Efficient Competitive Equilibria with Adverse Selection

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Abstract

Do Walrasian markets function orderly in the presence of adverse selection? In particular, is their outcome efficient? This paper addresses these questions in the context of a Rothschild and Stiglitz insurance economy. We identify an externality associated with the presence of adverse selection as a special form of consumption externality. Consequently, we show that while competitive equilibria always exist, they are not typically incentive efficient.

However, as markets for pollution rights can internalize environmental externalities, markets for consumption rights can be designed so as to internalize the consumption externality due to adverse selection. Markets for consumption rights amount to requiring that firms offering contracts designated for the ‘low risk types’ acquire the right to do so, at market-determined prices, from agents declaring to be of ‘high risk’. With such markets competitive equilibria exist, are always incentive efficient and any incentive efficient allocation can be decentralized as a competitive equilibrium.

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1 Introduction and Motivation

We study competitive exchange economies with adverse selection. Agents have private information regarding the probability distribution of their endowments. Firms offer trade contracts providing the agents with insurance against the realization of their individual idiosyncratic uncertainty. Agents’ trades can be fully monitored, so that firms can enforce exclusive contractual relationships.\(^1\) The agents’ private information is the only ‘friction’ to the operation of markets.\(^2\) We look at Walrasian equilibria, where both consumers and firms act as price takers.

We intend to address the following questions: Do Walrasian markets function orderly in the presence of adverse selection? What are the properties of allocations attainable as Walrasian equilibria of economies with adverse selection and exclusive contracts? And in particular, are Walrasian equilibria incentive efficient?

Our analysis is motivated by the fundamental contribution of Prescott and Townsend (1984), (1984b). They analyze Walrasian equilibria of economies with moral hazard and with adverse selection when exclusive contracts are enforceable. While for moral hazard economies they prove existence and constrained versions of the first and second theorems of welfare economics,\(^3\) their method does not succeed in the case of adverse selection economies. They conclude (p. 44) that “there do seem to be fundamental problems for the operation of competitive markets for economies or situations which suffer from adverse selection.”

We start by identifying a special form of consumption externality which arises in economies with adverse selection and exclusive contracts: the level of trades chosen by agents of one type influences in fact, via its effect on the incentive compatibility constraints, the admissible trades of agents of other types. We proceed then in the spirit of the approach pioneered by Arrow (1969) and Lindahl (1919), and introduce an appropriately enlarged structure of markets which includes markets for consumption rights. Each agent must acquire a sufficient amount of rights to the other types’ consumption so as to satisfy the incentive compatibility constraints. For instance, a higher level of consumption of the agents with a high risk type might exert a positive externality on the agents with a low risk type by relaxing the incentive constraints they face. In this case,

\(^{1}\)Exclusivity is clearly a strong assumption. It provides however an important benchmark as it is the case typically considered in contract theory, as well as in general equilibrium analyses of economies with asymmetric information, e.g., in Prescott-Townsend (1984). The properties of competitive equilibria when this condition is violated, and only non-exclusive contractual relationships are available, are investigated in Bisin-Gottardi (1999).

\(^{2}\)The price taking behavior may be justified in our economy by the fact that the each consumer is small in terms of the level of his trades in the market and the firms’ technology has constant returns to scale. Proper game theoretic foundations are however difficult in this framework, also because the properties of strategic models of adverse selection economies are not well established (see the discussion in Section 4.1.1).

\(^{3}\)See also Bennardo-Chiappori (2003), Kehoe, Levine and Prescott (2002), and Kocherlakota (1998).
the agents with high risk will sell their consumption rights at a positive price determined in the market to the low risk agents, which in turn will buy them to be able to increase the level of their own consumption.

We call Walrasian equilibria with such market structure ALPT for Arrow-Lindahl-Prescott-Townsend. This expanded structure of markets allows to internalize the consumption externality induced by the presence of adverse selection and therefore to fully decentralize incentive efficient allocations. We show that ALPT equilibria exist and incentive constrained versions of the first and second theorem of welfare economics hold. In other words, we successfully replicate, for adverse selection economies, the results that Prescott-Townsend (1984),(1984b) obtain for moral hazard economies, thereby offering a solution to the problem their papers opened.

Evidently, the implementation of the structure of markets which guarantees the decentralization of incentive constrained efficient allocations in adverse selection economies requires an enforcement mechanism which prevents agents from acquiring commodities for their own consumption without buying also rights to the other types’ consumption. However, we will argue that markets for consumption rights can be designed along similar lines to, e.g., markets for pollution rights, or markets for club goods. In particular, we argue that an efficient decentralization scheme can be designed where the enforcement mechanism operates on insurance firms rather than directly on agents, and where consumption rights take the simple form of the ‘right’ to trade in the market designated for ‘low risk’ agents. At equilibrium firms offering contracts exclusively to ‘low risks’ will acquire the ‘right’ to do so from ‘high risk’ agents at market-determined prices. When at equilibrium the prices of the rights are positive an efficient allocation is decentralized at which ‘low risks’ subsidize ‘high risks’ by compensating them for selling their ‘rights’ to trade in the ‘low risk’ insurance market.

We also consider economies where the markets for consumption rights are not available. In this case, each agent is only able to trade claims to his own consumption. He has then to satisfy the incentive compatibility constraints with respect to the level of trades chosen by the other types of agents, taken as given. We call Walrasian equilibria with such structure of markets EPT, for Prescott-Townsend with externality. There are no fundamental problems associated with this Walrasian equilibrium concept either. We show that an EPT equilibrium always exists and provides a useful - and somewhat robust - prediction to the outcome of competitive markets in such economies. Evidently, EPT equilibria are not ensured to be incentive efficient. However, they satisfy an appropriately defined notion of third best efficiency, and the second welfare theorem also holds for this equilibrium concept.

The analysis of this paper is developed for a simple insurance economy with adverse selection as the one considered by Rothschild and Stiglitz (1976). This constitutes a very important test case for any equilibrium notion for adverse selection economies. In addition, by exploiting the simple structure of the economy, we are able to clearly illustrate the features and to provide a complete characterization of the various notions of
Walrasian equilibrium which we study. In particular, the allocation we obtain as unique EPT equilibrium corresponds to the Rothschild-Stiglitz separating candidate equilibrium (this is so even for those parameter values for which Rothschild-Stiglitz (1976) found non-existence). On the other hand, ALPT equilibria typically entails a nonzero level of cross-subsidization (i.e., some degree of ‘pooling’ among types).

The paper is organized as follows. The structure of the economy is presented in the next Section; incentive efficient allocations are then characterized in Section 3. In section 4, first EPT equilibria and then ALPT equilibria are defined, their existence established and their efficiency properties characterized. Proofs are collected in the Appendix.\footnote{A more detailed and complete presentation of the proofs of all the results in the paper can be found in Bisin and Gottardi (2003).}

## 2 The Economy

Consider an economy with adverse selection. There is a continuum of agents of two different types, \(b\) and \(g\). Let \(\xi^b\) denote the fraction of agents of type \(b\) and \(\xi^g\) the fraction of agents of type \(g\) in the population. We assume \(\xi^b, \xi^g > 0\).

There is a single consumption good. Uncertainty enters the economy via the level of the agents’ endowment and is purely idiosyncratic. There are two possible states, \(H, L\), for every individual, and his endowment when \(H\) (resp. \(L\)) is realized is \(\omega_H\) (resp. \(\omega_L\)). Let \(\pi^i_s\) be the probability that individual state \(s, s \in S \equiv \{H, L\}\), is realized for an agent of type \(i, i \in \{g, b\}\). These random variables are independently distributed across all agents and identically distributed across agents of the same type.

Each agent is privately informed about his type. On the other hand, the realization of individual states is commonly observed.

With no loss of generality, let \(\omega_L < \omega_H\) (state \(H\) is then the good state), and \(0 < \pi^b_H < \pi^g_H < 1\) (thus type \(b\) is the ‘high-risk’ type, and type \(g\) the ‘low-risk’).

The preferences of each agent are described by von Neumann-Morgernstern utility function, with type independent utility index \(u: \mathbb{R}_+ \to \mathbb{R}_+\), defined over consumption in each idiosyncratic state \(s \in S\). Let then \(U^i(x^i) \equiv \sum_{s \in S} \pi^i_s u(x^i_s)\), for \(i \in \{g, b\}\).

We assume:

**Assumption 1.** Endowments are strictly positive for all agents: \(\omega_L, \omega_H > 0\). Preferences are strictly monotonic, strictly concave, twice continuously differentiable, and \(\lim_{x \to 0} u'(x) = \infty\).

Note that the economy is the same as the insurance economy with adverse selection considered by Rothschild and Stiglitz (1976).
3 Incentive Efficient Allocations

Let \( x^i \equiv (x^i_H, x^i_L) \), \( i \in \{g, b\} \). A (symmetric) feasible allocation is a pair \( \{x^b, x^g\} \) which satisfies the following resource feasibility constraint:

\[
\sum_{s \in S} \left[ \xi^g \pi^g_s (x^g_s - \omega_s) + \xi^b \pi^b_s (x^b_s - \omega_s) \right] \leq 0 \tag{1}
\]

where the purely idiosyncratic nature of the uncertainty and the Law of Large Numbers have been used to take the sum of the excess demand for the commodity contingent on each individual state, weighted by its probability.

