Estimating the Severity of the WikiLeaks United States Diplomatic Cables Disclosure*

Michael Gill† Arthur Spirling‡

December 19, 2014

Prepared for PA Letters

Abstract

In November 2010, the WikiLeaks organization began the release of over 250,000 diplomatic cables sent by US embassies to the US State Department, uploaded to its website by (then) Private Manning, an intelligence analyst with the US Army. This leak was widely condemned, including by the then Secretary of State, Hillary Clinton. We assess the severity of the leak by considering the size of the disclosure relative to all diplomatic cables that were in existence at the time—a quantity that is not known outside of official sources. We rely on the fact that the cables that were leaked are internally indexed in such a way that they may be treated as a sample from a discrete uniform distribution with unknown maximum; this is a version of the well known “German Tank Problem”. We consider three estimators that rely on discrete uniformity—maximum likelihood, Bayesian and frequentist unbiased minimum variance—and demonstrate that the results are very similar in all cases. To supplement these estimators, we employ a regression-based procedure that incorporates the timing of cables’ release in addition to their observed serial numbers. We estimate that, overall, approximately 5% of all cables from this timeframe were leaked, but that this number varies considerably at the embassy-year level. Our work provides a useful characterization of the sample of documents available to international relations scholars interested in testing theories of ‘private information’, while helping to inform the public debate surrounding Manning’s trial and thirty-five year prison sentence.

*December 19, 2014. Jonathan Bennett and Michael Egesdal provided helpful comments on an earlier draft. We are grateful to the Office of the General Counsel at Harvard University for legal advice. Replication materials (code, not data) for the estimators we describe can be found in the Political Analysis archive.

†Department of Government, Harvard University. mzgill@fas.harvard.edu

‡Department of Government, Harvard University. aspirling@gov.harvard.edu
1 Motivation

For scholars of international relations, the WikiLeaks US diplomatic cable disclosure of 2010 is a promising source of data on the American government’s beliefs and strategies as they pertain to interactions with other states. A mainstay of the now “dominant” (Lake, 2010) ‘rationalist’ approach to war (Fearon, 1995), this type of ‘private information’ is by its very nature difficult to obtain for analysts: it may never be released on the grounds that to do so would damage national interests, endanger its citizens (though see Shapiro and Siegel, 2010, for discussion) or more cynically, because it might allow popular challenge to elite control of foreign policy (Gibbs, 1995). While this has not necessarily prevented data-driven scholarship on the canonical bargaining model of conflict (e.g. Fearon, 1994b; Werner, 1999; Reed, 2003; Smith and Stam, 2004; Ramsay, 2008; Reiter, 2009) or historical case studies of important incidences of decision-making (e.g. Goemans, 2000; Snyder and Borghard, 2011), it does mean that such work cannot directly examine all the incentives faced, and actions taken, by leaders in the contemporary period. Precisely because it contains documents that were not designed for public release and thus are surely more candid about particular political relationships, the WikiLeaks disclosure potentially represents a trove of data that might be tapped to test subtle theories in international relations. These include hypotheses regarding ‘audience costs’ (Fearon, 1994a) and the signaling of resolve (see e.g. Smith, 1998; Schultz, 1999; Slantchev, 2006).

As with any ‘new’ data source, it is helpful to understand the nature of the sample available for researchers before they conduct statistical analysis. A fundamental question in this respect is the size—i.e., the ‘severity’—of the leak (sample) relative to all cables available (the population) at the time. If this sample is ‘small’—and far from the universe of cases—then some caution is required in interpreting results using the documents. This is especially true
when examining particular embassies (in particular countries) in particular years for which the data is necessarily more finely sliced. For example, if researchers have an interest in recent US diplomatic policy in certain areas of the world (e.g. Christensen, 2006), the utility of studying the disclosed cables presumably varies according to the density of their coverage: here, knowing that one has access to 5% of all instructions given to ambassadors is very different to having 95% or 100% of those orders. Furthermore, knowledge of population size is helpful since it prompts further investigation of the sampling process itself: that is, it encourages analysts to think carefully about possible selection biases—especially for data such as these that were not systematically released according to well known bureaucratic rules—and adjust for them in their statistical approaches.

Quite separate to these methodological concerns, the Manning case received widespread public attention and broader interest, including accusations of inhumane treatment from the United Nations special rapporteur on torture.\(^1\) Whether or not Manning’s sentence was just depends in part on the purported severity of the crimes she committed, and in part on the normatively appropriate relationship between the state and secrecy—a topic of recent interest to political theorists (see Sagar, 2013). Since this Letter may be seen as an attempt to measure the first of these issues, we see it as a contribution to the public debate surrounding the trial.

