Informal Communication

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Abstract

The typical sender-receiver game studied in the literature assumes that the receiver is uninformed. In reality, the receiver usually relates the sender's message to her own information and obtains a signal about their truthfulness. I analyze a sender-receiver model where both agents have private information and the sender cares to be perceived as honest. If the sender's reputation concerns are strong enough, the model predicts truthful information revelation as a unique equilibrium. This uniqueness result contrasts with the multiplicity of uninformative equilibria in cheap-talk games with an uninformed receiver. To achieve truthful information revelation as a unique equilibrium, the receiver may need to ask the sender for information, which per se is extraneous for her decision. If the sender also cares about the receiver's action, he usually randomizes between telling the truth and lying. Then extraneous information may make communication more as well as less honest.

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1 Introduction

Most people consult with others before making important decisions. They discuss their problem with friends, and elicit opinions from various advisors or experts. The messages of a consultant (sender) rarely have direct payoff implications, and the decision-maker (receiver) often has no way to verify the truthfulness of the information. The fact that opinions are routinely solicited and our intuition both suggest that consultants often respond truthfully to decision-makers' inquiries. However, the cheap-talk literature establishes that when messages are costless, truthful communication is one of many possible outcomes, even if incentives are perfectly aligned.

The purpose of this paper is to answer the following questions: Why do economic agents expand resources on discussing and soliciting opinions from others who do not have strong incentives to give them accurate information? What is the role of having information about others' information in facilitating honest communication? Does the receiver make the sender more honest by asking for details, which per se seem to be extraneous for her decision? I focus on what I call *informal communication*, that is, situations with incomplete contracting and unverifiable messages. In this setting, the correctness of the sender's message cannot be verified even after the receiver implements the decision. The sender has an interest in maintaining his reputation as an honest person and may also care about the receiver's decision, but contracting on either this decision or on messages themselves is not feasible. Here are some concrete examples involving informal communication: (a) an employer is to select one of two applicants for a job, and she has an advisor with a superior expertise to her own regarding the qualifications of both candidates, (b) a consumer wants to buy a used car, but she knows little about the construction of cars, so she wants a mechanic to estimate its current condition, (c) an attorney makes the decision to take a case; she needs information about an event from the past and speaks to potential witnesses.

In the typical sender-receiver game studied in the literature (see for example, Crawford-Sobel (1982)), the sender has a private signal, which he may convey to the uninformed receiver who has to take an action. The payoff of both players depends on the receiver's action and the sender's signal. These are known as "cheap-talk" games. In some cheap-talk models, as in my games of informal communication, the sender has an interest in maintaining his reputation for honesty (see Sobel (1985)). The most significant difference between my sender-receiver games and the ones studied in this literature is that the receiver also has private information[1].

The main result of the paper is that when messages are unverifiable - or equivalently, they can be verified only after the interaction is completed - this private information, no matter how small, facilitates honest communication. More specifically, Theorem 1 asserts that if the sender is motivated only by the reputation concerns, then he reports his signal truthfully in a unique equilibrium iff for every pair of the sender's possible signals there is a pair of the receiver's possible signals that result in the different likelihood ratios of these two signals of the sender. In the unique equilibrium, the sender reports truthfully also per se extraneous information. For example, he may explain why one candidate is more qualified than another, or why the consumer should not buy the car, without being asked for details.

In the more general case where the sender cares about the receiver's action, my game of informal communication may have multiple equilibria. Those equilibria are usually in mixed strategies: the sender randomizes between telling the truth and lying. Hence, messages convey some information, but there are bounds on how much information gets transmitted. In this more general case, I identify situations where extraneous information makes communication more (less) honest - that is, situations

[1] In the literature, there are two instances where the receiver's private information plays a role. In Seidmann (1990), the receiver's private information results in the existence of meaningful equilibria even when the sender's preferences over the receiver's actions are independent of the sender's signal. The reason is that any message induces a distribution of the receiver's actions across her signals, and the sender's preferences over those distributions may depend on his signal, even though the preferences over single actions do not.

In Prendergast (1993), the receiver's private information enables her to motivate the sender for collecting costly information. Information is represented by correlated draws from a normal distribution, and the parties may write contracts rewarding the sender for reporting draws close to the receiver's private draw.

where the chance of telling the truth becomes higher (lower) when the receiver asks for details.

A few themes are familiar from earlier work. A number of papers, beginning with Crawford and Sobel (1992), have studied "cheap-talk" games. In addition to some very important insights, cheap-talk models also provide two counter-intuitive predictions. First, cheap-talk games always have equilibria in which messages convey no information. Even in the games of pure common-interest, i.e. where the payoff functions of both agents coincide, there are equilibria where the sender communicates his true signal, as well as babbling equilibria where the sender sends each possible signal with the same probability. Second, truthful communication appears as an equilibrium in games, where the sender would achieve the highest payoff if his message conveyed no information.

These implausible equilibria - and especially, the impossibility of obtaining meaningful communication as a unique outcome - led to a search for refinements: Farrell (1993), Matthews, Okuno-Fujiwara and Postelwaite (1991), Rabin (1990), Rabin-Sobel (1996), Zapater (1997), Blume, Kim and Sobel (1993), and Warneryd (1993), Blume and Sobel My analysis shows that the sender's reputation concerns combined with the receiver's private information create an alternative mechanism that may prevent babbling and guarantee meaningful communication. However, there are two important differences. First, I show why meaningful communication may occur even if some conflict of interests arises (Proposition 3), but I offer no insight into why no-communication outcome prevails in games, where the sender prefers not to reveal his information. Second, I rather point out missing elements of cheap-talk modeling which may be responsible for counter-intuitive predictions, instead of refining the equilibrium concept itself.

Like other reputation models: Kreps and Wilson (1982), Milgrom and Roberts (1982), and Sobel (1985) in the case of honesty, I study incomplete information about the sender's type, where a fraction of senders committed to telling truth. This fraction of senders plays an important role not only because it enables other senders to build reputation that will play like committed senders in the future. It prevents the receiver from ignoring her private information. Without committed senders, there exist situations where the receiver's signal is correlated with the sender's signal, but receivers with all signals may assign to all massages the same probability of being truthful.

In a broader sense, the paper belongs to the literature, where messages (or actions) reveal not only the sender's information but also the sender's type. Therefore the sender faces a trade-off between information transmission and building his reputation for being some type. Examples include Sobel (1985), Benabou and Laroque (1992), Branderburger and Polak (1996), Morris (1999), Levy (2000), Ottaviani and Sorensen (1998) and (1999) (and in the cases of actions, Holmström (1999), Scharfstein and Stein (1990), Bernheim (1994), Prendergast and Stole (1996)).

The next section contains the details of the model. Section 3 characterizes and discusses equilibria: both when the sender is motivated purely by the reputation concerns and when he also cares about the receiver's action. Section 4 analyzes the role of payoff-irrelevant information. Proofs are relegated to Appendix.

2 Model

I study the following game: The sender S gets a signal s from set $T_S = \{s_1, ..., s_m\}$. The receiver R gets a signal r from $T_R = \{r_1, ..., r_n\}$. The probability that S obtains $s = s_i$ and R obtains $r = r_k$ is equal to p_i^k. This prior information structure can be represented by matrix

$$\begin{array}{cc} & \begin{array}{cccc} r_1 & r_2 & \cdots & r_n \end{array} \\ \begin{array}{c} s_1 \\ s_2 \\ \cdots \\ s_m \end{array} & P = \begin{bmatrix} p_1^1 & p_1^2 & \cdots & p_1^n \\ p_2^1 & p_2^2 & \cdots & p_2^n \\ \cdots & \cdots & & \cdots \\ p_m^1 & p_m^2 & \cdots & p_m^n \end{bmatrix} \end{array}$$

I denote rows of P by $\mathbf{p}_i$ and columns by $\mathbf{p}^k$. I assume that the prior information structure is common knowledge.