An incentive compatible allocation is a pair \( \{x^b, x^g\} \) which satisfies the following constraints:

\[
- \sum_{s \in S} \pi^g_s u(x^g_s) + \sum_{s \in S} \pi^b_s u(x^b_s) \leq 0, \tag{2}
\]

\[
- \sum_{s \in S} \pi^b_s u(x^b_s) + \sum_{s \in S} \pi^g_s u(x^g_s) \leq 0 \tag{3}
\]

**Definition 1** An allocation \( \{x^b, x^g\} \) is incentive efficient if it is feasible, incentive compatible (i.e., satisfies ((1-3))) and there does not exist another allocation \( \{\hat{x}^b, \hat{x}^g\} \), also feasible and incentive compatible, such that:

\[
U^b(\hat{x}^b) \geq U^b(x^b), U^g(\hat{x}^g) \geq U^g(x^g), \text{ with at least one inequality being strict.} \tag{5}
\]

For the simple adverse selection economy under consideration, Prescott and Townsend (1984) have provided a complete characterization of the set of incentive efficient allocations.\(^6\) At any incentive efficient allocation at least one of the two agents’ types is fully insured (has a deterministic consumption bundle). There is then a unique constrained efficient allocation where both types are fully insured and consume the same amount. This is the allocation induced by the pooling contract in Rothschild and Stiglitz (1976). At the pooling allocation (illustrated in Figure 1) no incentive constraint binds, the allocation is also Pareto efficient.

All other incentive efficient allocations are of one of the two following types: (i) allocations where the agents of type \( b \) are fully insured, while the type \( g \) agents are only partially insured (their consumption is higher in state \( H \) than in state \( L \)), and only the second of the two incentive constraints, (3), binds; (ii) allocations where type \( g \) agents

\(^5\)As shown by Prescott and Townsend (1984) (see, also, Cole (1989)), in the presence of asymmetric information it may be desirable to expand the commodity space so as to allow for random allocations of contingent commodities, or lotteries over consumption bundles. This is not the case for our simple adverse selection economy, for which allocations involving nondegenerate lotteries are always suboptimal (see Prescott and Townsend (1984)). To keep the notation simpler, definitions are then stated for the case of non random allocations.

\(^6\)See also Crocker and Snow (1985) and Jerez (2003).
are fully insured, agents of type $b$ are overinsured (their consumption is higher in state $L$) and only the first incentive constraint, (2), binds. Rothschild-Stiglitz’s separating pair of contracts induces an allocation of type (i) with the additional property that agents of type $b$ are insured at fair odds, hence agents of type $g$ do not subsidize agents of type $b$; see Figure 2 for a graphical representation. It is well known that, when the fraction $\xi^b$ of type $b$ agents is sufficiently small, the incentive efficiency of allocations of type (i) require a nonzero level of subsidy from the type $g$ agents to the $b$ types; in this case, the separating allocation is not incentive efficient.

Figure 1: The Pooling Allocation. $\omega$ denotes the individual endowment, $x^P$ the pooling allocation, $U^b$ and $U^g$ the indifference curves of types $b$ and $g$, respectively, at the pooling allocation. Both types of agents are fully insured and consume the aggregate per capita endowment, $x^P = \sum_{s \in S} [\xi^g \pi^g_s \omega_s + \xi^b \pi^b_s \omega_s]$.

As in Rothschild and Stiglitz, the pooling and the separating allocations will play a central role in our analysis.
Figure 2: The Separating Allocation. \((x^i,S)_{i=g,b}\) denotes the separating allocation, \(U^b\) and \(U^g\) the indifference curves respectively of type \(b\) and \(g\) at this allocation. Agents of type \(b\) are fully insured at fair odds, \(x_{b,S} = \sum_{s \in S} [\pi_b^s \omega_s]\) for each \(s\); type \(g\) agents are only partially insured, again at fair odds, so as to satisfy the type \(b\) incentive compatibility constraint, \(x_{g,S}\) lies at the intersection of \(U^b\) and \(g\)'s fair odds line.

4 Walrasian Equilibria

Various competitive equilibrium concepts have been used in the analysis of adverse selection economies with exclusive contracts. The standard strategic analysis of such economies, due to Rothschild and Stiglitz (1976), considers the Nash equilibria of a game in which insurance companies simultaneously choose the contracts they issue, and the competitive aspect of the market is captured by allowing the free entry of insurance companies. Such equilibrium concept does not perform too well. Equilibria in pure strategies do not exist for robust examples (Rothschild and Stiglitz (1976)). Equilibria in mixed strategies exist (Dasgupta and Maskin (1986)) but, in this set-up, are of difficult interpretation. Even when equilibria in pure strategies do exist, it is not clear that the way the game is modelled is appropriate for such markets, since it does not allow for dy-
namic reactions to new contract offers (Wilson (1977) and Riley (1979); see also Maskin and Tirole (1992)). Moreover, once dynamic reactions are introduced, equilibria are not robust to ‘minor’ perturbations of the extensive form of the game (Hellwig (1987)).

Our approach consists instead in studying Walrasian equilibrium concepts, where both agents and insurance firms act as price takers in competitive markets. At equilibrium a price is quoted for all possible contracts. We consider the case where agents’ trades are fully observable, so that exclusive contracts can be implemented. Following the approach used by Prescott and Townsend (1984) for moral hazard economies, we will model exclusivity of contracts by imposing incentive compatibility directly as a constraint on the set of admissible trades of every agent, allowing then trades contingent on the agent’s (declared) type. Prices are then linear over such restricted domain.

Differently from the case of moral hazard economies, with adverse selection each incentive compatibility constraint, as we see from (2-3), relates the consumption levels of the two types of agents: when agents say of type $b$ vary their consumption, this affects the set of values of consumption of type $g$ agents that satisfy incentive compatibility. Thus, when the incentive compatibility constraints are imposed the set of contracts that agents of one type can trade depends on the trades of the agents of the other type; an externality arises so in the specification of the set of admissible trades of each agent.

Our first contribution is to show that once the presence and the nature of the externality in adverse selection economies with exclusive contracts is clearly identified, Walrasian equilibria can be defined and their properties analyzed as in other competitive economies with externalities in consumption.

We will consider first the notion of EPT equilibrium, where each agent takes as given the level of trades made by the other types, i.e. the externality is not internalized. Next, the notion of ALPT equilibrium will be examined, where markets allowing to internalize the externality are introduced. We will show that, in contrast to the strategic approach by Rothschild and Stiglitz, Walrasian equilibria always exist.

### 4.1 EPT Equilibria

Consider first the case in which each agent can trade, at linear prices, claims contingent on the realization of his individual uncertainty as well as on his declared type. Let $q_{i,s} \in \mathbb{R}_+$ be the unit price at which any agent who claims to be of type $i \in \{g, b\}$ can trade the consumption good for delivery in his individual state $s \in S$, $q \equiv \{q_{g,H}, q_{g,L}, q_{b,H}, q_{b,L}\} \in \mathbb{R}_+^4$. To fix notation, note that here and in what follows a subscript $i \in \{g, b\}$ denotes the type declared by the agent, while a superscript denotes his actual unobservable type.

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7 An alternative way of modelling exclusive contractual relationships consists in allowing for non-linear price schedules for contracts. In this case suitable 'refinements' restricting admissible equilibrium prices are necessary to select between the large set of equilibria one obtains; see Gale (1992), (1999) and Dubey, Geanakoplos, Shubik (1995).
The set of admissible trades in such markets is restricted then by imposing incentive compatibility constraints. More precisely, the specification of the set of admissible trades of every agent is constructed as follows. The agent has to choose a vector \( z \equiv \{z_g, z_b\} \in \mathbb{R}^4 \), where \( z_i \equiv \{z_{i,H}, z_{i,L}\} \) denotes the net trades made when he declares to be of type \( i \in \{g, b\} \). Every agent can claim to be of type \( g \) and trade in the market designated for the type \( g \) (at the prices \( q_g \)); alternatively, he can claim to be of type \( b \) and trade in the market designated for \( b \) (at \( q_b \)). If he declares to be of type \( g \) and hence trades in the market for \( g \), i.e., \( z_g \neq 0 \), then he cannot trade in the market for \( b \), \( z_b = 0 \). Moreover, his net trades in the market for \( g \) have to be incentive compatible with respect to the net trades made in the market by agents who claim to be of type \( b \), taken as exogenously given: we will denote such trades by \( \bar{z}_b \equiv (0, \bar{z}_b) \). Type \( b \) agents must prefer then \( \bar{z}_b \) (at least weakly) to \( z_g \). Symmetric restrictions hold if the agent chooses instead to trade in the market for the \( b \) types, i.e., \( z_b \neq 0 \): type \( g \) agents must prefer \( \bar{z}_g \) to \( z_b \), where \( \bar{z}_g \equiv (\bar{z}_g, 0) \).

Let \( \omega \equiv \{\omega_H, \omega_L\} \in \mathbb{R}_+^2 \). The set of admissible net trades for each agent is then defined as follows:

\[
Z(\bar{z}_g, \bar{z}_b) = \left\{ z + (\omega, \omega) \in \mathbb{R}_+^4 : \begin{align*}
\forall i \in \{g, b\}, & \quad z_i \neq 0 \implies z_j = 0 \text{ for } j \neq i \text{ and} \\
\sum_{s \in S} \pi_s^i u (\bar{z}_{j,s} + \omega_s) & \geq \sum_{s \in S} \pi_s^i u (z_{i,s} + \omega_s)
\end{align*} \right\}
\]  

It depends on the level of trades made in the market \( \bar{z}_g, \bar{z}_b \) via the incentive compatibility constraints imposed in the specification of this set; thereby the externality.