2 Data

The WikiLeaks data consist of 251,237 diplomatic cables sent by the US State Department to US embassies and missions around the world. Machine readable versions of these documents are available at various websites, but what is not known to researchers is the fraction

\(^1\)“Bradley Manning’s treatment was cruel and inhuman, UN torture chief rules”, *The Guardian* March 12, 2012.
of all cables in existence that the leak represents. The original data date range is from 1966 to 2010 but we focus on all cables released on or after January 1, 2005. We do this because coverage prior to the year 2005 is sparse (there are approximately as many cables from 2005–2010 as there from 1966–2005); second, because we wished to guard against any change in protocols—concerning the nature of the cables—that the terrorist attacks of September 11, 2001 may have ushered in and that we assume were in full force by 2005. This leaves 163,958 cables.

Cables are classified into three categories, depending on the degree of damage to national security that “the unauthorized disclosure of which reasonably could be expected to cause.”\(^2\) In descending order of the purported balefulness of unauthorized release, these categories are ‘Top Secret’,\(^3\) ‘Secret’ and ‘Confidential’. Any cable not meeting the criteria for such restricted access is ‘Unclassified’.

From inspection we note that the cable titles, known internally as the ‘Reference ID’s, follow a consistent naming convention. Examples of this pattern can be seen in Table 1: a sample from the US Embassy in Tokyo, for 2009. Consider the Reference ID 09TOKYO13: the first two digits connote the year of creation (2009) (confirmed by the date ‘Created’ column—information that we culled from the original cable texts); this is followed by the issuing embassy (TOKYO); followed by a number (13). This particular cable was unclassified, as can be seen in the third column. Our immediate concern is whether the ‘13’ demarcates that this was, indeed, the 13th document sent by the Embassy that year, and thus whether we can treat all such numbers as indicative of their position in the creation order. Our primary check on this logic was to plot the document numbers against the dates of issue

\(^2\)Executive Order 13526, 2009.

\(^3\)None of our data has this classification.
Table 1: Example of Reference ID naming convention: two digit year, followed by Embassy origin, followed by cable number.

<table>
<thead>
<tr>
<th>Reference.ID</th>
<th>Date Created</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>09TOKYO4</td>
<td>1/2/09</td>
<td>CONFIDENTIAL</td>
</tr>
<tr>
<td>09TOKYO8</td>
<td>1/5/09</td>
<td>UNCLASSIFIED</td>
</tr>
<tr>
<td>09TOKYO12</td>
<td>1/5/09</td>
<td>UNCLASSIFIED</td>
</tr>
<tr>
<td>09TOKYO13</td>
<td>1/5/09</td>
<td>UNCLASSIFIED</td>
</tr>
<tr>
<td>09TOKYO15</td>
<td>1/6/09</td>
<td>UNCLASSIFIED</td>
</tr>
<tr>
<td>09TOKYO17</td>
<td>1/6/09</td>
<td>UNCLASSIFIED</td>
</tr>
<tr>
<td>09TOKYO20</td>
<td>1/6/09</td>
<td>UNCLASSIFIED</td>
</tr>
</tbody>
</table>

(column 2) for a sample of embassies, and verify that higher numbers appear on later dates. With some rare exceptions, this is the case.\(^4\) Where it is not the case, the reason is likely one or more of the following: multiple cables are sent from the same embassy on the same day; cables of different secrecy statuses are processed or cleared at different rates; it takes longer for some cables to be started and completed than others; there is some mild seasonality in the rate at which embassies write cables. We do not know why some cables are missing from the leak; for example, the sample implied by Table 1 is \{4, 8, 12, 13, 15, 17, 20\}, implying that cables with numbers 5, 6, 7, 9, 10 etc were written but not leaked. Two possibilities seem plausible: Manning had limited security clearance, and the ‘unobserved’ cables were classified as being unavailable at his grade (e.g., this includes those classified as ‘top secret’); alternatively, Manning was not systematic in gathering the cables and thus we receive a *de facto* random sample of those available.

If the cable numbers are a random sample from a discrete uniform distribution (an assumption we assess below), then there are well known techniques for estimating the total number of such cables in existence at the time of disclosure. The next section explains how.

