Semantics I
Anna Szabolcsi
Fall 2015
MW 2:00 - 3:15

The goal of Semantics I is to introduce some of the foundational questions of formal semantics, to build skills necessary for appreciating logical and mathematical approaches, and to nourish an interest in linguistic generalizations. Recent contributions by members of the department will regularly serve as illustrations of the claims and techniques. In this way, the course aims to prepare participants to read the literature, to engage with research conducted in the department, and to go on to both broaden and deepen their expertise in Semantics II and in seminars.

This course presupposes a good command of set theory, propositional logic, and predicate logic. Prospective participants will have returned a problem set to me by August 26. [The problem set can be found at the end of this consolidated file.]

The topics to be discussed form six blocks:

- What is meaning? Answers guided by the principle of compositionality
- Partially ordered sets and generalized quantifiers
- Categorial grammar and type theory, the lambda operator and combinators, variable-ful and variable-free semantics
- Techniques of quantification, binding, continuations
- Indefinites, universals, counting quantifiers, and quantifier particles cross-linguistically
- Events at the syntax/semantics interface: part-whole structures and argument structure

A preliminary calendar and list of readings is found on the next page. We may speed up or slow down on some of the topics, so the calendar serves for orientation only. Participants should count on having both a written and a reading assignment every week. Materials will be shared in a dropbox folder.

Participants are asked to always study the required readings in advance of class discussion, and to review them again afterwards. Active participation in the discussion is encouraged and expected. The required readings are typically excerpts from scholarly articles or books. They are supplemented by recommended readings, and several specialized articles will also be suggested along the way. All participants should study a selection of the recommended or specialized readings, in addition to the required ones; please negotiate the selection with me, based on your interests.

Course requirements include a squib or a term paper. Co-authored projects are welcome.

Start talking with me about your projects in mid-October. I’ll be happy to help with developing a topic or to suggest topics.
Preliminary calendar -- the time dedicated to each topic may be a bit shorter or longer.

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Semantics I
Szabolcsi, 9-2-2015

Compositionality: The meaning of a complex expression is a function of the meanings of its constituent parts and how they are assembled.

Gottlob Frege (1848-1925), Richard Montague (1930-1971)

Most sentences are not idioms; holistic speculations about meaning are just preparatory exercises.

Context -- aspects of “pragmatics” to be figured in judiciously
indexicals I am here today. Ja skazala, chto u menja bolit golova.
implicatures John has read two poems. John hasn’t read two poems.
speech acts Could you pass the salt? I congratulate you. You may take an apple or a pear.

What size parts? Phrases, words, morphemes, ...? dono-hito-mo ‘every person’, A-mo B-mo ‘A as well as B’
What kind of parts? Overt elements, null elements, type shifters, syncategorematic expressions, ...

Ways of assembling constituents
Top-down PSG versus bottom-up categorial / merge-style grammar,...? Movement rules?
How varied can the interpretations of the rules of assembly be?

How much ambiguity and homonymy are to be tolerated? What phenomena form natural classes and require uniform accounts? Big lessons from Carlson’s 1977 account of bare plurals, as well as its critics.

Explicit interpretations for sub-sentential expressions
Every cat knows Fido vs. ∀x[cat’(x) → know’(x, F)]
What is the meaning of every?

Compositionality à la Montague, UG: Meaning assignment is a homomorphism (=structure preserving mapping)

Syntax algebra, S
<expressions, syntactic operations>

Meaning algebra, M
<meanings, semantic operations>

expression a
↓ syn.op. P_i
expression b

meaning δ
↓ sem.op. Q_i

M and S are similar in a well-defined sense. A.o.t., each syntactic operation P_i is matched with a semantic operation Q_i of the same arity. Meaning assignment M-ASS is a homomorphism iff M-ASS commutes with the matching operations: M-ASS (P (a)) = Q (M-ASS (a))

But, what are the meanings in M that make all this viable in the case of natural languages?
This is the question to be pondered in the upcoming meetings.
Reading assignment for 9-9

General background for compositional semantics.

The origins of intensional logic. In Gamut vol. II, Chapter 1. (course folder)
In preparation for next week’s discussion.

The scanned Gamut file contains this plus further excerpts that we discuss in this block. We probably won’t get to DRT next week though.

Pointer

For Montague’s definition of compositionality, see p. 227 of UG (Theory of meaning) in the course folder.
Semantics I
Szabolcsi, 9-9-2015 (after continuing with the 9-2 handout)

What is "meaning"?

The following three assumptions are obtained from the Greats’ originals by replacing their terms by a “conceptual variable”. We are asking what “meaning” has to be for the three assumptions to work together.

Compositionality: The “meaning” of a complex expression is a function of the “meanings” of its constituent parts and how they are assembled. (Cf. Frege’s principle.)

Interchangeability of identicals: Suppose an expression C has A and B as its immediate constituents:

A

→

C

→

B

If the “meaning” of B is the same as the “meaning” of some D, then B and D must be interchangeable within C, without affecting the “meaning” of C. (Cf. Leibniz’s salva veritate.)

Reality check: If you know the “meaning” of an expression, and you know all the facts, then you are able to apply that expression referentially correctly. (Cf. Wittgenstein’s meaning as use.)

* Could “meaning” be just reference (extension)?

Syntax: <expressions, syntactic operations> (expressions = lexical + constructed by operations)
Semantics: <meanings, semantic operations> + a meaning assignment I for lexical expressions

Define “meanings” based on a first-order logical model M of the familiar kind: a particular set of individuals with particular sets and relations on them, plus two distinguished objects \{0,1\} for sentence “meanings”.

(If either the domain were different (e.g. numbers, not people), or the sets/relations were different (e.g. Britta were not a dancer), we would have a different model, and thus a different semantics for our syntax.)

Most Swedes are dancers. iff Most Swedes are carpenters.
Britta is a Swedish dancer. iff Britta is a Swedish carpenter.
At least one dancer is angry at Britta. iff At least one carpenter is angry at Britta.
Every dancer is bored by someone. iff Every carpenter is bored by someone.  

Success!
Is “meaning” = reference good enough in general? What are the intuitive judgments?

Cecil wants to be a dancer. ? iff Cecil wants to be a carpenter.
Cecil is a good dancer. ? iff Cecil is a good carpenter.
Cecil is a former dancer. ? iff Cecil is a former carpenter.
Cecil is looking for dancers. ? iff Cecil is looking for carpenters.
Cecil promised Britta a dodo. ? iff Cecil promised Britta a unicorn.

The new examples fail Interchangeability, because the truth values of these sentences do not depend on just who the dancers and the carpenters are in one particular state of affairs, but also on who they could be, if the state of affairs was modified in some reasonable way.

Also, do we want to say that the “meaning” of angry_at changes if Britta makes peace with Cecil?

So, let each model M contain an infinite set of imaginable states of affairs (possible worlds w1, ...). Assume the worlds are all accessible from each other in the sense that they are reasonable modifications of each other.

Let the meaning assignment now associate, with each expression, a function that picks out the reference (extension) of that expression in each world. This function is called the intension of the expression.

M1

We do not expect dancer and carpenter to have the same extensions (denotations) in all possible worlds, and so their intensions will be different.

I (dancer) = \{ <w1, {a,b,c}>, <w2, {a,b,c,d}>, <w3, \emptyset>, ... \}
I (carpenter) = \{ <w1, {a,b,c}>, <w2, {a,c}>, <w3, {a,c}>, ... \}

The intension of a sentence will be, analogously, that function which specifies its truth value in each possible world, depending on the intensions of the parts and how they are assembled.

(Intensional propositional logic is modal logic. The intension of a sentence is called a proposition. In intensional predicate logic, the intension of a name is called an individual concept, the intension of a predicate a property.)

With “meaning” = intension, Compositionality, Interchangeability, and Reality check will all be in order even in the second set of examples.

Montague’s notation: ^dancer = the intension of the expression dancer.
For any expression A, ^A is the intension of A, a function from possible worlds to A’s extension there.
If B is an intension function, ^B is the value of B in the actual world w*.

For any A, ^(^A) = A but ^(^B) ≠ B in general, only for ^(^A) = ^A.
(Contemporary notation: \( \forall A = \lambda w[A'(w)] \) and \( \exists B = \lambda w[B(w)] \) \((w^*)\). To be introduced when we have \( \lambda . \))

Example: Let POTUS be an intension-expression. Today, \( \forall \text{POTUS} = \text{Obama} \).
But \( \forall \text{Obama} \) picks out Obama in every world, not the president-in-that-world.
So \( \forall (\forall \text{POTUS}) \neq \text{POTUS} \), it is \( \forall \text{Obama} \).

A linguistic context is called **intensional wrto a particular position in it** if it is sensitive to the intension of the expression in that position.
E.g. `former ___` is intensional wrto its nominal complement, but `Swedish ___` is extensional wrto its nominal complement.

A linguistic context is **extensional wrto a particular position in it** if it is sensitive only to the extension of the expression in that position.
E.g. `Swedish ___` and `every ___` are extensional wrto their nominal complements.

In general, the semantics of determiner quantification is overwhelmingly extensional. Therefore, even though we cannot identify “meaning” with reference (extension), extensional semantics is hugely useful in the study of natural language.

* 

The next question to be investigated is whether “meaning” = intension is good enough in general. What may be problematic examples, and what solutions have been considered for those problems?

The one after that is whether the application of the Interchangeability test calls for the enrichment of “meaning” in yet other dimensions; if yes, what dimensions?
Assignment for 9-14

Written

You are welcome to work in pairs or triples; these tasks aren’t really meaningful in larger groups. If you collaborate, indicate who you collaborated with and write up the results individually. Please type the responses, using a 12 pt. font and 1-inch margins. Brings the responses to class on Monday, 9-14.

1. We have seen want to VP, good NP, former NP, look for DP, and promise someone DP form intensional contexts with respect to the argument places bearing the given category labels, whereas we implied that DP is a dancer, DP is a carpenter, Swedish NP, angry at DP, bored by DP, every NP, most NP, and at least one NP are intensional.

Are there predicates that are intensional wrto their subjects? Are adjectives typically extensional or intensional wrto their NP complements? Are verbs that take infinitival complements typically extensional or intensional wrto those VPs? Are there intensional prepositions? What kind of verbs are intensional wrto their DP complements? Are there intensional determiners? Collect examples and try to find some rhyme and reason in the way they pattern. If the examples come from literature that you independently know, note the source, but don’t go about this assignment by Googling “intensional XP.”

2. Are there systematic counterexamples to the “meaning”=intension assumption? I.e. are there principled ways to generate solid counterexamples, as opposed to finding some strange phenomenon whose problematic status heavily rests on the fact that we don’t really know how to analyze it? If you find productive sources of counterexamples, explain their nature and give a few convincing examples.

Reading

Review “Origins of intensional logic” after the 9-9 discussion.

In preparation for 9-14 and 9-16, read the following (both are in the scanned Gamut file in the folder)

Gamut II, 6.5 Compositionality, Logical Form, and Grammatical Form
Gamut II, 7.4.1, 7.4.2, 7.4.3, pp. 285-289 of 7.4.5., and 7.4.6.

In other words, are welcome to skip 7.4.4, as well as the second, technical part of 7.4.5 starting with “First we consider the simple donkey sentence (160)” . The first part of 7.4.5, Compositionality and DRT, is important!

The DRT section presupposes that you are comfortable with first order predicate logic. If that was a difficult area, please brush it up and also let me know if I can be of assistance.

Optional -- recommended

Gamut II, Ch. 2 Intensional propositional logic

A very nice introduction to modal logic and tense logic, including models.

If you don’t read it now, come back to it if and when you want to tackle modals and/or tense.
Is “meaning” = intension good enough in general?

Counterexamples can be constructed if we find expressions whose intensions are identical (in every possible world, the two expressions have identical extensions) and yet, there is a significant set of linguistic environments in which they are not interchangeable.

Tautologies are true in every world, so all tautologies have the same intension. Contradictions are false in every world, so all contradictions have the same intension.

Mary is either French or not French (p ‚Ä— p) and If Bill is Irish, then Bill is Irish (p ⊃ p) are both tautologies:

J is aware that Mary is either French or not French. ≠ J is aware that if Bill is Irish, then Bill is Irish.
J mentioned that Mary is either French or not French. ≠ J mentioned that if Bill is Irish, then Bill is Irish.

Essentially all predicates that take that-complements (propositional attitudes) are sensitive to more than just the intension of that complement (it is true that... may be the one exception). Likewise, tautological and contradictory properties can be defined:

Mary wanted to be French or not French. ≠ Mary wanted to belch fire or not to belch fire.

Interchangeability tests two-way entailment; one-way entailment is also a useful test. In classical logic, p entails p T, and p entails p V q. The failure of expected entailments is especially heart-breaking to the semanticist, since we check the correctness of our semantic analyses by investigating whether they predict the intuitively expected entailments.

John mentioned that Mary is tall. ⊢ John mentioned that Mary is tall and if Bill is French, then Bill is French.
John saw Mary post the letter. ⊢ John saw Mary post the letter or burn it.

Contexts in which logically equivalent expressions cannot be interchanged preserving truth, and classical logical entailments are not preserved, are sometimes called hyperintensional.

From this point on, philosophers and semanticists are less confident about where to go. Several different approaches have been proposed to define synonymy so as to prevent arbitrary logically equivalent expressions from qualifying as having the same meanings. For example:

Intensional isomorphism (Carnap 1947, Meaning and Necessity) and structured meanings (Cresswell 1985, Structured Meanings: The Semantics of Propositional Attitudes)
Partiality and situations instead of truth-values (Barwise 1981, Scenes and other situations; Barwise & Perry 1983, Situation Semantics)
Algorithms that compute the reference of expressions (Muskens 2005, Sense and the computation of reference; Lappin 2012, An operational approach to fine-grained intensionality [course folder])
Non-monotonic reasoning starting from vagueness and the Sorites (Cobreros, Egré, Ripley, & van Rooij 2014)

Unfortunately, these proposals often overkill. Instead of attempting to establish an empirically justifiable borderline between synonymous and non-synonymous expressions, they by and large prevent an expression from being synonymous with anything but itself. Partiality and non-monotonic reasoning may be interestingly different.
Discourse representations; meaning as context change potential

A different type of problem arises from considering what type of information about a complex expression a subsequent rule may rely on. Consider the following examples:

(i) Smith is a provost <=> Smith heads an academic institution
   a. Smith is a provost. *It_anaphoric has 3,000 students.
   b. Smith heads an academic institution. It_anaphoric has 3,000 students.

(ii) A man left <=> Not every man did not leave
    a. A man left. He_anaphoric was tired.
    b. Not every man did not leave. *He_anaphoric was tired.

(iii) A man was smiling and humming <=> A man who was smiling was humming
    a. A man was smiling and humming. A woman was, too [=smiling and humming].
    b. A man who was smiling was humming. A woman was, too [=humming].

We have three pairs of texts (a)-(b). The first sentence in (a) is always logically equivalent to the first sentence in (b). If meaning is something like intension, these have the same meanings. Then, the second sentence in (a) is always identical to the second sentence in (b). Thus, if the meaning of the two-sentence text is a function of the meanings of its component sentences, then the (a) texts are expected to mean the same thing as the corresponding (b) texts. But they do not. In the first two pairs, (b) has an anaphoric reading for the pronoun and (a) does not. In the third pair, VP-anaphora has different readings in (a) and (b).

The explanation can be given as follows. The presence or absence of certain kinds of syntactic units, not just the meaning of the whole first clause, determines whether pronominal or verbal anaphor can have the desired antecedent. This shows that the meaning of the two-sentence text was a function of the meaning PLUS some syntactic aspects of the first sentence.

The existence of such data led Kamp to assume that grammar has to relax compositionality to some extent. Specifically, he postulated a level of discourse representation at which anaphora is determined, among other things. At this level, indefinites like an academic institution and a man introduce individual discourse referents that pronouns can refer back to, but predicates like provost and quantifiers like not every man (not) do not. This is an indispensable level of representation between `traditional" syntax and semantic interpretation.

This view contrasts with that of Montague Grammar, where the relation between English syntax and meanings is mediated by a logical syntax (that of intensional logic), but this intermediate level is merely convenient and is fully eliminable, as established in the translation theory of Montague's UG.

Groenendijk & Stokhof (Gamut II, 7.4.5) argue that Kamp's conclusion is unwarranted. Look at the situation in the following way. We have two two-sentence sequences. The second sentence is the same in both sequences, nevertheless, the sequences as wholes are not synonymous. This can only be because the meanings of the first members were not identical, contrary to what we assumed up till now. The task is to devise a notion of meaning that bears this out.

This is the same kind of move as the switch from “meaning”=reference to “meaning”=intension, etc.

Following Heim, G&S propose to identify “meaning” with context change potential, arriving at what they call Dynamic Semantics. There are at least two ways in which the addition of a new sentence to a context changes that context. One, it increases plain information content. Two, it restricts, in various ways, how the context can
be expanded. Supporting anaphora comes under this latter heading. Now both have to be part of "meaning." Specifically, the context change potential of (sentences containing) a man will be different from that of (sentences containing) not every man (not); thus the contrast above will be accounted for.

* 

Although hyperintensionality and the issues arising in connection with anaphora are rather different, notice that both have lead to theories of meaning in which not only the denotation but also the syntactic or computational aspects of an expression are recognized as part of its meaning.

Clearly, the question arises how far we are ready to go in enriching meanings.

* 

It will be good to grab this opportunity to get acquainted with the basics of Kamp-style Discourse Representation Theory (DRT), as summarized in Gamut II, 7.4.
Assignment for 9-21

Reading

Gamut II, 6.5 Compositionality, Logical Form, and Grammatical Form
Gamut II, 7.4.1, 7.4.2, 7.4.3, pp. 285-289 of 7.4.5., and 7.4.6.
In other words, are welcome to skip 7.4.4, as well as the second, technical part of 7.4.5 starting with “First we consider the simple donkey sentence (160)”. The first part of 7.4.5, Compositionality and DRT, is important!

The DRT section presupposes that you are comfortable with first order predicate logic. If that was a difficult area, please brush it up and also let me know if I can be of assistance.

Written

Exx. 14 and 15 (p.277).
I don’t want to prevent anyone from creating boxes in Word or LaTeX, but if I were you, I’d do all of this by hand.

Recommended reading from the course folder

Lappin, An operational approach to fine-grained intensionality, 7 pages.
Schlenker, A plea for monsters, pp. 29-39.
Semantics I  
Szabolcsi, 9-21/23-2015

**Operations in partially ordered sets, and some linguistic applications**

(1) union: \( A \cup B \)  
    disjunction: \( p \lor q \)  
    conjunction: \( p \land q \)  
    complement: \( \neg A \)  
    negation: \( \neg p \)

What other operations are these related to? On what kind of entities can such operations be performed? What kind of structures do these entities form?

- **Partially ordered set (poset)**  
  \( \langle A, \geq \rangle \) where \( \geq \) is reflexive, transitive, and anti-symmetrical.

- **Lower bounds, upper bounds**  
  Let \( \langle A, \geq \rangle \) be a poset. For any subset \( X \) of \( A \),
  - \( c \) is a lower bound for \( X \) iff for every \( x \in X \), \( c \leq x \),
  - \( d \) is an upper bound for \( X \) iff every \( x \in X \), \( d \geq x \).

  **Greatest lower bound (glb, infimum), least upper bound (lub, supremum)**
  The greatest lower bound for \( X \), if it exists, is the greatest one of the lower bounds. The least upper bound for \( X \), if it exists, is the smallest one of the upper bounds.

- **Join, \( \lor \)** the least upper bound of a two-element set: \( a \lor b \)
- **Meet, \( \land \)** the greatest lower bound of a two-element set: \( a \land b \)

- **Join semi-lattices**: posets that are closed under join (viz., for any \( a, b \in A \), \( a \lor b \in A \)):

  ![Diagram of join semi-lattices]

  A poset that is not closed under join (e.g. there is no \( x \lor y \) in \( A \)) is not a join semi-lattice:

  ![Diagram of a poset that is not closed under join]

  **Mereological structures are free join semi-lattices.**

- **Meet semi-lattices**: posets that are closed under meet (same, upside down):

  ![Diagram of meet semi-lattices]

- Similarly for **meet semi-lattices** (same, upside down).
• A poset that is closed under both meet and join is a lattice.

A lattice that has a top, T and a bottom, \( \bot \) is bounded. (All finite lattices are bounded.)

• If the meet operation preserves non-empty finite joins, and the join-operation preserves non-empty finite meets, the lattice is distributive.

\[
a \lor (b \land c) = (a \lor b) \land (a \lor c) \quad \text{and} \quad a \land (b \lor c) = (a \land b) \lor (a \land c)
\]

The first two lattices above are distributive, the diamond and the pentagon are not.

• Complement, \( \neg \) for any \( a \in A \), \( \neg a \) is another element of \( A \) for which both of these hold:

\[
a \land \neg a = \bot \quad \text{and} \quad a \lor \neg a = T
\]

(Relative pseudo-complement, \( \rightarrow \) \( c \in A \) is \( a \rightarrow b \), the pseudo-complement of \( a \) relative to \( b \), iff \( c \) is the largest element of \( A \) for which \( a \land c \leq b \).)

• If a poset is closed under meet, join, unique complement, and distributive, it is a Boolean algebra. (If a poset is closed under meet, join, rel. pseudo-complement, and distributive, it is a Heyting alg.)

Every Boolean algebra is a distributive lattice. (Every Heyting algebra is a distributive lattice.)

(Every Boolean algebra is a Heyting algebra with \( a \rightarrow \bot = \neg a \).)