The choice problem of an agent of type \( i \in \{g, b\} \) has then the following form:

\[
\max_{z \in Z(\bar{z}_g, \bar{z}_b)} \sum_{s \in S} \pi_s^i u \left( \sum_{j \in \{g, b\}} z_{j,s} + \omega_s \right) 
\]  

\[ (P^{EPT,i}) \]

\[ ^8 \text{It is not explicitly required that type } g \text{ agents prefer } z_g \text{ to } \bar{z}_b; \text{ this is the second of the two incentive constraints appearing in the definition of incentive constrained allocations, (2). Such constraint would in fact be redundant: each agent (trading in market) } g \text{ will always choose his most preferred allocation, hence one he prefers to } \bar{z}_b \text{ if such allocation were available (and at equilibrium it will always be available, since type } g \text{'s incentive constraint is imposed to the problem of agents trading in market } b \text{ and at equilibrium we require } \bar{z}_b \text{ to be the actual choice made by agents trading in market } b).}

\[ ^9 \text{We do not explicitly exclude from the set of admissible trades in market } b \text{ those trades that reveal the agent cannot be of type } b \text{ (similarly for market } g); \text{ these are all the trades such that type } b, \text{ but not } g, \text{ can find in market } b \text{ another trade that is better. Adding such constraint on the set of admissible trades would be redundant here: when } \bar{z}_i, \text{ the trades made in market } i, \text{ coincide with the actual choice made by agents in that market, as we require at equilibrium, agents will correctly sort themselves through the different markets according to their true type (type } g \text{ agents cannot find in fact anything in market } b \text{ they prefer to } \bar{z}_g, \text{ which are trades they can make in market } g, \text{ and similarly for } b).}
\begin{align*}
\text{s.t.} & \quad \mathbf{q} \cdot \mathbf{z} \leq 0
\end{align*}

In addition to consumers there are firms who supply insurance contracts, defined by purchases and sales of claims contingent on the realization of the individual uncertainty and the agents’ declared type. Moreover, firms can construct aggregates - or ‘pools’ - of such contracts and the Law of Large Numbers provides, in the economy under consideration, a mechanism - or a technology - for transforming such aggregates into riskless claims. Let \( \mathbf{y} \equiv \{y_{g,H}, y_{g,L}, y_{b,H}, y_{b,L}\} \) denote the vector describing the supply of net trades of contingent commodities, on a per capita basis. Firms are then characterized by the following constant returns to scale technology:

\begin{align*}
Y = \{ \mathbf{y} \in \mathbb{R}^4 : & \sum_{i \in \{g,b\}} \sum_{s \in S} \pi^i s y_{i,s} \leq 0 \}. 
\end{align*}

The firms’ problem consists in the choice of a vector \( \mathbf{y} \), lying in the set \( Y \) (i.e., subject to the constraints that contracts offered are, in the aggregate, self-financing), so as to maximize profits:

\begin{align*}
\max_{\mathbf{y} \in Y} \mathbf{q} \cdot \mathbf{y} & \quad (P^{EPT,f}) 
\end{align*}

We restrict our attention here, and in what follows, to symmetric equilibria, where all agents of the same type make the same choice.

**Definition 2** An EPT is given by a collection of net trades for each type of consumers \( z^g, z^b \), a production vector \( \mathbf{y} \), a price vector \( \mathbf{q} \) and a pair \( \bar{z}_g, \bar{z}_b \) such that:

(i) for each \( i \in \{g,b\} \), \( z^i \) solves the optimization problem \( (P^{EPT,i}) \) of consumers of type \( i \), given \( (\mathbf{q}, \bar{z}_g, \bar{z}_b) \);

(ii) \( \mathbf{y} \) solves the firms’ profit maximization problem \( (P^{EPT,f}) \), given \( \mathbf{q} \);

(iii) markets clear:

\begin{align*}
\sum_i \xi^i z^i & \leq \mathbf{y}, \quad (5)
\end{align*}

(iv) the level of trades in each market, taken as given by agents, is consistent with the agents’ actual choice:

\begin{align*}
\bar{z}_g &= z^g \\
\bar{z}_b &= z^b
\end{align*}

Note that condition (iv) requires that at equilibrium each agent chooses to declare his true type. Agents of type \( g \) choose to trade in the market designated for \( g \) while agents of type \( b \) prefer to trade in the market designated for \( b \).

The formulation of the agents’ set of admissible trades in \( (P^{EPT,i}) \), together with the consistency condition (iv), ensure that the equilibrium allocation \( z^g, z^b \) is mutually incentive compatible, that is, satisfies (2-3).
The presence of incentive compatibility as a restriction on consumers’ admissible trades can be equivalently interpreted, rather than literally as consumers ‘self-imposing’ such constraints, as a restriction on the set of contracts firms can offer to consumers. The firms’ choice of $y$ can in fact be viewed as the choice of the contract to offer the agents, as insurance firms do in Rothschild and Stiglitz (1976) except that in our analysis firms are price-takers. Even though the set $Y$ of contracts that are admissible for the firms is quite large, only incentive compatible contracts will be offered at equilibrium: supply has in fact to equal demand, and agents’ demand is subject to the incentive compatibility constraints. Hence the set $Y$ of admissible contracts for the firms could have been restricted by imposing the incentive compatibility constraints without affecting the results.$^{10}$

The following result can be immediately derived from the first order conditions of the firms’ choice problem:

**Lemma 1** At an EPT equilibrium, prices of contingent commodities have to be ‘fair’:

$$q_{i,s} = \pi_{i,s}^i, \ i \in \{g, b\}, s \in S$$

(6)

On this basis, we are able to completely characterize the EPT equilibria of the economy under consideration. First of all, as already argued (see footnote 9), it is easy to verify that the specification of the set of admissible trades in (4) ensures that each agent will choose to trade in the market designated for his own type. Agents of type $b$ face then fair prices and no binding incentive constraint. They will fully insure, and choose the $b$’s component of the separating allocation, $x^{b,S}$.

The choice set (the set of budget feasible and incentive compatible allocations) of type $g$ agents at the equilibrium prices can be seen from Figure 2: it is given by the area lying below the lower contour of the fair odds line for type $g$ (which, by Lemma (1), coincides with the budget constraint for type $g$) and the indifference curve $U^b$ passing through $x^{b,S}$. The $g$’s component of the separating allocation $x^{g,S}$ is the consumption level preferred by agents of type $g$ in this choice set. We conclude that the only EPT equilibrium of the economy is Rothschild and Stiglitz separating allocation.

**Theorem 1** Under Assumption 1, a unique EPT equilibrium allocation always exists and coincides with Rothschild-Stiglitz separating allocation.

Thus, Rothschild and Stiglitz pooling allocation is never an EPT equilibrium. To understand why, it is important to note at the outset that this allocation is incentive compatible and hence it is included in the commodity space. Nonetheless no vector of prices supports the pooling allocation as an EPT equilibrium. Consider first the case

$^{10}$See also Jerez (2003) on this point.
of fair prices. In this case the pooling allocation cannot be an equilibrium since it is not budget feasible for the type $b$ agents. This is just a consequence of the fact that the pooling allocation requires cross-subsidization from types $g$ to types $b$, and cross-subsidization requires prices not to be fair. But the pooling allocation cannot be an equilibrium even if prices are not fair. Suppose prices were not fair, and in particular $q_g = q_b$ so that the pooling allocation is budget feasible for both types. In this case firms would have a profitable deviation (as argued in Lemma 1): since prices are different from probabilities, in particular $\pi_{H}^b > q_L > \pi_{L}^g$, firms could achieve an unboundedly large positive profit by selling commodity $(g, L)$ and buying $(b, L)$, i.e. by selling only insurance to the $g$ types. In other words, firms can make positive profits by ‘breaking’ the pooling and introducing some separation. This is indeed the argument that Rothschild and Stiglitz (1976) use to explain why the pooling allocation is not supported as an equilibrium. It holds unchanged in our analysis even though our Walrasian notion of equilibrium restricts the set of possible deviations by firms to those which are profitable at the given prices.

It is interesting to observe that in our economy the Rothschild-Stiglitz separating allocation constitutes the unique EPT equilibrium as well as the unique Walrasian equilibrium allocation when general non-linear price schedules are allowed and a refinement in the spirit of Kohlberg-Mertens (1986)’s notion of strategic stability is imposed (this can be seen from the analyses of Gale (1992) and Dubey, Geanakoplos and Shubik (1995) of related economies). In this sense, the Rothschild and Stiglitz separating allocation appears to represent a robust prediction of Walrasian equilibrium concepts for the economies under consideration when exclusive contracts are enforceable and, our analysis suggests, markets to internalize the consumption externality do not exist.

4.1.1 EPT and the Strategic Analysis of Competition

The separating allocation is always an EPT equilibrium. In the same economy Rothschild and Stiglitz (1976) find instead robust instances where the separating allocation is not an equilibrium (and no other allocation is). When all agents prefer the pooling allocation to the separating one, their argument goes, a firm can offer a pooling contract and profitably sell it to all agents, thereby ‘breaking’ the separating as a candidate equilibrium allocation. To understand why this argument does not apply in our context, we need to better examine the difference between the strategic approach to competitive markets used by Rothschild and Stiglitz and the Walrasian approach we have adopted in EPT.

In Rothschild and Stiglitz’s analysis a contract $y$ is a pair $(y_H, y_L)$. Each contract is traded at an implicit state contingent pricing kernel $q$ such that $qy = 0$; in other words, all contracts offered are by definition budget feasible for both agents. As a consequence, and because of adverse selection, the profitability of a contract depends on the composition of the agents acquiring the contract: a contract inducing the pooling allocation makes
zero profits if all agents buy it, but makes negative profits if only agents of type $b$ buy it. A set of contracts $\bar{y} = \{y^k\}_{k \in K}$ is an equilibrium if each contract $y^k$ makes non-negative profits and $\bar{y}$ is immune to ‘deviations’; that is, if one cannot find another contract $y'$, which makes profits when offered in the market together with $\bar{y}$.

A pair of contracts inducing the separating allocation is not an equilibrium in Rothschild and Stiglitz, when all agents prefer the pooling, because the introduction of a pooling contract constitutes a profitable deviation, since all agents will acquire it. It has been noted by Wilson (1977) and Riley (1979) that this notion of ‘deviation’, which only considers the effects of introducing a single new contract, is unduly strong. This is because, in the presence of adverse selection, when the introduction of other contracts or the withdrawal of some of the existing contracts are allowed (i.e., when we allow for reactions and counter-reactions to the deviation), the deviating contract may end up making negative profits. In particular, when the deviating contract is a pooling contract, there exists another contract which ‘skims’ type $g$ agents away from the pooling, can be profitably introduced, and makes the pooling no longer profitable. The equilibrium notions proposed by Wilson and Riley require then that a ‘deviating’ firm anticipate the possible contracts which can be offered as a reaction or counter-reaction to its ‘deviation’.