After getting a signal, the sender sends a message $\overline{s} \in T_S$ to the receiver who takes an action. It is essential that the set of messages contains all signals of the sender; the assumption that there are no other messages is made only for the sake of simplicity. The set of all possible actions A is finite with its generic element denoted a. Players are allowed to use mixed strategies. A mixed strategy of the sender is denoted $\sigma : T_S \times T_S \longrightarrow [0,1]$, where $\sigma(\overline{s}_j, s_i)$ stands for the probability that the sender with signal $s = s_i$ sends message $\overline{s} = s_j$. A mixed strategy of the receiver is denoted $\alpha : A \times T_S \times T_R \longrightarrow [0,1]$, where $\alpha(a, \overline{s}_j, r_k)$ stands for the probability of taking action a by the receiver with signal r_k when the sender sends message $\overline{s} = s_j$. There are two types of senders. With probability ε, the sender is a truth-teller, and with probability $1-\varepsilon$, the sender is a strategic agent. Truth-tellers always reveal their signals honestly, and strategic senders maximize their payoff. The payoff of a strategic sender with signal s_i is a weighted sum of two components: the expected decision payoff U_S and the reputation payoff V, with the weights ρ and $1-\rho$, respectively. The decision payoff is represented by u_S - a real-valued function of the receiver's action and the signals of both players. Thus, the expected decision payoff is given by

$$U_S(\sigma, \alpha, s_i) = \sum_{a \in A} \sum_{j=1}^{m} \sum_{k=1}^{n} \alpha(a, \overline{s_j}, r_k) \sigma(\overline{s_j}, s_i) \pi(r_k \mid s_i) u_S(a, s_i, r_k),$$

where $\pi(r_k \mid s_i)$ stands for the probability assigned by a sender with signal s_i to the event that the receiver has obtained signal r_k. Notice that

$$\pi(r_k \mid s_i) = \frac{p_i^k}{\sum_{l=1}^{n} p_i^l}.$$

The reputation payoff is assumed to be the expected posterior probability assigned by the receiver to the event that the sender is a truth-teller; it will be defined precisely later. The payoff of the receiver is represented by u_R - a real-valued function of his action and the signals of both players; thus, the expected payoff of the receiver with signal r_k when the sender sends message $\overline{s} = s_i$ is given by

$$U_R(\sigma, \alpha, r_k \mid \overline{s_i}) =$$

$$= \sum_{a \in A} \alpha(a, \overline{s_i}, r_k) \left[\varepsilon u_R(a, s_i, r_k) + (1 - \varepsilon) \sum_{j=1}^{m} \sigma(\overline{s_i}, s_j) \pi(s_j \mid r_k) u_R(a, s_j, r_k) \right],$$

where $\pi(s_i \mid r_k)$ stands for the probability assigned by a receiver with signal r_k to the event that the sender obtains signal s_i. Notice that

$$\pi(s_i \mid r_k) = \frac{p_i^k}{\sum_{j=1}^{m} p_j^k}.$$

Finally, let us define the reputation component of the sender's payoff. Let $v(\overline{s_i} \mid \sigma, r_k)$ denote the probability assigned to the event "S is a truth-teller" by R with signal r_k if S plays according to strategy σ and S sends message $\overline{s_i}$. Notice that

$$v(\overline{s_i}, \sigma, r_k) = \frac{\varepsilon \pi(s_i \mid r_k)}{\varepsilon \pi(s_i \mid r_k) + (1 - \varepsilon) \sum_{j=1}^{m} \pi(s_j \mid r_k) \sigma(\overline{s_i}, s_j)}.$$

Hence, the reputation payoff of S with signal s_i when he sends message $\overline{s_j}$, which I will call the reputation payoff to sending $\overline{s_j}$, is given by

$$V(\overline{s_j}, \sigma, s_i) = \sum_{k=1}^{n} \pi(r_k \mid s_i) v(\overline{s_j}, \sigma, r_k),$$

and the total payoff of S with signal s_i is given by

$$\rho U_S + (1 - \rho) V.$$

I denote this game by $G(P, \rho)$. I assume that either $\rho = 0$ or $p_i^k > 0$ for all i and k, and study its Nash equilibria: σ, α. Both assumptions guarantee that the concept of Nash equilibrium can be used. The assumptions has been made solely for this purpose. The analysis extends to the general case and sequential equilibria. It is important to notice that the sender's reputation payoff to sending a message is determined by his strategy itself. Of course, the sender takes the payoffs of his messages as given when he picks up his strategy. Therefore in a Nash equilibrium of $G(P, \rho)$, the sender's strategy has to be the best response to both the receiver's strategy and to itself.

Unlike traditional reputation models, my model has only one period. In order for reputation to have an effect, the value of establishing a reputation is incorporated as a component into the sender's payoff function. Then the reputation payoff may reflect either some intrinsic preference for being truthful or the sender's expectation to benefit in the future. In the latter case, one can view my model as a reduced form of a multi-period model. Further, I assume the correctness of the sender's messages can never be verified, even after the receiver implements her decision. I have basically in mind situations where there is a delay between when the message is received and the information about its truthfulness becomes available. Then the sender may extract the benefits of his reputation during this delay. However, there also exist situations of interest where the receiver may literally never be able to verify messages at any later date. For example, think of a situation where an advisor recommends to an employer one of two candidates to perform a specific task.

3 Equilibria

3.1 Pure Reputation Concerns - Uniqueness of Truth-Telling Equilibrium

Consider the case of $\rho = 0$, i.e. the sender is motivated purely by the reputation concerns. I shall give necessary and sufficient conditions for the uniqueness of truth-telling equilibria.

Definition 1 *The strategy $\sigma(\overline{s_i}, s_i) = 1$ for $i = 1, .., m$ will be called truth-telling.*

Theorem 1 ***(i)*** *Truth-telling is an equilibrium.*

(ii) *The equilibrium is unique if and only if*

$$\forall_{i,j=1,\ldots,m} \quad \mathbf{p}_i \text{ and } \mathbf{p}_j \text{ are linearly independent.} \tag{1}$$

Proof. See Appendix. ■

In other words, condition (1) means that for every pair of the sender's signals, there is a pair of the receiver's signals that result in different likelihood ratios of the two signals of the sender. In the interpretation, it means that different private signals of the receiver may provide different information regarding the truthfulness of different messages of the sender . Note that it is a mild condition. For example (cf Introduction): (a) An employer may have some general impression concerning each candidate. (b) A consumer might have driven the car, and in this way, have acquired some information about how the car works (c) Although the attorney does not know whether something has happened, she may have some information about related events or people involved. However, if the receiver is uninformed, as in cheap-talk games, every pair of rows of matrix P are linearly dependent, that is condition (1) is not satisfied.

The following example is an application of Theorem 1, helps to understand its proof.