A Heyting algebra that is not a Boolean algebra: Some (Heyting algebras that are also) Boolean algebras:

\[
\begin{array}{cccccc}
z & & & v & & \\
| & | & & | & | & \\
y & & & y & & \\
| & | & & | & | & \\
x & & & x & & \\
\end{array}
\]

\[
\begin{array}{cccccc}
z & & & v & & \\
| & | & & | & | & \\
\emptyset & x & y & y & z & \\
| & | & & | & | & \\
x & & & x & & \\
\end{array}
\]

among others, all finite powerset algebras
Some logico-linguistic applications:

- **Classical propositional logic** is a Boolean algebra (connectives: meet, join, and complement).

  \[
  \begin{align*}
  T & \text{ closed under meet?} & T \land T = & T \land F = & F \land F = \\
  \bot & \text{ closed under join?} & T \lor T = & T \lor F = & F \lor F = \\
  F & \text{ closed under unique complement?} & \neg T = & \neg F = & 
  \end{align*}
  \]

- Logics with no excluded middle, hence no double negation cancellation, are modeled with Heyting algebras. Intuitionistic logic, and the logics of Dynamic Semantics and Inquisitive Semantics.

- **Events** and **collectives** form mere join semi-lattices (no \( \perp \), no lattice).


- We’ll study generalized quantifiers using Hasse-diagrams that represent the powerset of the universe of discourse:

  ![Hasse-diagram](image)

- An algebraic semantics of **scope-taking** (Szabolcsi & Zwarts 1993):

  If the meaning of a scopal element is (at least in part) defined in terms of Boolean operations, cash out its contributions by performing those operations on the denotation of its scope.

  - What didn’t you see? \[ \text{compl. of } \{ x: \text{you saw x} \} \]
  - What did every girl see? \[ \text{meet of } \{ x: \text{Mary saw x}, \{ x: \text{Susan saw x}, \ldots \} \}
  - What did any girl see? \[ \text{join of } \{ x: \text{Mary saw x}, \{ x: \text{Susan saw x}, \ldots \} \]
  - What did two girls see? \[ \text{[distributive two requires both meets and joins]} \]

  Predicts trouble when the scope of the operator denotes an element of some A that is not closed under the pertinent operations: Weak Island effects. E.g. collectives and events (and hence <event,object> pairs) form mere join semi-lattices: not closed under meet or complement.

  - Which relatives of yours did you show every one of your rings to? \( \text{OK } \text{wh > every} \)
  - Which relatives of yours did you get every one of your rings from? \( \# \text{ wh > every} \)

  4,000 people visited the Rijksmuseum last year \( \text{OK 4,000 events, altogether 40 persons} \)
  4,000 people didn’t visit the Rijksmuseum last year \( \# \text{ 4,000 events, altogether 40 persons} \)
Assignment for 9-28

Reading

1. Pp. 1-7 of Background notions in lattice theory... (or some of Landman, Ch 6).
2. We’ll use pp. 5-12 and pp. 45-70 of my Quantification book for generalized quantifiers. Read these parts in preparation for next week. (They are based on old lecture notes, but especially some of the discussion of the literature pertaining to determiners goes way beyond what we’ll cover at this point.) -- Please bring printed or electronic copies to serve as class handouts.

Written

1. Take the set E = {a,b,c} and the relation Rn on it, as defined below. Is each <E,Rn> a poset? To answer, check whether each Rn is reflexive, transitive, and anti-symmetrical. It’s probably convenient to do that by drawing 4 models with arrows for the pertinent relation. If one <E,Rn> is not a poset, point out which property fails and how.

   R1 = {<a,c>, <b,c>, <c,a>, <b,b>}
   R2 = {<a,b>, <b,c>, <a,c>, <b,b>}
   R3 = {<a,b>, <b,c>, <a,a>, <b,b>, <c,c>}
   R4 = {<a,b>, <b,c>, <a,c>, <a,a>, <b,b>, <c,c>}

2. Similar questions will have (probably) been discussed in the Wednesday class. Recall the discussion and replicate or extend it for the cases below. Make the explanations reasonably precise.

   a. Is this a meet semi-lattice? Why/why not? Does each element have a unique complement?

   b. Is this lattice distributive? Why/why not? Does each element have a unique complement?

   c. Is this lattice distributive? Why/why not? Does each element have a unique complement?

   d. Is this lattice distributive? Why/why not? Does each element have a unique complement?

3. And finally, linguistics! Read the first part of Fromkin, ed., Ch 9, Cross-categorial parallelisms. It talks about mass nouns and atelic predicates having cumulative reference. Can you express “cumulative reference” in terms of the notions introduced this week? Bonus question: That chapter talks about telic verbs mapping the structures of their direct objects to the structures of the VP-events in a particular way. What is the official name of that kind of mapping, based on material in the 3 weeks of this course? Explain your choice.
Generalized quantifiers

Handout: Szabolcsi 2010, Quantification, pp. 5-12 and pp. 45-62

A uniform interpretation for different kinds of DP

Fig. 2.1 The set of properties Smaug has: all the sets that have \( a \) as an element.

Fig. 2.2 The set of properties every dragon has: all the sets that have \( \{a, b, c\} \) as a subset.

Fig. 2.3 The set of properties more than one dragon has: all the sets whose intersection with \( \{a, b, c\} \) has more than one element.

Generalized quantifiers and their elements: operators and their scopes
Boolean operations

Semantic properties, e.g. monotonicity

Useful further notions, e.g. witness sets

Fig. 4.1 The intersection of two generalized quantifiers

every dragon' = \{\{a, b\}, \{a, b, c\}, \{a, b, d\}, \{a, b, c, d\}\}

at least one serpent' = \{\{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\},
\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\}\}

every dragon and at least one serpent' = \{\{a, b, c\}, \{a, b, d\}, \{a, b, c, d\}\}

Fig. 4.5 An increasing generalized quantifier

more than one robot' = \{\{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\},
\{b, c, d\}, \{a, b, c, d\}\}

Fig. 4.8 The witnesses of more than one robot'

= \{\{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}

NB not the same dragons
Assignment for 10-5

Reading

(i) For the material covered this week, pp. 5-12 and pp. 45-62 of Quantification. The endnotes should be read as part of the text -- lots of good stuff there.

(Consult the literature surveyed in these sections if you are interested! I’m happy to help if I have paper or electronic copies of something you can’t easily lay your hands on.)

(ii) In preparation for next week, Determiner denotations (pp. 60-70). Much of this will be familiar from Determiners and nouns (=section 7.3.3 of Compositionality). Supplement it with Determiners and adverbs (=section 9.2 of Cross-categorial parallelisms).

(iii) Optional: In Background notions..., Quantifiers and negation in Boolean terms and Generalized quantifiers significantly overlap with (i), but they go into some details that you may want to be alerted to. This is a recommended reading, if you plan to play more with quantifiers.

Written

Make copies of the blank Hasse-diagrams on the previous page and use them for this assignment. Do the shadings clearly, using pencils or highlighters, so that both you and I can see at a glance what’s going on in each diagram. If comments are required, write them next to the diagram. -- Do (1) and (2) on your own. Collaboration on (3) is encouraged.

(1) "Quantification" contains 5 Hasse-diagrams in Chapter 4 where the membership of the sets is given in the caption, but the shading is left to the gentle reader. Take your own diagrams and do the shading for each of these. Reason out how the shading has to go, but check your results against the caption in the book, and make corrections as needed.

(2) Let the humans in the universe be {Andy, Belinda, Carl}. Using altogether 4 Hasse-diagrams, 
(i) Shade each of the GQs {P: P(a)}, {P: P(b)}, and {P: P(c)}. In Keenan’s terminology, these are the “Montagovian individuals” .
(ii) Shade the GQ [[everyone]], and concisely say by what operation(s) it is obtained from the Montagovian individuals.
(iii) Shade the GQ [[someone]], and concisely say by what operation(s) it is obtained from the Montagovian individuals.
(iv) Shade the GQ [[no one]], and concisely say by what operation(s) it is obtained from the Montagovian individuals.
(v) You don’t need to do the shading for this, but say how you’d obtain the GQ [[two people]].
Various linguistic tests set apart indefinite and definite-or-specific noun phrases. One is relational have:

(a) Belinda has {a son / (at least) two sons / no sons}.
(b) # Belinda has {the son / every son / Carl}.

The same kind of noun phrases are set apart in there-insertion contexts. But there one must be careful to discount so-called “list readings” that serve to draw attention, rather than assert existence. Negation and interrogation can be used for that purpose:

(a’) There won’t be {a film / (at least) two films} in the program. There will be no films in the program. Will there be {a film / (at least) two films / no film} in the program?
(b’) # There won’t be {the film / every film / Fury} in the program. # Will there be {the film / every film / Fury} in the program?

Your task is to investigate the two classes of noun phrases (not the relational or existential contexts). In doing so, assume that the universe is big enough to make all the relevant linguistic distinctions. E.g. if there is just one film in the universe, many of these GQs collapse -- but that’s characteristic of the given universe, not of the meanings of the noun phrases.

(c) Could you characterize the two classes in terms of the Boolean operations by which the GQs in them are obtained from the Montagovian individuals? The characterization could refer to all the operations used to define the members of each class, or the last (=main) operation used to define them. You might be able to characterize one of the two classes and say that the other doesn’t fit that bill, or you might be able to give a positive characterization for each of the two classes.

(d) Could you characterize the actual GQs in each of the two classes (not how they are obtained from Montagovian individuals) with reference to reasonably simple notions or properties, for example:

Witness: The set of individuals X is a witness of a given QG iff X is an element of QG and X is a subset of the “topic set” of QG. (Section 4.1.3)

Minimal element: The set of individuals Y is a minimal element of a given QG iff Y is an element of QG, and no proper subset of Y is an element of QG.

Principal filter: A QG is a principal filter iff it is monotonically increasing and there is a particular set of individuals Z such that QG is \( \{ P : Z \subseteq P \} \). Z is called the QG’s generator set.

(e) If you had some success with both (c) and (d), do you find some connection between their results?
Semantic determiners (DET)

Terminological caveat: DETs are studied as part of GQ theory but they are not GQs, in careful parlance. DETs are Lindström’s <1,1> quantifiers, GQs are Lindström’s <1> quantifiers.

- **Determiners as relations between two sets**

  ![Venn Diagram](image)

  DET is symmetrical iff \( \text{DET}(N)(P) = \text{DET}(P)(N) \)
  
e.g. \( a(n), \) two, no

  DET is intersective iff \( \text{DET}(N)(P) = \text{DET}(N \cap P)(P) \)
  
e.g. \( a(n), \) two, no

  DET is co-intersective iff \( \text{DET}(N)(P) = \text{DET}(N-P)(P) \)
  
e.g. every

  DET is proportional iff \( \text{DET}(N)(P) \) iff \( n/m \) of \( N \) are in \( P \)
  
e.g. most of, few of

- **Determiners as functions from restrictor-denotations to GQs**

  ![Diagram](image)

  Proposition

  where \([[[\text{restrictor}]]]\) is typically, but not always, the denotation of the NP complement of \( D \)

  Restrictedness = Conservativity + Extension

  DET is conservative iff \( \text{DET}(N)(P) = \text{DET}(N \cap P) \)
  
  DET has extension iff for all universes \( E_1, E_2, \)
  
  if \( N_{E_1} = N_{E_2} \) and \( P_{E_1} = P_{E_2}, \)
  
  then \( \text{DET}(N_{E_1})(P_{E_1}) = \text{DET}(N_{E_2})(P_{E_2}). \)
  
  = area (iii) can be ignored

- **Empirical claim**: Natural language determiners are (overwhelmingly) restricted.
  
  Significance: syntax--semantics match
  
  computation, learnability
  
  inherently sortal DETs are possible
• DET is **sortally reducible** if DET(N)(P) can be expressed in terms of DET quantifying over the whole universe E. DET is **inherently sortal** if must be defined as quantifying over the restrictor N.

DET is sortally reducible iff there is a function # into \( \{0,1\} \) expressible in terms of \( \land, \lor, \neg \) such that for all \( N, P \subseteq E \), \( \text{DET}(N)(P) = D(E)(#(N,P)) \).

If there is no such #, DET is inherently sortal.

Compare: There is no Boolean connective # such that

\[ \text{Most dragons are asleep} \text{ iff } Mx[\text{dragon} # \text{ asleep}] \text{, where } Mx[\varphi] \text{ is ”most things in } E \text{ are } \varphi”. \]

(Sortally reducible \( \not= \) definable in first order logic. The DET *most* is not sortally reducible and not first-order definable. But *(in)finitely many* is sortally reducible, while not definable in first order logic. Presuppositional *neither, both, and the ten* are first-order definable, while not sortally reducible.)

• **Not only DETs are restricted.** Adverb-meanings like [[always]], [[for the most part]] and [[only]] are too, although their restrictors may be more difficult to compute.

• **Possibly unrestricted DETs:** This election could have two winners.
  `there are two individuals who could win this election`

• **Covert quantifier domain (=restrictor) restriction**

  Every **bottle** is empty.
  The **syntacticians** campaigned for the **syntacticians**.
  More than one child broke every **balloon**.

• **Challenges for generalized quantifier theory, typically for building GQ from DET and [[restrictor]]**
  (see also Ch 5 of Quantification)

  John has all **SISTERS**.
  Few **INCOMPETENT** cooks applied.
  JOHN read the fewest books.
  Every boy read a different book/the same book.
  Four thousand ships passed through the lock last year.
  The rabbit in the hat is white.

  At least four men arrived late  vs.  More than three men arrived late
  You can borrow at most five books  vs.  You can borrow fewer than six books
  At least two men shook hands  vs.  ?? More than one man shook hands

  How many books should I read?  vs.  How many books should I read?
  At least five.  vs.  At least the following five: A, B, C, D, and E.

  È cinque donne che (*non) ho invitato  vs.  Sono cinque donne che (non) ho invitato
  Combien as-tu (*beaucoup) consulté de livres?  vs.  Combien de livres as-tu (beaucoup) consulté?
Assignment for Oct. 13

Reading

Go back and review Ch 2.1-2 and Ch 4 of Quantification, supplemented with Ch 5 (Potential challenges), endnotes included. (Section 2.3 will be covered in a few weeks.) We have discussed most of this material; but read the rest for pointers to further literature and perhaps for ideas to work on, in this semester or later.

The syllabus asks you to choose and read a few things going beyond the required readings. As we progress, and already in two weeks when we have types and , more and more primary literature will become readable. For the time being, let me point out some good items that are available online in Bobst (in addition to the Carlson, Lappin, and Schlenker papers in the course folder).


[The Handbook is a gold mine! It’s recent, the articles are good, and the coverage is wide.]


In preparation for thinking about squibs, I’m in the process of obtaining permissions from members of the Fall 2011 class, to share their squibs with you, to give you some idea of what people in the past have done.

If you want to know what Semantics II will cover, here is CB’s Spring 2011 syllabus: http://linguistics.as.nyu.edu/docs/IO/16267/barkersemantics2syllabusspring2011.pdf
I doubt that the 2016 offering will be the same, but I suspect it will be somewhat similar.

Written: Two small “convince yourselves” tasks pertaining to only and most

The case of only

English has sentences of the form Only dogs bark. These raise the following questions:
(i) Is only a determiner?
(ii) Is only conservative?

Based on the literature and classroom discussion, you may already know the answers to both, so there isn’t a huge mystery here. This exercise asks you to think through the questions and carefully spell out the answers, and to draw a conclusion regarding the claim that natural language determiners are conservative.
The case of most (of the)

Let $Mx[f] \text{ be true iff more than 50\% of universe has property } f$. Now compare:

Model #1: the universe contains just
10 dogs, 8 of which bark
20 cats, which do not bark

Model #2: the universe contains just
3 dogs, which do not bark
20 cats, which do not bark

Carefully examine whether (i) the English sentence Most (of the) dogs bark is true or false in the two models, and (ii) whether the formulae below, including $M$ as defined above, are true or false. What do you find? Draw a conclusion from the result. (If you want, you can also examine further unrestricted first-order-ish formulae.)

<table>
<thead>
<tr>
<th>True or false</th>
<th>In #1:</th>
<th>In #2:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Most of the dogs bark</strong></td>
<td>[Blank]</td>
<td>[Blank]</td>
</tr>
<tr>
<td>$Mx[\text{dog}(x) \rightarrow \text{bark}(x)]$</td>
<td>[Blank]</td>
<td>[Blank]</td>
</tr>
<tr>
<td>$Mx[\text{dog}(x) &amp; \text{bark}(x)]$</td>
<td>[Blank]</td>
<td>[Blank]</td>
</tr>
<tr>
<td>$Mx[\text{dog}(x) \lor \text{bark}(x)]$</td>
<td>[Blank]</td>
<td>[Blank]</td>
</tr>
</tbody>
</table>
Predicate logic treats quantifiers syntagmatically. We remedy this and also introduce a recursive structure into categories and into the universe of the model.

Categorial grammar has two main ideas:

(1) Syntactic category labels are nothing but explicit encodings of what combinations are possible. E.g. to say that walks belongs to category e\t is to say that it can combine with a member of category e that occurs to its left (its specifier) and the result will be a member of category t. On this approach walks cannot belong to the same category as sees, since the first takes one and the second two arguments.

(2) Syntactic categories are mapped to semantic types in a trivially simple way. An expression of category e\t is a function from entities to propositions. This makes a strong commitment to syntax and semantics being parallel. (What the type of propositions is depends on our theory of meanings.)

Syntax:

Categories:  e, t ∈ Cat;
if A and B ∈ Cat, A/B and B\A ∈ Cat;
nothing else is in Cat.

Lexicon:  Cat e = {Bill1, Mary1}
Cat t = Ø
Cat e\t = {walks}
Cat ((e\t)/e) = {sees}
Cat (t/(e\t)) = {Bill2, Mary2, everyone, something}

Rules:  If α ∈ Cat A/B and β ∈ Cat B, then αβ ∈ Cat A.  \[A/B \text{ looks for a } B \text{ on the right}\]
If β ∈ Cat B and α ∈ Cat B\A, then βα ∈ Cat A.  \[B\A \text{ looks for a } B \text{ on the left}\]

Semantics:

The universe is now typed. Types are “categories of things in the model”. We imagine the universe to contain two sorts of things: entities (of type e) and worlds (of type s), plus two designated objects, 1 and 0 (of type t). Functions of any kind can be recursively defined from these. E.g., functions of type ⟨e,e⟩ (from entities to entities) or ⟨e,t⟩ (from entities to truth values) or ⟨s,t⟩ (from worlds to truth values). Recognizing all the types so defined does not make any commitment regarding which of them have linguistic or philosophical significance. Using categories and types is very natural in linguistics, since words do not combine arbitrarily: they combine with particular kinds of other words to yield particular kinds of phrases. In logic, the use of types is a way to avoid the Russell paradox.
A model M is \langle D, W, I \rangle, where D is a set of individuals and W is the set of worlds. I is an interpretation function from constant expressions to things in the universe.

The \text{Dom}(ain) of a type with respect to D and W is the set of things that belong to that type (entities, or truth values, or worlds, or various kinds of functions).

Types: \quad e, t, s \in \text{Type};
if B, A \in \text{Type}, \langle B, A \rangle \in \text{Type};
nothing else is in \text{Type}.

\text{Dom}_e, D, W = D, \quad \text{Dom}_s, D, W = W, \quad \text{Dom}_t, D, W = \{0, 1\}

\text{Dom}_{\langle B, A \rangle}, D, W = \{ f: \text{Dom}_{B, D, W} \rightarrow \text{Dom}_{A, D, W} \}

Syntactic categories are related to semantic types by the following simple map f. Expressions of category e are mapped to things of type e(ntity). Expressions of category t are mapped to propositions. To maintain the match with the Muskens text, propositions will be sets of worlds, of type \langle s, t \rangle. Function categories are mapped to appropriate functions. Syntactic left/right directionality becomes irrelevant.

f(e) = e
f(t) = \langle s, t \rangle
f(A/B) = f(B\setminus A) = \langle f(B), f(A) \rangle

The world of evaluation is notated as w. Let D = \{b, m, f\} and let I work as follows:

\[ [[\text{Bill1}]]^{M, w} = b \]
\[ [[\text{Mary1}]]^{M, w} = m \]
\[ [[\text{walks}]]^{M, w} = \text{that function from individuals to propositions which assigns } 1 \text{ to an individual in the world of evaluation, } w \text{ iff it walks } w, \text{ viz. the characteristic function of the set of walkers in } w \]
\[ [[\text{sees}]]^{M, w} = \text{that function from individuals to } [ \text{a function from individuals to propositions} ] \text{ which assigns } 1 \text{ to a pair of individuals in } w \text{ if the first sees the second in } w. \text{ NB: sees(x)(y) = sees(y,x).} \]
\[ [[\text{Bill2}]]^{M, w} = \text{that function from sets of individuals to propositions which assigns } 1 \text{ to a set } P \text{ in } w \text{ iff } b \in P \text{ in } w, \text{ viz. the characteristic function of the set of } \text{ those sets that contain } b \text{ in } w \]
\[ [[\text{Mary2}]]^{M, w} = \text{that function from sets of individuals to propositions which assigns } 1 \text{ to a set } P \text{ in } w \text{ iff } m \in P \text{ in } w, \text{ viz. the characteristic function of the set of } \text{ those sets that contain } m \text{ in } w \]
\[ [[\text{everyone}]]^{M, w} = \text{that function from sets of individuals to propositions which assigns } 1 \text{ to a set } P \text{ in } w \text{ iff } \text{human} \subseteq P \text{ in } w, \text{ viz. the characteristic function of the set of } \text{ those sets that contain everyone in } w \]
\[ [[\text{something}]]^{M, w} = \text{that function from sets of individuals to propositions which assigns } 1 \text{ to a set } P \text{ in } w \text{ iff } \text{thing} \cap P \neq \emptyset \text{ in } w, \text{ viz. the characteristic function of the set of } \text{ those sets that contain something in } w \]

Concatenation (in current generative syntax: merge) is interpreted as functional application; who applies to whom depends on who is the function:

\[ [[\alpha \beta]] = [[\alpha]] \left( [[\beta]] \right) \quad \text{or} \quad [[\alpha \beta]] = [[\beta]] \left( [[\alpha]] \right) \]
Everyone and something now belong to a proper category and they receive exactly the kind of interpretations that the theory of generalized quantifiers assigned to them. Notice that we preserve all the results that we had when studying GQs. We don’t revise anything; we merely build a grammar that accommodates those results. To simplify, propositions on this page are of type \( t \).