In a Walrasian equilibrium all technologically feasible contracts $y \in Y$ are simultaneously available for trade in a competitive market. This includes the pooling, the separating, as well as the contracts which ‘skim’ $g$ types from the pooling. All such contracts are offered at a single state contingent price kernel $q$ at which all markets clear. Market clearing implies that many, in fact most, contracts will not be traded at equilibrium: at the equilibrium prices firms selling them would make either zero or negative profits and consumers do not demand them.\footnote{The notion of Walrasian competitive equilibrium we adopt, which requires market clearing, possibly at no-trade, for all technologically feasible commodities has been introduced by Makowski (1980) to study product innovations, and has become standard in the general equilibrium literature; see Allen-Gale (1988), Pesendorfer (1995) for the application of this notion to the analysis of competitive models of financial innovation.} It is our claim that the consistency requirement of Walrasian equilibria, that all contracts are offered and are supported by a single state contingent kernel, captures the chain of reaction and counter-reactions that one can argue (following the work of Wilson (1977) and Riley (1979)) plagues the strategic notion of equilibrium in economies with adverse selection.

More specifically, at the EPT equilibrium a single state contingent price kernel given by a set of fair prices is quoted for all contracts (Lemma 1), including in particular any contract inducing a pooling allocation and its associated 'skimming' contract. In terms of Rothschild and Stiglitz strategic logic, the pricing at the EPT of the pooling allocation reflects the belief that only agents of type $b$ would buy it; in this case, as we noted, the pooling contract does not constitute a profitable deviation. In terms of our Walrasian model this means that the pooling allocation $i$ is not budget feasible at the Walrasian equilibrium prices for agents of type $b$, and $ii$) it does not satisfy incentive compatibility.
for the agents of type $g$ when all contracts (and not just the pooling) are available for trade in the competitive market. At EPT equilibrium prices therefore, even though firms are indifferent between offering the pooling and the separating contracts, the pooling is not demanded by consumers of either type.

4.1.2 Welfare Properties of EPT Equilibria

As we noticed earlier, when the fraction $\xi^b$ of type $b$ agents is sufficiently small the separating allocation is not incentive efficient. The characterization of EPT equilibria obtained in Theorem 1 therefore reveals that the first welfare theorem does not hold, that EPT equilibria may not be incentive efficient. This should not come as a surprise: our formulation of Walrasian equilibria in adverse selection economies, EPT, clearly identifies an externality, which is not internalized by the structure of markets considered and may preclude the incentive efficiency of equilibrium allocations.

![Figure 3: The Second Welfare Theorem.](image)

$\omega$ denotes the initial endowment, $t = \{t_H, t_L\}$ the transfer, $U^b$ and $U^g$ the indifference curves of types $b$ and $g$ at the EPT equilibrium after the transfer. The initial EPT equilibrium allocation is $(x^{i,S})_{i=g,b}$. The EPT equilibrium after transfer $t$ is $(x^{i'})_{i=g,b}$.

It is useful however to examine more closely what is the precise source of the ineffi-
ciency. As shown by Lemma 1, at an EPT equilibrium the prices of contracts traded by each type are always fair. Thus at equilibrium there is never cross-subsidization across types: the contracts traded by each type satisfy a separate zero profit condition. We show next that not only such lack of cross-subsidization is clearly responsible for the possible inefficiency of EPT equilibria, but, in the economy considered here, this is in fact the only source of inefficiency: EPT equilibrium allocations are efficient within the restricted subset of allocations which are incentive efficient and satisfy an additional condition requiring that there is no cross-subsidization across types.\footnote{See Gale (1996) for a similar result for (appropriately refined) competitive equilibria of adverse selection economies with fully nonlinear price schedules.} Thus the following third best version of the first welfare theorem holds:\footnote{It is also of interest to observe that, when the fraction of the high risk types is very small, so that the separating allocation is mostly inefficient, it can be Pareto improved by the competitive equilibria obtained when firms are not allowed to discriminate agents with respect to the level of their trades (firms have to offer insurance at a price independent of the quantity traded).}

**Proposition 1** Under Assumption 1, all EPT equilibrium allocations are efficient within the restricted set of feasible allocations which are incentive compatible and, in addition, satisfy the condition

\[
\sum_{s \in S} \pi_s^i (x_i^s - \omega_s) \leq 0, \quad i \in \{g, b\}. \tag{7}
\]

On the other hand, the second welfare theorem holds for the present structure of markets: any incentive efficient consumption allocation can be decentralized as an EPT equilibrium with transfers (possibly dependent on the individual state, which is observable, but not the agents’ type, not observable).

**Proposition 2** Under Assumption 1, for any incentive efficient (and individually rational) consumption allocation \((x^b, x^g)\) there exists a set of transfers \((t_H, t_L)\) which are feasible, i.e., \(\sum_{s \in S} \left[ \xi^g \pi_s^g t_s + \xi^b \pi_s^b t_s \right] \leq 0\), and such that \((x^b, x^g)\) is the consumption allocation at the EPT equilibrium of the economy under consideration when each agent receives a transfer \((t_H, t_L)\).

Figure 3 illustrates this result. For the given initial endowment \(\omega\), the EPT equilibrium is the corresponding separating allocation \((x_i^s)_{i=g,b}\) Consider now the class of incentive efficient allocations characterized by full insurance of type \(b\) agents and partial insurance of type \(g\). They have the property that the consumption level of type \(g\) satisfies the incentive constraint of type \(b\) with equality (lies on \(U^b\)). Take the incentive efficient allocation \((x_i')_{i=g,b}\) as an example. Transfers \(t = (t_H, t_L)\) can be constructed as in the Figure so that the EPT equilibrium (the separating allocation) associated to the endowment \(\omega + t\) is in fact \((x_i')_{i=g,b}\). The formal proof is contained in the Appendix.
4.2 ALPT Equilibria

In the definition of EPT equilibria, the set of admissible trades of each agent is restricted by incentive compatibility constraints which relate the level of net trades an agent can make in a market to the net trades made in the other market. This fact generates, as we noticed, an externality in consumption. To ensure the efficiency of Walrasian equilibria it is then necessary to introduce markets where agents can purchase the ‘commodity’ which generates the externality. This is in accord with the approach proposed by Arrow (1969) and Lindahl (1919) for general economies with externalities and public goods.

Consider for example the case in which an increase in the consumption of the $b$ types relaxes the incentive compatibility constraints which are restraining $g$’s consumption and hence increase the utility of type $g$ agents. If a market is established where agents of type $g$ can purchase at given prices the ‘rights to the consumption of agents of type $b$’, type $g$ agents would be able to expand their consumption by acquiring the appropriate amount of rights for $b$’s consumption that guarantees the incentive compatibility constraints they face are satisfied. These rights are in turn sold by the type $b$ agents, who can increase the supply of such rights simply by increasing their consumption level. Whenever the incentive compatibility constraints is binding for the $g$ types, the equilibrium price of $b$’s consumption rights will be positive and hence the proceeds of their sale will allow agents of type $b$ to afford a higher consumption level. The externality can so be internalized. The same route can then also be followed to deal with the possible external effects of type $g$’s consumption on $b$’s admissible trades.

We proceed with the formal description of the structure of markets that decentralizes incentive efficient allocation in the adverse selection economies under consideration. The commodity space of every agent is expanded so as to have ‘complete markets’ in consumption rights. We will refer then to the competitive equilibrium notion we obtain as Arrow-Lindahl - Prescott-Townsend (ALPT) equilibrium.

Every agent has now access to markets where, in addition to trading claims to the consumption good contingent on his individual state and declared type, he can also sell rights for the consumption of the type he declared to be and purchase rights for the other type’s consumption. The introduction of markets for consumption rights also requires us to specify the agents’ initial endowment of such rights. We assume each agent is endowed with $\alpha \omega$ units of rights for his own declared type’s consumption and $(1 - \alpha)\omega$ units of rights for the other type’s consumption; $\alpha$ is a constant lying in $(0, 1]$, describing the proportion of the initial endowment given by rights for own consumption relative to the other type’s consumption.\(^{14}\) The higher is $\alpha$, the higher will be the amount of both the rights for the other type’s consumption an agent has to buy to satisfy incentive compatibility and the rights for own consumption the agent can supply in the market;\(^{14}\) The total amount of rights for consumption of any of the two types that is distributed as initial endowment is then equal to $\omega$, the initial endowment of the corresponding consumption good.

\(^{14}\)
thus the larger is $\alpha$ the larger will be the level of trades of consumption rights.\(^{15}\)

Let $\mathbf{z} \equiv \{z_g, z_b\} \in \mathbb{R}^4$ be, as before, the vector of net trades of contingent claims, where $z_i \equiv \{z_{i,H}, z_{i,L}\}$ denotes the trades of an agent when he declares to be of type $i \in \{g, b\}$. Consider an agent choosing net trades $z_g \neq 0$ in market $g$, hence declaring to be of type $g$; he cannot trade in the market designated for type $b$ agents (must choose $z_b = 0$). Moreover, such agent now also chooses a level of net trades of rights for type $b$’s consumption, $\zeta(b) \equiv \{\zeta_H(b), \zeta_L(b)\} \in \mathbb{R}^2$, as well as for type $g$’s consumption, $\zeta(g) \equiv \{\zeta_H(b), \zeta_L(b)\} \in \mathbb{R}^2$. Symmetric conditions apply to agents declaring to be of type $b$.