Example 1

The receiver, a friend of Mrs. Brown would like to know if Mrs. Brown was in Princeton yesterday to visit her relative. She asks the sender if he has seen Mrs. Brown. The sender can offer one of three responses: $\overline{A1}$="I have seen Mrs. Brown with an elderly woman" or $\overline{A2}$="I have seen Mrs. Brown with a young man" or $\overline{N}$="I did not see Mrs. Brown". The receiver has met the relative of Mrs. Brown's, has fifty-fifty prior that Mrs. Brown was in town yesterday, and believes that if she were in Princeton, there would be a .5 chance that the sender would have encountered her. The sender also knows that Mrs. Brown has a relative in town, and assigns probability .5 that the relative is a nephew (M) and probability .5 that it is her sister (W). There is an $\varepsilon > 0$ prior chance that the sender always tells the truth; otherwise his payoff is the expected receiver's posterior assessment that he is an honest type.

The following table describes the information structure above:

$$\begin{array}{c} \\ A1 \\ A2 \\ N \end{array}\begin{array}{c} \begin{array}{cc} W & M \end{array} \\ \left[\begin{array}{cc} 1/4 & 0 \\ 0 & 1/4 \\ 1/4 & 1/4 \end{array}\right] \end{array}.$$

The argument behind Theorem 1 consists of three steps.

Step 1. The payoff of a sender who truthfully reveals Ai exceeds the payoff to sending $\overline{Ai}$ of a sender with signal Aj or N.

Indeed, if a sender with signal Aj sends $\overline{Ai}$, then with probability 1, it gets detected that he lies, i.e. he is certainly assigned probability 0 of being a truth-teller, whereas a sender who reveals honestly signal Ai is assigned a positive probability of being an honest type. Consider now a sender with signal N. If he sends $\overline{Ai}$ his payoff equals the payoff of a sender who truthfully reveals Ai if the receiver has signal W; if the receiver, however, has signal M, his payoff is 0. Since he assigns fifty-fifty chance that the receiver has W, his expected payoff equals half of that for a sender who truthfully reveals Ai.

(a) The payoff of a sender who truthfully reveals N equals to the average of the payoffs to sending $\overline{N}$ for senders

with signals Ai.

(b) If a sender with signal Ai sends $\overline{N}$ with a lower probability than a sender with signal Aj, then a sender

with Ai obtains a higher payoff to sending $\overline{N}$ than a sender with signal Aj.

Indeed, if the receiver has signal W (M) the payoff of a sender who truthfully reveals N equals to the payoff to sending $\overline{N}$ for a sender with signals $A1$ (respectively $A2$). Thus (a) follows from the fact that a sender with signal N assigns fifty-fifty chance that the receiver has signal W (M). To see (b) notice that when the receiver with signal W (M) hears message $\overline{N}$, she knows that it can come either from a sender with signal N or strategic senders with signal $A1$ $(A2)$. If strategic senders with signal $A1$ use message $\overline{N}$ more frequently than strategic senders with signal $A2$, then hearing message $\overline{N}$ conditional on W being the receiver's own signal yields a lower assessment of a sender being a truth-teller than that conditional M being the receiver's own signal.

Step 2. In an equilibrium, the payoff of the sender equals ε independently of his signal.

In equilibrium, the receiver cannot on average be misled. That is, the average payoff of the sender equals ε, where the "averaging" is being done over the three possible signals of the sender. Suppose that some signal yields the payoff lower than ε, and consider signal s which yields the lowest payoff. Note first that, without loss of generality, I can assume that s equals to some Ai. Indeed, suppose that $s = N$. If only a sender with signal N sends $\overline{N}$, then his payoff cannot be lower than ε. Thus, there must be some signal Ai such that a sender with Ai sends $\overline{N}$ with a positive probability. Therefore, by Step 1 (a) and (b), there must be also some Ai such that a sender with signal Ai sends $\overline{N}$ with a positive probability and his payoff to sending $\overline{N}$ does not exceed that of a sender with signal N. Assume, without loss of generality, that $s = A1$. Then by Step 1, senders with $A2$ and N send $\overline{A1}$ with probability 0, which implies that the payoff of a sender with $A1$ cannot be lower than ε, a contradiction.

Step 3. From Steps 1 and 2 it follows that in equilibrium, messages $\overline{A1}$ and $\overline{A2}$ can come only from senders who tell the truth. This in turn implies that message $\overline{N}$ can also come only from a sender who tells the truth, because if a sender with signal Ai used $\overline{N}$ with a positive probability, then his payoff to sending $\overline{Ai}$ would exceed ε.

Finally, note that Example 1 illustrates how the receiver's private information facilitates honest communication. It enables the receiver to discriminate truth from lie. If the receiver had not met the relative of Mrs. Brown's, then truthful communication could not be guaranteed. One readily checks that the game would have a continuum of equilibria, including one in which messages convey no information.□

In the proof of Theorem 1, Step 1 generalizes to the assertion that for any message $\overline{s_i}$ and strategy σ, S' payoff, conditional on honestly revealing s_i is not lower that the payoff conditional on sending $\overline{s_i}$ and lying. Moreover, it is strictly higher unless S' truthfulness and R's signal are independent; Step 3 shows first that

$$\sigma \cdot P = P,$$

where σ represents the $m \times m$ matrix whose entry in row i and column j equals $\sigma(\overline{s_j}, s_i)$. Then linear algebra arguments yield that

$$\sigma = \mathbf{I}$$

iff P satisfies (1), $\mathbf{I}$ stands for the $m \times m$ identity matrix.

Remark 1

Consider now an arbitrary matrix P whose rows may be linearly dependent. Then the rows of P can be segregated into sets whose members are linearly dependent. Let $S_i = \{s_j : \mathbf{p}_i \text{ and } \mathbf{p}_j \text{ are linearly dependent}\}$. Then $\{S_i : i = 1, ..., m\}$ forms a partition of T_S, and if $\mathbf{p}_i$ and $\mathbf{p}_j$ are linearly independent for some i and j, then the partition consists of at least two elements. Further let

$$\sigma^G(\overline{s_i}, s_j) = \sum \left\{ \sigma(\overline{s_{j/}}, s_{i/}) : s_{j/} \in S_j \text{ and } s_{i/} \in S_i \right\};$$

that is, $\sigma^G(\overline{s_i}, s_j)$ stands for the probability that the sender with a signal from S_j sends a message from S_i.

Slightly modifying the proof of Theorem 1, one can show:

Theorem 2 *In every Nash equilibrium of* $G(P, 0)$*,*

$$\sigma^G(\overline{s_i}, s_i) = 1 \textit{ for } i = 1, ..., m.$$

That is, the sender always sends a message from the S_i that contains his signal. If the S_i consists of one element, then he simply tells the truth. If, however, it consists of at least two elements, there is a continuum of equilibria that correspond to each level of information transmission from sending every member of S_i with the same probability to revealing the true element of S_i with probability 1.

Finally, note that there is also a puzzling implication of Theorem 1. If the sender is motivated exclusively by reputation concerns and the receiver is sure about that, then he communicates, and she fully believes even in messages that report almost unbelievable signals (signals that had very low prior probability). Think, for example, about someone telling you "I saw an UFO". It seems plausible that the receiver would rather assign to such messages a large margin of doubt, and in anticipation of that, the sender, who tries to build the reputation for reporting his information truthfully, would rather avoid sending unbelievable messages.

3.2 Other Motives

In the case $\rho > 0$, i.e. when the sender has motives other than reputation, my sender-receiver game may have several equilibria.[2] These equilibria are usually in mixed strategies, i.e. the sender randomizes between telling the truth and lying. Indeed, whenever a conflict of interests arises, truth-telling is no longer an equilibrium. If it were, the reputation payoffs of all messages would be equal to ε, and the sender would send the message that maximizes his decision payoff. On the other hand, as the reputation concerns get stronger, the probability of telling the truth eventually approaches 1.