\(^4\)We also verified for each the ID number for a given embassy resets to 0 at the start of a new year (e.g. moving from 2008 to 2009 for Tokyo cables).
Estimating the Maximum of a Discrete Uniform Distribution: The German Tank Problem

Popular in introductory statistical texts (e.g. Larsen and Marx, 2011), the German Tank Problem considers the position of a commander of Allied Forces in the Second World War seeking to estimate the total number of enemy tanks \( N \) in existence (see Ruggles and Brodie, 1947, for a detailed historical discussion). The commander has observed the serial numbers of a (without replacement) random sample of such tanks, of size \( k \), and has only this information on which to base his estimate. It may be assumed that the serial numbers are sequentially numbered in the population, starting with 1 for the first tank produced by the enemy, 2 for the second tank and so on. Of course, the commander does not necessarily observe any consecutive numbers in his sample and a typical (for \( k = 6 \)) sample might be as follows: \( x = \{4, 30, 14, 5, 55, 41\} \).

Generally, the problem is to estimate the maximum of a discrete uniform distribution using sampling without replacement. The maximum likelihood estimate, \( \hat{N}_{MLE} \), is well known, and is simply the maximum of the sample itself, denoted \( m \). So \( \hat{N}_{MLE} = 55 \) tanks for our running example. Derivations may be found elsewhere, but the intuition is straightforward: clearly, the maximum likelihood estimate cannot be less than the maximum of the sample, since this implies that at least one of the sample values is impossible. More subtly, \( \hat{N}_{MLE} \) also cannot be a number larger than the maximum of the sample because that number was not observed, and thus plugging it in to the likelihood must return a lower value than would be seen if \( m \) was used.\(^5\) We note that \( m \) is also a sufficient statistic for \( N \).

\(^5\)Using \( m \) as the estimate for the maximum of a discrete uniform, the probability of seeing any particular element of \( x \) is \( \frac{1}{m} \); if anything larger than \( m \) is used, say \( m + 1 \), that probability—and the value of the likelihood—is reduced. But anything smaller than \( m \) is impossible, thus \( m \) is \( \hat{N}_{MLE} \).
While $\hat{N}_{MLE}$ is straightforward to calculate, and consistent, it is biased. Formal proofs are not complicated, but the intuition suffices for current purposes: because $m$ is always less than or equal to the true value of $N$, the sample maximum will not, in small samples, reveal the truth ‘on average’. Goodman (1952) suggests a minimum variance unbiased estimator $\hat{N}_{Goodman} = m + \left(\frac{m}{k}\right) - 1$. The key component, $\frac{m}{k}$, represents the “average gap between...the serial numbers” (Goodman, 1952, 622–623), and adjusts the maximum likelihood estimate upwards in a way that is consistent with the information in the sample. Goodman (1952) then gives an unbiased variance estimator; formulae for confidence intervals are given in Goodman (1954). For our running example, $\hat{N}_{Goodman} \approx 63$.

Höhle and Held (2006) give a Bayesian estimator, wherein the goal is to obtain a posterior distribution, $\Pr(N|m)$. From Bayes’ rule, we have that $\Pr(N|m) = \frac{\Pr(m|N)\Pr(N)}{\Pr(x)}$, where $\Pr(x)$ is the normalizing constant and $\Pr(N)$ is our prior distribution for $N$. One important possibility for $\Pr(N)$ is the improper uniform which reflects the idea that any particular value of $N$ up to $\infty$ seems as plausible as any other (subject to the caveat that it must be greater than or equal to the maximum of the sample). Höhle and Held (2006) use binomial series theory to show that a straightforward closed form expression then exists for the mean of the resulting posterior: $E(N|x) = k\frac{k-1}{k-2}(m-1)$ for $k > 2$. In our running example, $\hat{N}_{Bayes} \approx 68$.

The estimators are of varying complexity and have distinct philosophical origins. Related to this, the estimates they produce may differ, especially in small samples, and it is not immediately obvious that one is ‘superior’ to the others. We assume therefore that readers will be interested in seeing the results from each, and will be comforted if these results are similar across estimators (in the sense that the estimates are not simply a function ‘cherry picking’ a particular approach). These estimators perform optimally when the observed cables are, on a per embassy-year basis, a random sample of the available reference numbers $1, 2, \ldots N$. 

7
Goodman (104 1954) suggests two easily implemented procedures (see Online Appendix A) to establish whether this is indeed the case for a given sample. For what follows, we use a Kolmogorov-Smirnov test in which every set of embassy-year cable reference numbers is compared to a uniform distribution. If the test does not return a statistically significant $p$-value (i.e., $p > 0.05$) we cannot reject the null of non-uniformity for that sample. Applying this logic, some 641 embassy-year samples were plausibly uniform; this constitutes 63,853 cables in the data altogether.\footnote{Readers may be concerned as to the power of the tests of uniformity employed. In Online Appendix B, we consider this issue and conclude that for most of our embassy-years, for most departures from the uniform, this is not a concern.}