Again, the set of admissible trades in market $g$ is restricted by appropriate incentive constraints. We require the trades of contingent claims made in such market to be incentive compatible, now with respect to the trades of rights for $b$’s consumption made by the same agent: type $b$ agents must prefer the total amount held of such rights, on a per capita basis, equal to the initial endowment $(1 - \alpha)\omega$ plus the quantity acquired through trades, $\frac{\xi^g}{\xi} \zeta(b)$, to the total amount held of contingent claims, $\omega + z_g$ (condition (ii) in the definition of admissible trades, (8), below).\(^{16}\) In addition, we require that an agent cannot make in market $g$ trades that only a type $b$ agent would choose, as these trades would clearly signal the agent has lied when declaring his type to be $g$ (condition (iii) in (8) below).\(^{17}\) In particular, we exclude all those trades that only an agent of type $b$ would prefer to the trades made in the market by agents who claim to be of type $g$, $\bar{z}_g$, taken as given; as we will see, this guarantees that agents declare their true type.

Naturally, the agent’s consumption, and hence his utility, only depends upon the amount of contingent claims $z_g$ traded by the agent. The rights for $b$’s consumption do not enter therefore the utility of the agent directly, but determine the level of contingent claims he can buy, and hence his consumption.

In addition, an agent declaring to be of type $g$ can supply rights for type $g$’s consumption. His endowment of such rights is $\alpha \omega$. Moreover, by acquiring an amount $z_g$ of contingent commodities for his own consumption, the agent generates an equal amount of rights for $g$’s consumption.\(^{18}\) The agent has no direct use for such rights, as they do

\(^{15}\)The main consequence of the level of $\alpha$ for the properties of equilibrium allocations, as we will see later, is that the greater the level of trades of rights, the greater the payment needed from agents of one type to the other to internalize the externality.

Also, the condition $\alpha > 0$ is needed to ensure that agents can find some trades in the interior of their budget set at all prices and hence the existence of an equilibrium.

\(^{16}\)As for EPT, we omit imposing the other incentive constraint, requiring that type $g$ prefers $\omega + z_g$ to $(1 - \alpha)\omega + \frac{\xi^g}{\xi} \zeta(b)$, which is redundant.

\(^{17}\)As argued in footnote 9, an analogous constraint could have been imposed also for EPT, but was in that case redundant. This is no longer true for ALPT where the set of admissible trades in each market is larger (incentive compatibility is required in fact with respect to the amount of rights chosen by the same agent). Rustichini and Siconolfi (2002) do not impose this condition and find that equilibria may not exist.

\(^{18}\)The total amount outstanding of such rights will then be equal to type $g$’s consumption $z_g + \omega$. 

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not enter his utility nor affect the incentive constraints he faces. Hence, if they trade at a positive price he will sell the entire amount he has to the agents trading in the $b$ market; in general, the net supply of such rights cannot exceed the amount of rights possessed by the agent: $-\zeta(g) \leq z_g + \alpha \omega$.

The set of admissible trades of every agent is then:

$$
\mathcal{Z} = \left\{ \mathbf{z} + (\omega, \omega) \in \mathbb{R}_+^4; \ \zeta \in \mathbb{R}^4 : 
\begin{align*}
\forall i \in \{g, b\}, \quad z_i \neq 0 \Rightarrow z_j = 0 \quad \text{for} \ j \neq i \ \text{and}:

(i) \quad \frac{\xi_i^c}{\xi_j^c} \zeta(j) + (1 - \alpha)\omega \geq 0, \ \zeta(i) + z_i + \alpha \omega \geq 0,

(ii) \quad \sum_{s \in S} \pi_s^j u\left(\frac{\xi_i^c}{\xi_j^c} \zeta(j) + (1 - \alpha)\omega_s\right) \geq \sum_{s \in S} \pi_s^i u\left(z_{i,s} + \omega_s\right),

(iii) \quad \begin{cases}
\sum_{s \in S} \pi_s^j u\left(\omega_s + z_{i,s}\right) < \sum_{s \in S} \pi_s^i u\left(\omega_s + \bar{z}_{i,s}\right) \\
\sum_{s \in S} \pi_s^j u\left(\omega_s + z_{i,s}\right) < \sum_{s \in S} \pi_s^i u\left(\omega_s + \bar{z}_{i,s}\right)
\end{cases}
\right. 
\right\}
$$

(8)

Condition (i) requires the non-negativity of the amount held of consumption rights, both for the own and the other type's consumption, while conditions (ii) and (iii) are the incentive compatibility constraints restricting admissible trades. We should contrast these expressions to the corresponding ones for EPT. In EPT the admissible trades for an agent declaring to be of type $g$ depend on the level of trades made in the $b$ market, $\bar{z}_b$, taken as exogenously given. In ALPT they depend instead (via (ii)) on the level of trades of rights for $b$'s consumption, that the agent himself chooses, and (via (iii)) on the level of trades made in the same $g$ market, $\bar{z}_g$.

The set of admissible and budget feasible trades for the agent’s own consumption implied by these constraints is typically larger than in EPT. This is illustrated in Figure 4 below. The set of admissible and budget feasible trades of claims for type $g$’s consumption at an ALPT equilibrium is represented by the area below the solid line. On the other hand, at the EPT equilibrium, as we saw, the set of admissible and budget feasible trades for type $g$ agents is given by the smaller area lying below the lower contour of the fair odds line for type $g$ and the indifference curve of type $b$ at the separating allocation.

Also, in EPT the presence of the incentive compatibility constraints in the specification of the set of admissible trades generate, as we argued, an externality. On the other hand in ALPT, where markets for consumption rights are introduced, no externality is induced by the presence of these constraints, in condition (ii). At the same time, condition (iii) in $\mathcal{Z}$, where $\bar{z}_g$ appears, seems to introduce again an externality in the specification of the trading set. However we will show that condition (iii) only guarantees that, at equilibrium, agents sort themselves correctly through markets according to their true type, when this is only privately observable, but does not affect the equilibrium…
allocation and prices in any other way. More precisely, we will show (see the proof of Theorem 2 in the Appendix) that the agents’ choice is the same as the one we would obtain if each type $i$ were free to choose any incentive compatible (i.e., subject only to conditions (i) and (ii) above) and budget feasible allocation in the market designated for his type $i$. As a consequence, even though formally the set $\mathcal{Z}$ of feasible trades of every agent still depends on the level of trades made in the market, we can argue that in this case the externality does not matter, as the market for consumption rights allows to properly internalize it.

The vector of prices at which consumption rights can be traded is $p \equiv \{p_H(g), p_L(g), p_H(b), p_L(b)\} \in \mathbb{R}_+^4$, while the vector of prices of state contingent commodities is again denoted as $q \equiv \{q_{g,H}, q_{g,L}, q_{b,H}, q_{b,L}\} \in \mathbb{R}_+^4$. The choice problem of an agent of type $i \in \{g, b\}$ has now the following form:

$$
\max_{(z, \zeta) \in \mathcal{Z}} \sum_{s \in S} \pi^i_s \left( \sum_{j \in \{g, b\}} z_{j,s} + \omega_s \right)
$$

subject to

$$
q \cdot z + p \cdot \zeta \leq 0
$$

Firms are characterized by the same technology $Y$ as before and their choice problem is also the same:

$$
\max_{r \in Y} \quad q \cdot y
$$

**Definition 3** An ALPT equilibrium is a collection of net trades for each type of consumers $(z^i, \zeta^i)_{i \in \{g, b\}}$, a production vector $y$, a price vector $(p, q)$ and a pair $\bar{z}_g, \bar{z}_b$ such that:

(i) for each $i \in \{g, b\}$, $(z^i, \zeta^i)$ is a solution of the optimization problem of type $i$ consumers ($P^{\text{ALPT},i}$), given $(p, q)$ and $\bar{z}_g, \bar{z}_b$;

(ii) $y$ solves the firms’ profit maximization problem ($P^{\text{ALPT},f}$), given $q$;

(iii) markets clear, both for contingent commodities:

$$
\sum_i \xi^i z^i \leq y;
$$

and for consumption rights:

$$
\sum_i \xi^i \zeta^i \leq 0;
$$

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(iv) the level of trades in each market is consistent with the actual choice made by agents in that market:

\[ \bar{z}_g = z^g \]

\[ \bar{z}_b = z^b. \]

As the analogous condition in EPT, (iv) requires that agents declare their type truthfully. Also, the specification of the set of admissible trades \( Z \) together with the consistency condition (iv) and the market clearing conditions again imply that the equilibrium consumption allocation, \( z^b, z^g \) is mutually incentive compatible.

We show next that, for the economy under consideration, an ALPT equilibrium always exists.\(^{19}\)

**Theorem 2** Under Assumption 1, an ALPT equilibrium always exists.

Prescott and Townsend (1984, p. 44) briefly discuss a formulation of competitive equilibrium for adverse selection economies. A more extensive discussion appears in the working paper version of that paper, where three different unsuccessful attempts to the decentralization of efficient allocations are examined. The first attempt consists in analyzing competitive markets for insurance where agents can trade at linear prices, equal to the average odds in the population, and face no incentive compatibility constraint restricting their set of admissible trades; in this case, as Prescott and Townsend notice, competitive equilibria typically fail to exist and, we can add, are not efficient.\(^{20}\) Their second attempt is equivalent to allowing fully non-linear price schedules. It produces an existence result but a disturbing multiplicity of equilibria.\(^{21}\) The third attempt in Prescott and Townsend is related to ALPT in that it aims at decentralizing incentive efficient allocations by expanding markets in the spirit of Arrow and Lindahl. In their formulation, each type of agent chooses directly the consumption allocations for the other type, rather than rights to the other type’s consumption. In our context this is equivalent to the requirement that the price an agent, say of type \( g \), faces for the rights for \( b \)’s consumption, \( p(b) \) is the same as the price that agents of type \( b \) face for claims for their own consumption, \( q_b \) (or that the two are traded in the same market). At

\(^{19}\)Standard arguments cannot be directly applied here. We refer to the Appendix for a discussion of the difficulties encountered to establish existence, and for the proof of the result.

\(^{20}\)Prescott and Townsend (1984) draw then the conclusion that the source of the difficulties with adverse selection is that "agents with characteristics which are distinct and privately observed at the time of initial trading enter the economy-wide resource constraints in a heterogeneous way" (p. 23). We should point out that, as shown by Bisin and Gottardi (1999), this feasibility problem indeed arises when exclusive contracts are not enforceable, and might indeed preclude the existence of competitive equilibria at any price.