Proposition 3 *Let $\mathbf{p}_i$ and $\mathbf{p}_j$ be linearly independent for any i and j. For every $\sigma^* < 1$, there exists $\rho^* > 0$ such that if $\rho < \rho^*$, then $\sigma(\overline{s_i}, s_i) > \sigma^*$ for $i = 1, ...m$ in every Nash equilibrium.*

Proof. See Appendix. ■

[2] Even pure common-interest games usually have other equilibria than truth-telling when ρ is sufficiently large.

In the interpretation, informal messages convey some information, but there are bounds on how much information gets transmitted, or individuals usually leave some margin of doubt for information acquired in informal communication. However, as the reputation concerns get stronger, this margin of doubt eventually approaches 0. Another feature of the equilibria in the general case is that the probability assigned by the receiver to the event "the sender tells the truth" depends on her private information. In the interpretation, some individuals trust more and some trust less what the sender says. Again, as the reputation concerns get stronger, the probabilities assigned by the receiver with different signals eventually approaches 1.

4 Extraneous Information

In this section, I show how per se extraneous information may be used for strategic reasons, and I explore whether asking for extraneous information leads to more or less honest communication. Note that Example 1 provides an illustration. Asking "With whom did you encounter Mrs. Brown?" may be extraneous for the receiver's purposes. It, however, enables the receiver to discriminate truth from lie. If she asked the sender to use only one of the two responses: $\overline{Y} =$"I saw Mrs. Brown", $\overline{N} =$"I did not see Mrs. Brown", then the game would have a continuum of equilibria, including one in which messages convey no information, and truthful communication could not be guaranteed.

More precisely, for each game $G(P, \rho)$, the set of the sender's signals T_S can always be partitioned into subsets T_S^q, $q = 1, ..., M$, such that if $s_i, s_j \in T_S^q$ for some $q = 1, ..., M$, then

$$u_S(a, s_i, r_k) = u_S(a, s_j, r_k) \text{ and } u_R(a, s_i, r_k) = u_R(a, s_j, r_k) \tag{2}$$

for any $a \in A$ and $r_k \in T_R$. Of course, the interesting case is when some sets T_S^q consist of more than one signal. The set T_S^q containing the sender's signal represents his payoff-relevant information. This set is what I call a *conclusion.* Single elements of

T_S^q represent the sender's payoff-relevant and extraneous information. I call elements of T_S^q, i.e. single signals, *conclusions augmented with extraneous information.*

To see whether asking for details, which per se are extraneous for the receiver's decision, makes the sender more honest I will compare equilibria of two games: $G(P, \rho)$ defined as in Section 2 and the same game, where the set of messages consists of $\overline{T_S^q}$, $q = 1, ..., M$. A truth-teller sends the $\overline{T_S^q}$ that T_S^q contains his signal. Let $G_{NE}(P, \rho)$ denote this modified game. In the interpretation, $G_{NE}(P, \rho)$ represents a situation, where the receiver asks the sender to communicate only his conclusion, while $G(P, \rho)$ from Section 2 represents a situation, where the receiver asks the sender to augment the conclusion with extraneous information. Note that $G_{NE}(P, \rho)$ can be identified with $G(P^{NE}, \rho)$, where the rows of P^{NE} are

$$\mathbf{p}_q^{NE} = \sum_{s_i \in T_S^q} \mathbf{p}_i$$

for $q = 1, ..., M$. I assume that both P and P^{NE} satisfy condition (1).

There are two effects, which make the sender more or, respectively, less honest when he is asked for extraneous information, and which seem to be robust. Although I will describe these effects by means of examples, it should become clear that they appear in a large class of games.[3]

Consider a game $G(P, \rho)$, where T_S can be partitioned into two sets: $T_S^1 = \{A1, A2\}$ and $T_S^2 = \{N\}$, and T_R can be partitioned into two sets: $T_R^1 = \{B1, ..., Bl\}$ and $T_R^2 = \{C1, ..., Cl\}$ such that the information structure is represented by the following matrix:

[3] No general result should be, however, expected. First, there are several non-equivalent measures of honesty. Think for example about two equilibria: In one equilibrium, the sender lies rarely, but when he lies, the receiver gets very low payoff. In another equilibrium, the sender lies more often, but lying does not have substantial payoff consequences. Second, both $G(P, \rho)$ and $G(P^{NE}, \rho)$ usually have multiple equilibria. Finally, the answer to the question whether the sender becomes more or less honest is sensitive to the parameters of the model.

$$\begin{array}{c} \\ A1 \\ A2 \\ N \end{array}\begin{array}{c} \begin{array}{cccccc} B1 & ... & Bk & C1 & ... & Ck \end{array} \\ \left[\begin{array}{cccccc} p^1+\gamma^1 & ... & p^l+\gamma^l & p^1-\gamma^1 & ... & p^l-\gamma^l \\ p^1-\gamma^1 & ... & p^l-\gamma^l & p^1+\gamma^1 & ... & p^l+\gamma^l \\ p_3^1 & ... & p_3^l & p_3^1 & ... & p_3^l \end{array}\right]. \end{array}$$

The above information structure assumes that extraneous information $A1$ and $A2$ is symmetric from the perspective of the sender who has signal N.

Suppose further that: (1) The receiver's payoff depends only on her action and the sender's conclusion, $u_R = u_R(a, T_S^q)$. Moreover, there exist $0 = \pi_0 < \pi_1 < ... < \pi_L = 1$ and actions $a_0, ...a_L$ such that a_l is a unique maximizer of the receiver's expected payoff when she assigns probability $\pi \in (\pi_{l-1}, \pi_l)$ to event "$s \in T_S^1$", a_l, a_{l+1} are the only two maximizers for $\pi = \pi_l$, and a_0 (respectively, a_L) is a unique maximizer for $\pi = \pi_0$ ($\pi = \pi_L$). (2) The sender's payoff depends only on the receiver's action, $u_S = u_S(a)$, and is increasing in the probability assigned by the receiver to event "$s \in T_S^1$", $u_S(a_1) < ... < u_S(a_L)$.

It is easy to check that:

(1) $G(P^{NE}, \rho)$ has an equilibrium, where the sender with $A1$ and $A2$ sends $\overline{T_S^1}$ with probability 1, and the sender with N sends $\overline{T_S^2}$ with probability $\sigma^{NE} \in [0, 1]$ and $\overline{T_S^1}$ with probability $1 - \sigma^{NE}$. The equilibrium of this form is unique.

(2) $G(P, \rho)$ has an equilibrium, where the sender with $A1$ or $A2$ reveals his signal truthfully, and the sender with N sends $\overline{N}$ with probability $\sigma \in [0, 1]$ and each $s \in T_S^1$ with probability $(1 - \sigma)/2$. The equilibrium of this form is unique.[4]

Let E^{NE} and E stand for the two equilibria. Let $\pi(r)$ denote the probability assigned to to event "$s \in T_S^1$" in equilibrium E^{NE} by the receiver with signal r who receives message $\overline{T_S^1}$.

Proposition 4 *Suppose that there exists a non-empty set of signals $\emptyset \neq T_R^* \subset T_R$ such that: (1) if $r \in T_R^*$, then $\pi(r) = \pi_l \in \{\pi_1, ..., \pi_{L-1}\}$, (2) if $r \in T_R - T_R^*$, then*

[4] For some parameters of the model, both $G(P^{NE}, \rho)$ and $G(P, \rho)$ may have, however, other equilibria.