When a function \( \alpha \) is a characteristic function of some set, the application of \( \alpha \) to some argument \( \beta \) is equivalent to saying that \( \beta \in \alpha \). Assume that Bill and Mary walk. Then \( [[\text{walk}]]([[\text{Bill1}}]) = b \in \{m,b\} \). We will ignore worlds for this demonstration. Using this shortcut:

\[
\begin{align*}
\text{Bill1 walks, } t & \quad \text{everyone walks, } t \\
\{m,b\} = & \quad \{m,b\} \in \{P : \text{human }\subseteq P\} = \text{human }\subseteq \{m,b\} = \\
& \quad = \{m,b,f\} \subseteq \{m,b\} = 0
\end{align*}
\]

Alternatively, Bill walks might be built in analogy to Everyone walks, using Bill2. Bill walks will come out interpreted identically both ways. In one derivation Bill1 is the argument and walks is the function, and in the other, Bill2 is the function and walks is the argument.

\[
\begin{align*}
\text{Bill2 walks, } t & \quad \text{everyone walks, } t \\
\{m,b\} \in & \quad \{P : b \in P\} = \\
\{m,b\} = & \quad \{P : b \in P\} = 1
\end{align*}
\]

We have just implemented the claim that all noun phrases denote generalized quantifiers. Everyone denotes the set of properties that every individual entity has, and Bill denotes the set of properties that the individual entity called Bill has. The denotation of Bill can also be “simplified” to the individual entity called Bill, but the denotation of everyone cannot be “simplified” to any individual entity. Similarly for few kids, more than five kids, etc.

Bill1 is useful in explaining the possibility of coreference between the name and a singular pronoun. Bill2 is useful in explaining the possibility of coordinations like Bill and every plumber.

In our grammar, the interpretations of syntactic expressions are given directly, without invoking a level of logical form. E.g., everyone is directly correlated with a particular function. It is of utmost importance to see that this can be done. At the same time, it is very convenient to take advantage of a simple and transparent notation like \( \{P : \text{human }\subseteq P\} \) when doing our calculations. Therefore, we might want to translate English expressions first into some convenient formal language, letting its interpretation serve as the interpretation for English. This is what Montague did in UG and PTQ. But the usefulness of the set theoretic notation is limited. What we need is a comparably simple and flexible all-purpose functional notation. The lambda calculus offers one.
Lambda

We start with some of the basic reasons why semanticists use lambda abstraction and functions that are so conveniently defined using the lambda calculus, and then go on to define the notation, independently of linguistic applications.

We may want to
(i) extend the use of sentential connectives to subsentential cases,
(ii) assign explicit interpretations to arbitrary coherent parts of sentences, as per compositionality, and
(iii) define arbitrary functional operations (combinators), e.g. type-lifter and function composer.
[Notation: “*” indicates that the expression is syntactically ill-formed.]

(i) run and sing * run' ∧ sing' \( \lambda x [\text{run}'(x) \land \text{sing}'(x)] \)
Bill and Joe * Bill' ∧ Joe' \( \lambda P [P(\text{Bill}') \land P(\text{Joe}')] \)
(ii) every man * \( \forall x [\text{man}'(x) \rightarrow \ldots(x)] \)
\( \lambda P \forall x [\text{man}'(x) \rightarrow P(x)] \)
(iii) type-lifter
\( \lambda x \lambda P [P(x)](\text{Bill}') = \lambda P [P(\text{Bill}')] \)
(iv) function composer
\( \lambda f \lambda g \lambda x [f(gx)](\text{refrain}'(\text{from}') = \lambda x [\text{refrain}'(\text{from}') x] \)

Re (i), Step 1: To make the use of the sentential connective \( \land \) legitimate, pad out * run' ∧ sing' with variables: run'(x) ∧ sing'(x). This amounts to saying that if run' and sing' had argument x, their conjunction would be run'(x) ∧ sing'(x).

Step 2: But they do not in fact have that argument (run and sing is not a sentence). The assumption of x must be withdrawn. This is indicated by the prefix \( \lambda x. \). \( \lambda \) is the abstraction operator.

Re (ii), Step 1: Every man runs would be \( \forall x [\text{man}'(x) \rightarrow \text{run}'(x)] \). Everything in this formula, save for run', is the contribution of every man. Hence, to represent every man, get rid of run' by replacing it with a predicate variable \( P. \)

Step 2: To withdraw the assumption of \( P, \) prefix \( \lambda P. \)

Definitions:

Syntax: If \( \alpha \) is a well-formed expression and \( x \) a variable, \( \lambda x [\alpha] \) is a well-formed expression.

Semantics: \( \lambda x [\alpha] \) denotes a function. When this function is applied to some \( b \) of the same type as \( x \), the function value is computed by replacing every substitutable occurrence of \( x \) in \( \alpha \) with \( b \).

This replacement process is called beta reduction.

E.g, \( \lambda x [x^2] \) denotes that function which assigns each number its square: \( \lambda x [x^2](3)=3^2 \).

Re: substitutibility

One: Only those \( x \)'s in \( \alpha \) can be replaced that are bound by the initial lambda operator. In \( \lambda x [f(x) \land \forall x [h(x)]] \), only the \( x \) in \( f(x) \) can be replaced; the one in \( h(x) \) is bound by the universal. Note: \( \lambda x [f(x) \land \forall x [h(x)]] = \lambda x [f(x) \land \forall y [h(y)]] \). To prevent mis-applications, it may be useful to reletter the “homonymous” variables that are bound by an operator other than the lambda (you don’t have to, if you are sure you are careful enough – in contrast to Case Two, below).
Two: If the argument to which the lambda-defined function applies is described by an expression that is, or contains, a free variable, care must be taken to ensure that this variable remains free in the course of computing the function value. Suppose \( \lambda x[\forall y[f(x) \to h(y)]] \) is applied to the argument \( y \), a free variable. Let the current assignment \( g \) of values to variables have \( g(y) = \text{bill} \). Then, \( \lambda x[\forall y[f(x) \to h(y)]](y) \) must be the same as \( \lambda x[\forall y[f(x) \to h(y)]](\text{bill}) \), i.e. \( \forall y[f(\text{bill}) \to h(y)] \). If we had mechanically replaced \( x \) by \( y \), we would have gotten \( \forall y[f(y) \to h(y)] \), which is an entirely different thing. To prevent misapplications, we must reletter those bound variables in \( \alpha \) that happen to be "homonymous" with the free variable in the argument. That is, \( \lambda x[\forall y[f(x) \to h(y)]] \) is not applied to \( y \). It is first relettered as \( \lambda x[\forall z[f(x) \to h(z)] \). Note that a free variable can never be relettered.

Since the lambda operator is an all-purpose device for defining functions, any functions, there are no restrictions on what its domain and co-domain might be. Above, we assumed for simplicity that \( x \) was an individual variable, but in fact it might be a variable over any domain. Likewise, the \( \alpha \) in \( \lambda x[\alpha] \) may be anything: (a) a truth value, (b) a function, (c) an individual that varies with \( x \), (d) a fixed object; etc.

(a) \( \lambda x[\text{run}'(x)] \) is the characteristic function of the set of runners.
\[ \lambda x[\text{run}'(x)] = \text{run}’ \quad \text{because for every argument } b, \quad \lambda x[\text{run}'(x)](b) = \text{run}'(b) \]
A characteristic function is a function from some D to \{0,1\}. It "characterizes" some subset C of D by assigning 1 to d's that are also in C, and 0 to those that are not.

(b) \( \lambda x[\lambda y[\text{employ}'(x)(y)]] \) is a function from potential employees \( x \) to VP-denotations, viz., functions from potential employers \( y \) to \{0,1\}.

(c) \( \lambda x[\text{mother-of}'(x)] \) is a function from individuals to their mothers.

(d) \( \lambda x[\bullet] \) is a constant function that maps everything to \( \bullet \).

Vital conventions on representing argument order:

The notation \( f(a)(b) \) is short for \( (f(a))(b) \). The function \( f \) is first applied to \( a \) and then to \( b \). (This is called left-associativity.) The order of the lambda-prefixes represents the inviolable order of how the function can be applied to arguments:

\[
\begin{align*}
\lambda x \lambda y [f(x)(y)](a) (b) &= \lambda x \ [\lambda y \ [f(x)(y)](a) (b)] = \lambda y[f(a)(y)](b) = f(a)(b) \quad \text{and never } f(b)(a)!
\end{align*}
\]

Naturally, explicit bracketing may indicate that things are otherwise:

\[
\begin{align*}
\lambda x \ [\lambda y[f(x)(y)](a)] (b) &= \lambda y[f(b)(y)](a) = f(b)(a)
\end{align*}
\]

How to find the type of a lambda-expression:

\[
[ \lambda x_e \ [ \lambda P(e,t) \ [ P(e,t)(x_e)]_t \ ] ] \quad \text{is of type } \langle e, \langle (e,t), t \rangle \rangle
\]

\[
\langle e, \langle (e,t), t \rangle \rangle
\]

\[
\langle \text{arg1, arg2, value} \rangle
\]
Different ways of writing the same thing:
\[ \lambda x [\lambda y f(x)(y)] = \lambda x \lambda y f(x)(y) = \lambda x. y f(x) = \lambda x y f(y, x) \]

Heim–Kratzer and many linguists after them follow the "dot convention" to indicate the scope of the lambda operator. Moreover, they always explicitly indicate the domain of the function:
\[ \lambda x : x \in D. f(x) \quad \text{or, for short, } \lambda x \in D. f(x) \]

* 

Can a function be applied to an arbitrary function, including itself? (Optional)

Some functions applied to themselves yield a perfectly well-behaved and useful new function. But some other self-applications allow us to replicate the Russell paradox. The set theoretic version of the Russell paradox goes as follows:

Assume that for every property P, there is a set \( \{x : P(x)\} \).
Now ask the reasonable question: What sets are elements of themselves?
Both \( \{x : x=x\} \in \{x : x=x\} \) and \( \emptyset \not\in \emptyset \) would make good sense.
But let \( R \) be \( \{x : x \not\in x\} \), i.e. the set of those things that are not elements of themselves.
Is \( R \) an element of \( R \)? If yes, ..., if no, ... Paradox.
\( R \) is an element of itself iff it is not an element of itself.

The functional version replicates this (from Curry—Feys 1958):

Let \( N \) be negation. We define a function \( Y \) such that \( Y(Y) = N(Y(Y)) \).
Applying \( Y \) to itself is the same as the negation of applying \( Y \) to itself. \( Y \) is called the paradoxical combinator. How to define a \( Y \) that behaves in this paradoxical way?

Let \( Y \) be \( W(B(N)) \), with \( W(h)(z) = h(z)(z) \) and \( B(f)(g)(x) = f(g(x)) \).
This will do, because \( Y(Y) = W(B(N))(Y) = ((B(N))(Y))(Y) = N(Y(Y)) \).

Written with lambdas, let \( W = \lambda h \lambda z [h(z)(z)] \) and \( B = \lambda f \lambda g \lambda x [f(g(x))] \). Then
\[ Y(Y) = W(B(N))(Y) = \lambda h \lambda z [h(z)(z)][\lambda f \lambda g \lambda x [f(g(x))](N)(Y)] = \]
\[ \lambda h \lambda z [h(z)(z)][\lambda g \lambda x [N(g(x))](z)(z)](Y) = \lambda z [\lambda g \lambda x [N(g(x))](z)(z)](Y) = \]
\[ \lambda g \lambda x [N(g(x))](Y)(Y) = N(Y(Y)) \]

The typed versions of the B and W combinators are extremely well-attested even in the grammars of natural languages. It is purely the absence of typing that leads to the paradoxical result.

* 

Abstraction and conditionalization (Optional)
Van Benthem 1989 (Logical Constants Across Varying Types, *Notre Dame Journal of Formal Logic* 30/3, 315-342) explores some analogies between the typed lambda calculus and implicational logic. See the excerpt below.

For implicational logic, and especially conditionalization, see any discussion of Natural Deduction. For example, Gamut I, 4.3.3 *Implication*:

“In order to derive \( \phi \rightarrow \psi \), we first take \( \phi \) as an assumption and then try to derive \( \psi \). If we can do this, we may draw the conclusion that \( \phi \rightarrow \psi \), but as of that moment we may no longer proceed on the assumption that \( \phi \). We say that assumption \( \phi \) is *dropped* (or *withdrawn*).”

And finally, Boolean operations in higher types can be derived from their base meanings in the truth tables. A case in point is the metamorphosis from sentence negation to predicate negation:

\[ \lambda x_{(e,t)} \cdot \lambda y_{e} \cdot \text{NOT}_{(t,t)}(x(y)). \]

There is a system to such changes, as will be seen now.

In fact, *type changing* is a general phenomenon in natural language that shows many systematic traits (see Chapter 7 of [7], and [10]). We shall outline a few points that will be necessary for our further investigation of logical constants.

Generally speaking, expressions occurring in one type \( a \) can move to another type \( b \), provided that the latter type is *derivable* from the former in a logical calculus of *implication* (and perhaps conjunction). The basic analogy operative here is one discovered in the fifties: Functional types \( (a, b) \) behave very much like implications \( a \rightarrow b \). Then, transitions as mentioned above correspond to derivations of valid consequences in implicational logic.

(...) Thus, the derivational analysis shows a common pattern in all three examples, being a form of Transitivity:

\[ (x, y) \Rightarrow ((y, z), (x, z)). \]

In general, again, admissible type changes in natural language correspond to valid derivations in a *constructive* implicational logic, given by the usual natural deduction rules of modus ponens and conditionalization. Also frequent, in addition to the above inference of Transitivity (often called ‘Geach’s Rule’ in this context), are the so-called rules of Raising (also called ‘Montague’s Rule’):

(...) Moreover, these derivations are not purely syntactic. For they correspond one-to-one with terms from the typed lambda calculus, explaining how denotations in the original type are changed into denotations in the new type. Here is an illustration for Boolean negation:

<table>
<thead>
<tr>
<th>Proof Tree</th>
<th>Lambda Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ((e, t))</td>
<td>(x_{e} )</td>
</tr>
<tr>
<td>2 ((t, t))</td>
<td>(y_{(e, t)})</td>
</tr>
<tr>
<td>(t)</td>
<td>(y(x))</td>
</tr>
<tr>
<td>((e, t))</td>
<td>(\text{NOT}_{(t, t)}(y(x)))</td>
</tr>
<tr>
<td>((e, t))</td>
<td>(\lambda x_{e} \cdot \text{NOT}(y(x)))</td>
</tr>
<tr>
<td>((e, t))</td>
<td>(\lambda y_{(e, t)} \cdot \lambda x_{e} \cdot \text{NOT}(y(x)))</td>
</tr>
</tbody>
</table>

Note how application encodes modus ponens, and lambda abstraction encodes conditionalization.
Assignment for Oct. 19

Problems (A)-(B)-(C) have solutions at the end of this assignment. Check your work against those and make corrections as appropriate. Feel free to check in small steps, even after every sub-problem, but don’t consult the solutions before making your own attempts. -- Alert me if you are not sure why a solution is what is given. (D)-(E) are left for you. As usual, collaboration is encouraged.

(A) Provide the simplest (but correct) categorial grammar derivations for the sentences below, written in the style of the example in (a). If the list contains a category for some word, stick with that category; if it doesn’t, devise a category for it that is compatible everything in the list and enables you to derive the sentences. Also specify the type in both notations.

<table>
<thead>
<tr>
<th>sentence</th>
<th>category</th>
<th>type, our notation</th>
<th>type, Muskens</th>
</tr>
</thead>
<tbody>
<tr>
<td>jumped, sneezed</td>
<td>e, t</td>
<td>&lt;e, &lt;s, t&gt;&gt;</td>
<td>est</td>
</tr>
<tr>
<td>saw, kicked</td>
<td>(e\t)/e</td>
<td>&lt;e, &lt;e, &lt;s, t&gt;&gt;&gt;</td>
<td>eest</td>
</tr>
<tr>
<td>everyone, Kim</td>
<td>t/(e\t)</td>
<td>&lt;&lt;&lt;e, &lt;s, t&gt;&gt;, &lt;s, t&gt;&gt;</td>
<td>(est)st</td>
</tr>
<tr>
<td>and, or</td>
<td>(x\x)/x</td>
<td>&lt;x, &lt;x, x&gt;&gt;</td>
<td>xxx</td>
</tr>
<tr>
<td>believed</td>
<td>(e\t)/t</td>
<td>&lt;&lt;&lt;s, t&gt;, &lt;e, &lt;s, t&gt;&gt;&gt;</td>
<td>(st)est</td>
</tr>
<tr>
<td>a(n), the, every</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mayor</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>gracefully</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>from</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>refrained</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Kim jumped.  

(b) Kim jumped gracefully.  

(c) Kim jumped from Nalumasartoq.  

(d) The mayor refrained from criticism.  

(e) Kim sneezed and kicked Pat.  

(f) Kim saw and kicked Pat.  

(g) Kim or Pat sneezed.

(B) Assuming the types below, are the logical expressions in (h)-(l) well-formed? (Don’t look for linguistic equivalents, they needn’t have any.) If yes, give their types. If not, where is the clash?

<table>
<thead>
<tr>
<th>expression</th>
<th>type</th>
</tr>
</thead>
<tbody>
<tr>
<td>a(g(d))</td>
<td>a&lt;e, t&gt; b&lt;t, t&gt; c_e d_e f&lt;e, &lt;e, t&gt;&gt; g&lt;e, e&gt;</td>
</tr>
<tr>
<td>b(g(c))</td>
<td></td>
</tr>
<tr>
<td>f(c)(d)</td>
<td></td>
</tr>
<tr>
<td>f(g)</td>
<td></td>
</tr>
<tr>
<td>b(f(a))</td>
<td></td>
</tr>
</tbody>
</table>
(C) Consider the following three examples:

(m) Kim saw Pat.
(n) Everyone saw Pat.
(o) Kim saw everyone.

We already know how to derive (m) and (n), but (o) is a problem. Explain why, comparing it with (m) and (n).

(D) Our categorial grammar currently has only one rule (in two versions): functional application (leftward-looking and rightward-looking).

Application: If $\alpha \in \text{Cat } A/B$ and $\beta \in \text{Cat } B$, then $\alpha \beta \in \text{Cat } A$. \text{[A/B looks for a B on the right]}
If $\beta \in \text{Cat } B$ and $\alpha \in \text{Cat } B\backslash A$, then $\beta \alpha \in \text{Cat } A$. \text{[B/A looks for a B on the left]}

Let us add two more rules. Lifting is the rule that productively turns the category of $\text{Kim1}$ into the category of $\text{Kim2}$. It also comes in lefward-looking and rightward-looking versions, and it is generalized to arbitrary categories $A$ and $B$. It reverses function-argument relations within one constituent.

Lifting: If $A, B \in \text{Cat}$, and $\alpha \in \text{Cat } A$, then also, $\alpha \in \text{Cat } B/(A\backslash B)$ and $\alpha \in \text{Cat } (B/A)\backslash B$.

For example:
Pat and Pat
-------- Lift ----------Lift
t/(e\t) (t/e)\t

Composition is a re-bracketing operation, i.e. it makes the syntax associative. Notice the critical condition: the value-category $B$ of one of the functions is the same as the argument-category $B$ of the other, and the order $\alpha \beta$ reflects the slanting of the slashes in the input categories.

Composition: If $\alpha \in \text{Cat } A/B$ and $\beta \in \text{Cat } B/C$, then $\alpha \beta \in \text{Cat } A/C$.
If $\alpha \in \text{Cat } C\backslash B$ and $\beta \in \text{Cat } B\backslash A$, then $\alpha \beta \in \text{Cat } C\backslash A$.

For example:
refrained from
(e\t)/((e\t)(e\t)) ((e\t)(e\t))/e
----------------------------------------Compose
(e\t)/e (as in What did Kim refrain from?)

(p) Can you now derive $\text{Kim saw Pat}$ with the constituent structure $[\text{Kim saw}][\text{Pat}]$? If yes, derive it; if not, explain why the three rules plus the lexical categories given, don’t make it possible.

(q) Pretend that $\text{she}$ is a proper name in the nominative and $\text{her}$ is a proper name in the accusative. Can you now derive $\text{Her saw she}$ and $\text{Her she saw}$? If yes, derive them; if not, explain why the three rules plus the lexical categories given, don’t make it possible.
Let us now return to Kim saw everyone. We can _almost_ derive it now. Show how far we can go and what remains amiss. Determine whether a very modest and conservative new assumption would enable you to complete the derivation. If yes, derive it and explain the new assumption. If not, explain why the situation is hopeless, or why the derivation would require something radically new.

**Solutions to (A)-(B)-(C) start on the next page.**

**Reading**

Muskens’ handbook article is a short and excellent background reading. Everything he says is important and insightful, but I’m not sure how practical it is as a hands-on learning tool if you are encountering types and lambdas for the first time. Perhaps read it as a summary once you have worked through the contents of my handout.

I also uploaded excerpts from Gamut II/4 for precise definitions. You may or may not find their style of discussion easy, so this is entirely optional. (The middle section on categorial grammar has not aged well, plus it contains confusing typos, so it’s intentionally left out.)

Steedman’s 1991 “Structure and intonation” as an optional reading. It’s a wonderful paper pertaining to the claim that it is good for syntax to be (greatly, though not fully) associative – prosody and coordination need it! I highly recommend reading it. Steedman has much further work on intonation.