Ultimately, we suspect that \textit{all} the embassies in \textit{all} the years use the same numbering process, and below we will report both sets of results—for the embassy-years that fulfill the uniform sample tests, and separately for \textit{all} the data together. When uniform assumptions are not met, estimates should be interpreted with more caution.\footnote{In Online Appendix C we provide extensive simulation results exploring the precise role between non-uniformity and the nature of the conclusions we can draw about the estimates.} With this in mind, in addition to the Goodman, Bayes, and MLE estimators, we implement a separate regression-based estimator of population size that is less dependent on uniformity (though does rely more heavily on assumptions about the expected number of cables created per day being relatively fixed over time). In words, for each embassy-year, for any given date in the Manning sample, we regress the maximum serial number observed upon that date on the numeric day of year (e.g. January 1 is 1, and December 31 is 365) on which it was written. More precisely, within each embassy-year, the relevant equation is $M_t = \beta_0 + \beta_1 \cdot t + \varepsilon$, where $M_t$ is the maximum serial observed on day $t$, $t$ is the numeric calendar day, and $\varepsilon$ is the error. For a 365-day calendar year, the regression-based estimate of the total number of cables produced is then $\hat{N} = \hat{M}_{365} = \hat{\beta}_0 + \hat{\beta}_1 \cdot 365$. When the expected rate of cables generated per day is fixed over time, the quantity $\hat{\beta}_1$ closely approximates the daily rate of cable generation.
Additional details of this model can be found in Online Appendix C.

**Results**

The upper panel of Table 2 displays our results for the sample that meets the uniformity assumption. Each row refers to a different estimator, but our immediate observation is that the estimates are very similar in every case, no doubt in part because we have generally large samples to work with. The column labelled ‘(total) $\hat{N}$’ is the sum of the estimates for $\hat{N}$ for every given embassy-year: i.e., the estimate for Bangkok in 2010 plus the estimate Tokyo in 2009 plus the estimate for New Delhi in 2009 and so on (for all 710 satisfying embassy-years in this sample). This is estimated to between 1,051,073 and 1,361,770 cables. Of these, the 63,784 cables leaked constitute an estimated proportion of roughly 5% to 6%. That is, for those embassy years appearing to satisfy the discrete uniformity assumption, we estimate that the WikiLeaks cable leaks represent approximately 5% to 6% of all cables sent within the period under study.

From studying the right-most columns of the table, we see that there is a fair degree of variation in terms of leaking: the mean proportion leaked is around 13%, but for some embassy-years it is much larger or smaller. Recalling that there are no ‘Top Secret’ cables in our data, one explanation for this variation is that documents at that classification level are not evenly distributed over embassy-years (literally, some embassies send more high level material at certain times than others). Consequently, regardless of the $\hat{N}$ estimated for the embassy-year in question, the numerator in the proportion calculation is lower in some cases than others. Figure 1 plots the proportion leaked for all embassy-years in this sample, with some ‘interesting’ cases highlighted.
Table 2: Table shows the total number of cables ($\hat{N}$), as well as the corresponding proportion of leaked cables as estimated by our various approaches for the period 2005–2010. Upper panel refers to embassy-years which meet random uniform sample requirement (total sample size= 63,784); lower panel refers to all embassy-years (total sample size= 163,958). The column “overall” is estimated total share of cables leaked, pooling across all embassies; “mean” and “median” denote the average and median proportions leaked in a given embassy year; and “25th” and “75th” mark the 25th and 75th percentiles, respectively.

If we do not restrict to the random uniform samples, and include all embassy-years, we obtain the results in the bottom panel. Again these are similar across estimators, and the proportion leaked overall is estimated to be around 5% of all cables.

**Discussion**

We estimate that, overall, approximately 5% of all US diplomatic cables in existence between 2005 and 2010 were disclosed by WikiLeaks. Exactly how ‘severe’ this is in terms of absolute size is debatable. Obviously, the damage done to US interests is at least as dependent on the contents of the cables as it is on the sheer volume of the leak, and it is only the latter that we have investigated here. Generally, our Letter provides methods by which researchers
Figure 1: Plot compares the estimated share of leaked documents at the embassy-year level against the estimated total number of documents in an embassy year. A solid LOESS regression line is plotted [in red], and the horizontal axis uses a log-10 scale. Overall there is a negative association between the size of the embassy, as measured in total cables produced, and the proportion of cables leaked. Although untestable, a story consistent with this association is that larger embassies have on average higher rates of ‘sensitive material’ produced than smaller embassies (i.e., communications inaccessible to Private Manning, given her security clearance).
can be aware of the sample size available to them, overall and by embassy-year should they choose to work with such data. Assuming the official sources (either leaked or declassified) use numbering conventions as discussed here, the same techniques can be used for other projects. Future work might consider the content of the cables, and utilize our estimates in regression-type analyses.
Online Appendix A  Verifying Samples are Random from Discrete Uniform