$\pi(r) \notin \{\pi_1, ..., \pi_{L-1}\}$. *There exists* $\overline{\gamma} > 0$ *such that for* $0 < \gamma_1, ..., \gamma_l < \overline{\gamma}$,[5]

(a) If, in equilibrium E^{NE}*, the receiver with signal* $r \in T_R^*$ *who receives message* $\overline{T_S^1}$ *takes action* a_l*, then* $\sigma < \sigma^{NE}$ *and, for every* $r \in T_R$*, the receiver's expected payoff is lower in equilibrium* E *than in equilibrium* E^{NE}*.*

(b) If, in equilibrium E^{NE}*, the receiver with signal* $r \in T_R^*$ *who receives message* $\overline{T_S^1}$ *takes action* a_{l+1}*, then* $\sigma > \sigma^{NE}$ *and, for every* $r \in T_R$*, the receiver's expected payoff is higher in equilibrium* E *than in equilibrium* E^{NE}*.*

Proof. See Appendix. ■

Proposition 4 concerns situations, where if the sender communicates only his conclusion, then the receiver is indifferent between two actions. The sender strictly prefers one to the other. If the receiver takes the action that the sender likes less (Proposition 4(a)), then asking for extraneous information makes the sender less honest, and makes the receiver worse off. If the receiver takes the action that the sender likes more (Proposition 4(b)), then asking for extraneous information makes the sender more honest, and makes the receiver better off. Heuristically, the reason is that the receiver from part (a) who hears extraneous information "consistent" with her own signal (i.e. she hears $\overline{Ai}$ when $r \in T_R^i$), increases the probability assigned to the event "$s \in T_S^1$". This creates for the sender an additional incentive for lying. Indeed, if it happens that the sender picks up a "consistent" message, he may convince the receiver to take the action that he likes more. Similarly, the receiver from part (b) who hears extraneous information "inconsistent" with her own signal (i.e. she hears $\overline{Ai}$ when she has $r \in T_S^j$), decreases the probability assigned to the event "$s \in T_S^1$". This creates for the sender an additional incentive for telling the truth. If it happens that the sender picks up an "inconsistent" message, the receiver may take the action that he likes less.

[5]The purpose this assumption is to isolate the effects of asking for extraneous information described in this proposition from potential other effects.

5 Appendix

5.1 Proof of Theorem 1

I need the following notation: **0** and **1** stand for vectors whose all coordinates are equal to 0 and 1, respectively; that is, **0**=(0,...,0) **1**=(1,...,1). **I** denotes the identity matrix.

Let

$$V_j(\sigma) = \max_{i=1,...,m} V(\overline{s_i}, \sigma, s_j), \text{ and } H_j(\sigma) = V(\overline{s_j}, \sigma, s_j).$$

denote the highest payoff that can be achieved by a strategic sender with signal s_j and a truth-teller with the same signal, respectively, given strategy σ. Let $\sigma_i^T = (\sigma(\overline{s_1}, s_i), ..., \sigma(\overline{s_m}, s_i))$, $i = 1, ..., m$, be the strategy of a sender with signal s_i represented as a row-vector, and σ_i the same strategy represented as a column-vector. The strategy of the sender σ can be represented as matrix $[\sigma_1\sigma_2...\sigma_m]$; I will simply write that σ=$[\sigma_1\sigma_2...\sigma_m]$.

I will prove the theorem by a sequence of lemmata. I begin with a straightforward algebraic result.

Lemma 5 *If* $\mathbf{v} = (v_1, ..., v_n)$ *satisfies* $0 \leq v_1 \leq ... \leq v_n$*, and* $\mathbf{x} = (x_1, ..., x_n)$ *satisfies* $\sum_{k=1}^{n} x_k = 0$*, and there exists* l *such that* $x_1, ..., x_l \leq 0$ *and* $x_{l+1}, ..., x_n \geq 0$*, then* $\mathbf{x} \bullet \mathbf{v} \geq 0$*. Moreover,* $\mathbf{x} \bullet \mathbf{v} = 0$ *only if* $v_1 = ... = v_n$ *or* $x_1 = ... = x_n = 0$*.*

Lemma 6 *Suppose that* $\sigma(\overline{s_i}, s_j) > 0$ *for some* $j \neq i$*. Then:*

(a)

$$V(\overline{s_i}, \sigma, s_i) \geq \sum_{j \neq i} w_j V(\overline{s_i}, \sigma, s_j),^{6} \tag{3}$$

where

$$w_j = \frac{\sigma(\overline{s_i}, s_j)\mathbf{p}_j \bullet \mathbf{1}}{\sum_{j \neq i} \sigma(\overline{s_i}, s_j)\mathbf{p}_j \bullet \mathbf{1}},$$

(b) *the inequality is strict unless* $\mathbf{p}_i$ *and* $\sum_{j \neq i} \sigma(\overline{s_i}, s_j)\mathbf{p}_j$ *are linearly dependent.*

Proof. Note that

$$V(\overline{s_i},\sigma,s_i)-\sum_{j\neq i}w_jV(\overline{s_i},\sigma,s_j)=$$

$$=\sum_{k=1}^{n}\left[\pi(r_k\mid s_i)-\sum_{j\neq i}w_j\pi(r_k\mid s_j)\right]v(\overline{s_i},\sigma,r_k),$$

and apply Lemma 5 to $v_k=v(\overline{s_i},\sigma,r_k)$ arranged in increasing order and

$$x_k=\pi(r_k\mid s_i)-\sum_{j\neq i}w_j\pi(r_k\mid s_j).$$

Since $\sum_{j\neq i}w_j=1$ and $\sum_{k=1}^{n}\pi(r_k\mid s_i)=\sum_{k=1}^{n}\pi(r_k\mid s_j)=1$, $\sum_{k=1}^{n}x_k=0$. To complete the proof of (**a**) I shall show that

$$\text{if } v_k\leq v_l \text{ and } x_k\geq 0\text{, then } x_l\geq 0. \tag{4}$$

Note that

$$\begin{aligned} v(\overline{s_i},\sigma,r_k) &= \frac{\varepsilon}{\varepsilon+(1-\varepsilon)\sum_{j=1}^{m}\dfrac{\pi(s_j\mid r_k)}{\pi(s_i\mid r_k)}\sigma(\overline{s_i},s_j)}= \\ &= \frac{\varepsilon}{\varepsilon+(1-\varepsilon)\sum_{j=1}^{m}\dfrac{p_j^k}{p_i^k}\sigma(\overline{s_i},s_j)}, \end{aligned}$$

so $v_k\leq v_l$ means that

$$\sum_{j=1}^{m}\frac{p_j^k}{p_i^k}\sigma(\overline{s_i},s_j)\geq\sum_{j=1}^{m}\frac{p_j^l}{p_i^l}\sigma(\overline{s_i},s_j). \tag{5}$$

Suppose that $x_k\geq 0$, but $x_l<0$. Note that $x_k\geq 0$ means that

$$\pi(r_k\mid s_i)\geq\sum_{j\neq i}\frac{\sigma(\overline{s_i},s_j)\mathbf{p}_j\bullet\mathbf{1}}{\sum_{j\neq i}\sigma(\overline{s_i},s_j)\mathbf{p}_j\bullet\mathbf{1}}\pi(r_k\mid s_j),$$

or equivalently,

$$\sum_{j\neq i}\frac{p_j^k}{p_i^k}\sigma(\overline{s_i},s_j)\leq\frac{\sum_{j\neq i}\sigma(\overline{s_i},s_j)\mathbf{p}_j\bullet\mathbf{1}}{\mathbf{p}_i\bullet\mathbf{1}};$$

similarly, $x_l < 0$ means that

$$\sum_{j \neq i} \frac{p_j^l}{p_i^l} \sigma(\overline{s_i}, s_j) > \frac{\sum_{j \neq i} \sigma(\overline{s_i}, s_j) \mathbf{p}_j \bullet \mathbf{1}}{\mathbf{p}_i \bullet \mathbf{1}}.$$