Be aware that Steedman uses the backslash differently than how my handout does, which follows the Lambek tradition. For Steedman, the argument category is always on the right. So he writes $(S\backslash NP)/NP$ for our $(e\backslash t)/e$. I suppose the switch from $e$ to $NP$ and from $t$ to $S$ in the categories (not in the types) will not be confusing.
Solutions:

(A)

Kim1
jumped, sneezed  e \ t  e \ t  e \ t  <e, <s, t>>  est
saw, kicked      (e \ t)/e  <e, <e, <s, t>>>  eest
everyone, Kim2   t/(e \ t)  <<e, <s, t>>, <s, t>>  (est)st
and, or          (x \ x)/x  <x, <x, x>>  xxx
believed         (e \ t)/t  <<s, t>, <e, <s, t>>>  (st)est
a(n), the, every  (t/(e \ t))/(e \ t)  <<e, <s, t>>, <<e, <s, t>>, <s, t>>>  (est)(est)st
mayor            e \ t  <e, <s, t>>  est
gracefully       (e \ t)/(e \ t)  <<e, <s, t>>, <e, <s, t>>>  (est)est
from             ((e \ t)/(e \ t))/e  <e, <e, <s, t>>, <e, <s, t>>>  e(est)est
refrained        (e \ t)/((e \ t)/(e \ t))  <<e, <s, t>>, <e, <s, t>>, <e, <s, t>>>  ((est)est)est

(a) Kim jumped
e  e \ t
---------------
 t

(b) Kim jumped gracefully
 e  e \ t  (e \ t)/(e \ t)
---------------
 e \ t
---------------
 e \ t
---------------
 t

(c) Kim jumped from Nalumasartoq
 e  e \ t  ((e \ t)/(e \ t))/e  e
---------------
(e \ t)/(e \ t)
---------------
 e \ t
---------------
 t

(d) The mayor refrained from criticism
(t/(e \ t))/(e \ t)  e \ t  (e \ t)/(e \ t)/(e \ t)/(e \ t)/e  e
---------------
 t/(e \ t)
---------------
(e \ t)/(e \ t)
---------------
 e \ t
---------------
 t
(e) Kim sneezed and kicked Pat where $x = e \langle t \rangle$

(Math)

(f) Kim saw and kicked Pat where $x = (e \langle t \rangle) / e$

(Math)

(g) Kim or Pat sneezed. where $x = t / (e \langle t \rangle)$

(Math)

Why not Kim and Pat? Well, is there an individual Kim or Pat?

(B) $a_{e,t}$ $b_{\langle t, t \rangle}$ $c_e$ $d_e$ $f_{e, \langle e, t \rangle}$ $g_{e,e}$

(h) $a(g(d))$ well-formed: type $t$

(i) $b(g(c))$ no: $b_{\langle t, t \rangle}$ cannot apply to $g_c$

(j) $f(c)(d)$ well-formed: type $t$

(k) $f(g)$ no: $f_{e, \langle e, t \rangle}$ cannot apply to $g_{e,e}$

(l) $b(f(a))$ no: $f_{e, \langle e, t \rangle}$ cannot apply to $a_{e,t}$
Neither Kim and saw, nor saw and everyone can combine by application. Even if we assumed that everyone has a second category, (t/e)\(t\), for the direct object position, it couldn’t combine with saw.

There are multiple ways to remedy this situation. The one explored in (D)-(E) introduces useful new elements and also paves the way to QR and related scope assignment techniques, although it is _not_ the same as QR.
Assignment for Oct. 26

Reading

Next week’s discussion will center on the status of variable binding, i.e. the transition from an expression with a free, assignment-dependent variable \((fx)\) to one with a bound, assignment-independent one \((\lambda x[fx], \forall x[fx])\), in logic and in grammar. We will see that all the authors below have problems with it; interestingly, they complain about different aspects of it, and they draw somewhat different conclusions for how to write a grammar. In preparation for the discussion, read at least the following excerpts:

Szabolcsi 1987, Sections 0-1-2 (= 8 unnumbered pages)
Steedman 1991, Sections 1-2, pp. 260-270. [much of this is summarized in my 1987]
Heim & Kratzer 1998, Ch 5.1-2, pp. 86-106.

Our focus will not be on comparing different versions of categorial grammar or different theories of pronoun/reflexive binding, but the abstract issues arise in this context. -- None of these papers uses our Lambek-style slashes :(. However, that’s what more recent literature uses (e.g. Barker & Shan 2014).

Written

Of the problems below, (A)-(F) are required. (A)-(B)-(C) have solutions, hopefully without typos, in the back. Please check your work and make corrections as needed. Alert me if something is not crystal clear! (G)-(H) are optional (collaboration encouraged).

To enable me to correct this assignment efficiently, please either print out the PDF version and write the answers neatly by hand in the spaces next to or below the questions, or use the Word version and type in your answers in a similar fashion, respecting the spaces and pagebreaks.

(A) Is any of these statements ill-formed? Is any of them false? If yes, very briefly say why.

Note that whatever was true about free and bound variables in predicate logic remains true here. You can assume that types match properly (i.e. nothing will be ill-formed due to a type mismatch).

(a) \(\lambda a \forall x[a(x)] (\lambda y[f(y)(z)]) = \forall x[f(x)(z)]\)

(b) \(\lambda a \forall x[a(x)] (\lambda y[f(y)(x)]) = \forall x[f(x)(x)]\)

(c) \(\lambda a \lambda x[a(x)] (\lambda y[f(y)(z)]) = \lambda x[f(x)(z)]\)

(d) \(\lambda a \lambda x[a(x)] (\lambda y[f(y)(x)]) = \lambda x[f(x)(x)]\)

(e) \(\lambda x[f(x)] = f\)
(B) Determine the types of the following \(\lambda\)-expressions; the method is given in the handout.

(i) \(\lambda a_{c,e} \lambda b_{c,t} \lambda c_e \ [b(a(c))]\)

(ii) \(\lambda d_e \lambda f_{c,e,t} \lambda g_{c,e} \ [f(g(d))]\)

(C) To practice paying close attention to brackets and parens, work through the lambda conversion (beta reduction) below. The end result should be \(N(Y(Y))\). The colors play no role in the conversions. (Apologies, Russell and Curry...)

\[
Y(Y) = W(B(N))(Y) = \lambda h \lambda z [h(z)](z) \ (\lambda f \lambda g \lambda x [f(g(x))](N)) \ (Y)
\]

Should you want more practice, the last section of this paper has many \(\lambda\)-conversions written out step-by-step: http://www.nyu.edu/projects/szabolcsi/szabolcsi_combinatory_grammar_1992.pdf.

(D) The joys of reverse engineering: Give the interpretations of the underlined words/phrases using the \(\lambda\)-notation. Indicate the types of the variables in the quantifier and lambda prefixes, as in problem (B), using not-too-tiny subscripts and our non-abbreviated notation for types. \(i\) is a variable of type \(s\).

1. If The train left is \(\lambda i \exists x \forall y [(\text{train}(y)(i) \leftrightarrow x=y) \ \& \ \text{left}(x)(i)]\), what is the?

2. If Bill smokes and snores is \(\lambda i [\text{smoke}(b)(i) \ \& \ \text{snore}(b)(i)]\), what is smokes and snores?

3. If smokes and snores is as you proposed, what is the pertinent (specific) version of and?
(E) More reverse engineering: Reflexives

4. If Every cat licks itself is $\lambda i \forall x [cat(x)(i) \rightarrow lick(x)(x)(i)]$, what is lick itself?

5. If lick itself is as you proposed, what is itself?

(F) Following up on (E):

(a) According to your result, does itself denote a GQ?

(b) Does the result seem satisfactory, considering the syntactic and semantic behavior of the word?

(G) Optional. Explain in plain English what each of the following expressions mean. $i^*$, $j^*$, $i, j$ are variables of type $s$, $x$ is of type $e$, and $\text{walk}$ is of type $\langle e, \langle s, t \rangle \rangle$.

6. $\lambda x[\text{walk}(x)(i^*)] = \lambda x[\text{walk}(x)(j^*)]$

7. $\lambda i[\lambda x[\text{walk}(x)(i)]] = \lambda x[\text{walk}(x)(j^*)]$

8. $\lambda j \lambda i[\lambda x[\text{walk}(x)(i)]] = \lambda x[\text{walk}(x)(j)]$

(H)  **Optional.** In PTQ, Montague offered a uniform analysis for be in equative (a) and predicative (b) sentences. This in turn allowed him to interpret indefinites like a dragon using the same QG interpretation in their predicative (b) and argumental (c, d) roles.

(a) Bilbo is Mr. Baggins.  
(b) Smaug is a dragon.  
(c) A dragon tricked Bilbo.  
(d) Bilbo tricked a dragon.

This be has come to be called «the transitive verb be», and Partee 1987 reincarnated it as the type-shifter BE. A type-shifter is a disembodied operation that adjusts the type of an expression in a mathematically predictable way. (Partee, B. H. 1987. Noun Phrase interpretation and type-shifting principles. In Groenendijk et al. eds., *Studies in Discourse Representation Theory and Generalized Quantifiers*, Foris, 115-141. -- Reprinted in various collections.)

Here is what the transitive verb be looks like, in an extensional shape:

$$\lambda \varphi \lambda x[\varphi(\lambda y[x=y])]$$  
where $$\varphi$$ is a variable of type $$(e,t,t)$$

(1) Build and interpret (a)-(b) using a plain [S [V O]] constituent structure, Montague’s be, and the usual GQs for Bilbo, Mr. Baggins, and a dragon. Spell out the interpretations of the three GQs using the $$\lambda$$-notation, and write out the $$\lambda$$-conversion steps at each stage of the derivation. (Don’t worry about be vs. is.)

(2) Explain how this be is capable of serving in both equative and predicative sentences. To do this, you have to discover and invoke a simple semantic fact.

(3) How is this be a transitive verb? Assume that the vanilla transitive verb trick has the same category and type as be above. If so, its interpretation cannot be just $$\lambda x_0 \lambda y_0 [\text{trick}(x)(y)]$$ -- what should it be? Spell out the new interpretation for trick, and demonstrate how it works with our GQs, by deriving (c) and (d). The final interpretations of the sentences should end up being what we always thought they were. I.e. we are not reinterpreting (c) and (d), we are rethinking the composition process.

BTW, PTQ’s be and trick were not quite what I am suggesting here, because PTQ modeled the treatment of all transitive verbs after that of intensional seek, whereas our treatment is purely extensional. But the be–trick connection is valid, and having thought this through will be very useful when we come to the scope theories of Hendriks and Barker&Shan.

Add new pages for (1)-(3).
Solutions for (A)-(B)-(C)

(A) Is any of these statements ill-formed? Is any of them false? If yes, very briefly say why.

(a) \( \lambda a \forall x[a(x)] \left( \lambda y[f(y)(z)] \right) = \forall x[f(x)(z)] \) Well-formed and true.

(b) \( \lambda a \forall x[a(x)] \left( \lambda y[f(y)(x)] \right) = \forall x[f(x)(x)] \) Well-formed and true. x occurs free on the left-hand side and bound on the right-hand side. The truth is, \( \lambda a \forall x[a(x)] \left( \lambda y[f(y)(x)] \right) = \forall x[f(x)(x)] \) Well-formed and true.

(c) \( \lambda a \lambda x[a(x)] \left( \lambda y[f(y)(z)] \right) = \lambda x[f(x)(z)] \) Well-formed and true.

(d) \( \lambda a \lambda x[a(x)] \left( \lambda y[f(y)(x)] \right) = \lambda x[f(x)(x)] \) Well-formed and true. x occurs free on the left-hand side and bound on the right-hand side. The truth is, \( \lambda a \lambda x[a(x)] \left( \lambda y[f(y)(x)] \right) = \lambda x[f(x)(x)] \) Well-formed and true.

(e) \( \lambda x[f(x)] = f \) Well-formed and true, by the definition of \( \lambda \)-abstraction.

(B) Determine the types of the following \( \lambda \)-expressions; the method is given in the handout.

(i) \( \lambda a_{<e,t>} \lambda b_{<e,t>} \lambda c_{e} \left[ b(a(c)) \right] \)

\( <<e,t>, <<t,t>, <e,t>> > > > > \) (cf. the function value, \( b(a(c)) \) is of type t)

(ii) \( \lambda d_{e} \lambda f_{<e,e,t>} \lambda g_{<e,e>} \left[ f(g(d)) \right] \)

\( <e, <<e,<e,t>>, <<e,e>, <e,t>>, > > > > \) (cf. the function value, \( f(g(d)) \) is of type \( <e,t> \))

(C) To practice paying close attention to brackets and parens, work through the lambda conversion (beta reduction) below.

As in the last page of the handout:

\( Y(Y) = W(B(N))(Y) = \lambda h \lambda z[h(z)(z)] \left( \lambda f \lambda g \lambda x[f(g(x))] \right) (N) ) (Y) = \)

\( \lambda h \lambda z[h(z)(z)]\left( \lambda g \lambda x[N(g(x))])\right)(Y) = \)

\( \lambda z[\lambda g \lambda x[N(g(x))])(z)(z)](Y) = \)

\( \lambda g \lambda x[N(g(x))])\right)(Y) = N(Y(Y)) \)
Variable binding: complaints and remedies

Variable binding: the transition from an expression with a free, assignment-dependent variable (fx) to one with a bound, assignment-independent variable (λx.fx, ∀x.fx), through the addition of a syntcategorematic operator (λx, ∀x).

Jacobson 1999:

under the standard theory, the meaning of any expression is relative to assignment functions. Put differently – and in somewhat non-standard terms – the meaning of an expression can be seen as a function from assignment functions to some (other) model-theoretic object. Note then that under the view that the meaning of an expression is a function from assignment functions, it follows that the assignment functions themselves are model-theoretic objects. And since these are functions from variable names to something else, the variable names themselves are model-theoretic objects (see Landman and Moerdijk, 1983, for very relevant discussion). Under the variable-free approach, there are no such things as assignment functions or variable names as model-theoretic objects, nor is the meaning of an expression relative to assignment functions.²

theoretic objects, then they are of course distinct objects. And yet, when they find themselves in forms which are alphabetic variants, they never make a different semantic contribution. The same point can be made for expressions containing variables which are unbound within those expressions: John likes x₁ and John likes x₂ never seem to function like different semantic objects.⁸ In other words, there is an obvious sense in which x₁ and x₂ really are not different semantic objects – unlike other distinct model-theoretic objects.

Szabolcsi 1987:

The standard solution to this problem goes as follows. In accordance with the FA-strategy, the gap is filled by a placeholder interpreted as a variable and, in necessary deviation from the FA-strategy, the extracted constituent is affixed to the sentence in a syncategorematic fashion resembling the introduction of binding operators in logic. In view of what such sentences mean, the procedure seems semantically correct.

Notice, however, that this solution makes one expect that the possibilities for gaps and extracted constituents to occur in natural
language are the same as those for variables and operators in logical syntax. This expectation is not borne out. Consider the following paradigmatic cases of divergence noted in the literature:

(2)a. Free variables: \( fx \)
   * --- saw Bill.

b. Vacuous operators: \( \lambda x[a] \)
   * What did Mary see Bill?

c. Crossed binding: \( \lambda x\lambda y[fx(gy)] \)
   * What_1 do you wonder who_2 to talk about \(-1\) to \(-2\)?

d. Binding over arbitrary domains: \( \lambda x[...x...] \)
   * Who_1 did you meet John, who likes \(-?\)?
   ? Who_1 did you go home without meeting \(\_1\)?
Two syncategorematic operators in propositional/predicate logic: $\land$ and $\forall x$

1. If $\phi$, $\psi$ are formulas, $\phi \land \psi$ is a formula.
   
   If $\phi$, $\psi$ are both true in model $M$ at assignment $g$ of values to variables, then $\phi \land \psi$ is true in $M$ at $g$. It is false otherwise.

2. If $\phi$ is a formula, $\forall x. \phi$ is a formula.
   
   $\phi$ is true in $M$ at assignment $g$ iff the values $g$ assigns to the free variables in $\phi$ satisfy the requirements imposed by the “rest of $\phi$”. E.g. if $\phi$ is $fx$, and $g(x)$ is Bill, $\phi$ is true iff $f(Bill)$ holds in $M$.
   
   $\forall x. \phi$ is true in $M$ at assignment $g$ iff for every assignment $g'$ that differs from $g$ at most in the value they assign to $x$, $g'(x)$ satisfies the requirements imposed by the “rest of $\phi$”.

In (1), the interpretation of $\phi \land \psi$ completely preserves and builds on the interpretations of $\phi$ and of $\psi$. $\land$ can be recast as a categorematic operator, interpreted as $\{<p,q>: \text{both } p \text{ and } q \text{ are true}\}$.

In (2), the interpretation of $\forall x. \phi$ does not preserve and build on the interpretation of $\phi$, if $\forall x$ binds a variable $x$ in $\phi$. It says, forget the value $g$ assigns to $x$. Go back to the raw syntactic unit $fx$ and define an interpretation for $\forall x. fx$ directly. With this behavior, $\forall x$ cannot be recast as a categorematic operator. (Mutatis mutandis, the same holds for the abstraction operator $\lambda x.$)

There are two possibilities for a categorematic treatment here.

(A) The expression $\forall x. fx$ is not built from the expression $fx$. So the interpretation of $\forall x. fx$ does not need to be built from the interpretation of $fx$.

(B) The expression $\forall x. fx$ is built from $fx$, but $fx$ is not interpreted in the classical way, cf. (2). Instead, $fx$ is interpreted in a new, assignment-independent way (and its “deictic” use is obtained in a separate step). Then it is possible to build the interpretation of $\forall x. fx$ from that, in a meaning preserving fashion. For example, $fx$ is interpreted as the set of possible alternatives for it to be true: $\{fa, fb, fc, \ldots\}$, and $\forall x. fx$ as asserting that each of those alternatives is actually true.

The above remarks pertain to the grammar of predicate logic (same for the lambda calculus).

What about the grammar of natural language? If we want natural language sentences to have a step-by-step compositional interpretation, we cannot use a syncategorematic technique. In addition to the problems above, Szabolcsi and Steedman point out that a syncategorematic treatment of extraction and binding makes bad empirical predictions about what sentences are grammatical, and so requires stipulations about well-formedness.

The variable-free, combinatory grammar of Steedman and Szabolcsi uses strategy (A). The variable-free, combinatory grammar of Jacobson uses strategy (B). See the specific POSITIVE PROPOSALS in the papers.

Heim & Kratzer do not propose a variable-free grammar. But they work top-down, rather than bottom-up. This way they avoid having to forget the values that the current $g$ assigns to the variables in the input.

You will find that people often use syncategorematic bottom-up techniques for quantification and $\lambda$-abstraction. They are either unaware of the problems, or choose to ignore them, or think that well-defined though not really compositional is good enough.
Assignment for Nov. 2

Check your $\lambda$ assignment against the solutions of (D)-(E)-(F)-(G), now added to the updated assignment file. The discussion of quantifier scope assignment next week will presuppose a mastery of that material.

Read pp. 10-19 of *Quantification* in preparation for next week. Be sure to actually and carefully read it in advance of class discussion.

Heim & Kratzer Ch 7, Quantification and Grammar, will be the other reading pertaining to the topic.

There is no new written assignment for this weekend.
Frege discovered that asymmetrical scope relations can be derived by introducing quantifiers one by one. Earlier logics, e.g. syllogisms, basically couldn’t deal with sentences with more than one quantifier. Two semantically distinct Fregean strategies for asymmetrical scope:

- **Quantifier phrases denote generalized quantifiers** (type $<e,t>,t>$) and expressions corresponding to their scopes denote properties (type $<e,t>$):

  $$[[\text{quantifier phrase}]]\ ([[\text{scope}]] \text{ or, equivalently, } [[\text{scope}]] \in [[\text{quantifier phrase}]].$$

  This strategy has been implemented in various ways, using quantifying-in (Montague), Quantifier Raising (May), in-situ scope via higher-order verb types (Hendriks), in-situ scope via continuations (Barker), etc.

- **Quantifier phrases denote arity-reducers**: functions from the set of all $n+1$-ary relations (over a domain $E$), mapping each to an $n$-ary relation. Type notation: $(p_{n+1}, p_n)$, where $p_0$ is the type of 0-place predicates, i.e. sentences. The denotation of their scope is presented in the shape of a relation:

  $$[[\text{quantifier phrase}]]\ ([[\text{scope}]] \text{ e.g. } S>O \text{ someone saw everything } t/(e\ t)\ (e\ t)/e\ ((e\ t)/e)\ (e\ t))$$

  $$\text{-----------------------------------------------}$$

  $$O>S \text{ someone saw everything } (t/e)/(e\ t)/e\ (e\ t)/e\ (t/e)\ t$$

  $$\text{-----------------------------------------------}$$

  See Keenan 2014 (JoS) for the best discussion. This is what H&K call the “flexible types” approach in 7.2.1.

  The arity-reducer approach is great for semantically dependent expressions, but its utility as a general scope assignment mechanism is syntactically limited to cases where the expressions a quantifier phrase is supposed to scope over are contiguous, as observed in Hendriks 1993.

  John admires himself.
  John thinks that Susan invited everyone but himself. 
  John will read every book that Susan will.
  ... because every boy some book read (doesn’t derive O>S in SOV, e.g. Dutch and German)

- **Frege’s solution doesn’t cover irreducibly polyadic quantification.** Some potential candidates:

  Different people have different tastes.
  Five women gave birth to seven babies between them.
  A relative of every townsman and a relative of every villager hate each other.