\(^{21}\)The recent work of Gale (1992) and Dubey-Geanakoplos-Shubik (1995), can be interpreted as following up on this attempt by introducing appropriate refinements; see footnote 7.
these prices, the choices of the two types of agents will typically not be consistent (in our notation, \( \xi^g \zeta(b) + (1 - \alpha) \omega \neq z_b + \omega \)), hence, as Prescott and Townsend notice, an equilibrium will generally fail to exist. In other words, Prescott and Townsend expand the commodity space to \( \mathbb{R}^4_+ \) while we expand it to \( \mathbb{R}^8_+ \). In our case, therefore, trades by different types are not aggregated into a single market clearing condition, which, as noticed by Prescott and Townsend (1984) would interfere with the functioning of competitive markets.

4.2.1 Characterization and Welfare Properties of ALPT Equilibria

The following result shows that the structure of markets considered here indeed allows to solve the problem of decentralizing incentive efficient allocations, or (an incentive constrained version of) the First Theorem of Welfare Economics holds:

**Proposition 3** All ALPT equilibria are incentive efficient.

We proceed now with the characterization of ALPT equilibria in our economy. We show in the proof of Theorem 2 in the Appendix that there is always an ALPT equilibrium where prices are as follows:

\[
q_{i,s} = \pi^i_s, \, s \in S, \, i \in \{g, b\} \tag{9}
\]

\[
p_s(b) = \beta \pi^b_s, \, p_s(g) = 0, \, s \in S \tag{10}
\]

for some \( \beta \in (0, 1) \). The form of these equilibrium prices clearly illustrates how incentive efficient allocations are sustained as ALPT equilibria. The prices \( q \) at which agents can trade the contingent claims for the consumption good are fair (as in EPT). But then the type \( g \) agents have to pay a positive price to acquire the rights for \( b \)'s consumption needed to guarantee the incentive compatibility of their trades, while the type \( b \) agents receive a positive revenue from the sale of their own consumption rights. On the other hand, the rights for \( g \)'s consumption trade at a zero price. As a consequence, agents of type \( b \) do not have to pay any cost to ensure the incentive compatibility of their allocation, and agents of type \( g \) receive nothing from the sale of their own rights. At the equilibrium allocation obtained at the above prices, \( b \)'s incentive constraint (3) is binding, hence agents of type \( b \) exert a positive externality on the type \( g \) agents, and are subsidized by them.

Figure 4 illustrates an ALPT equilibrium. Using the characterization of the equilibrium prices in (9-10) we can compute the boundary of the set of admissible and budget feasible trades of claims for own consumption of type \( g \) (see the caption of the figure for details), that is, the solid curve in the figure. The equilibrium allocation for agents of type \( g \), \( x^{g,E} \), corresponds to their preferred allocation on this curve. The equilibrium allocation of agents of type \( b \) (not in the figure) is just their highest budget feasible full
insurance allocation, where their budget includes the possible income due to the sale of consumption rights.

Is the presence of some level of cross-subsidization an intrinsic feature of ALPT equilibria? To answer this question, we can examine how the equilibrium varies with the initial distribution of consumption rights, as described by \( \alpha \). Let us denote by \( \beta(\alpha) \) the equilibrium value of \( \beta \), associated to \( \alpha \), in the expression (10) of equilibrium prices; analogously, let \( x(\alpha) \) be the corresponding equilibrium allocation. The next proposition reveals an important property of ALPT equilibria and in particular of the level of cross-subsidization which obtains at equilibrium. As \( \alpha \) tends to 0, the sequence of ALPT equilibrium allocations \( x(\alpha) \) converges to the incentive efficient allocation \( x \) characterized by the minimum level of subsidization from agents of type \( g \) to agents of type \( b \).
More formally, $x$ is obtained as a solution of the problem of maximizing the expected utility of type $g$ agents, $U^g(x^g)$, subject to the resource feasibility, incentive compatibility constraints and the additional constraint that $\sum_s \pi^b_s(x^b_s - \omega_s) \geq 0$, i.e., that $b$ is not subsidizing $g$. It is immediate to see, given the characterization provided of incentive efficient allocations, that $x$ coincides with the Rothschild and Stiglitz separating allocation, whenever this is incentive efficient, and is otherwise (when the constraint $\sum_s \pi^b_s(x^b_s - \omega_s) \geq 0$ is not binding) given by the incentive efficient allocation at which $b$'s welfare is minimal. \(^{22}\)

**Proposition 4** Under Assumption 1, $\lim_{\alpha \to 0} x(\alpha) = \bar{x}$.

When $\alpha$ is close to 0, the initial distribution of rights is the most favorable to the type $g$ agents, as they receive almost the entire initial endowment of rights for $b$'s consumption, and hence the additional amount of such rights they need to purchase will be the lowest. Since the way type $b$ agents receive a subsidy is through the sale of rights for their type’s consumption, the equilibrium will be the incentive efficient allocation $\bar{x}$ where the subsidy to the $b$ agents is minimal.

Proposition 4 shows that, for the economies in which Rothschild-Stiglitz separating allocation is incentive efficient, this allocation is decentralized as an ALPT equilibrium with $\alpha = 0$, and the equilibrium prices satisfy (10) with $\beta = \lim_{\alpha \to 0} \beta(\alpha) < 1$. On the other hand, for the economies in which Rothschild-Stiglitz separating allocation is not incentive efficient an ALPT equilibrium does not exist when $\alpha = 0$, \(^{23}\) but the sequence of ALPT equilibria allocations converges, as $\alpha \to 0$, to $\bar{x}$, the allocation with the minimum level of subsidization from agents of type $g$ to agents of type $b$, and equilibrium prices in the limit satisfy (10) with $\lim_{\alpha \to 0} \beta(\alpha) = 1$.

Using such a characterization of ALPT equilibria with $\alpha = 0$, it is now straightforward to extend the second welfare theorem obtained for EPT equilibria, Proposition 2, to ALPT equilibria: any incentive efficient consumption allocation can be decentralized as an ALPT equilibrium with an initial distribution of rights given by $\alpha = 0$ and with transfers (possibly dependent on the state but not the agents’ type).

**Proposition 5** Under Assumption 1, for any incentive efficient consumption allocation $(x^b, x^g)$ there exists a set of transfers $(t_H, t_L)$, which are feasible, i.e., $\sum_{s \in S} [\xi^g \pi^g_s t_s + \xi^b \pi^b_s t_s] \leq 0$, and such that $(x^b, x^g)$ is an ALPT equilibrium allocation, with $\alpha = 0$, when each agent receives the transfer $(t_H, t_L)$.

### 4.2.2 The Implementation of ALPT Equilibria

While we have shown that by appropriately designing markets for consumption rights incentive Pareto optimal allocations can be decentralized as Walrasian equilibria, such

\(^{22}\)Note that $\bar{x}$ is also the allocation induced by what is sometimes referred to as the Wilson-Miyazaki pair of contracts.

\(^{23}\)See footnote 15.
markets might appear to be difficult to implement in actual economies. The objective of this section is to argue that, on the contrary, markets for consumption rights can in fact be designed along the lines, e.g. of markets for pollution rights, or of markets for the access to clubs, so that they can be implemented and support efficient equilibrium allocations when EPT equilibrium allocations are not constrained efficient.

Markets for consumption rights must necessarily be associated to an enforcement mechanism. In the case of pollution rights, the enforcement mechanism must guarantee that firms do not pollute without having acquired the necessary pollution rights from the consumers.\textsuperscript{24} In our adverse selection economy, the enforcement mechanism must prevent the type \( g \) consumers (the 'low risks') from trading contingent claims for their own consumption if they do not hold the appropriate amount of rights for type \( b \)'s (the 'high risks') consumption.\textsuperscript{25} In fact, a simpler design of markets for consumption rights with the properties just outlined, modelled after the existing markets for pollution rights, requires enforcement to operate on firms rather than consumers. Moreover, the enforcement mechanism is easier to implement if the units of the consumption rights are re-normalized so that every consumer is endowed with (a single unit of) the right to trade in market \( g \).\textsuperscript{26}

We now illustrate how such a decentralization scheme may work and give rise to the same equilibrium outcomes as the structure of markets considered in ALPT. There are again two markets, one for the agents (declaring to be) of type \( b \) and one for those (declaring to be) of type \( g \). If a consumer chooses to trade in market \( b \) he can sell the right to trade in market \( g \) he is endowed with, for a price determined in equilibrium, and is then free to trade his preferred amount of contingent claims to the consumption good at prices \( q_b \), which in equilibrium will be fair, \( q_{b,s} = \pi_{b,s}^b, s \in S \). He will therefore choose to fully insure (if his true type is \( b \)); also, when the price of the trading rights is positive he can use the proceeds from the sale of its right to afford a level of consumption which is in excess of the expected value of his endowment.

When firms operate in the \( b \) market, they are free to choose the composition of the contract they wish to offer, specifying the net payment to the agent in each state, and how many units of the contract to sell on the market, so as to maximize profits. The \( g \) market, on the other hand, constitutes in our set-up the high quality market. To be able to sell a contract in this market and make sure that only the "right fraction" of the

\textsuperscript{24}See Carlson et. al. (1993) and Ledyard and Szakaly (1994) for a discussion of the properties of enforcement mechanisms in the market for pollution rights which make them easily and effectively implementable.

\textsuperscript{25}Arrow-Lindahl equilibrium concepts are sometimes criticized on the basis of the fact that the price taking assumptions is inconsistent with individualized prices (as markets are too 'thin'); see for instance Chari-Jones (2000). But in the economy considered here there is a continuum of agents of each type, hence the presence of a market of rights to consumption for each type is still consistent with price taking behavior.