That is,

$$\sum_{j=1}^{m} \frac{p_j^k}{p_i^k} \sigma(\overline{s_i}, s_j) = \sigma(\overline{s_i}, s_i) + \sum_{j \neq i} \frac{p_j^k}{p_i^k} \sigma(\overline{s_i}, s_j) \leq$$

$$\leq \sigma(\overline{s_i}, s_i) + \frac{\sum_{j \neq i} \sigma(\overline{s_i}, s_j) \mathbf{p}_j \bullet \mathbf{1}}{\mathbf{p}_i \bullet \mathbf{1}} <$$

$$< \sigma(\overline{s_i}, s_i) + \sum_{j \neq i} \frac{p_j^l}{p_i^l} \sigma(\overline{s_i}, s_j) = \sum_{j=1}^{m} \frac{p_j^l}{p_i^l} \sigma(\overline{s_i}, s_j). \tag{6}$$

Of course, (5) and (6) contradict one another.

To prove (**b**) observe first that $v_1 = ... = v_n$ implies $x_1 = ... = x_n = 0$. Indeed, it follows directly from (4) and $\sum_{k=1}^{n} x_k = 0$. Therefore, Lemma 5, (3) holds with equality only if

$$\forall_{k=1,\dots,n} \quad \pi(r_k \mid s_i) - \sum_{j \neq i} w_j \pi(r_k \mid s_j) = 0,$$

or equivalently,

$$\frac{\mathbf{p}_i}{\mathbf{p}_i \bullet \mathbf{1}} = \sum_{j \neq i} \frac{\sigma(\overline{s_i}, s_j) \mathbf{p}_j \bullet \mathbf{1}}{\sum_{j \neq i} \sigma(\overline{s_i}, s_j) \mathbf{p}_j \bullet \mathbf{1}} \frac{\mathbf{p}_j}{\mathbf{p}_j \bullet \mathbf{1}} = \frac{\sum_{j \neq i} \sigma(\overline{s_i}, s_j) \mathbf{p}_j}{\sum_{j \neq i} \sigma(\overline{s_i}, s_j) \mathbf{p}_j \bullet \mathbf{1}},$$

which means that $\mathbf{p}_i$ and $\sum_{j \neq i} \sigma(\overline{s_i}, s_j) \mathbf{p}_j$ are linearly dependent. ■

Remark 2

Lemma 6 has been stated in the form that is more convenient for further analysis. It may also be stated, however, as a result from probability theory, and that form seems to be easier for interpretation purposes: Let s, $\overline{s}$, and r be random variables corresponding to the sender's signal, the sender's message, and the receiver's signal, respectively. Let τ be a random variable corresponding to the sender's type ($\tau = 1$ if the sender is a truth-teller, and let $\tau = 0$ if he is a strategic sender). Let $P(A)$ stand for the probability of event A, $P(A \mid B)$ for the probability of event A conditional on event B, $E(\zeta)$ for the expected value of a random variable ζ, and $E(\zeta \mid B)$ for the expected value of a random variable ζ conditional on event B.

Lemma 7 *Suppose that $\overline{s} = s_i$ is a positive-probability event. Then*

$$\begin{aligned} & E\left[P(\tau = 1 \mid r \;\&\; \overline{s} = s_i) \mid \overline{s} = s_i \;\&\; s = s_i\right] \\ \geq \;\; & E\left[P(\tau = 1 \mid r \;\&\; \overline{s} = s_i) \mid \overline{s} = s_i \;\&\; s \neq s_i\right], \end{aligned}$$

and the inequality is strict unless

$$\forall_{k=1,\dots,n} \quad P(r = r_k \mid \overline{s} = s_i \;\&\; s = s_i) = P(r = r_k \mid \overline{s} = s_i \;\&\; s \neq s_i). \tag{7}$$

Note that (7) simply says that S' truthfulness and R's signal, both conditional on messages are $\overline{s} = s_i$ being used, are independent random variables.□

Lemma 8 *In every equilibrium, the expected payoff of the sender equals ε independently of his signal:*

$$V_j(\sigma) = \varepsilon \text{ for } j = 1, \dots, m.$$

Proof. In equilibrium,

$$\varepsilon \mathbf{p}_i \cdot \mathbf{1} H_i(\sigma) + \sum_{j=1}^{m} (1-\varepsilon) \mathbf{p}_j \cdot \mathbf{1} \sigma(\overline{s_i}, s_j) V(\overline{s_i} \mid \sigma, s_j) = \varepsilon \mathbf{p}_i \cdot \mathbf{1}. \tag{8}$$

Indeed,

$$\varepsilon \mathbf{p}_i \cdot \mathbf{1} H_i(\sigma) + \sum_{j=1}^{m} (1-\varepsilon) \mathbf{p}_j \cdot \mathbf{1} \sigma(\overline{s_i}, s_j) V(\overline{s_i} \mid \sigma, s_j) =$$

$$\sum_{k=1}^{n} \varepsilon \mathbf{p}_i \cdot \mathbf{1} \pi(r_k \mid s_i) \frac{\varepsilon \pi(s_i \mid r_k)}{\varepsilon \pi(s_i \mid r_k) + (1-\varepsilon) \sum_{j=1}^{m} \pi(s_j \mid r_k) \sigma(\overline{s_i}, s_j)} +$$

$$+ \sum_{k=1}^{n} \sum_{j=1}^{m} (1-\varepsilon) \mathbf{p}_j \cdot \mathbf{1} \sigma(\overline{s_i}, s_j) \pi(r_k \mid s_j) \cdot$$

$$\cdot \frac{\varepsilon \pi(s_i \mid r_k)}{\varepsilon \pi(s_i \mid r_k) + (1-\varepsilon) \sum_{j=1}^{m} \pi(s_j \mid r_k) \sigma(\overline{s_i}, s_j)}$$

$$
\begin{aligned}
&= \sum_{k=1}^{n} \varepsilon p_i^k \frac{\varepsilon p_i^k}{\varepsilon p_i^k + (1-\varepsilon)\sum_{j=1}^m p_j^k \sigma(\overline{s_i}, s_j)} + \\
&\quad + \sum_{k=1}^{n}\sum_{j=1}^{m} (1-\varepsilon) p_j^k \sigma(\overline{s_i}, s_j) \frac{\varepsilon p_i^k}{\varepsilon p_i^k + (1-\varepsilon)\sum_{j=1}^m p_j^k \sigma(\overline{s_i}, s_j)} = \\
&= \sum_{k=1}^{n} \varepsilon p_i^k = \varepsilon \mathbf{p}_i \cdot \mathbf{1}.
\end{aligned}
$$

Let A denote the set of agents with the lowest equilibrium payoff, i.e. $V_i(\sigma) \leq V_j(\sigma)$ for $i \in A$ and $j = 1, ..., m$. Say, $V_i(\sigma) = V$ if $i \in A$. I shall first show that $V \geq \varepsilon$.