- The Skolem-functional approach dissociates scope from constituent structure (even more than Hendriks’ and Barker&Shan’s), and in principle has wider coverage. See Steedman 2012 (Taking Scope, MIT Press).
Deriving asymmetrical scope based on generalized quantifiers à la PTQ

(19) Subject > Object reading

\[
\text{[More than one dragon spotted every man]} \quad \text{apply } \text{more than one dragon} \\
\exists >_1 z [\text{dragon}'(z) \land P(z)] \\
\lambda P \exists >_1 z [\text{dragon}'(z) \land P(z)] \\
\lambda x_2 \forall y [\text{man}'(y) \rightarrow \text{spot}'(y)(x_2)] \\
\lambda x_2 \forall y [\text{man}'(y) \rightarrow \text{spot}'(y)(x_2)] \\
\lambda \text{bind subject } x_2 \\
\forall y [\text{man}'(y) \rightarrow \text{spot}'(y)(x_2)] \\
\lambda x_1 [\text{spot}'(x_1)(x_2)] \\
\lambda x_1 [\text{spot}'(x_1)(x_2)] \\
\lambda \text{bind object } x_1 \\
\exists >_1 z [\text{dragon}'(z) \land \forall y [\text{man}'(y) \rightarrow \text{spot}'(y)(z)]] \\
\exists >_1 z [\text{dragon}'(z) \land \forall y [\text{man}'(y) \rightarrow \text{spot}'(y)(z)]]
\]

(20) \[
\lambda P \exists >_1 z [\text{dragon}'(z) \land P(z)] (\lambda x_2 \forall y [\text{man}'(y) \rightarrow \text{spot}'(y)(x_2)]) = \\
\exists >_1 z [\text{dragon}'(z) \land \lambda x_2 \forall y [\text{man}'(y) \rightarrow \text{spot}'(y)(x_2)](z)] = \\
\exists >_1 z [\text{dragon}'(z) \land \forall y [\text{man}'(y) \rightarrow \text{spot}'(y)(z)]]
\]

(21) Object > Subject reading

\[
\text{[More than one dragon spotted every man]} \quad \text{apply every man} \\
\forall y [\text{man}'(y) \rightarrow \exists >_1 z [\text{dragon}'(z) \land \text{spot}'(y)(z)]] \\
\forall y [\text{man}'(y) \rightarrow \exists >_1 z [\text{dragon}'(z) \land \text{spot}'(y)(z)]] \\
\lambda Q \forall y [\text{man}'(y) \rightarrow Q(y)] \\
\lambda x_1 \exists >_1 z [\text{dragon}'(z) \land \text{spot}'(x_1)(z)] \\
\lambda x_1 \exists >_1 z [\text{dragon}'(z) \land \text{spot}'(x_1)(z)] \\
\lambda \text{bind object } x_1 \\
[\text{more than one dragon spotted } e_c_1] \\
\exists >_1 z [\text{dragon}'(z) \land \text{spot}'(x_1)(z)]
\]

\[
\text{[more than one dragon]} \\
\lambda P \exists >_1 z [\text{dragon}'(z) \land P(z)] \\
\lambda x_3 [\text{spot}'(x_3)(x_2)] \\
\lambda x_3 [\text{spot}'(x_3)(x_2)] \\
\lambda \text{bind subject } x_3 \\
[\text{e}_c_2 \text{ spotted } e_c_1] \\
\exists >_1 z [\text{dragon}'(z) \land \text{spot}'(y)(z)]
\]

(22) \[
\forall y [\text{man}'(y) \rightarrow \lambda x_3 \exists >_1 z [\text{dragon}'(z) \land \text{spot}'(x_3)(z)]] = \\
\forall y [\text{man}'(y) \rightarrow \exists >_1 z [\text{dragon}'(z) \land \text{spot}'(x_3)(z)]] \\
\forall y [\text{man}'(y) \rightarrow \exists >_1 z [\text{dragon}'(z) \land \text{spot}'(y)(z)]]
\]
Quantifier Raising (QR) as in May 1977


Some questions:

• QR seems to be tensed-clause bounded. Why?

  A boy thought that every cat was hungry.  A boy saw that Susan fed every cat.
  # `every cat > a boy'  # `every cat > a boy'

• Quantifier phrases do not directly bind pronouns. What binds them?

  Every boy lost his temper.

• Does pronoun binding require a c-commanding quantifier phrase?

  Every boy’s behavior upset his mother.
  ‘every boy_i ... his_i mother’
  Every boy’s behavior upset a teacher.

  That every boy was hungry upset his mother.
  # `every boy_i ... his_i mother'
  That every boy was hungry upset a teacher.

(19) Subject > Object reading

\[ \lambda P \exists z [\text{dragon}(z) \land P(z)] \]

\[ \lambda x_2 \forall y [\text{man}(y) \rightarrow \text{spot}(y)(x_2)] \]

\[ \forall y [\text{man}(y) \rightarrow \text{spot}(y)(x_2)] \]

\[ \lambda x_1 [\text{spot}(x_1)(x_2)] \]

\[ \lambda \text{bind subject } x_2 \]

\[ \lambda \text{bind object } x_1 \]

\[ \text{build sentence with two free vbls} \]

(33) \( \lambda B \lambda A \left[ A \left( \lambda y \left[ B \left( \lambda x [Pxy]\right) \right] \right) \right] \]

\( x=\text{object}, y=\text{subject} \)

(21) Object > Subject reading

\[ \forall y [\text{man}(y) \rightarrow \exists z [\text{dragon}(z) \land \text{spot}(y)(z)]] \]

\[ \lambda x_1 [\lambda z \exists z [\text{dragon}(z) \land \text{spot}(x_1)(z)]] \]

\[ \lambda \text{bind object } x_1 \]

\[ \text{build sentence with two free vbls} \]

(34) \( \lambda B \lambda A \left[ B \left( \lambda x \left[ A \left( \lambda y [Pxy]\right) \right] \right) \right] \]

\( x=\text{object}, y=\text{subject} \)
References

Barker & Shan 2014. *Continuations and Natural Language*. OUP.
Cecchetto 2004. Explaining the locality conditions of QR: consequences for the theory of phases.
   *Natural Language and Linguistic Theory* 12: 345-397.
Hendriks 1993. *Studied Flexibility*. PhD, UvA.
Assignment for Nov. 9

Reading

Please all read Quantification, Chapter 2, up to 2.3.6 (Continuations and scope). In 2.3.5, feel free to skip the definitions (36) Argument Raising and (37) Value Raising, if you wish. It suffices to know that Hendriks uses these two type-change rules to obtain the desired categories for verbs. Even so, it is possible and useful to read the remaining part of 2.3.5 to familiarize yourself with (i) so-called semantic reconstruction and (ii) universals that only apparently scope out of their clauses/container DPs.

If you are planning to take Semantics II, please treat also Heim & Kratzer, Chapter 7 (Quantification and grammar) as a required reading. It almost completely overlaps with our discussion this week and earlier, but you should be familiar with their notation and style of argumentation.

(We will not discuss continuations, but if some of you are interested, we can meet and talk through 2.3.6 and some relevant parts of the Barker & Shan book.)

Next week’s topic is “different quantifiers, different scopes”. You will get a very clear idea of the issues if you read the first page of the handout. The corresponding required reading is Quantification, Chapters 6 (Scope is not uniform and not a primitive) and 7 (Existential scope versus distributive scope), with the exception of 7.1.2-3. Chapters 8 (Distributivity and scope) and 9 (Bare numeral indefinites) will be optional.

(The choice-functional analysis of indefinites discussed in 7.1.2-3 was very popular for about one decade after Reinhart introduced it, but its interaction with decreasing operators was found too problematic. It has been quietly abandoned by now. If you are planning to work with pertinent semantics literature, it is highly recommended to read 7.1.2-3, but otherwise it is rather safe to skip.)

Written

Derive both readings of this sentence, in full detail, using the almost-Montagovian method in 2.3.2, with the minimal extension that disjunction requires.

Every elf saw Smaug or Bilbo.
   (i) some saw Smaug, some Bilbo
   (ii) all saw the same but I don’t know/remember/tell you which

In addition, if you are planning to take Semantics II, derive reading (ii), using either Hendriks’ method in 2.3.5 or Heim & Kratzer’s method, as in their 7.5.1 (or both, if you wish).

Optional: Go back to the “transitive BE” problem (H) and solve it in full, with all the new wisdom you now have.

Squib statements and meetings; volunteers for presenting Kratzer and Champollion

Send me your squib statements (see my Oct 26 email) by Sunday evening (one per project). Especially those of you who have been working on your topics since the first round of meetings should meet up with me again next week; I’ll send out a Doodle poll.

In the weeks of November 30 and December 7 we’ll be venturing into event-semantics: some of Kratzer’s chapters and part of Lucas’s L&P article. It would be great if people who have an active interest in event semantics and/or argument structure volunteered to present them, e.g. a team of two per paper. Email will follow.
Different quantifiers, different scopes: indefinites and universals

Semantics I
Szabolcsi
Nov 9-11, 2015

The classical view
All quantifier phrases are equal in their internal structure and in the way they take scope.


The classical view

Quantifier phrases denote Generalized Quantifiers

\[ \lambda P[\text{every-dog}'(P)] \]
\[ \lambda x[\text{saw-a-cat}'(x)] \]
\[ \lambda x[\text{a-cat-saw}'(x)] \]

Scopes denote properties

\[ \{ \text{the set of properties that every dog has' } \} \]
\[ \{ \text{'the property of seeing a cat / being seen by a cat' } \} \]

Predictions of the classical view

Re: internal composition
When two quantifiers denote the same set of properties (are logically equivalent), differences in their internal composition do not matter.

Re: scope taking
All QPs have the same ability to scope over any other QP or operator, and over the same syntactic domains (with the possible exception where the result is incoherent gibberish).

• Are these predictions correct?

• If not, what classes of quantifiers and what syntactic/semantic mechanisms should be recognized?
Are these predictions correct?

Scope taking is not uniform (old observations)
If [two friends of mine come to visit], I'll have to skip class.
✓ `for two particular friends, if they visit, I'll skip class'
If [every friend of mine comes to visit], I'll have to skip class.
   # `for every friend, if he/she visits, I'll skip class'
More than one girl saw every film.
✓ `for every film, more than one girl saw it’
Every girl saw more than one film.
   # `there is more than one film that every girl saw’
She didn’t greet (more than) two guests / every guest.
✓ `(more than) two > not’ but  # `every > not’

Are these predictions correct?

Internal composition matters (more recent observations)
• At least two men left  IFF More than one man left
  ✓ At least two doctors shook hands.
  # More than one doctor shook hands.
• At most four men left  IFF Fewer than five men left
  Beryl had three sheries.
  OK ⇒ Beryl had fewer than five sheries.
  NO ⇒ Beryl had at most four sheries.
• Most of the men left  IFF More than 50% of the men left
  # The kids read most of the books each.
  ✓ The kids read more than 50% of the books each.

Is Montague’s version of compositionality wrong?

My response: The meaning assignment \( h \) can only be expected to be a homomorphism if Expressions are built in the right syntactic steps, and Meanings are of the right kind and have the right granularity.

Scopal diversity:
Three classes of “quantifier phrases”

1 Bare (numeral) indefinites
2 Distributive universals
Formal tools in the literature
choice functions,
Skolemization,
Dist, \( \delta \) operators

both have two kinds of scope:
unbounded existential scope &
clause-bounded distributive scope

3A,B Counting quantifiers
clause-bounded, intervention-sensitive split scope
amount vs. individual readings
quantification over degrees

Class 1 Bare (numeral) indefinites

Seem to have unbounded, island-free scope

• [If two friends of mine come to visit, I’ll have to skip class.]
  
  \[
  \text{adjunct island} \\
  \text{✓ ‘for two particular friends, if they visit, I'll skip class’}
  \]
• Each student has to hunt down [every paper which shows [that a certain claim by Chomsky is wrong]].
  ✓ ‘each student > a certain claim > every paper’
  ✓ ‘a certain claim > each student > every paper’
• [If some lady dies, Bill inherits a house.]
  ✓ ‘for some lady, if she dies, Bill inherits a house’

Bare (numeral) indefinites
But do they have island-free scope?

A student has to hunt down [every paper which shows [that certain claims by Chomsky are wrong]].
✓ ‘for certain claims, a student has to hunt down...’
BUT students cannot vary with claims
[If two ladies die], Bill inherits a house.
✓ ‘for two particular ladies, if they die...’
BUT only one house in total can be inherited

COMPARE
Two ladies left Bill a house.
✓ ‘two houses in total’
A contradiction!

The findings cannot be described using the classical scope vocabulary.

The first set of data shows that indefinites can be referentially independent of quantifiers and negation that c-command them outside islands \( \Rightarrow \) they have unbounded scope.

The second set shows that plural indefinites cannot induce variation in other, clause-external indefinites \( \Rightarrow \) they have clause-internal scope.

“Distributive scope” of indefinites

*Two ladies each left him a house.*

\[ \exists \text{cf} \left[ \text{cf(two-ladies')} \delta\left(\text{left-him-a-house}')\right) \right] \]

If \( \alpha \) is a plurality and \( \beta \) is a property, \( [\alpha \delta(\beta)] \) is true iff \( \forall x[\text{atom}(x, \alpha) \rightarrow \beta(x)] \).

The \( \delta \) operator (Link 1983), like adverbial *each*, is adjoined to the predicate, not to NP, and is thus unaffected by the extra-clausal existential scoping of the plural indefinite.

*If two ladies each die, Bill inherits a house.*

Using choice functions to avoid island-free movement of indefinites in LF syntax.

Two analyses of maximal “existential scope”

*If two ladies die, Bill inherits a house.*

- \( \exists \text{cf} \left[ \text{cf(two-ladies) die} \rightarrow \text{Bill inherits a house}\right] \)
  - `there is a choice function cf such that if the pair that cf chooses from the set of pairs of ladies dies, Bill inherits a house’
- \( \text{cf(two-ladies) die} \rightarrow \text{Bill inherits a house}\)
  - `if the pair that the contextually relevant cf chooses from the set of pairs of ladies dies, Bill inherits a house’

Solution

Distinguish two kinds of scope for indefinites.

*“Existential scope”* pertains to referential independence. Unbounded.

- Formal tool: choice function variable, existentially closed from a distance (Reinhart) or contextually given (Kratzer)
- *“Distributive scope”* pertains to the ability to induce variation in others. Clause-bounded.
- Formal tool: silent distributive operator on the predicate (a universal quantifier, \( \delta \) operator, Link)

Choice functions cf

A choice function cf looks at every set and chooses an element of that set. **dog(cf(dog)) always true**

\[
\begin{align*}
\text{cf}_1(\text{dog}) &= \text{Fido} & \text{cf}_2(\text{dog}) &= \text{Spot} \\
\text{cf}_1(\text{cat}) &= \text{Max} & \text{cf}_2(\text{cat}) &= \text{Tiger} \\
\text{cf}_1(\text{city}) &= \text{Paris} & \text{cf}_2(\text{city}) &= \text{LA} \\
\text{cf}_1(\text{two-dogs}) &= \{\text{Fido, Spot}\} & \text{cf}_2(\text{two-dogs}) &= \{\text{King, Spot}\} \\
& \ldots & \ldots & \ldots
\end{align*}
\]

also with sets whose elements are not individuals:

two-dogs* = \{ \{\text{Fido, Spot}, \{\text{King, Spot}, \{\text{Spike,King}, \{\text{Fido,King}, \{\text{Fido,Spike}, \{\text{Spike,Spot} \} \} \} \} \} \}

Two analyses of clause-external but dependent (=intermediate) readings

*Each student must hunt down [every paper which shows [that a certain claim is wrong]].*

- `each student > a certain claim > every paper’
- with intermediate \( \exists \)-closure of cf:
  \( \forall x[\text{student'}(x) \rightarrow \exists \text{cf} \forall y[\text{paper'}(y) \land \text{show'}(y, \text{wrong'}(\text{cf(claim'))} \rightarrow \text{hunt-down'}(x, y))] \)
  - with Skolemized contextual choice function, cf(x):
    \( \forall x[\text{student'}(x) \rightarrow \forall y[\text{paper'}(y) \land \text{show'}(y, \text{wrong'}(\text{cf(x)(claim'))} \rightarrow \text{hunt-down'}(x, y))] \)
Class 2 Universals: Are they all alike?

*each* vs. *every* vs. *all the*

Some tourist or other thought that ... sight(s) was/were boring.
Can tourists vary with sights?

*each:* yes  *every:* no  *all the:* no

Some tourist or other visited ... sight(s).
Can tourists vary with sights?

*each:* yes  *every:* yes  *all the:* no

... tourist(s) lifted up the van.
Can the tourists have acted collectively?

*each:* no  *every:* no  *all the:* yes

[although: ✓ It took every tourist to lift up the van.]

Unbounded existential scope for universals?

*You cannot list every prime number.*

⇒ There is a set, the one containing all primes, such that you cannot list every element of it.

*I don’t believe that you listed every prime number.*

⇒ There is a set, the one containing all primes, such that I don’t believe that you listed every element of it.

*If every prime number is divisible by 1, then ...*

⇒ There is a set, the one containing all primes, such that if every element of it is divisible by 1, then ...

Domain restriction and co-variation

Context: There are 3 empty vinegar bottles and 4 full wine bottles in the cupboard. We need vinegar. I look in the cupboard and report,

*Every bottle is empty.*

Can this be true? Not every bottle in the world, not even every bottle in the cupboard is empty!

Context: “Syntax” is a course that every student must complete at some point. Head of department notices,

*Every “Syntax” teacher failed every first-year student.*

Can this be true? Did they all teach all the first-years?

Parallelism with indefinites

The prime numbers examples show that sentences with universals entail the maximal-scope existence of the (non-empty) restrictor set. *(Every NP is a principal filter.)*

The bottles example shows that the restrictor set of *every* can be further delimited by context.

The first-years example shows that the restrictor sets can co-vary with a c-commanding quantifier.

\[ \text{cf(pow(bottle'))} = \text{a contextually relevant subset of the set of bottles} \]

\[ \text{cf(x)(pow(first-year'))} = \text{contextually relevant subsets of the set of first-years, chosen in variation with a quantifier that binds the Skolem parameter x.} \]

Why is dual scope news?

Traditional examples inspired by predicate logic:

- *Every student read a book.*

Critical new examples:

- *Two students read a book.*

- *Every prof failed every first-year student.*

*Every student* can induce variation, but does not exhibit variation.

*A book* can vary, but does not induce variation.

“What is the scope of this QP?”

*used to be a different question in each case.*

For *every student*, “What is its distributive scope?”

For *a book*, “What is its existential scope?”
Assignment for Nov. 16

Reading

Relating to this week, the required readings are Quantification, Chapters 6 (Scope is not uniform and not a primitive) and 7 (Existential scope versus distributive scope), with the exception of 7.1.2-3 (=optional).

Chapters 8 (Distributivity and scope) and 9 (Bare numeral indefinites) are on the whole optional; but see 8.3 and 9.3 below.

Next week’s discussion will focus on some of the puzzles from the Oct. 7 handout, below, and some of their relatives, fresh from Dylan Bumford’s ongoing dissertation work. Either both lectures, or at least the Wednesday lecture will be given by Dylan.

JOHN read the fewest books.
Every boy read the same book.
The rabbit in the hat is white.

How many books should I read? vs. How many books should I read?
At least five. At least the following five: A, B, C, D, and E.

In preparation, read Section 9.3 (Cardinal vs. individual readings of numeral indefinites) and 10.3-4 (The split-scope analysis of comparative quantifiers; More than half, most of the, and the most). Budget time for this, these are not breezy sections; but familiarity with the material will likely make next week smoother. – I’ll let you know if Dylan recommends different preparatory readings.

Written

Read the first part of Section 8.3 (Distributive singular quantifiers), pp. 121-125, preceding 8.3.4. In one to two pages, discuss how the data and analyses reviewed there bear on the general claims that scope is not uniform and that scope not a primitive.
Class 3A Counting quantifiers

Unlike indefinites and every NP-type universals, counters do not take existential scope outside their own clause.

*Some tourist or other thought that more than ten sights were boring.*

# there are more than ten sights which ...

Counters do not take inverse scope over the subject (at most, they take inverse scope over another counter in subject).

- *Every girl read more than ten books.*
  # there are more than ten books read by every girl
- *Some girl or other read more than ten books.*
  # girls vary with books
- *At least one girl read more than ten books.*
  ? girls vary with books

How many patients must Dr. X visit?

(a) The following four: A, B, C, D.
(b) Four.

Dr. X must visit more than three patients.

(a) The following ones: A, B, C, D.
(b) ..., not just three.

Dr. X must visit the sickest patients.

(a) Those who are sicker than the other patients.
(b) He must visit sicker ones than how sick ones the other doctors visit.
(c) He must visit sicker ones than how sick ones the other doctors must visit.

Dr. X must visit the most patients.

(a) # most of the patients
(b) He must visit more than how many the other doctors visit.
(c) He must visit more than how many the other doctors must visit.

How many patients must Dr. X visit?

- For what number n, there are n patients whom Dr. X must visit?’ (individual reading)
- For what number n, it must be that there are n patients whom Dr. X visits?’ (cardinal reading)

How many patients did few doctors visit?

- For what number n, there are n patients whom few doctors visited?’ (individual reading)

# For what number n, for few doctors are there n patients whom they visited?’ (cardinal reading)
Degree comparison

d is a variable over degrees, D over degree intervals

More than three people smile.

\[ \lambda d. \exists \text{people}'(x) \land |x| > 3 \land P(x) \]

-er than 3  
d-MANY people smile

\[ \lambda d. \lambda D' [\max(D') > \max(D)] \]

[than] 3  
\[ \lambda d. d = 3 \]

\[ \max(\lambda d. \exists \text{people}'(x) \land \text{smile}'(x) \land |x| > d) > \max(\lambda d. d = 3) \]

iff \[ \exists \text{people}'(x) \land \text{smile}'(x) \land |x| > 3 \]

Inverse scope: over subject vs.
over another VP-internal quantifier

Every student read more than one paper.
# more than one NP > every NP

John submitted more than one paper to every 
journal.
✓ more than one NP > every NP
John submitted every paper to more than one 
journal.
✓ more than one NP > every NP

Subject QP and splitting counter

Surface scope

Subject QP

-er than 3  
t-MANY NP ...

Intervention

-er than 3  
Subject QP  
violates Intervention (f)

t-MANY NP ...

Inverse scope

-er than 3  
t-MANY NP  
Subject QP ...

violates Shortest (d)

Degree operator ... intervener ... restriction

✓ Modal or intensional operator scopally intervenes 
between the degree operator and its restriction 
d-many/much NP (but see Lassiter, SALT 22).
✓ Name or non-distributively interpreted plural 
indefinite intervenes.

