taxes) include the following:

\[
\beta v'(g) = \beta u'(c_{t+1})
\]

(check this! convince yourself, this is important). In this equation, also, the left-hand-side represents the marginal benefit of \( g \) and the right-hand-side
its marginal cost. Therefore we have shown above that the marginal cost of $g$ in the government problem is greater than the marginal cost associated to the planner’s problem, $\beta u'(c_{t+1})$, if $S'(g) < 0$ and $\tau > 0$. We conclude that that the provision of the public good in the government problem is less than the efficient amount (resulting from the planner’s problem). [This is not straightforward, because the levels of consumption $c_t$ and $c_{t+1}$ in the two problems are also different; in other words, it is not enough to compare the first order conditions we did compare, but we need to look at the whole set of first order conditions. The result is true, however, because savings is smaller in government’s problem, and hence $c_{t+1}$ smaller. Try to prove this if you feel strong.] Equivalently, the choice of public provision $g$ by the government with taxes on savings is smaller than the choice that would derive from a government problem with lump-sum taxes. [Perhaps it is worth convincing yourself of this; write down the problem with lump-sum taxes and solve it; show the solution coincides with the solution of the planning problem.]

Note that $\beta u'(c_{t+1})(1 + r) - u'(c_t)$ is a measure of the distortion due to taxes on savings; that is, when taxes are lump-sum it is always true that $\beta u'(c_{t+1})(1 + r) = u'(c_t)$, while with taxes on savings $\beta u'(c_{t+1})(1 + r) = \frac{u'(c_t)}{1 - \tau} > u'(c_t)$, and hence $\beta u'(c_{t+1})(1 + r) - u'(c_t)$ increases with $\tau$.

1.3 Time Inconsistency

Remember we considered the case in which at time $t$ the government decides the amount of the public good to be provided and the tax rate to finance it. Suppose now we consider the case in which at time $t+1$, after savings decisions have been made by the representative agent on the basis of the levels of $g$ and $\tau$ which we derived in the previous section, the government can choose possibly a different level of public good provision $G$ and associated tax $T$ to satisfy the financing constraint. What is the government problem now?

The government at time $t+1$ takes as given the amount of saving $S(g)$, and chooses $G$ as the solution to the following problem:

$$
\max_G \beta \left( u \left( S(g) (1 + r) \left( 1 - \frac{G}{(1 + r) S(g)} \right) \right) + v(G) \right)
$$

The first order condition of this problem is:

$$
u'(c_{t+1}) = v'(g)
$$
Convince yourself that the $G$ which solves this problem is greater than $G$. (Now, this is easy, because $S(g)$ does not change, and hence $c_{t+1}$ only changes in account of $T$.)

We conclude that, if the government would be allowed to change its decision at time $t+1$, it would. Even though the government is always maximizing the utility of the representative agent. The intuition is that at time $t$ the government limits the provision of the public good to limit the distortions that taxes impose on savings. At time $t + 1$ instead savings has already happened, and hence raising the public good provision (and taxes) does not distort savings.

Of course the agents at time $t$ might not trust the government not to raise taxes at time $t + 1$, and hence might reduce savings nonetheless. What do you expect it will happen? You should be able to answer this informally. For a formal answer we need *Game Theory* - coming in the next classes.