Notice that

$$
\sigma(\overline{s_i}, s_j) = 0 \text{ for every } j \notin A \text{ and } i \in A. \tag{9}
$$

Otherwise $V_j(\sigma) = V(\overline{s_i} \mid \sigma, s_j) \leq V$ for some $j \notin A$ such that $\sigma(\overline{s_i}, s_j) > 0$ by Lemma 6. Further, notice that $H_i(\sigma) \geq V_i(\sigma)$ for any $i \in A$. Indeed, if $\sigma(\overline{s_i}, s_j) = 0$ for every $j = 1, .., m$, then $H_i(\sigma) = 1$; if $\sigma(\overline{s_i}, s_j) > 0$ for some j, then by Lemma 6, there is also such a j that $V_i(\sigma) \leq V_j(\sigma) = V(\overline{s_i} \mid \sigma, s_j) \leq V(\overline{s_i} \mid \sigma, s_i) = H_i(\sigma)$. Of course, $H_i(\sigma) \leq V_i(\sigma)$ for every $i = 1, .., m$. Therefore

$$
H_i(\sigma) = V_i(\sigma) = V \text{ for every } i \in A. \tag{10}
$$

Thus, summing up (8) for all $i \in A$, and applying (9) and (10),

$$
\sum_{i \in A} \varepsilon \mathbf{p}_i \cdot \mathbf{1} = \sum_{i \in A} \left[\varepsilon \mathbf{p}_i \cdot \mathbf{1} H_i(\sigma) + \sum_{j=1}^{m} (1-\varepsilon) \mathbf{p}_j \cdot \mathbf{1} \sigma(\overline{s_i}, s_j) V(\overline{s_i} \mid \sigma, s_j) \right] =
$$

$$
\begin{aligned}
&= \sum_{i \in A} \left[\varepsilon \mathbf{p}_i \cdot \mathbf{1} H_i(\sigma) + \sum_{j \in A} (1-\varepsilon) \mathbf{p}_j \cdot \mathbf{1} \sigma(\overline{s_i}, s_j) V(\overline{s_i} \mid \sigma, s_j) \right] = \\
&= V \sum_{i \in A} \varepsilon \mathbf{p}_i \cdot \mathbf{1} + V \sum_{j \in A} \mathbf{p}_j \cdot \mathbf{1} (1-\varepsilon) \left(\sum_{i \in A} \sigma(\overline{s_i}, s_j) \right) \leq \\
&\leq V \sum_{i \in A} \varepsilon \mathbf{p}_i \cdot \mathbf{1} + V \sum_{j \in A} \mathbf{p}_j \cdot \mathbf{1}^T (1-\varepsilon) = V \sum_{i \in A} \mathbf{p}_i \cdot \mathbf{1},
\end{aligned}
$$

and so $V \geq \varepsilon$.

Now I can show that $V_i(\sigma) = \varepsilon$ for $i = 1, ..., m$. First, note that $H_i(\sigma) \geq \varepsilon$ for $i = 1, ..., m$ and $V(\overline{s_i} \mid \sigma, s_j) \geq \varepsilon$ whenever $\sigma(\overline{s_i}, s_j) > 0$. Indeed, $V(\overline{s_i} \mid \sigma, s_j) = V_j(\sigma) \geq \varepsilon$; if $\sigma(\overline{s_i}, s_j) > 0$ for some j, then $H_i(\sigma) \geq V_j(\sigma) \geq \varepsilon$ for some such a j by Lemma 6; if $\sigma(\overline{s_i}, s_j) = 0$ for every j, then $H_i(\sigma) = 1$. Summing up (8) for all $i = 1, ..., m$, and applying $\sum_{i=1}^{m} \sigma(\overline{s_i}, s_j) = 1$ and $\sum_{i=1}^{m} \mathbf{p}_i \cdot \mathbf{1} = 1$

$$\sum_{i=1}^{m} \varepsilon \mathbf{p}_i \cdot \mathbf{1} H_i(\sigma) + \sum_{j=1}^{m} (1-\varepsilon) \mathbf{p}_j \cdot \mathbf{1} V_j(\sigma) = \varepsilon.$$

Thus, by Lemma 6, $V_j(\sigma) = \varepsilon$ for $j = 1, ..., m$. ■

Lemma 9 *In every equilibrium,*

$$\sigma \cdot P = P.$$

Proof. By Lemma 8, $V(\overline{s_i}, \sigma, s_j) = V_j(\sigma) = \varepsilon$ for every j such that $\sigma(\overline{s_i}, s_j) > 0$, and $V(\overline{s_i}, \sigma, s_i) = H_i(\sigma) \leq V_i(\sigma) = \varepsilon$. Therefore (3) in Lemma 6 holds with equality. This yields that

$$c_i \mathbf{p}_i = \sum_{j \neq i} \sigma(\overline{s_i}, s_j) \mathbf{p}_j$$

for some constant c_i. Thus,

$$\begin{aligned} V(\overline{s_i}, \sigma, s_i) &= \sum_{k=1}^{n} \frac{p_i^k}{\mathbf{p}_i \cdot \mathbf{1}} \frac{p_i^k \varepsilon}{p_i^k \varepsilon + p_i^k (1-\varepsilon) \sigma(\overline{s_i}, s_i) + (1-\varepsilon) c_i p_i^k} \qquad (11) \\ &= \frac{1}{\mathbf{p}_i \cdot \mathbf{1}} \sum_{k=1}^{n} \frac{p_i^k \varepsilon}{\varepsilon + (1-\varepsilon)[\sigma(\overline{s_i}, s_j) + c_i]} = \frac{\varepsilon}{\varepsilon + (1-\varepsilon)[\sigma(\overline{s_i}, s_j) + c_i]}. \end{aligned}$$

Observe that $V(\overline{s_i}, \sigma, s_i) = \varepsilon$. Indeed, if $\sigma(\overline{s_i}, s_i) > 0$, then $V(\overline{s_i} \mid \sigma, s_i) = \varepsilon$ by Lemma 8. Suppose that $\sigma(\overline{s_i}, s_i) = 0$. If also $\sigma(\overline{s_i}, s_j) = 0$ for every $j \neq i$, then $V(\overline{s_i}, \sigma, s_i) = 1$ which contradicts Lemma 8. If $\sigma(\overline{s_i}, s_j) > 0$ for some $j \neq i$, then by Lemma 6, $V(\overline{s_i}, \sigma, s_i) \geq V(\overline{s_i}, \sigma, s_j)$ for some $j \neq i$ such that $\sigma(\overline{s_i}, s_j) > 0$, and $V(\overline{s_i}, \sigma, s_j) = \varepsilon$, $V(\overline{s_i}, \sigma, s_i) \leq \varepsilon$ by Lemma 8.

Solving equation (11) for c_i, I obtain $c_i = 1 - \sigma(\overline{s_i}, s_i)$, so

$$\mathbf{p}_i = \sum_{j=1}^{m} \sigma(\overline{s_i}, s_j) \mathbf{p}_j,$$

or equivalently $\sigma \cdot P = P$. ■

Definition 2 *Let σ be an equilibrium strategy of the sender.*

(i) A set $\varnothing \neq A \subset \{1, ..., m\}$ is called separable if $\sigma(\overline{s_i}, s_j) = 0$ for every $j \notin A$ and $i \in A$.

(ii) A separable set A is called minimal if no proper subset of A is a separable set.

In words, it means that a sender with a signal s_j, $j \notin A$, never sends a signal s_i, $i \in A$; notice, however, that the definition allows a sender with a signal s_i, $i \in A$, to send a signal s_j, $j \notin A$. The next lemma lists these properties of separable sets that I need in the sequel.