# Every NP, few NP, only XP, or negation scopally 
intervenes (Honcoop 1998, Kennedy 1999, Pesetsky 
Caveat: Sometimes a quantifier linearly intervenes,
but does not scope, between the degree operator 
and its restriction, e.g. ✓ pair-list reading. (Szabolcsi & 
Zwarts 1993).

Account in terms of split and 
intervention (Takahashi 2006)

a. The decomposition of more than n NP into -er than n and 
d-many NP.
b. QR forced by type mismatches, subject to Shortest Move.
c. Optional Quantifier Lowering, subject to Shortest Move.
d. Shortest: QR/QL targets the closest node of type t.
e. VP-internal XPs are equidistant from vP of type t.
f. Intervention constraint: A quantificational DP cannot 
intervene between DegP and its trace in d-many NP.
g. Scope Economy: Covert QR/QL cannot be semantically 
vacuous.
h. Scope commutativity facts of comparative quantifiers.

Interface Transparency

“Extending other work, our conclusion is that competent speakers 
associate sentences with canonical specifications of truth 
conditions, and that these specifications provide default verification 
procedures. From this perspective, examining how a sentence 
constrains its verification can provide clues about how speakers 
specify the truth condition in question. More generally, our data 
support an Interface Transparency Thesis (ITT), according to which 
speakers exhibit a bias towards the verification procedures provided 
by canonical specifications of truth conditions. In conjunction with 
specific hypotheses about canonical specifications, the ITT leads to 
substantive predictions, because given available information, the 
canonical procedure may have to rely on (noisy) input representa-
tions that lead to less accuracy in judgment, compared with an 
alternative strategy that is cognitively available to speakers.” 
(Lidz et al. 2011)
More on **superlatives**

*Who climbed the highest mountain?*

**Absolute** reading, ABS

'Who climbed the mountain that is higher than any other mountain [in the area]?'

**Relative** readings, REL

'Who climbed a higher mountain than how high a mountain anyone else climbed?'


In ABS, `-est` has DP-internal scope.

In REL, `-est` has sentential scope.

Calls for setting aside word boundaries for scope assignment.

---

**Most** as a superlative: **many-est**

- **Like highest, most and fewest** have relative readings:
  *Who climbed the most/fewest mountains?*
  'more/fewer than anyone else climbed'

- **Like highest, most** has an absolute reading, which is equivalent to the classical proportional reading:

  Most (of the) men snore = |MEN \ (∩) SNORE| > |MEN \ (∩) NOT SNORE|

  But fewest doesn’t:  
  *Fewest (of the) men snore*

  Hackl 2009: A decompositional analysis can explain these; one that takes most, fewest to be lexical primitives cannot.

---

The Comparative-Superlative Generalizations, Bobaljik 2012

| ABB       | good – better – best |
| ABC       | bonus – melior – optimus |
| AAB       | unatt.  | good – gooder – best |
| ABA       | unatt.  | good – better – goodest |

**The Containment Hypothesis:**

The representation of the superlative properly contains that of the comparative.

```
[[ [adj] comparative ] superlative ]
`Adj + more than + all others`
```

Hung.

<table>
<thead>
<tr>
<th>sok</th>
<th>több</th>
<th>legtöbb</th>
</tr>
</thead>
<tbody>
<tr>
<td>many/much</td>
<td>more</td>
<td>most</td>
</tr>
</tbody>
</table>

Need to decompose even more than Hackl does!
References
in addition to those in Quantification

• Lassiter 2012, Quantificational and modal interveners in degree constructions. *SALT* 22, 
  http://journals.linguisticsociety.org/proceedings/index.php/SALT/article/view/2649

• Nouwen 2010, Two kinds of modified numerals. *Semantics & Pragmatics* 3, Article 3: 1–41. doi: 10.3765/sp.3.3
• Szabolcsi 2012, Compositionality without word boundaries: (the) more and (the) most. *SALT* 22, 
  http://journals.linguisticsociety.org/proceedings/index.php/SALT/article/view/2629
1 Classic split-scope effects

1.1 Factoring meanings

(1) John published \{exactly, fewer than, at least\} three papers this year
   a. John published n-many papers this year
   b. The (maximum) n that makes (a) true is \{exactly, less than, at least\} 3

(2) How many papers did John publish this year?
   a. John published n-many papers this year
   b. The (maximum) n that makes (a) true is what?

(3) John published no papers this year
   a. John published n-many papers this year
   b. The (maximum) n that makes (a) true is 0

1.2 Splitting their scopes

1.2.1 Modal interveners

(4) You need to publish \{exactly, fewer than, at least\} three papers this year
   a. It needs to be the case that you publish n-many papers this year
   b. The (maximum) n that makes (a) true is 3

(5) How many papers do you need to publish this year?
   a. It needs to be the case that you publish n-many papers this year
   b. The (maximum) n that makes (a) true is what?

(6) You need publish no papers to get tenure here
   a. It needs to be the case that you publish n-many papers this year
   b. The (maximum) n that makes (a) true is 0.

1.2.2 No quantificational interveners!

(7) Everyone published exactly three papers this year
   a. Everyone published n-many papers this year
   b. The (maximum) n that makes (a) true is 3
1.2.3 Self-interveners

(8) This year in our department, exactly three students published exactly seven papers
   a. n-many students published m-many papers (between them)
   b. The (maximum) n and m that make (a) true are 3 and 7 (respectively)

2 Definite descriptions

(9) The student who published a paper
   a. n-many students published a paper
   b. The (maximum) n that makes (a) true is 1

And indeed, definites sometimes self-intervene!

(10) The student who published the paper on de_nites
    a. n-many students published m-many papers on de_niteness
    b. The (maximum) n and m that make (a) true are 1 and 1 (respectively)

• Split scope not limited to pure size tests.

Ordinality tests:

(11) Out of our cohort, I bet John will publish the second/next paper
    a. I expect a member of our cohort x will publish a paper
    b. I expect the second/next x to make (a) true will be John

Distributional tests:

(12) John finally learned to relax this year; he taught his classes and published the occasional paper, but mostly took it easy.
    a. John published a paper at time t
    b. The distribution of the ts that make (a) true is “occasional”

(13) The average paper of John’s was 22.4 pages long
    a. John’s paper is d pages long
    b. The average of the ds that make (a) true is 22.4

Identity tests:

(14) John and Bill published papers on the same topic
    a. John and Bill published papers on topics x and y (respectively)
    b. The x and y that make (a) true are the same

(15) John published a paper with the wrong analysis
    a. John published a paper with analysis x
    b. The xs that makes (a) true was wrong
Quantificational tests:

(16) John published the only paper this year
   a. The members of X published a paper this year
   b. The X that makes (a) true is \{John\}

(17) John published the longest paper this year
   a. x published a d-good paper this year
   b. The biggest d s.t. that <x, d> makes (a) true has x = John

3 Superlatives

(18) Bring me the student with the most expensive iphone
   a. Bring me the student with \[Absolute\] the iphone 6 or whatever
   b. Bring me the student who has an iphone \[Relative\] that is more expensive than any other student’s

Zooming in on the relative reading, several things to note:

- The superlative DP itself is interpreted indefinitely!
  – Compare with all of the classic split-scope effects of numerals
  – Same goes for all the other the Adj effects above
  – A few classic diagnostics:
    There’s the most bugs in the SEMANTICS lab
    The semantics lab has the most bugs

- Superlatives quantify over two variables at once, and only one of them is in their own DP!
  – In (18) we look at <s, d> pairs, where s 2 student and d is the price of (one of) s’ s iphones
  – Also true of ‘only’ and the ordinals
  – We can also get modal intervention

(19) John needs to write the longest term paper
   a. x needs to write a term paper of length nx
   b. The biggest n s.t. <x, n> that makes (a) true has x = John

(20) Professor John requires the only term paper this semester
   a. The members of X require a term paper this semester
   b. The X that makes (a) true is \{Professor John\}

- But no quantificational intervention

(21) John scored the most points in every game
   a. x scored d-many points in every game
   b. # The biggest d s.t. <x, d> makes (a) true has x = John
Quantifier particles cross-linguistically
(based on Szabolcsi 2015)

In many languages, the same particles that form quantifier words also serve as connectives, additive and scalar particles, question markers, roots of existential verbs, and so on. Do these have a unified semantics, or do they merely bear a family resemblance? Are they aided by silent operators in their varied roles -- if yes, what operators?

Today: The basic data and the basic problem

(1) a. mind-en-ki dare-mo ‘everyone/anyone’ MO-family
    b. mind A mind B A-mo B-mo ‘A as well as B, both A and B’
    c. A is (és) B is A-mo ‘A too/even A’

(2) a. vala-ki dare-ka ‘someone’ KA-family
    b. (vagy) A vagy B A-ka B(-ka) ‘A or B’
    c. vagy száz hyaku-nin-toka ‘some one hundred = approx. 100’
    d. val-, vagy- -- ‘∃ be’ participial & finite stems
    e. -- dare-ga V...-ka ‘Who Vs?’
    f. S-e S-ka ‘whether S’
    g. vaj-on S-ka-na ca. ‘I wonder’

Table 1.1: Distribution of Q-particles in Sinhala, Malayalam, Tlingit, and Japanese

Slade 2011
First stab: MO expresses lattice-theoretic meet (\(\cap\)), KA expresses lattice-theoretic join (\(\cup\)).

Can be correctly implemented in Inquisitive Semantics, where

\[\Box [\text{Joe dances}] \] is the powerset of the set of worlds in which Joe dances, \(\text{pow}\{w: \text{dance}_w(joe)\}\)

(4) a.  \(\begin{align*}
\text{Joe dances} \\
\Box [\text{Joe dances}] 
\end{align*}\)

b.  \(\begin{align*}
\text{Joe and Kate and Mary dance, Everyone dances} \\
\Box [\text{Kate dances}] \cap \Box [\text{Mary dances}] \cap \Box [\text{Joe dances}] 
\end{align*}\)

(5) a.  \(\begin{align*}
\text{common core of Kate or Mary or Joe dances, Someone dances, Who dances?} \\
\Box [\text{Kate dances}] \cup \Box [\text{Mary dances}] \cup \Box [\text{Joe dances}] 
\end{align*}\)

b.  \(\begin{align*}
\text{whether Joe dances} \\
\Box [\text{Joe dances}] \cup \Box [\neg \text{Joe dances}] 
\end{align*}\)

Problem: Reiterated MO/KA, unary MO/KA [ not: MO/KA\{A, B, ...\}, and MO/KA\{A\}\neq A ]

(6) Schematically

John MO Mary MO danced.

'John danced and Mary danced'

Hungarian

János is Mari is táncolt.

'John danced and Mary danced'

(7) Schematically

John KA Mary KA danced.

'John or Mary danced'

Sinhala (Slade 2011)

Gunapāla hari Chitra hari gamaṭa giyā.

'G or C went to the village'

(8) Schematically

Mary danced. John MO danced.

'John, too, danced'

Hungarian

János is táncolt.

'John, too, danced'

(9) Mary is at home. -- Or (perhaps), John is at home.

(10) Schematically

John bought 100 KA books.

'John bought some 100 books'

Hungarian

János vett vagy száz könyvet.

'John bought some 100 books'

(11) a.  \(\begin{align*}
\text{if/whether Mary danced} \\
\{w: \text{dance}_w (m)\},\{w: \text{not} \text{dance}_w (m)\} 
\end{align*}\)

b.  \(\begin{align*}
\text{if/whether Mary danced or not} \\
\{w: \text{dance}_w (m)\},\{w: \text{not} \text{dance}_w (m)\} 
\end{align*}\)

Gist of the proposal:

MO and KA inhabit contexts interpreted as meets and joins, but they are not meet and join operators themselves. Instead, MO and KA impose semantic requirements that are satisfied when their contexts are interpreted, respectively, as the meet (greatest lower bound) and the join (least upper bound) of the contribution of their hosts and something else.
Assignment for Nov. 30

Nov 30 and Dec 2 are about event semantics. On Monday I'll be giving a general background; on Wednesday the team Mary & Haoze & Philip (physically represented by Mary and Haoze, Philip being away at a conference) will present some of the core ideas in Kratzer's The event argument (2002).

The required reading is Chapters 1 and 2 of Kratzer.

The optional supporting readings are two handbook articles: (i) Maienborn, Event semantics, and (ii) Champollion & Krifka, Mereology. All three items are in the course folder.
Why events?

Pronominal anaphora (Davidson 1967)
Jones buttered the toast. He did it slowly, deliberately, in the bathroom.

Perception reports (Higginbotham 1983)
John saw Mary/someone leave.
Veridicality: If John saw Mary leave, then Mary left.
Exportability of existential: If John sees someone leave, then there’s someone whom John sees leave.
\[ \exists x [x \text{ is an event } \& \text{ leave}(Mary, x)] [\text{John sees } x] \]
Contrast with: John saw that Mary/someone left.

Causal relations (Parsons 1990)
Mary’s leaving made Susan angry / angered Susan.
\[ \exists x \exists y [x \text{ is an event } \& \text{ leave}(Mary, x) \& y \text{ is an event } \& \text{ angry}(Susan, y)] [\text{CAUSE } (x, y)] \]

If events are “things,” how are they individuated?
The sphere rotated quickly and (in the process) heated up slowly. (Parsons 1990)
The event of the sphere’s rotating quickly isn’t the same as the event of the sphere’s heating up slowly (even though they occupy the same space-time coordinates).

Pat came home late last night because of a traffic jam. She started cooking spaghetti at 11 P.M.
The traffic jam caused Pat’s cooking spaghetti (false)
The traffic jam caused Pat’s cooking spaghetti late (true)
The event of Pat’s cooking spaghetti is not the same event as Pat’s cooking spaghetti late. (?)

Stage level preds (have event argument) vs. individual level preds (no event argument) (Kratzer 1995)
a. When Mary speaks French, she speaks it well. (SLP)
b. * When Mary knows French, she knows it well. (ILP)
c. When Mary knows a foreign language, she knows it well. (ILP+indef)
d. When a Moroccan knows French, she knows it well. (ILP+indef)

though * When Anil died, his widow usually killed herself (de Hoop & de Swart 1989)

Adverbs of quantification and the proportion problem (de Swart 1991)
For the most part, if a knight admires a lady, he sings her praise. (seemingly counts knight-lady pairs)
For the most part, if a knight meets a lady, he bows to her. (counts events of knight-lady meetings)

Event-related readings (Krifka 1990)
Four thousand ships passed through the lock last year.

Big event consisting of small events (Schein 1993)
Unharmoniously, every organ student sustained a note on the Wurlitzer for 16 measures.
\[ \exists e : \text{unharmonious}(e) [\text{every } x : \text{student}(x) \& \exists e' : e' < e \& \forall z [\text{INFL}(e', z) \leftrightarrow z = x]] \]
Aspectual classes (state, activity, achievement, accomplishment), telic/atelic (Vendler, Dowty, Krifka)
Romeo wandered for an hour. Gertrude consumed ale for 5 mins. = cumulative reference
Romeo woke within an hour. Gertrude consumed a poisoned drink within 5 mins. = quantized ref.

part-whole structures: free join semi-lattice, mereology (*X)

Collective vs. distributive readings (Carlson, Krifka, Schein, Landman, ...)
Two men lifted up the table Two men built a table
one event one event, one table
two events two events, two tables

Cumulative readings are scopeless distributive readings (Schein 1993, Landman 2000)

Landman implements Schein’s crucial observation that distributive but scopeless plural co-arguments yield a cumulative reading. That cumulative readings can be captured in this way is a benefit of neo-Davidsonian event-semantics. In that theory the properties of an event are specified in a series of conjoined propositions, and the arguments of the predicate appear in separate conjuncts: hence their scopal independence. Distributivity is anchored in the interpretation of the thematic roles *Agent and *Theme.

Three boys invited four girls (between them)
(sum subject in-situ + sum object in-situ)

\[ \exists e \in *\text{INVITE} : \exists x \in *\text{BOY} : |x|=3 \quad \& \quad *\text{Agent}(e)=x \quad \& \]
\[ \exists y \in *\text{GIRL} : |y|=4 \quad \& \quad *\text{Theme}(e)=y \]

where *INVITE is a join semi-lattice of events, with INVITE prodividing its set of atoms,
*BOY is a join semi-lattice of individuals, with BOY providing its set of atoms,
*Agent(e)=x iff \( \forall z \in \text{atoms}(x) : \exists e \in \text{VERB} : \text{Agent}(e)=z, \)
and similarly for *GIRL and *Theme.

‘there is a sum of inviting events e, a sum of three boys x, and a sum of four girls y, such that every atomic part of x is the Agent of an atomic part of e, and every atomic part of y is the Theme of an atomic part of e’

I. Background

Davidson (1967): in addition to syntactic arguments of the verb, argument structure also contains an event argument

(1) We purchased those slippers in Marrakesh.

Ordered argument method (Davidson 1967)

(1') \( \exists e [\text{purchase}(\text{those slippers})(\text{we})(e) \land \text{in}(\text{Marrakesh})(e)] \)

'There is a past event which is the purchase of those slippers by us, and which takes place in Marrakesh'

- Purchase is a three-place predicate
- Secondary predicate in introduces the location

Davidson's view: events are things in the world that can exist in space and time

Neo-Davidsonian method (Parsons 1990)

(1'') \( \exists e [\text{purchase}(e) \land \text{agent}(\text{we})(e) \land \text{theme}(\text{those slippers})(e) \land \text{in}(\text{Marrakesh})(e)] \)

'There is an event that is a purchase, whose agent is us, whose theme are those slippers, and which takes place in Marrakesh'

- Purchase is a one-place predicate
- Event arguments are introduced through secondary predicates denoting thematic roles

Neo-Davidsonian view: every verb has an event argument, and other argument positions are related to the verb through thematic roles

Kratzer claims that external arguments are associated by the Neo-Davidsonian method, but themes are not, or not necessarily

II. Syntactic motivation for Kratzer's claim

How can we sever the external argument from the verb? Some options:

(a) \( \exists x \exists e [\text{purchase}(e) \land \text{theme}(x)(e)] \)

'e is a purchase with theme x'

(b) \( \exists x \exists e [\text{purchase}(x)(e)] \)

'e is a purchase of x'
kramer (2002)

(c) Σ[e[purchase(e)]

'e is a purchase'

Focusing on (a):

• The theme argument is, but the agent argument is not a syntactic argument of purchase
• There is thematic decomposition at logical-conceptual structure
• Is there genuine lexical decomposition at the logical-conceptual structure that is not matched by parallel decomposition in the morphology or syntax?


• Argument structure is literally identified with syntactic structure it projects

Hale and Keyser, Talmy: cross-linguistic lexicalization patterns

• Can provide evidence for logical-conceptual decomposition
• Should be possible to reconstruct universal logical-conceptual representations by comparing how they are sounded out in different languages

Other syntactic arguments for Kratzer's claim:

Optionality of arguments in event nominals

• possible syntactic diagnostic for Neo-Davidsonian association
• only shows external arguments to be Neo-Davidsonian

(2)

a. Gifts of books from John to Mary would surprise Helen.
b. Gifts of books from John would surprise Helen.
c. Gifts of books to Mary would surprise Helen.
d. Gifts from John to Mary would surprise Helen.
e. Gifts from John would surprise Helen.
f. Gifts of books would surprise Helen.
g. Gifts to Mary would surprise Helen.
h. Gifts would surprise Helen.

Compare with:

(3)

a. *The constant assignment is to be avoided.
b. The constant assignment of unsolvable problems is to be avoided.

Why are gift and assignment different?

• Gift is not compositionally derived from give
• Verbs must realize their arguments syntactically even when they are part of a nominalization, if the nominalization is compositional
• If assignment must have a direct object when it is part of a compositional event nominalization, then having an object is part of its meaning
• If external arguments were true arguments of their verbs, they should have to be realized in compositional event nominalizations

Passives complicate matters:

(4) Too many unsolveable problems were assigned.

The external argument can still be realized as an implicit impersonal pronoun

How do we know if the external argument is present in verbal passives?

No self-action permitted (from Baker, Johnson and Roberts 1989)

• diagnoses presence of external argument in syntax

(5)  
   a. The children are being dressed.
   b. The climbers are being secured with a rope.

It is not possible for (5a) or (5b) to permit self-action. Why? Compare (5) and (6).

(6)  
   a. They are dressing the children.
   b. They are securing the climbers.

No self-action permitted with nominalizations:

(7)  
   a. The report mentioned the painfully slow dressing of the children.
   b. The article praised the expeditious securing of the climbers

Self-action only permitted with adjectival passive:

(8)  
   a. The climbers are secured with a rope.
   b. The climbers are being secured with a rope.

Problem: Why is the external argument present in some constructions and not others?

Kratzer's answer: “When external arguments are missing...we are at a stage of the syntactic derivation where they are not yet there. We just haven't yet projected enough functional structure.”

Manner adverbs (low), time adverbs (high)

(9)  
   a. Die schön gekämmten Kinder
   the nicely combed children
   (Compatible with self-action)
b. Die gestern gekämmtten Kinder
   the yesterday combed children
   'The children who were combed yesterday'
   (Imcompatible with self-action)

Manner adverbs must be low and temporal adverbs must be high

(10)   a. Ich hab' dich gestern schön gekämmt.
       I have you yesterday nicely combed
       'I combed you nicely yesterday'

   b. *Ich hab' dich schön gestern gekämmt.
      I have you nicely yesterday combed

Only manner adverbs can be topicalized

(11)   a. Schön gekämmt hat er dich nicht
       nicely combed has he you not
       'He didn't comb you nicely yesterday'

   b. *Gestern gekämmt hat er dich nicht
      yesterday combed has he you not
      'He didn't comb you yesterday'

III. Relevance to syntax

Pre-Minimalism:

   **Projection Principle** says that all lexical information must be syntactically represented
   • What about event nominalizations like *gift*?
   • What about verbal passives?

   **Theta Criterion:** Each theta-role is assigned to one and only one argument.
   Each argument is assigned one and only one theta-role
   • Can you think of any counter examples?