Goodman (1954) suggests several ways to check that a given sample of serial numbers is a random sample from a discrete uniform distribution. First, denoting $g$ the largest number in the sample, divide each of the remaining $k-1$ observations in the sample by $g$. These $k-1$ observations should then be statistically indistinguishable from a uniform on $[0,1]$, with a Kolmogorov-Smirnov test an appropriate way to check this. Second, one may break the data into equally spaced ordinal categories, and then conduct a $\chi^2$-test on this coarsened data: if the null cannot be rejected, then the data is at least consistent with a discrete uniform. Both tests are straightforward to implement, and we use the former.

Online Appendix B  Power of the Uniformity Tests

The techniques applied in this Letter work optimally when the sample is one drawn from a discrete uniform distribution of serial numbers. This does not mean that any bias induced by non-uniformity invalidates the general thrust of the results presented in the Letter, but for completeness, we analyze this first order concern here.

We verified uniformity with the Komologorov-Smirnov (KS) test noted above, but one may be concerned as to its power and want to know the circumstances under which that test is able to correctly reject the null of non-uniformity. While there is a theoretical literature on the general issue of the power of the KS test (e.g. Durbin, 1961; Lewis, 1965), we wanted to obtain specific results for our data. Thus we set up a series of simulation experiments in which we allowed discrete serial numbers to be generated from a set of Beta distributions (including the uniform as a special case) and considered the performance of the KS test
therein. The basic idea is to verify that in ‘reasonable’ sample sizes, the test can distinguish (i.e., return a \( p < 0.05 \)) between the simulated Beta sample and an actually discrete random uniform sample.

In what follows, consider a Beta distribution \( \mathcal{B}(\alpha, \beta) \); for clarity in the printing of our plots, we refer to \( \alpha \) as shape1 and \( \beta \) as s2. Our simulation sample size varies from 5 (around the minimum in our empirical study) to 1300 (close to the maximum in our empirical study) at intervals of 40—thus it increases \( \{5, 45, 85 \ldots 1220, 1260, 1300\} \). For each sample size, we fix \( \alpha \), and then iterate between values of \( \beta \) (1, 2, 3, 4, 5), before iterating \( \alpha \) (again 1, 2, 3, 4, 5) thus covering all 25 possible combinations: \( \mathcal{B}(1, 1), \mathcal{B}(1, 2), \mathcal{B}(1, 3), \ldots, \mathcal{B}(5, 4), \mathcal{B}(5, 5) \)). For each value of \( \alpha \) and \( \beta \), we conduct the drawing of the simulated sample and the uniform with which it is paired, a total of 50 times. We then take the mean of these 50 \( p \)-values. Pseudo-code is as follows:

1. for given \( s \) {
2. for \( \alpha \in \{1, 2, 3, 4, 5\} \) {
3. for \( \beta \in \{1, 2, 3, 4, 5\} \) {
4. for \( i \in \{1, \ldots, 50\} \) {

   (a) draw a random sample of size \( s \) from \( \mathcal{B}(\alpha, \beta) \)

   (b) conduct a Kolmogorov-Smirnov test of this sample against a uniform sample of size \( s \)

   (c) record the \( p \)-value of this test

5. take the mean \( p \)-value of these 50 trials, store}

6. iterate the value of \( \beta \) (i.e., \( \beta + 1 \)), do the 50 trials keeping value of \( \alpha \) fixed}
7. iterate the value of $\alpha$ (i.e., $\alpha + 1$), iterate through values of $\beta$ for that value of $\alpha$ 

8. iterate the size of the sample to the next entry in the sample size vector 

Recall, we would like to see that the $p$-values are below 0.05 for any ‘reasonable’ sample size: this would imply that the test is correctly distinguishing between cables from a simulated (non-uniform) distribution versus a ‘truly’ uniform one. For our various sample sizes and Beta distributions, our results are displayed in Figure 2. In each of the plots, the broken line represents $p < 0.05$. Thus, points (that is, particular Beta distribution samples) falling below the line are successfully differentiated from a uniform sample by the KS test. As a sanity check, we included a simulation set for $B(1,1)$ (i.e., a uniform) in the top panel: helpfully, it is not differentiable from the uniform it is paired with for the tests (i.e., the black squares are everywhere above the broken line).