\textsuperscript{26}Relatedly, see Cole-Prescott (1997) and Ellickson-Grodal-Scotchmer-Zame (1999) for the study of competitive equilibria of economies with clubs.
agents in the economy will buy it, firms have to acquire an appropriate amount of trading rights from the agents who chose to trade in market $b$. More precisely, firms operating in the $g$ market have to satisfy an enforcement constraint, requiring each contract sold to be backed by at least $\xi^b_g$ units of the trading rights bought from agents in market $b$, as well as an incentive compatibility constraint, requiring the specification of the contract to be such that only type $g$ agents prefer it to the contracts that can be traded in market $b$.\footnote{As in the case of ALPT, a self-selection constraint also needs to be imposed, prescribing firms not to offer contracts in market $g$ that only type $b$ agents would buy.} The larger the amount of trading rights backing a contract, the better the contract can be since the incentive compatibility constraint will be looser, but also the higher will be its price, which in equilibrium naturally includes the cost of the rights.

The consumers trading in market $g$ face then a given set of contracts among which they can choose, which at equilibrium coincide with the set of contracts firms are willing to offer in this market. The prices they face is given by the cost, evaluated at the fair price $q_{g,s} = \pi^g_s$, $s \in S$, of the amount of contingent claims included in a contract, to which the cost of the trading rights backing the contract has to be added.

At a competitive equilibrium every agent chooses to trade in the market designated for his type and markets clear. In particular, the price of trading rights is such that the number of rights bought by firms operating in the $g$ market is equal to the number of agents of type $b$ in the economy, who all sell the right they are endowed with. It is possible to show that the competitive equilibrium allocation is the same as the one at an ALPT equilibrium, with $\alpha = 0$;\footnote{Or, more precisely, when $\alpha \rightarrow 0$.} hence the structure of markets described decentralizes allocation $x$, the incentive efficient allocation which is preferred by the type $g$ agents.

With the objective of exploring some strategic foundations for our notion of ALPT equilibrium, we have also studied an extension of the definition of the core proposed by Marimon (1988) (see also Boyd-Prescott-Smith (1988)) for adverse selection economies.\footnote{It should be pointed out though that various other notions have been proposed and an agreements has not yet been reached on what is an appropriate notion of the core for economies with adverse selection.} Such notion is characterized by the fact that blocking coalitions cannot tax, only subsidize agents outside the coalition, but otherwise agents of one type can separate at no cost. It is fairly immediate to see that a single allocation is in the core according to this notion, and is $x$. But suppose instead that the high quality types ($g$ in our set-up) can form a coalition excluding the low quality types ($b$) only if they pay a given amount $C$, which we can interpret as the cost of acquiring the ‘right’ to separate from the low quality types (of course the allocation proposed by the deviating coalition must also be incentive compatible). This clearly parallels the role of consumption rights in ALPT. It can be shown that, by varying $C$, we can obtain as core allocations the set of ALPT equilibrium allocations corresponding to different values of $\alpha$.\footnote{Or, more precisely, when $\alpha \rightarrow 0$.}
5 Conclusions

We have studied in this paper how Walrasian markets work in economies with adverse selection when exclusive contracts are available and whether their outcome is efficient. In particular, we have identified a form of externality as a potential problem for the operation of Walrasian markets in this set-up. We have then shown that an enlarged structure of markets, which includes markets for consumption rights, allows to internalize this externality and hence to 'resolve' the problem of decentralizing incentive efficient allocations. All Walrasian (ALPT) equilibrium allocations are incentive efficient in this case. We have also shown that when such enlarged set of markets is not available, a Walrasian (EPT) equilibrium always exists but may fail to be incentive efficient, and inefficiency is a robust phenomenon.

Our results have been derived for a class of simple insurance economies with adverse selection where agents can be of two possible types, and the - privately observed - type of each agent only concerns the probability structure of the idiosyncratic shocks affecting the agent. Such economies provide an important benchmark for the analysis of markets and equilibria with asymmetric information, at least since the work by Rothschild and Stiglitz (1976). However, the equilibrium concepts we introduced can be extended to more general classes of economies with adverse selection.
Appendix

We omit the proof of Theorem 1, showing the existence of EPT equilibria. We refer the interested reader to Bisin and Gottardi (2003).

Proof of Theorem 2. Some steps of the proof of this Theorem, establishing the existence of ALPT equilibria, are useful to better understand our specification of ALPT equilibria and their characterization given in the paper. We therefore provide here a brief sketch of such steps, while referring again the reader to Bisin and Gottardi (2003) for a detailed and complete proof.

Formally, ALPT equilibria are Arrow-Lindahl equilibria for economies with externalities in consumption. However, the presence of markets indexed on the agents’ types when types are privately observed and the peculiar form of the externality induced by the presence of incentive compatibility in the specification of the set of the agents’ admissible trades imply that standard arguments for the existence of Arrow-Lindahl equilibria cannot be used to establish the existence of ALPT equilibria.

The first difficulty of the proof consists in dealing with condition (iii) in the definition of the set of feasible trades, $Z$ in (8). As noted in the text, this condition appears to introduce an externality. However we will show that it only ensures that, at equilibrium, agents choose the market designated for their type, with no effect on equilibrium allocations and prices.

To this end, we consider the following alternative specification of type $i$’s choice problem:

$$\max_{(z_i, \zeta) \in \mathbb{R}^2 \times \mathbb{R}^4} \sum_{s \in S} \pi^i_s u (z_{i,s} + \omega_s) \quad (AP^{ALPT,i})$$

subject to:

\[
\begin{aligned}
& (i) \quad z_i + \omega \in \mathbb{R}^2_+; \quad \zeta(i) + z_i + \alpha \omega \geq 0; \quad \frac{\xi^i}{\xi^j} \zeta(j) + (1 - \alpha) \omega \geq 0, \text{ for } j \neq i, \\
& (ii) \quad \sum_{s \in S} \pi^i_s u \left( \frac{\xi^i}{\xi^j} \zeta_s(j) + (1 - \alpha) \omega_s \right) \geq \sum_{s \in S} \pi^i_s u (z_{i,s} + \omega_s),
\end{aligned}
\]

and

$$\sum_s q_{i,s} z_{i,s} + p \cdot \zeta \leq 0.$$

This differs from $(P^{ALPT,i})$ for the fact that type $i$ agents are required to trade in market $i$, at the prices $(q_i, p)$, but are otherwise free to choose any budget feasible and incentive compatible bundle in such market (i.e., the restriction imposed by condition (iii) in the specification of $Z$ is no longer imposed).
When \((PALPT,i)\) is replaced by \((APALPT,i)\) in definition 3 we say we have an auxiliary equilibrium. To show the relationship between ALPT and auxiliary equilibria we establish some properties of the allocation of the contingent commodities and consumption rights at auxiliary equilibria.

Consider the optimal level of the net supply of rights for own consumption, obtained from \((APALPT,i)\). As long as the price of such rights is positive, the supply will clearly be at the maximal level, e.g., for an agent of type \(g\) will be \(\zeta^g(g) = -(z^g_g + \alpha \omega)\). Substituting this expression into the market clearing condition for consumption rights, \(\xi^b \zeta^b(g) + \xi^g \zeta^g(g) = 0\), we get:

\[
\frac{\xi^b}{\xi^g} \zeta^b(g) + (1 - \alpha) \omega = z^g_g + \omega,
\]

thus the amount of rights for \(g\)’s consumption held by the type \(b\) agents has to equal the actual consumption level of the \(g\) types.

On the other hand, when \(p(g) = 0\) the supply of rights for \(g\)’s consumption is indeterminate: any level \(\zeta^g(g) \geq -(z^g_g + \alpha \omega)\) is optimal. The demand of such rights by the \(b\) types is also indeterminate. When \(p(g) = 0\) the incentive compatibility constraint can be ignored to find the consumption level of the \(b\) types \(z^b_b\) solving \((AP^{ALPT,b})\); \(\zeta^b(g)\) can then be set at any level such that \(\sum_{s \in S} \pi^g_s u \left( \frac{\xi^b}{\xi^g} \zeta^b(g) + (1 - \alpha) \omega_s \right) \geq \sum_{s \in S} \pi^g_s u \left( z^b_{b,s} + \omega_s \right)\).

These features imply that, even though the market clearing condition for consumption rights requires in this case demand to be less or equal than the supply, we can always, without any loss of generality, set the supply at its maximal level \(-(z^g_g + \alpha \omega)\) and require that at equilibrium demand equals supply.

Thus we can say that, both when \(p(g)\) is zero or is positive, at equilibrium the amount of rights for \(g\)’s consumption held by \(b\) will be equal to the actual consumption level of \(g\). A symmetric argument then holds for \(\zeta^b(b)\) and \(z^b_b\). Since \(z^g_g\) has to be incentive compatible with respect to \(\zeta^g(b)\) and \(z^b_b\) with respect to \(\zeta^b(g)\), this finding implies that the consumption allocation \(z^g_g + \omega, z^b_b + \omega\) will also have to be mutually incentive compatible.

On this basis we can show:

**Lemma A. 1** The set of ALPT equilibria and auxiliary equilibria coincide.

Thus at an ALPT equilibrium the choice of any agent is the same as the one obtained when the agent is free to choose any incentive compatible and budget feasible bundle in the market designated for his own type.

The problem of finding an ALPT equilibrium can then be reduced to the problem of finding an auxiliary equilibrium, where the consumers’ optimization problem is simpler. Establishing their existence still poses some difficulties. The main difficulty arises from the fact that the agents’ choice problem is not convex: the set of allocations satisfying incentive compatibility is in fact typically not convex. The existence of Arrow-Lindahl equilibria with non-convexities is problematic (and, as far as we know, has not been addressed in the literature). The source of the difficulty is that the usual way of dealing
with non-convexities by exploiting the presence of a continuum of agents (i.e., let agents of the same type choose different bundles - or lotteries - and ensure that average demand satisfies market clearing) cannot be exploited in the case of Arrow-Lindahl equilibria. In the presence of markets for consumption rights, to require that market clearing only holds for average demand does not ensure the feasibility (in our case, the incentive compatibility) of the induced consumption allocation. For this, the whole lottery chosen by agents of one type has to coincide with the one chosen by the other type.