   **The Uniformity of Theta Assignment Hypothesis (UTAH) (Baker 1988):**
   Identical thematic relationship between items are represented by identical structural relationships
   between those items at the level of Deep-Structure
   • But what about double object constructions?
Minimalism:

Pylkkänen (2008)
- Argues that the v head by itself does not introduce an argument
- Uses Kratzer’s proposal that a VoiceP introduces external arguments
- Assumes that the functional head Voice denotes a thematic relation between the external argument and the event described by the verb

Wood and Marantz (2015)
- Syntax is truly autonomous—syntactic representations and their relations do not carry semantic values
- There is an argument-introducing head *i*
  - This head takes the place of v, voice, p, and appl (both high and low)
  - Can be null (expletive)
  - Interpretation of *i* is subject to contextual allosemy at the syntactic interface
- What does this mean for semantics?

III. Schein’s Argument

Kratzer uses Schein’s argument to show that the usual translation of every cannot account for cumulative interpretations

(1) Three copy editors caught every mistake in the manuscript.
(2) Three video games taught every quarterback two new plays.

- The sentence (1) can have a ‘cumulative’ interpretation:
  ‘We hired three copy editors to look at a manuscript independently of each other, and between them, they caught all the mistakes in the manuscript’.

  This reading is surprising because every is an otherwise distributive quantifier.
- The sentence (2) can have a following reading:

  ‘Three video games were responsible for the fact that every quarterback learned two possibly different plays’.

  every quarterback and three video games are related cumulatively, while every quarterback behaves as a distributive quantifier with respect to two new plays: every quarterback could learn two different plays.

  In the sentence (3) below a similar problem arises:

(3) Five insurance associates gave a $25 donation to several charities.
This sentence can have a following reading:

‘Taken together, the contributions of five insurance associates amounted to a gift of $25 to each of several charities’.

Formalization of such a reading is difficult.

- The straightforward formalization (1) is given below, but it does not give a satisfactory result:

\[ (4) \exists x [3 \text{ copy editors}(x) \land \forall y [\text{mistake}(y) \rightarrow \exists e [\text{catch}(y)(x)(e)]] ] \]

Q. Why? Who caught the mistake?
A. Always the same three copy editors as a plurality—which is not the reading we want. Under the reading we want, mistakes were found by subparts of that plurality.

We could think of the following formalizations then.

\[ (5) \exists x [3 \text{ copy editors}(x) \land \forall y [\text{mistake}(y) \rightarrow \exists e \exists z [z < x \land \text{catch}(y)(z)(e)]] ] \]

(5) does fix one problem of (4): now the mistakes were found by a subset of the plurality

However, this formalization is not adequate either: it can be true in a situation where a single copy editor found all the mistakes without others contributing at all.

We can introduce an agent argument into the formalization:

\[ (6) \exists e \exists x [3 \text{ copy editors}(x) \land \text{agent}(x)(e) \land \forall y [\text{mistake}(y) \rightarrow \exists e' [e' < e \land \text{theme}(y)(e')]] ] \]

- The three editors were the agents of an event
- In this event, every mistake was caught
- But, each editor is not necessarily involved in a catching event
- In fact, the editors could have been involved in different activities
- If only one editor was the agent of the catching events in which all mistakes were caught, the formalization is right
- Even though the formalization is right, it is not what we intend

How can we capture in intuitive meaning of (1)?

We could introduce the theme argument:

\[ (7) \exists e \exists x [3 \text{ copy editors}(x) \land \text{agent}(x)(e) \land \text{catch}(e) \land \forall y [\text{mistake}(y) \rightarrow \exists e' [e' < e \land \text{theme}(y)(e')]] ] \]
This formalization is also wrong: this only means that the event in which all the editors were involved is a catching event. Suppose only one editor has caught all the mistakes and the two others were engaged in different catching activities.

We could say that the editors were the agents of a minimal event in which every mistake was caught:

\[(8) \exists e \exists x [3 \text{ copy editors}(x) & \text{agent}(x)(e) & \text{Min}(P)(e)],\]

where \( P = \lambda e \forall y [\text{mistake}(y) \rightarrow \exists e'' [e'' < e & \text{catch}(y)(e'')]] \), and

\( \text{Min} = \lambda Q \lambda e [Q(e) & \neg \exists e' [e' < e & Q(e')]] \)

This formulization excludes some scenarios we do not want to exclude. Suppose there are two mistakes in the manuscript: A and B. One of the editors found A but not B. The other two both found B but not A. In such situation (1) is true while (8) is false.

We ultimately want to say that the three copy editors were the agents of an event that was a completed event of catching every mistake in the manuscript. It is an event in which every mistake in the manuscript was caught, and which does not contain anything that is irrelevant to the enterprise of catching mistakes in the manuscript, i.e. it is an event in which every mistake in the manuscript was caught, and which is a catching of mistakes in the manuscript.

The intuition is captured in a formula below:

\[(9) \exists e \exists x [3 \text{ copy editors}(x) & \text{agent}(x)(e) & \forall y [\text{mistake}(y) \rightarrow \exists e' [e' \leq e & \text{catch}(y)(e')]] & \exists z [\text{mistakes}(z) & \text{catch}(z)(e)]] \).

But this is exactly how event descriptions are understood in Davidsonian semantics.

**IV. Cumulativity**

Unlike agent arguments, **theme arguments are not cumulative**.

(1) Al, Bill and Carl planted a rosebush.

**Scenario:**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( e_1 )</td>
<td>Al dug a hole</td>
</tr>
<tr>
<td>( e_2 )</td>
<td>Bill inserted a rosebush</td>
</tr>
<tr>
<td>( e_3 )</td>
<td>Carl covered soil</td>
</tr>
<tr>
<td>( e_1 \oplus e_2 \oplus e_3 )</td>
<td>Al, Bill and Carl planted a rosebush</td>
</tr>
</tbody>
</table>

a. The agent argument is cumulative

- \( \text{Agent}(e_1 \oplus e_2 \oplus e_3) = \text{Agent}(e_1) \oplus \text{Agent}(e_2) \oplus \text{Agent}(e_3) = Al \oplus Bill \oplus Carl \)
b. The theme argument is not cumulative
   • Theme($e_1 \oplus e_2 \oplus e_3$) = a rosebush
   • Theme($e_1$) $\oplus$ Theme($e_2$) $\oplus$ Theme($e_3$) = a hole $\oplus$ a rosebush $\oplus$ soil
   • Theme($e_1 \oplus e_2 \oplus e_3$) $\neq$ Theme($e_1$) $\oplus$ Theme($e_2$) $\oplus$ Theme($e_3$)

   “Themes lack the conceptual independence of agents. Theme arguments seem to be tightly linked to
   their verbs. Agents are different. Actions seem to have agents independently of how we describe them.”

(2) The relation that holds between a planting event and an individual is cumulative:
   • $\forall e \forall e' \forall x \forall y[[plant(x)(e) \land plant(y)(e')] \rightarrow plant(x \oplus y)(e \oplus e')]

**Problem** The premise of this argument is to grant an equation: the event of planting is identical to the
sum of the three events, $e_1 \oplus e_2 \oplus e_3$. That is, the sum of digging, inserting and covering is planting.
However, intuitively, this equation is hard to accept (Williams 2009)

(3) John killed Peter.

   **Scenario:**
   
   $e_1$ | John bought a knife  
   $e_2$ | John went to Peter’s house  
   $e_3$ | John stuck the knife into Peter’s breast  
   $e_4$ | John killed Peter

   buy $\oplus$ go $\oplus$ stick = kill ??

**References**


Compositional semantics and event semantics

Lucas Champollion
Presented by Paloma Jeretic and Haoze Li

1 Introduction

• Bringing together Davidsonian event semantics and compositional semantics.
• Verbs are existential quantifiers over events.
• Quantificational arguments are given a semantic account.

2 Verbs as existential quantifiers over events

• New definition of verbs: Verbs are interpreted as predicates that hold of sets of events

  • Scope Domain Principle (Landman 1996): Non-quantificational NPs can be entered into
    scope domains. Quantificational NPs cannot be entered into scope domains.

  In other words, the existential closure cannot scope out of a quantificational noun phrase.

  (1) John kissed every girl.
    a. \( \forall x[\text{girl}(x) \rightarrow \exists e[\text{kiss}(e) \land \text{ag}(e, j) \land \text{th}(e, x)]] \)
    b. \# \( \exists e[\text{kiss}(e) \land \text{ag}(e, j) \land \forall x[\text{girl}(x) \rightarrow \text{th}(e, x)]] \)

  In (1b), the individual kissing events are not available, which is problematic.

  • Champollion generalizes the Scope Domain Principle: “In general, the event quantifier always
    takes lowest possible scope with respect to other scope-taking elements.”

  (2) John didn’t laugh.
    a. \( \neg \exists e[\text{laugh}(e) \land \text{ag}(e) = \text{john}] \)
    b. \# \( \exists e[\neg[\text{laugh}(e) \land \text{ag}(e) = \text{john}] \)

  (2b): “There exists an event such that John does not laugh.” This is trivially true, and not the
  meaning of the sentence.

  • A theory-internal reason for giving low scope to the event quantifier: the Unique Role Re-

  (1b) is a contradiction whenever there is more than one girl, because we can only have one
  theme of the kissing event.

  • Existential closure of the event should be included in the lexical entry of the verb so that all
    the other quantifiers will have to take scope above it.
3 Composition

3.1 Heim and Kratzer-style Quantifier Raising in event semantics

(3) John kissed every girl.

(4) Lexicon
a. \([\text{John}]^g = \text{john}\)

b. \([\text{kiss}]^g = \lambda y \lambda x \lambda e [\text{kiss}(e) \land \text{ag}(e) = x \land \text{th}(e) = y]\)

c. \([\text{every}]^g = \lambda P \lambda Q \forall x [P(x) \rightarrow Q(x)]\)

d. \([\text{girl}]^g = \lambda x [\text{girl}(x)]\)

e. \([\exists\text{-closure}]^g = \lambda R \exists e [R(e)]\)

(5) Composition

[Diagram of Composition]

a. \([t_1]^g = g(1)\)

b. \([\text{VP}]^g = \lambda x \lambda e [\text{kiss}(e) \land \text{ag}(e) = x \land \text{th}(e) = g(1)]\)

c. \([\text{S1}]^g = \lambda e [\text{kiss}(e) \land \text{ag}(e) = \text{john} \land \text{th}(e) = g(1)]\)

d. \([\text{S2}]^g = \exists e [\text{kiss}(e) \land \text{ag}(e) = \text{john} \land \text{th}(e) = g(1)]\)

e. \([\text{g}]^g = \lambda x [\text{S2}]^g[1 \rightarrow x] = \lambda x \exists e [\text{kiss}(e) \land \text{ag}(e) = \text{john} \land \text{th}(e) = x]\)

f. \([\text{DP}]^g = \lambda Q \forall x [\text{girl}(x) \rightarrow Q(x)]\)

g. \([\text{S3}]^g = \forall x [\text{girl}(x) \rightarrow \exists e [\text{kiss}(e) \land \text{ag}(e) = \text{john} \land \text{th}(e) = x]]\)

3.2 Champollion’s proposal

• New definition of verbs: Verbs are interpreted as predicates that hold of sets of events

(6) \([\text{kiss}] = \lambda f \exists e [\text{kiss}(e) \land f(e)]\)

• Thematic roles: Type lifting

(7) a. \([\text{th}]^g = \lambda Q \lambda V \lambda f [Q(\lambda x [V(\lambda e [f(e) \land \text{th}(e) = x])])]\)
b. 

\[ [\text{ag}]^g = \lambda Q \lambda V \lambda f [Q(\lambda x[V(\lambda e[f(e) \land \text{ag}(e) = x)])] \quad \langle \langle e, t \rangle, \langle \langle v_t, t \rangle, \langle v_t, t \rangle \rangle \rangle \]

• New closure (similar to lower operation in Barker and Shan and Downarrow operator in Dynamic Semantics)

(8) 

\[ [\text{closure}] = \lambda e.\text{true} \]

• Composition without QR

(9) John kissed every girl.

\[
\begin{array}{c}
\text{CP} \\
\downarrow \text{closure} \\
\text{IP} \\
\downarrow \text{DP2} \\
\text{VP} \\
\downarrow \text{John} \\
\text{ag} \\
\text{kiss} \\
\text{DP1} \\
\text{every girl} \\
\text{th}
\end{array}
\]

Lexicon, FA

a. \[ [\text{every girl}] = \lambda P_{(et)} \forall x[\text{girl}(x) \rightarrow P(x)] \]

b. \[ [\text{th}] = \lambda Q_{(et,t)} \lambda V_{(vt,t)} \lambda f_{(vt)}[Q(\lambda x[V(\lambda e[f(e) \land \text{th}(e) = x)])] \]

c. \[ [\text{DP1}] = \lambda V_{(vt,t)} \lambda f_{(vt)} \forall x[\text{girl}(x) \rightarrow V(\lambda e[f(e) \land \text{th}(e) = x])] \]

d. \[ [\text{kiss}] = \lambda f_{(vt)} \exists e[\text{kiss}(e) \land f(e)] \]

e. \[ [\text{VP}] = \lambda f_{(vt)} \forall x[\text{girl}(x) \rightarrow \exists e[\text{kiss}(e) \land f(e) \land \text{th}(e) = x]] \]

Lexicon

f. \[ [\text{John}] = \lambda P_{(et)}[P(\text{john})] \]

Lexicon

g. \[ [\text{ag}] = \lambda Q_{(et,t)} \lambda V_{(vt,t)} \lambda f_{(vt)}[Q(\lambda x[V(\lambda e[f(e) \land \text{ag}(e) = x)])] \]

Lexicon

h. \[ [\text{DP2}] = \lambda V_{(vt,t)} \lambda f_{(vt)}[V(\lambda e[f(e) \land \text{ag}(e) = \text{john}] \]

Lexicon

i. \[ [\text{IP}] = \lambda f_{(vt)} \forall x[\text{girl}(x) \rightarrow \exists e[\text{kiss}(e) \land f(e) \land \text{ag}(e) = \text{john} \land \text{th}(e) = x]] \]

Lexicon

j. \[ [\text{closure}] = \lambda e.\text{true} \]

Lexicon

k. \[ [\text{CP}] = \forall x[\text{girl}(x) \rightarrow \exists e[\text{kiss}(e) \land \text{true} \land \text{ag}(e) = \text{john} \land \text{th}(e) = x]] \]

FA

\[ = \forall x[\text{girl}(x) \rightarrow \exists e[\text{kiss}(e) \land \text{ag}(e) = \text{john} \land \text{th}(e) = x]] \]

FA

Inverse scope

• Lifted ‘theme’ (Hendrick-style)

(10) \[ [\text{th-lift}] = \lambda Q_{(et,t)} \lambda V_{(vt,t)} \lambda M_{(vt,t,t)} \lambda f_{(vt)}[Q(\lambda x[M(V)(\lambda e[f(e) \land \text{th}(e) = x)])] \]

(11) A diplomat visited every country.
Distributive shift

(12) John invited four girls.

a. $\exists X \subseteq girl, |X| = 4 \land \exists e[\text{invite}(e) \land ag(e) = \text{john} \land th(e) = X]$  Group reading

b. $\exists X \subseteq \text{girl}, |X| = 4 \land \\
\forall y, y \in X \rightarrow \exists e[\text{invite}(e) \land ag(e) = \text{john} \land th(e) = y]$  Distributive reading

(13) dist-shift = $\lambda Q_{\langle et, t \rangle} \lambda P_{\langle et \rangle}. Q(\lambda x. \forall y. y \in X \rightarrow P(y))$

(14) Deriving the distributive reading
Negation

(15)  John didn’t laugh for two hours.

   a. For two hours, it was not the case that John laughed.
   b. It is not the case that John laughed for two hours.

(16)  Deriving the reading *for two hours > not*
a. \[PP = \lambda V_{(ut,t)} \lambda f_{(ut)} \exists t[\text{hours}(t) = 2 \land t \subseteq T \land \forall t'[t' \subseteq t \rightarrow V(\lambda e[f(e) \land \tau(e) = t']))] \]

b. \[\text{not} = \lambda V_{(ut,t)} \lambda f_{(ut)} \cdot \neg V(f) \]

c. \[\text{[laugh]} = \lambda f_{(ut)} \exists e[\text{laugh}(e) \land f(e)] \]

d. \[\text{[VP1]} = \lambda f_{(ut)} \exists t[\text{hours}(t) = 2 \land t \subseteq T \land \forall t'[t' \subseteq t \rightarrow \neg \exists e[\text{laugh}(e) \land f(e) \land \tau(e) = t'])] \]

e. \[\text{[VP2]} = \lambda f_{(ut)} \exists t[\text{hours}(t) = 2 \land t \subseteq T \land \forall t'[t' \subseteq t \rightarrow \neg \exists e[\text{laugh}(e) \land f(e) \land \tau(e) = t'])] \]

f. \[\text{[DP]} = \lambda V_{(ut,t)} \lambda f_{(ut)} [V(\lambda e[f(e) \land \text{ag}(e) = \text{john})]) \]

g. \[\text{[IP]} = \exists t[\text{hours}(t) = 2 \land t \subseteq T \land \forall t'[t' \subseteq t \rightarrow \neg \exists e[\text{laugh}(e) \land f(e) \land \tau(e) = t'])] \]

h. \[\text{[past-closure]} = \lambda V_{(ut,t)}[t_r \ll \text{now} \land V(\lambda e[\tau(e) \subseteq T \land t_r])] \]

i. \[\text{[CP]} = t_r \ll \text{now} \land \exists t[\text{hours}(t) = 2 \land t \subseteq T \land \forall t'[t' \subseteq t \rightarrow \neg \exists e[\text{laugh}(e) \land \text{ag}(e) = \text{john} \land \tau(e) = t')]] \]

(17) Deriving the reading not > for two hours

\[
\begin{array}{c}
\text{CP} \\
\text{past-closure} \\
\text{IP} \\
\text{DP} \\
\text{VP2} \\
\text{John} \\
\text{ag} \\
\text{not} \\
\text{VP1} \\
\text{laugh} \\
\text{PP} \\
\text{for two hours}
\end{array}
\]

a. \[PP = \lambda V_{(ut,t)} \lambda f_{(ut)} \exists t[\text{hours}(t) = 2 \land t \subseteq T \land \forall t'[t' \subseteq t \rightarrow V(\lambda e[f(e) \land \tau(e) = t'])] \]

b. \[\text{[laugh]} = \lambda f_{(ut)} \exists e[\text{laugh}(e) \land f(e)] \]

c. \[\text{[VP1]} = \lambda f_{(ut)} \exists t[\text{hours}(t) = 2 \land t \subseteq T \land \forall t'[t' \subseteq t \rightarrow \exists e[\text{laugh}(e) \land f(e) \land \tau(e) = t'])] \]

d. \[\text{[not]} = \lambda V_{(ut,t)} \lambda f_{(ut)} \cdot \neg V(f) \]

e. \[\text{[VP2]} = \lambda f_{(ut)} \exists t[\text{hours}(t) = 2 \land t \subseteq T \land \forall t'[t' \subseteq t \rightarrow \exists e[\text{laugh}(e) \land f(e) \land \tau(e) = t'])] \]

f. \[\text{[DP]} = \lambda V_{(ut,t)} \lambda f_{(ut)} [V(\lambda e[f(e) \land \text{ag}(e) = \text{john})]) \]

g. \[\text{[IP]} = \lambda f_{(ut)} \exists t[\text{hours}(t) = 2 \land t \subseteq T \land \forall t'[t' \subseteq t \rightarrow \exists e[\text{laugh}(e) \land f(e) \land \tau(e) = t'])] \]

h. \[\text{[past-closure]} = \lambda V_{(ut,t)}[t_r \ll \text{now} \land V(\lambda e[\tau(e) \subseteq T \land t_r])] \]

i. \[\text{[CP]} = t_r \ll \text{now} \land \neg \exists t[\text{hours}(t) = 2 \land t \subseteq T \land \forall t'[t' \subseteq t \rightarrow \exists e[\text{laugh}(e) \land f(e) \land \text{ag}(e) = \text{john} \land \tau(e) = t'])] \]
Semantics I presupposes a solid working knowledge of

set theory, propositional logic, and predicate logic.

You may use this problem set in combination with the first five chapters of Allwood, Anderson, and Dahl, *Logic for Linguistics* (Cambridge UP) or any other textbook that you have already studied. I will be happy to work with you, using email or Skype, over the summer, if you need assistance with any aspect of the material. Please return the problem set to me at least one week before the start of the semester.
Set Theory

(1) Let A be \{a, b, c\}.
(a) Is \emptyset an element of A? ______ Explain why.
(b) Is \emptyset a subset of A? ______ Explain why.

(2) (a) Let K be the set \{a, b\}. Give the power set of K. (How many elements should Pow(K) have? Did you get that many? If not, go back and reconsider.)

_____________________________________________________________________

(b) Let L be the set \{a, \emptyset\}. Give the power set of L. (How many elements should Pow(L) have? Did you get that many? If not, go back and reconsider.)

_____________________________________________________________________

(c) Let M be the set \{a, \{\emptyset\}\}. Give the power set of M. (How many elements should Pow(M) have? Did you get that many? If not, go back and reconsider.)

_____________________________________________________________________

(3) Assume a universe U = \{a, b, c, d\}, A = \{a, b, c\}, B = \{b, c\}; as always, \emptyset is the empty set.
(a) What set is A \cap B? ________________________________________________

(b) What set is A \cup B? ________________________________________________

(c) What set is A \cap \emptyset? ___________________________________________

(d) What set is A \cup \emptyset? ___________________________________________

(e) Complete the following: Whenever set X is smaller than or equal to set Y, their union is the set _____ and their intersection is the set _____.
(4) Assume the same U, A, and B as in (3).

What are (all) the sets Z such that $A \cap Z = A$? Give the set of all such sets using the {...} notation.