Our immediate observation from the the bottom panels (shape1=$\alpha=4$ and shape1=$\beta=5$) is that at essentially anything above a small number of cables in the sample (45), the KS test can differentiate between a uniform sample and a Beta. For the top three panels, the evidence is more complicated. Basically, for small sample sizes, say fewer than 100 cables, the KS test sometimes commits type II errors: e.g. for a sample size of 5, the (‘average’) KS test reports $p > 0.05$ for any Beta distribution with shape1=$\alpha=1$ (top panel). The KS test does worst when the Beta is symmetric and its parameters take low values: i.e when shape1=$\alpha=\beta=s2=2$ or 3. That is, when the distribution of serial numbers is quite close to uniform around its median. This can be seen by the [red] circles above the plot on the second panel, and the [green] triangle in the third panel for a sample size around 45. Of course, it is not obvious that such unimodel symmetric distributions of cables are likely in practice; more importantly, the bias induced by non-uniformity in e.g. the Goodman estimator is not necessarily troubling per se: in Online Appendix C we give much more discussion of this
(potential) issue. Finally, notice that when we get up to our mean sample size (around 100) the KS test generally gets it right and has the power we need, with the exception of the case where $\alpha = \beta = 2$ and we need a sample size of around 300 to be confident we have a uniform.

**Online Appendix C  Simulation Study**

We assume the sample of cable serial numbers observed in each embassy year to be draws from a discrete uniform distribution, in keeping with the practice in Goodman (1952) and Goodman (1954) and other studies. In the context of our applied research question, the discrete uniformity assumption means that each serial number, within a given embassy year, has the same ex ante probability of being included in the final Manning sample.

In this appendix, we briefly discuss conditions under which the discrete uniformity assumption is appropriate to estimate the cable population size for all cables originating from a particular embassy in a given year, and how the Goodman estimator tends to perform in settings where discrete uniformity is violated. To perform these analyses, we simulate cables being written at the daily level (and being released over the course of a year) and observe how non-constant probabilities of cables arriving in the final Manning sample may bias estimates of cable population sizes. In brief, we find that temporal shifts in the probability cables are excluded from the Manning sample are more likely to bias estimates of the total population size than vicissitudes in the daily rate of cables being written.

We close this document with a replication of our serial number analysis using a regression-based approach that incorporates information on the timing of each cable observed in the sample to help inform our estimates of the cable population size in each embassy-year. In
Figure 2: Power of the Kolmogorov-Smirnov test (literally, y-axis is a mean p-value) for various values of a Beta distribution versus a uniform, at various sample sizes (x-axis). Top plot fixes first Beta parameter at 1 (‘shape1=1’), varies the second (s2; see legend). Second plot fixes first Beta parameter at 2, varies the second. Third plot fixes first Beta parameter at 3 and so on. Broken line represents $p = 0.05$, and thus all points below this line are successfully distinguished from the uniform by the KS test.
general, when the discrete uniformity assumption is satisfied, both Goodman-type estimates and regression-based approaches provide unbiased estimates of the population size; under some conditions, however, regression-based techniques may be preferable to the Goodman estimator if there are sharp changes in the probability is excluded from the Manning sample near the end of a calendar year, or if reasonable assumptions can be made about a fixed expected rate of cable generation across periods.

C.1 Overview

Our objective is to observe how various population size estimators perform on simulated data when (a) there may be seasonality in the rate at which cables are written, (b) there may exist seasonality in the sensitivity of cables being written. We also seek to inspect how such biases may manifest in large versus small sample settings. Evaluating such concerns through simulations, however, will require our making somewhat stylized assumptions about the data generation process of our sample. The main conclusions of our simulation studies are as follows: large shifts in the probability cables are excluded from the Manning sample are more likely to bias Goodman-type estimates of population size than shifts in the number of cables created per day. For the Goodman estimator, the bias introduced is greater as the probability of inclusion in the Manning sample is decreasing over time.

To reach these conclusions, in first set of simulations, we will assume the number of cables written on a particular day is a draw from a Poisson distribution with a fixed rate parameter. In a second set, we model the number of cables written per day as realizations of a Hawkes process (e.g., Hawkes, 1971; Ogata, 1988), which allows the instantaneous rate of cable generation to vary as a part of a “self-exciting” point process, where the occurrence of any event (i.e., a cable being written) increases the short term probability of another cable being written. In our applied context, the simulated Hawkes process will lead to clustered
periods of time with higher than baseline (i.e., random) patterns of cable generation. For one set of Poisson simulations, we will set the rate parameter to equal $\lambda = 5$. For one set of Hawkes simulations, we will set the initial conditions to equal $\mu = 10/3$, $\alpha_1 = 1$, and $\beta_1 = 3$, and simulate events in continuous time for $T = 365$. These parameters were selected because they generate, in expectation, equal totals of cables over the course of an entire year, but vary in terms of their temporal clustering and variance.\(^8\) The virtue in maintaining approximately equal yearly sample sizes in the Poisson and Hawkes study conditions is that it allows for easy inspection of how clustered periods of higher cable generation rates—rather than sample size on its own, or variation in the probability serial numbers are out of sample—influence population size estimates.\(^9\) As the next section will show, however, the ‘burstiness’ of cables being generated over time may be more likely to bias regression-based estimates of population size when such periods of time are correlated with large shifts in the probability cables are excluded from the Manning sample. Goodman-type estimators (which rely more heavily on the observed value of the sample maximum serial number) may be less sensitive to burst-induced biases if the probability that cables appear in the Manning sample is sufficiently high near the end of a calendar year.

