1 Missing Markets

Consider an economy with $s = 1, \ldots, S$ states of uncertainty. Consider an asset or firm $j$ with payoff $x^j$. For any agent $i$ with utility $u(c^i)$, the price $p^j_i$ can be written

$$p^j_i = E_t(MRS^i_{s,t+1}x^j_{s,t+1}) = \sum_{s=1}^{S} \pi_s MRS^i_{s,t+1}x^j_{s,t+1}$$

(1)

where the index $s$ refers to the state, $\pi_s$ is the objective probability of state $s$, and we denote $MRS^i_{s,t+1} = \beta \frac{u'(c^i_{s,t+1})}{u'(c^i_{t})}$. The right-hand-side of equation (1) is interpreted as the marginal valuation of agent $i$ for asset $j$; that is, the price agent $i$ is willing to pay (equivalently, to be payed) to buy (equivalently, to sell) an infinitesimal extra amount of asset $j$. In equilibrium, the market price of the asset $p^j_i$ must be such that each agent $i$ is indifferent between buying or selling the asset; that is, every agent’s marginal valuation for the asset must be the same and equal to the equilibrium market price.

If markets are complete any payoff can be generated via a portfolio of the traded assets. (Form Exercise 1 on Class 1: In an economy with $S$ states of uncertainty, complete markets are obtained if $S$ or more assets with independent payoff are traded; that is, if the matrix generated by stacking each asset’s payoff as column has rank $S$). In particular, a payoff equal to 1 in state $s$ and 0 otherwise can be generated; and this can be done for any $s = 1, \ldots, S$. Using equation (1) to value these assets, for all the different agents $i$, one concludes that

$$MRS^i_{s,t+1} = MRS^{i'}_{s,t+1}, \text{ for all agents } i \text{ and } i'$$

(2)

Since equation (2) holds for any $s$, the whole $MRS^i_{s,t+1}$ is equalized across all agents $i$ in the economy.

If markets are missing and hence are incomplete, instead, a payoff equal to 1 in state $s$ and 0 otherwise cannot always be generated for any state $s$. As a consequence, in general, $MRS^i_{s,t+1}$ is not equalized across agents $i$ for all state $s$. (Only marginal valuations for the payoff that either are traded or can be generated with traded assets are equalized.)

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1You can as well assume that his/her utility is also indexed by $i$, that is, that agents have different utilities. Identical preferences are used only to justify using aggregate consumption as an argument on the marginal rate of substitution in asset pricing.
1.1 Capital Structure and Security Design

Suppose a firm has a payoff \( x_{t+1}^j \) to finance. If markets are complete, then the proceeds of equity financing (that is, selling directly the payoff \( x_{t+1}^j \) at time \( t \)) is:

\[
p_j^t = E_t(MRS_{t+1}^i x_{t+1}^j)
\]

and is independent of the agent \( i \) whose marginal valuation is used; each agent would pay the same amount for the equity of the firm. The same is true if markets are incomplete but the payoff \( x_{t+1}^j \) can be generated with traded assets.

If instead markets are incomplete and the payoff \( x_{t+1}^j \) cannot be generated with traded assets, the firm will be sold to the agent who values it the most, and the proceeds from equity financing is:

\[
\max_i \sum_{s=1}^S \pi_s MRS_{s,t+1}^i x_{s,t+1}^j
\]

Can the firm gain by controlling its capital structure, that is by using different financial instrument to finance itself than equity? In fact the firm has in this case an incentive to split its payoff state by state, to exploit the differences in \( MRS_{s,t+1}^i \) across agents due to market incompleteness, and sell each single peace to the agent who values it the most. Formally, the maximal proceeds from financing are obtained by selling assets with payoff equal to 1 in state \( s \) and 0 otherwise, for any \( s \); in this case the proceeds are:

\[
\sum_{s=1}^S \max_i \pi_s MRS_{s,t+1}^i x_{s,t+1}^j
\]

We conclude by noting that our analysis demonstrates once again that capital structure has no effect when markets are complete (Modigliani-Miller 1) but does have an effect when markets are missing and hence incomplete. In this case, firms have a tendency to split their payoff to finance more cheaply their operations.

2 Transaction Costs

Even if any payoff can be generated via a portfolio of the traded assets, markets are incomplete when transaction costs are associated to trading positions. For instance, trading very often to re-balance one’s portfolio, or trading very many different assets and stocks to diversify might entail a cost.

Idiosyncratic risk (see Class 1 for the precise definition) is often diversified by holding a large (in the mathematical limit, an infinite) number of independent assets or stocks. For instance, by holding many insurance contracts, insurance company diversify away the idiosyncratic risk of each single agent, and hold instead only the systematic risk of the aggregate position (the Law of Large
Number guarantees this). When markets are complete and there are no transaction costs, idiosyncratic risk is not valued in equilibrium, exactly because it can be diversified away by holding many independent sources of independent risk. When instead transaction costs make holding such portfolios (with many independent assets or stocks) costly, than there does not exists any purely idiosyncratic risk (idiosyncratic risk becomes in effect systematic and hence is valued, negatively, in equilibrium).

2.1 Capital Structure and Security Design

Transaction costs therefore generate the incentives for a firm to finance its operations by selling pools of its idiosyncratic risk. For instance, a firm could securitize its contractual positions with different agents hit by independent risks, if this can be done at lower cost inside the firm (otherwise the agent could construct portfolios of the different contractual positions, but this entails transaction costs by assumption).

Notice that, once the firm has reduced its risk only to systematic risk by pooling independent positions, if markets are incomplete the incentive to split this risk remains intact. An example are Mortgage-Backed-Securities, which pool independent idiosyncratic mortgage risks and then sell it by trenching systematic risk into different risk classes (see Allen-Gale, Security Design and Risk Sharing, MIT Press, 1993; Chapter 1.)