What are (all) the sets W such that $A \cup W = A$? Use the {...} notation.

(5) Venn-diagrams can be used to determine whether two sequences of set theoretic operations lead to the same result. For instance:

Does it make a difference whether (i) we first intersect A and B and then union the result with C, or (ii) we union B and C and then intersect the result with A? Determine by shading the relevant areas.

(a) \((A \cap B) \cup C =? = A \cap (B \cup C)\)

(b) \((A \cap B) \cap C =? = A \cap (B \cap C)\)
(c) \((A \cup B) \cap C =?= (A \cap C) \cup (B \cap C)\)

(d) \((A \cap B) \cup C =?= (A \cup C) \cap (B \cup C)\)

(6) Using Venn-diagrams, determine whether the following equivalences are valid or not. Draw clear diagrams. Complements are understood with reference to the universe of discourse, so don’t forget to enclose A and B in a box representing the universe. (I write the complement of A as \(\neg A\).)

(a) \(\neg(A \cup B) = (\neg A) \cup (\neg B)\)
(b) \[ --(A \cap B) = (--A) \cup (--B) \]

(7) Below you find the definitions of three functions in terms of ordered pairs:

\[
\begin{align*}
f &= \{<a,b>, <b,c>, <d,b>\} \\
g &= \{<b,a>, <c,b>, <d,b>\} \\
h &= \{<b,c>, <d,b>, <a,b>\}
\end{align*}
\]

Are these functions identical? Justify your answer in words and by representing functions graphically (a separate diagram for each function). Use arrows to indicate how the function maps each element of the domain to an element of the co-domain (range) on the next page.

<table>
<thead>
<tr>
<th>domain</th>
<th>co-domain (range)</th>
<th>domain</th>
<th>co-domain (range)</th>
</tr>
</thead>
<tbody>
<tr>
<td>f: a</td>
<td>a</td>
<td>g: a</td>
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(8) Make a one-letter change in the definition of function \( f \) in (7) so that the result is no longer a function (it will be merely a relation).
Propositional logic

(1) Which of the following expressions is/are ambiguous (and thus ill-formed) in propositional logic? How can the ambiguous one(s) be disambiguated using parentheses? Explain why the other(s) is/are unambiguous.

(a) \( p \land q \lor r \)

(b) \( \neg p \land q \)

(c) \( p \lor q \lor r \)

(2) Below you see the truth tables of two propositional connectives, notated as \# and $.

(a) Shade those areas of the Venn-diagrams where \( p \# q \) and \( p \$ q \) are true.

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>#q</th>
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<tbody>
<tr>
<td>1</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>2</td>
<td>T</td>
<td>F</td>
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<tr>
<td>3</td>
<td>F</td>
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<tr>
<td>4</td>
<td>F</td>
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</table>
(b) Is # identical to one of the connectives discussed in Allwood et al. 4.2? If yes, which?

(c) Is $ identical to one of the connectives discussed in 4.2? If yes, which?

(d) The contribution of the new connective can be defined using a combination of familiar connectives. Express the compound proposition involving the new connective using $p$, $q$, two or more of the familiar connectives, and parentheses, as necessary.

(3) Consider material implication. Try to define its contribution using
(a) disjunction+negation, and
(b) conjunction+negation.

Hint: you can think this way: “$p \rightarrow q$ is true iff $p$ is ….. and(?)/or(?) $q$ is …..”,
and this way: “$p \rightarrow q$ is false iff $p$ is ….. and(?)/or(?) $q$ is …..”.

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<tr>
<th>p</th>
<th>q</th>
<th>$p \rightarrow q$</th>
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<tr>
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<td>F</td>
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</table>

(c) Write out the truth tables for (a) and for (b), showing that they indeed yield the same T-F-T-T pattern in their final columns as the above definition of $p \rightarrow q$. 
(4) The following tautology can be easily explained using set theory
(t stands for 'tautology,' f for 'contradiction'):

(a) \( t \land p \equiv p \)
   The conjunction of a tautology \( t \) and some sentence \( p \) that may be either true or false has
   the same truth value as \( p \).
   Explanation: Let the set of worlds in which \( p \) is true be \( P \).
   Since \( t \) is a tautology, and a tautology is true in every world, \( P \subseteq T \).
   We know that whenever \( A \subseteq B \), \( A \cap B = A \).
   Hence, \( P \cap T = P \).

Translate (b)-(c)-(d) into English and explain them using set theory, as above:

(b) \( p \lor t \equiv t \)

(c) \( p \land f \equiv f \)

(d) \( p \lor f \equiv p \)
(5) **Optional (but very much encouraged):** Show how $\lor$ and $\land$ can be expressed using just Sheffer's stroke $|$ (which means 'not both'). Hint: notice that $p|p$ is equivalent to $\neg p$ (as below), and go on to get two (different) propositional variables into the picture.

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<tbody>
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<td>f</td>
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(6) **Optional (but very much encouraged):** Sheffer’s stroke by itself can do the same job as any combination of the widely-used propositional connectives. Why? – Sheffer’s stroke is not the only guy that can do this job, and do it elegantly. Can you guess what the meaning of the other such guy?
Some important tautologies (valid, or always true, propositions), cf. (8) on p.55 of Allwood et al.

(i) \( p \lor \neg p \)    (either \( p \) or its negation is true)
(ii) \( \neg (p \land \neg p) \)    (\( p \) and its negation can't be true together)
(iii) \( p \equiv p \)    (everything is identical to itself)
(xiv) \( \neg \neg p \equiv p \)    (two negations [reversals] cancel out)
(xv) \( p \lor p \equiv p \)    (the union of \( P \) with itself is \( P \))
(xvi) \( (p \equiv q) \equiv (\neg p \equiv \neg q) \)    (negating both sides of an equation preserves the equation)

De Morgan Laws (the laws about duals – conjunction and disjunction inherit their duality from intersection and union):

(vi) \( \neg (p \lor q) \equiv \neg p \land \neg q \)
(vii) \( \neg (p \land q) \equiv \neg q \lor \neg p \)

Expressing implication using negation+disjunction or negation+conjunction:

(xiii) \( (p \implies q) \equiv (\neg p \lor q) \)    (implication is true iff antecedent is false or consequent is true)
(x) \( \neg (p \implies q) \equiv (p \land \neg q) \)    (implication is false iff antecedent is true and consequent is false)

The above tautologies are conceptually basic. You need to know them by heart.

Some other tautologies that can be derived using the above are, e.g.,

(iv) \( (p \lor q) \equiv \neg (\neg p \land \neg q) \)
(v) \( (p \land q) \equiv \neg (\neg p \lor \neg q) \)
(ix) \( (p \land q) \equiv \neg (p \implies \neg q) \)
(x) \( (p \implies q) \equiv (p \land \neg q) \)
(xi) \( \neg (p \land q) \equiv (p \implies \neg q) \)
(xii) \( (p \lor q) \equiv (\neg p \implies q) \)
(xvii) \( p \equiv (p \land p) \)
Conversational Predicate Logic

**John walks:**  
$W(j)$

John walks and talks:  

John or Mary talks:  

John saw Mary:  

**Something barks:**  
$\exists x [B(x)]$

Something is a dog and it barks =  
Something that is a dog barks =  
**Some dog barks:**  
$\exists x [D(x) \land B(x)]$

Some dog barks or growls:  

Some dog does not bark:  

No dog barks:  

John saw some dog:  

**Everything changes:**  
$\forall x [C(x)]$

Everything changes if it is a rule =  
Everything that is a rule changes =  
**Every rule changes:**  
$\forall x [R(x) \rightarrow C(x)]$

Every rule changes or gets eliminated:  

Not every rule changes:  

No rule changes:
Every dog saw John:

John likes Mary.
Bill likes somebody else.  $\exists x [x \neq m \land L(b, x)]$

John likes Mary.
Bill likes everybody else.

John likes himself.

John likes only himself.

At least one dog barks.  $\exists x [D(x) \land B(x)]$

At least two dogs bark.

Exactly two dogs bark.

Two dogs bark and nothing else does.
Free vs. bound variables; duals

Free versus bound variables

- **blue(x) ≠ blue(y)**  
  Why?

Pointing with free variables:

**This is blue** 'the entity I am pointing to (with my left hand) is blue'

**That is blue** 'the entity I am pointing to (with my right hand) is blue'

The free variables x and y may accidentally point to the same object, but in general, you cannot assume that they do so. Free variables have their own individual identity.

- **∀x[blue(x)] ≡ ∀y[blue(y)]**  
  **∃x[blue(x)] ≡ ∃y[blue(y)]**  
  Why?

Making general statements with quantifiers using bound variables:

**Everything is blue** 'Whatever I point to, that entity is blue'

**Something is blue** 'I can point to an entity that is blue'

A bound variable is a mere placeholder. The sentence says nothing specifically about x or y.

Duals (= are related to each other via negation in a particular way)

- **∃ and ∀ are duals.**  
  Why?

Imagine a universe with 3 elements: a, b, c. Then:

**∃x[blue(x)]** ≡ **blue(a) ∨ blue(b) ∨ blue(c)**

**∀x[blue(x)]** ≡ **blue(a) & blue(b) & blue(c)**

The propositional connectives ∨ and & are duals -- see the de Morgan laws. Then ∃, which is disjunction over the universe and ∀, which is conjunction over the universe, are necessarily duals, too.

- **¬∀x [blue(x)]** ≡ **∃x¬ [blue(x)]** and  
  **¬∃x [blue(x)]** ≡ **∀x ¬ [blue(x)]**  
  in other words,

- **¬∀x ¬ [blue(x)]** ≡ **∃x [blue(x)]** and  
  **¬∃x ¬ [blue(x)]** ≡ **∀x[blue(x)]**

because

- **¬(blue(a) & blue(b) & blue(c))** ≡ **¬blue(a) ∨ ¬blue(b) ∨ ¬blue(c)** and

- **¬(blue(a) ∨ blue(b) ∨ blue(c))** ≡ **¬blue(a) & ¬blue(b) & ¬blue(c)**
Square of oppositions

all N and no N are each other's internal negations because
All men do NOT walk = No man walks and
No man does NOT walk = All men walk

all N and not all N are each other's external negations because
NOT all men walk = Not all men walk and
NOT not all men walk = All men walk

all N and an N are each other's duals because
NOT all men do NOT walk = A man walks and
It is NOT the case that a man does NOT walk = All men walk etc.

Re: Allwood et al, Ch 5.1-2-3, up to 5.4 (The semantics of predicate logic).

On p. 64 the sentence, “In (12), $K(y)$ can be substituted for $K(x)$ without changing the meaning of the expression” is not quite right. The variable $x$ that occurs in $K(x)$ is free (the scope of the universal quantifier, indicated by parentheses, ends before $K(x)$). It is true that replacing this $x$ with $y$ would not affect the binding situation. But given a particular assignment of values to variables, $x$ may be getting the value Jack and $y$ the value Jill, which does make a difference.

On p. 67 the book introduces restricted quantification. That is fine, but please do not use it. We’ll deal with the “counterintuitive” aspect of predicate logic later. Likewise, do not use variable symbols that indicate that the value comes from a particular noun-set, e.g. $\exists d [B(d)]$ for “Some dog barks”. This notation would also correspond to the idea of restricted quantification, but it is not legal in predicate logic.

On p. 70, reflects the state of Chomskyan syntax when the book was first written. It is no longer assumed that transformations preserve meaning (depending on what version of syntax you are learning, it may not even have “transformations”), so the fact that (28a) and (28b) are not synonymous is no longer a problem.

Finally, if you look through the syntax of predicate logic, you’ll notice that it does not include equality (the predicate constant “=” that features in “$x=y$”). This means that the version of the logic defined in the book can express fewer things than we want. So please add “=” to the vocabulary on p. 71, and the following clause to p. 72:

“If $t_1$ and $t_2$ are individual terms (constants or variables), $t_1= t_2$ is a well-formed formula.”
Abbreviate predicates with the first letter of the predicate; you don’t need to give keys.

(1) (a) Express the meaning of **No man flies**, using

the quantifier $\forall$ (and whatever else it takes): ____________________________________

the quantifier $\exists$ (and whatever else it takes): ____________________________________

(b) Express the meaning of **Not every man flies**, using

the quantifier $\forall$ (and whatever else it takes): ____________________________________

the quantifier $\exists$ (and whatever else it takes): ____________________________________

(c) Express the meaning of **Some man flies**, using

the quantifier $\forall$ (and whatever else it takes): ____________________________________

the quantifier $\exists$ (and whatever else it takes): ____________________________________

(2) (a) Express the meaning of **It is not the case that I didn’t compose a symphony** without using the symbol $\neg$:

(3) Express the meanings of the following English sentences using predicate logic. If you think the sentence is truth-conditionally ambiguous, try to give all the relevant formulae.

(a) John left and Mary stayed.

(b) John and Bill left.

(c) John sighed and left.

(d) Every man and woman left.
(e) Every man left, but some woman stayed.

(f) John didn’t greet any guests.

(g) John did not greet some guest.

(h) Some guest did not leave.

(i) Only John left.

(j) John, too, left.

(k) John saw only dogs.

(l) John has all sisters.

(m) The Dutchman left.

(4) Which of these equivalences are tautologies? I.e. when do the two sides say the same thing, no matter what? Briefly explain why (or why not). Pay very close attention to the choice of the variables and to the square brackets [...] that indicate the scopes of the quantifiers.

(a) \( \forall x \left[ F(x) & G(y) \right] \equiv \forall z \left[ F(z) & G(y) \right] \)

(b) \( \forall x \left[ F(x) \right] & G(x) \equiv \forall y \left[ F(y) \right] & G(y) \)

(c) \( \forall x \left[ F(x) & g(x) \right] \equiv \forall y \left[ F(y) & G(y) \right] \)

(d) \( G(x) & \forall y \left[ F(y) \right] \equiv G(x) & \forall x \left[ F(x) \right] \)

(e) \( \forall x \left[ F(x) & G(x) \right] \equiv \forall y \left[ F(y) & G(x) \right] \)
(5) Which of these equivalences are tautologies? (I.e. when do the two sides say the same thing, no matter what?) Briefly explain why (or why not).

(a) \( \forall x \neg[C(x) & B(x)] \equiv \neg \exists x[C(x) & B(x)] \)

(b) \( \forall x \neg[C(x) & B(x)] \equiv \forall x[C(x) \lor B(x)] \)

(c) \( \forall x \neg[C(x) & B(x)] \equiv \forall x[\neg C(x) \lor \neg B(x)] \)

(d) \( \neg \forall x[C(x) & B(x)] \equiv \exists x[\neg C(x) \lor \neg B(x)] \)

(e) \( \forall x \neg[C(x) \rightarrow B(x)] \equiv \forall x[C(x) \land \neg B(x)] \)

If you are new to predicate logic and you are confident about and comfortable with everything so far, you are doing well! Take a rest before you go on. If you find that the following segment is too difficult to tackle on your own, don’t despair, just let me know.
The semantics of predicate logic

Suppose the universe consists of three dogs and four cats, and arrows correspond to barking at something:

(1) Every dog barked at some cat (on the direct scope reading, S>O)

∀x[D(x) → ∃y[C(y) & B(x, y)]] is true,
because for every individual in the universe, if it is a dog, then we find a cat that it barked at, and if it is not a dog, it does not matter whether we find a cat that it barked at:

(i) D(a) and C(d) and B(a, d) and
(ii) D(b) and C(e) and B(b, e) and
(iii) D(c) and C(g) and B(c, h) and
(iv) D(d), D(e), D(f) and D(h) are false.

(2) Every dog barked at some cat (on the inverse scope reading, O>S)

∃y[C(y) & ∀x[D(x) → B(x, y)]] is false,
because we do not find any individual that is a cat and every dog barked at it. It would be true if, for example, each of a, b, and c had barked at f.
Strategy: Working “from outside in”, cash out each quantifier in terms of individuals.

If the quantifier is universal, check whether its scope holds true for every individual that can be assigned to the variable that the quantifier binds. Cf. $\forall x[F(x)] = F(a) \& F(b) \& ... \& F(h)$.

If the quantifier is existential, check whether its scope holds for at least one individual that can be assigned to the variable that the quantifier binds. Cf. $\exists y[G(y)] = G(a) \lor G(b) \lor ... \lor G(h)$.

Two ways of saying the same thing:
For every individual $x$, $F(x)$ is true = Every way of assigning an individual to variable $x$ makes $F(x)$ true.
For some individual $y$, $G(y)$ is true = There is at least one way of assigning an individual to variable $y$ that makes $G(y)$ true.

**Compositionality**: The truth conditions of a formula of predicate logic are uniquely determined by the truth conditions of its constituent parts.

$G(c)$ is true iff the individual that $c$ refers to is in the extension of $G$.
$\exists y[G(y)]$ is true iff at least one individual that can be assigned to variable $y$ is in the extension of $G$.
$\exists y[G(y) \& I(y) \& K(x)]$ is true iff at least one individual that can be assigned to variable $y$ is in the extension of $G$ and in the extension of $I$, and the individual that is currently assigned to $x$ (the individual that $x$ is “pointing at”) is in the extension of $K$.

**Model theoretic semantics**:

The actual truth of each formula is determined with respect to a particular model $M$ and an assignment $g$ of individuals to variables. In predicate logic, a model is a set $U$ of individuals (the universe of discourse) with a specification $I$ of what individuals the individual terms refer to, and what sets, relations, and functions are in the extensions of the predicates expressions. This specification $I$ is called an interpretation function for the constant (non-variable) expressions in the syntax.

The valid formulas of predicate logic (tautologies) are true in all models, or under all interpretations, irrespective of how big the universe is and what the extensions of the predicates happen to be in it.
Which of the following are valid? Construct models that falsify them, if you can.

(3) \((\forall x[D(x) \rightarrow F(x)] \& D(a)) \rightarrow F(a)\)

(4) \((\exists x[D(x) \& F(x)] \& D(a)) \rightarrow F(a)\)

(5) \((\forall x \forall y[H(x,y)] \rightarrow \forall y \forall x[H(x,y)]\)

(6) \((\exists x \exists y[J(x,y)] \rightarrow \exists y \exists x[J(x,y)]\)

(7) \((\forall x \exists y[L(x,y)] \rightarrow \exists y \forall x[L(x,y)]\)

(8) \((\exists y \forall x[L(x,y)] \rightarrow \forall x \exists y[L(x,y)]\)

(9) \((\forall x[A(x) \lor B(x)] \rightarrow (\forall x[A(x)] \lor \forall x[B(x)]))\)

(10) \((\forall x[A(x) \& B(x)] \rightarrow (\forall x[A(x)] \& \forall x[B(x)]))\)
(11) Work through Exs 14, 15, and 16 using the book’s solutions as a guide. You don’t need to hand these in, but don’t skip them; problems of this sort will come up on the midterm.

For Ex 15, note that the book represents the “like” relation as a set of pairs, where the first member of the pair is the liker and the second is the liked one. I have no idea why there are no commas between the pairs enclosed in curly brackets (typo?); there should be commas, just as there as commas in \{a,b\}. So the formal description of the model M (universe U plus interpretation function I) in Ex 15 corresponds to the following picture. I do not indicate Boy=\{a,b\} and Girl=\{c,d\} so as not to muddle up the drawing.

\[
\begin{align*}
L = \{&<a,a>, <b,b>, <c,c>, \\
&<a,c>, <a,d>, <b,c>, <c,a>, <d,a>\}
\end{align*}
\]

(12) A modification of Ex 17. All the sentences in Ex 1 are true in the following model:

\[
\begin{align*}
A = \{&<b,f>, <a,a>\} \\
B = \{&<f,b>, <b,b>\}
\end{align*}
\]

Explain why, for each sentence in (a) through (g).
(13) Translate these statements into predicate logic.

(a) \( \text{fish} \subseteq \text{yawn} \)

(b) \( \text{fish} \cup \text{yawn} = \emptyset \)

(c) \( \text{freddie} \in \text{fish} \cap \text{yawn} \)

(d) \( \text{fish} \cap \text{yawn} \neq \emptyset \)

(e) \( |\text{fish} \cap \text{yawn}| > 2 \)  

\( |A| \) signifies the cardinality of the set \( A \)

(14) Are the following valid, i.e. true in all models/interpretations? Why?

(a) \( \exists x [A(x) \lor B(x)] \rightarrow (\exists x [A(x)] \lor \exists x [B(x)]) \)

(b) \( (\exists x [A(x)] \lor \exists x [B(x)]) \rightarrow \exists x [A(x) \lor B(x)] \)

(c) \( \exists x [A(x) \& B(x)] \rightarrow (\exists x [A(x)] \& \exists x [B(x)]) \)

(d) \( (\exists x [A(x)] \& \exists x [B(x)]) \rightarrow \exists x [A(x) \& B(x)] \)

(15) Express these using predicate logic.

(a) Some dog barks and some cat does not.

(b) No dog is hungry.

(c) Every dog is both hungry and sleepy.

(d) Not every dog is ferocious.
(e) Fido and Spot resemble each other.

(f) Some boy hates every other boy.

(g) There is a chair in every corner.

(h) Exactly one cat meowed.

(i) One cat and nothing else meowed.

Let triangles be tapirs and boxes bison; let solid lines stand for ‘sees’ and dashed lines for ‘dread’.

Are the following sentences true in this world?

\[
\begin{align*}
\exists x \forall y [\neg S(x,y) & \land \neg S(y,x)] \\
\forall x \exists y [S(x,y)] \\
\forall x [S(x,x) & \lor \exists y [x \not= y \land S(x,y)]] \\
\forall x \forall y [(B(x) & S(x,y)) & \rightarrow \neg S(x,x)] \\
\forall x [S(x,x) & \rightarrow \neg \exists y [B(y) & S(x,y)]] \\
\forall x [D(x,x) & \rightarrow \neg \exists y [T(y) & S(x,y)]] \\
\forall x [B(x) & \rightarrow \exists y [S(x,y)]] \\
\exists x [D(c,x) \leftrightarrow D(x,x)]
\end{align*}
\]