C.2 Sensitivity of Assumptions for Goodman and Regression-based Estimators

Absent large shifts over time in the probability that serial numbers are excluded from the Manning sample, daily cable counts being produced from Poisson and Hawkes processes are

\(^8\)In addition to the “Large $N$” case where the expected number of cables written per year is 1825, we will also replicate our analysis on a “Small $N$” case when the expected yearly total is 365.

\(^9\)If the number of cables created on any given day is $n_t \sim \text{Pois}(\lambda)$, then the expected number of cables being created over the course of a year is simply $\sum_{t=1}^{365} E[n_t] = 365 \cdot 5 = 1825$. In a Hawkes process, the instantaneous rate parameter in time $t$ is $\lambda(t) = \mu + \sum_{t_i<t} \alpha e^{\beta(t-t_i)}$. Under the condition that the exponential rate of decay is greater than the self-excitation growth rate ($\beta > \alpha$), and as the number of periods $T \to \infty$, the expected value of the rate parameter is $E[\lambda] = \frac{\mu}{1 - \int_0^\infty e^{-\beta t} \, dt} = \frac{\mu}{1 - (\alpha/\beta)}$.  

19
both acceptable for the Goodman and regression-based estimators of cable population size at the embassy year level. This point can be demonstrated with a simple example. First let the number of cables observed on a given day be \( n_t \), and the probability any given cable is included in the Manning sample on day \( t \) be \( p_t = p = 0.5 \). If cables are given serial numbers in the order in which they are released, and the probability a cable is included in the sample is independent of the day of the year, one can imagine data being generated over a full year like those listed on the lefthand side of Table 3. As should be clear, despite the daily variation in cable counts across days, the probability any given serial number is included in the sample is orthogonal to the day of the year on which it was written. This implies that the sample of serial numbers \( \{1, 4, 5, 7, 8, 10, 11, \ldots, 1829\} \) is precisely a random sample from a discrete uniform distribution of size \( N = 1831 \), since each serial number has an equal probability of being drawn into the Manning sample. It is important this stylized example imposes no structure on how \( n_t \) is drawn. Regardless of whether the daily counts of cables result from Poisson or Hawkes processes (much less any stochastic process), the serial numbers included in the Manning sample are precisely a random draw from a discrete uniform distribution, which is guaranteed so long that \( p_t \) is fixed over time. On such a sample of data, to estimate the number of cables written in a given embassy year using Goodman estimator, therefore, would be a natural choice. In our simulation results we show that data generated and analyzed in such a fashion provide unbiased estimates of the population size.

The righthand side of Table 3 provides an example when the assumption of discrete uniformity (of serial numbers in the Manning sample, over the course of a full year) is not satisfied. The example provided is meant to be an extreme case in which there is a pronounced reduction in the probability of cables being included in the study sample in periods 4, \ldots, 365, moving from \( p_t = 0.5 \) to \( p_t = 0 \). If such censorship were to occur in the data—i.e., for a fixed \( N = 1831 \), relying on a study sample of \( \{1, 4, 5, 7, 8, 10, 11, 13, 16\} \) instead
Table 3: Serial numbers included in a hypothetical Manning sample over the course of a year, where $p_t$ denotes each cable’s probability of being drawn into the Manning sample in time $t$. The righthand column notes the maximum serial number observed in the Manning sample in period $t$, denoted $M_t$. Underlined serial numbers indicate cables included in a hypothetical Manning sample.