Lemma 10 ***(a)*** *$A = \{1, ..., m\}$ is a separable set.*

(b) *Every separable set contains a minimal separable set.*

(c) *If $\sigma(\overline{s_i}, s_i) < 1$ for some $i = 1, ..., m$, then there is a separable set A such that $\sigma(\overline{s_i}, s_i) < 1$ for every $i \in A$.*

(d) *If $\sigma(\overline{s_i}, s_i) < 1$ for every $i \in A$ and A is a separable set, then A contains at least two elements.*

Proof. **(a)** and **(b)** are straightforward. To obtain **(c)** put $A = \{i = 1, ..., m : \sigma(\overline{s_i}, s_i) < 1\}$. **(d)** follows from Lemma 8. Indeed, otherwise the equilibrium payoff of the sender with the only signal s_i, $i \in A$, would be greater than ε. ■

For any set $A \subset \{1, ..., m\}$ denote by m_A the number of elements of A. Given a strategy σ, let σ_A stand for the matrix obtained from σ by deleting all row and columns corresponding to signals s_i, $i \notin A$, and let P_A denote the matrix obtained from P by deleting all rows corresponding to signals s_i, $i \notin A$.

Lemma 11 *If A is a minimal separable set and $\sigma(\overline{s_i}, s_i) < 1$ for every $i \in A$, then*

$$rank(\sigma_A - \mathbf{I}) \geq m_A - 1.$$

Proof. I shall show that either $\lambda = 1$ is not an eigenvalue of σ_A, or the space of eigenvectors corresponding to the eigenvalue $\lambda = 1$ is 1-dimensional. Namely, each eigenvector must have all coordinates equal.

Suppose $\mathbf{v}$ is an eigenvector corresponding to $\lambda = 1$. Without loss of generality I can assume that the first coordinate of $\mathbf{v}$ is the largest coordinate, $v_1 \geq v_i$ for $i = 1, ..., m_A$. Suppose to the contrary that $v_1 > v_i$ for some i, and let i^* be the smallest such an i. Then each of the first $i^* - 1$ coordinates of $\mathbf{v}\cdot\, \sigma_A$ is not greater than

$$\sum_{j=1}^{m_A} \sigma(\overline{s_j}, s_i) v_j.$$

Since $\sum_{i=1}^{m_A} \sigma(\overline{s_i}, s_1) \leq 1$ and $\sigma(\overline{s_i}, s_1) \geq 0$ for every i, this expression can be equal to v_1 only if either $v_1 = v_i$ for every i or $\sigma(\overline{s_j}, s_i) = 0$ for every $j \geq i^* - 1$. The former case is excluded by assumption. In the latter case, I obtain a contradiction with the minimality of A; its proper subset obtained by removing $i = 1, ..., i^* - 1$ form A is a separable set as well. ■

Now I can prove Theorem 1.

Proof. **(a)** and the "only if" part of **(b)** are straightforward. To show "if" part suppose that $\sigma(\overline{s_i}, s_i) < 1$ for some i. Take a minimal separable set A with $\sigma(\overline{s_i}, s_i) < 1$ for every $i \in A$. By Lemma 10 such a set exists and contains at least two elements. Represent $\{s_1, ..., s_m\}$ as $S^1 \cup S^2$, where $S^1 = \{s_i : i \in A\}$ and $S^2 = \{s_i : i \notin A\}$. By Lemma 11, $rank(\sigma_A - \mathbf{I}) \geq m_A - 1$. Therefore $rank P_A \leq m_A - rank(\sigma_A - \mathbf{I}) = 1$ which completes the proof of "if" part of **(b)** as P_A consists of at least two rows of P. ■

5.2 Proof of Proposition 2

Proof. Let $NEG(P, \rho)$ denote the set of all Nash equilibria of $G(P, \rho)$. One readily checks that $NEG(P, \rho)$ is an upper hemi-continuous correspondence with respect to ρ. Thus Proposition 2 follows from Theorem 1. ■

5.3 Proof of Proposition 4

I shall prove (a); the proof of (b) is analogous.

By symmetry, $Bj \in T_R^*$ iff $Cj \in T_R^*$. Notice that, for $(\gamma^1, ..., \gamma^l) = (0, ..., 0)$ and any $r \in T_R$, the receiver with signal r when she hears message $\overline{Ai}$ takes in equilibrium E the same action as in equilibrium E^{NE} when she hears message $\overline{T_S^1}$. Notice also that E for $\gamma^1, ..., \gamma^l \geq 0$ converges to E for $\gamma^1, ..., \gamma^l = 0$ as $(\gamma^1, ..., \gamma^l) \to (0, ..., 0)$. Therefore there exists $\overline{\gamma} > 0$ such that for $\gamma^1, ..., \gamma^l < \overline{\gamma}$:

(**i**) the receiver with signal $r \in T_R - T_R^*$ who hears message $\overline{A1}$ or $\overline{A2}$ takes in equilibrium E the same action as in equilibrium E^{NE} when she hears message $\overline{T_S^1}$,

(**ii**) the receiver with signal $Bj \in T_R^*$ ($Cj \in T_R^*$) who hears message $\overline{A1}$ (respectively, $\overline{A2}$) takes action a_l with probability ν_j and action a_{l+1} with probability $1 - \nu_j$, where $\nu_j \in (0, 1]$,

(**iii**) the receiver with signal Bj (Cj) who hears message $\overline{A2}$ (respectively, $\overline{A1}$) takes action a_l with probability 1.

Property (**ii**) implies that

$$\frac{p^j + \gamma^j}{p^j + \gamma^j + p_3^j \left(\dfrac{1 - \sigma}{2} \right) (1 - \varepsilon)} \leq \pi_l, \tag{12}$$

and one has equality instead of inequality when $\nu_j < 1$. Since $\pi(Bj)$ - the probability assigned to to event "$s \in T_S^1$" in equilibrium E^{NE} by the receiver with signal Bj who receives message $\overline{T_S^1}$ - equals, by assumption, π_l, (12) holds with equality for $\gamma^j = 0$ and σ replaced with σ^{NE}. Thus, (12) cannot hold for $0 < \gamma^1, ..., \gamma^l < \overline{\gamma}$ and $\sigma \geq \sigma^{NE}$. This yields $\sigma < \sigma^{NE}$ when $0 < \gamma^1, ..., \gamma^l < \overline{\gamma}$.

Suppose that $r \notin T_R^*$. Since R takes a_l in both E and E^{NE} conditional on $\overline{T_S^1}$ and $\overline{A1}$ or $\overline{A2}$, respectively, and a_0 conditional on $\overline{T_S^2} = \overline{N}$, the difference between R's payoffs in E^{NE} and E equals $(\sigma^{NE} - \sigma)[u_R(a_0, T_S^2) - u_R(a_l, T_S^2)] > 0$, because $\sigma < \sigma^{NE}$ and a_0 is a unique maximizer of R's expected payoff when she assigns probability 0 to event "$s \in T_S^1$". Suppose that $r \in T_R^*$. It follows from properties (**ii**) and (**iii**) that whenever R takes action a_{l+1}, she has to be indifferent between a_{l+1} and a_l. Therefore

R's expected payoff is the same as if she took a_l conditional on $\overline{A1}$ and $\overline{A2}$ and a_0 conditional on $\overline{N}$. Thus, the same argument as that for $r \notin T_R^*$ establishes that the receiver's expected payoff is lower in equilibrium E than in equilibrium E^{NE}.

6 References

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