To address these difficulties, the strategy of our proof consists in identifying a subset of the set of possible prices, characterized by conditions (9) and (10) in the text, that is by the fact that contingent prices are fair, while the price of rights for type $g$’s consumption is zero, and the price of rights for $b$’s consumption is some positive fraction $\beta$ of the probability $\pi^b$. We then show that, when prices are restricted to lie in this set, agents’ demand is single-valued, i.e. that non-convexities do not matter over such domain. Moreover, we show that local nonsatiation holds, demand is always well-defined and a continuous function of prices in this set (which can be simply parameterized by $\beta$) and that it exhibits the needed boundary behavior. On this basis, the existence of a price vector in this set such that market clears (and hence of an auxiliary, as well as of an ALPT equilibrium) follows by a fairly standard fixed point construction.

**Proof of Proposition 1.** If, at a solution of the problem of maximizing a weighted average of the agents’ utility subject to (2-3) and (7), both incentive compatibility constraints hold as equalities, under the assumptions made on agents’ preferences (in particular, the single crossing property) we must have $x^g = x^b$. But then (7) implies $x^g = x^b = \omega$ and, since the separating allocation is always preferred by both agents to autarchy, this cannot be a solution.

On the other hand, if only one of the two incentive constraints is binding, say the one for type $b$ (constraint 3), then the optimal $x^b$ is simply obtained by maximizing $U^b(x^b)$ over (7); thus it will always be at the full insurance point $x^b_{H} = x^b$ satisfying (7). The level of $x^g$ is then determined by maximizing $U^g(x^g)$ subject to (7) and (3), taking $x^b$ as given at the full insurance level determined before. It is immediate to see that the pair $(x^b, x^g)$ is the same as the separating allocation of Rothschild and Stiglitz. If we apply a symmetric argument when (2) is the only constraint binding, we find that no solution exists in this case (when $x^g$ is at the full insurance level satisfying (7), no value exists for $x^b$ which also satisfies (7) and (2)).

**Proof of Proposition 2.** Let $(x^b, x^g)$ be an arbitrary incentive efficient allocation. By Lemma 1, at an EPT equilibrium, prices of contingent commodities are necessarily ‘fair’. If we consider then the budget equations $\pi^b \cdot z^b$ and $\pi^g \cdot z^g$ going through, respectively, the points $(x^b - \omega)$ and $(x^g - \omega)$, they will intersect at a single point; call it $t := (t_L, t_H)$. Since $(x^b, x^g)$ satisfies the resource feasibility condition (1) and $\pi^i \cdot (x^i - \omega - t) = 0$, 

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\( i \in \{g, b\} \), the allocation \((\omega + t)\) is feasible:

\[
\sum_{s \in S} \left[ \xi_g \pi_s^g (\omega_s + t_s) + \xi_b \pi_s^b (\omega_s + t_s) \right] \leq \sum_{s \in S} \left[ \xi_g \pi_s^g \omega_s + \xi_b \pi_s^b \omega_s \right],
\]

which implies that a transfer \((t_H, t_L)\) to all the agents is also feasible.

We show next that \((x^b, x^g)\) is the (unique in fact) EPT equilibrium consumption allocation of the economy with endowments \((\omega + t)\). Suppose not. There exists then another consumption bundle, say \(\hat{x}^g\) for \(g\), such that \((\hat{x}^g - (\omega + t))\) also lies in \(Z(x^g - (\omega + t), x^b - (\omega + t))\), i.e. is incentive compatible relative to \(x^b - (\omega + t)\) and budget feasible \((\sum_s \pi_s(x^g_s - (\omega_s + t_s)) \leq 0)\), and is such that \(\hat{x}^g\) is strictly preferred to \(x^g\) by type \(g\). But then \(\sum_{s \in S} \left[ \xi_g \pi_s^g (\hat{x}^g_s - (\omega_s + t_s)) + \xi_b \pi_s^b (x^b_s - (\omega_s + t_s)) \right] \leq 0\); thus \((\hat{x}^g, x^b)\) is also feasible, incentive compatible and Pareto dominates \((x^g, x^b)\), a contradiction. ■

Proof of Proposition 3. The proof is quite standard. Suppose not, i.e. there exists a feasible, incentive compatible allocation \((\hat{x}^b, \hat{x}^g)\) which Pareto dominates the equilibrium allocation \((x^b, x^g)\). Then, note that the following constitutes an admissible choice for agent \(i \in \{g, b\}\) (i.e. satisfies conditions (i) and (ii) defining the set of admissible trades of problem \((AP^{ALPT, i})\): \(\hat{z}_i^i = (\hat{x}^i - \omega), \hat{z}_i^j = -[\hat{x}^j - \omega(1 - \alpha)], \hat{z}_i^j = [\hat{x}^j - (1 - \alpha) \omega] / \xi_i, \) for \(j \neq i\). Given the equivalence between auxiliary and ALPT equilibria, established in Lemma A.1, and the fact that local nonsatiation holds, as argued in the Proof of Theorem 2\(^{30}\), it must be

\[
q_g \cdot (\hat{x}^g - \omega) + p(b) [\hat{x}^b - (1 - \alpha) \omega] \xi^b / \xi^g - p(g) [\hat{x}^g - \omega(1 - \alpha)] \geq 0
\]
\[
q_b \cdot (\hat{x}^b - \omega) + p(g) [\hat{x}^g - (1 - \alpha) \omega] \xi^g / \xi^b - p(b) [\hat{x}^b - \omega(1 - \alpha)] \geq 0
\]

one of the two inequalities being strict. Summing then the two inequalities, multiplied respectively by \(\xi^g\) and \(\xi^b\), we get:

\[
\xi^g q_g \cdot (\hat{x}^g - \omega) + \xi^b q_b \cdot (\hat{x}^b - \omega) > 0.
\]

Since \((\hat{x}^b, \hat{x}^g)\) is a feasible allocation, \((\xi^g \hat{x}^g, \xi^b \hat{x}^b)\) lies in the firms’ production possibilities’ set \(Y\) and, by the previous inequality, yields positive profits. This contradicts the fact that the production plan chosen by the firms at equilibrium, \((\xi^g x^g, \xi^b x^b)\), maximizes their profits at the prices \(q\), since its profits equal zero. ■

Proof of Proposition 4. At any ALPT equilibria where prices are of the form (9-10) type \(b\) agents face a zero price for the rights for \(g\)’s consumption. Hence their

\(^{30}\)See Bisin and Gottardi (2003) for the formal proof.
consumption choice is not constrained by incentive compatibility and they always choose to fully insure:

\[ z^b_{b,H} + \omega_H = z^b_{b,L} + \omega_L = \frac{(1 - \beta + \alpha \beta) \left( \sum_{s \in S} \pi^b_s \omega_s \right)}{1 - \beta}. \] (11)

The term on the right hand side is the income of a type \( b \) agent, given by the value of his endowment of contingent claims, evaluated at \( q_b = \pi^b \), plus the value of the amount of rights for type \( b \)'s consumption the agents has, evaluated at \( p(b) = \beta \pi^b \).

Since both sequences \( x(\alpha) \) and \( \beta(\alpha) \), giving the equilibrium allocation and prices for different values of \( \alpha \), lie in compact sets, they admit convergent subsequences; let \( \hat{x}, \hat{\beta} \) be their limit. On the basis of the above argument, the equilibrium consumption level of type \( b \)'s agents is given by (11), i.e.

\[ x^b_s(\alpha) = \frac{(1 - \beta(\alpha) + \alpha \beta(\alpha)) \left( \sum_{s \in S} \pi^b_s \omega_s \right)}{1 - \beta(\alpha)}, \quad s \in S. \]

The subsidy received by \( b \), \( \sum_{s \in S} \pi^b_s (x^b_s(\alpha) - \omega_s) \), is then equal to

\[ \frac{\alpha \beta(\alpha)}{1 - \beta(\alpha)} \sum_{s \in S} \pi^b_s \omega_s. \] If \( \hat{\beta} = \lim_{\alpha \to 0} \beta(\alpha) \) is strictly less than one, the value of the subsidy converges to full insurance at fair odds, i.e. to type \( b \)'s component of Rothschild and Stiglitz separating allocation \( x^{b,S} \). From the first welfare theorem (Proposition 3) \( x^{b,S} \) must be part of an incentive efficient allocation, hence \( x^g(\alpha) \) must converge to \( x^{g,S} \) and the separating allocation \( x^{g,S}, x^{b,S} \) coincides with \( x \) in this case.

On the other hand, if \( \hat{\beta} = 1 \), the problem of agents of type \( g \), (\( P'_{\text{ALPT}} \)), in the limit (for \( \alpha = 0, \beta = 1 \)), reduces to the problem of maximizing \( U^g(x^g) \) subject to the resource feasibility and incentive constraints (1-3) (since the budget constraint coincides with (1)). The solution of this problem is \( x \). The value of the subsidy to agents of type \( b \) in this case converges to a level which is positive or zero (but cannot be negative since it is strictly positive for all \( \alpha > 0 \)). ■

**Proof of Proposition 5.** Let \((x^b, x^g)\) be any incentive efficient allocation. By Proposition 2, there exists a set of feasible transfers \((t_H, t_L)\) such that \((x^b, x^g)\) is an EPT equilibrium allocation for the economy with endowments \( \omega + t = (\omega_L + t_L, \omega_H + t_H) \). Moreover, by Theorem 1, the economy with endowments \( \omega + t \) has a unique EPT equilibrium (which is then \((x^b, x^g)\)), coinciding with the Rothschild and Stiglitz separating allocation for that economy. Since \((x^b, x^g)\) is incentive efficient, from Proposition 4 it follows that it can be decentralized as an ALPT equilibrium with \( \alpha = 0 \) of the economy with endowments \( \omega + t \). ■
References