<table>
<thead>
<tr>
<th>$t$</th>
<th>$n_t$</th>
<th>Serials$_t$</th>
<th>$p_t$</th>
<th>$M_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>1 2 3 4 5</td>
<td>0.5</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>6 7 8 9</td>
<td>0.5</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>10 11 12 13 14 15 16 17</td>
<td>0.5</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>18 19 20 21 22 23</td>
<td>0.5</td>
<td>22</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>24 25 26 27 28</td>
<td>0.5</td>
<td>26</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>365</td>
<td>3</td>
<td>1829 1830 1831</td>
<td>0.5</td>
<td>1829</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$t$</th>
<th>$n_t$</th>
<th>Serials$_t$</th>
<th>$p_t$</th>
<th>$M_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>1 2 3 4 5</td>
<td>0.5</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>6 7 8 9</td>
<td>0.5</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>10 11 12 13 14 15 16 17</td>
<td>0.5</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>18 19 20 21 22 23</td>
<td>0.5</td>
<td>22</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>24 25 26 27 28</td>
<td>0.5</td>
<td>26</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>365</td>
<td>3</td>
<td>1829 1830 1831</td>
<td>0</td>
<td>NA</td>
</tr>
</tbody>
</table>

of \{1, 4, 5, 7, 8, 10, 11, \ldots, 1829\}—the Goodman estimator would severely underestimate the true population size. However, if instead one were to use only the first three days of observed data, a linear extrapolation that relies only on the maximum serial observed on each day in the observed Manning sample may provide a more plausible estimate of the total population size. Extrapolation is particularly merited if one is willing to make assumptions about the expected number of cables being written per day being relatively constant over time.

The aim here is to provide a sketch of this intuition. Assume the number of cables written on day $t$ is $n_t \sim g(\theta)$, with the expected number of documents written per day being $E[X]$. If a cable is written in time $t$, there is some $p_t$ probability that cable is included in the Manning sample. Each cable written (i.e., both those in the Manning sample and outside the Manning sample) are given a serial number according to the order in which it is written, in line with the process shown in Table 3. On the first day, the expected number of cables included in the sample is $E[X] \cdot p_1$, on the second day the expected number of cables included in the Manning sample is $E[X] \cdot p_2$, and on the $t$-th day the expected number of cables included in the Manning sample is $E[X] \cdot p_t$. This is by definition true so long as the distribution from
which daily cable counts are drawn is fixed over time. The expected Manning sample size
over \( T \) periods is simply \( E[X] \cdot \sum_{t=1}^{T} p_t \).

In general, however, the expected number of cables appearing in the Manning sample per
day is not the same as the expected value of the maximum serial number observed on day \( t \), which we will denote \( E[M_t] \). In a trivial case, for example, \( E[M_t] \) may be undefined if \( p_t = p = 0 \), even if \( E[X] > 0 \). The expected value of the sample maximum observed on day \( t \) will depend on several quantities: the number of periods that have passed prior to period \( t \) (accounting for the serial numbering pattern), the expected number of cables written per
day, and the expected number of cables entering into the Manning sample on day \( t \). More
formally, we denote the expected value of the maximum serial number observed on day 1 as
\( E[M_1] = f(p_1, \theta) \), the maximum serial number observed on day 2 as \( E[M_2] = E(X) + f(p_2, \theta) \),
and \( E[M_t] = (t - 1)E[X] + f(p_t, \theta) \). The first summand of \( E[M_t] \) accounts for the expected
starting point of serial numbers written in period \( t \), regardless of whether they appear in
the sample; the second summand adjusts directly for the expected maximum serial number
observed in the Manning sample, which is a function of the underlying daily count function
and \( p_t \). If we differentiate \( E[M_t] \) with respect to \( t \), we observe \( \frac{\partial E[M_t]}{\partial t} = E(X) + \frac{\partial f}{\partial p_t} \cdot \frac{\partial p_t}{\partial t} \). By
necessity the sign on \( \frac{\partial f}{\partial p_t} \) will always be positive, but \( \frac{\partial p_t}{\partial t} \) may positive, negative, or equal to
zero. Clearly, if \( \frac{\partial p_t}{\partial t} = 0 \), then \( \frac{\partial E[M_t]}{\partial t} = E(X) \).

\textbf{C.2.1 A Regression-based Estimator of Cable Population Size}

The observation that \( \frac{\partial E[M_t]}{\partial t} = E(X) + \frac{\partial f}{\partial p_t} \cdot \frac{\partial p_t}{\partial t} \) is valuable because it motivates a linear
regression-based approach to estimate the total number of cables written in a given year.
It also provides intuition on the bias that may be introduced through such an estimation
approach if \( \frac{\partial p_t}{\partial t} \neq 0 \). Let us first consider the case when \( \frac{\partial p_t}{\partial t} = 0 \). If changes over time in the
probability cables are included in the Manning sample are not linearly associated with time,
the subsequent regression-based approach will be appropriate to estimate the total number of cables produced at an embassy in a given year. Namely, for each embassy year, aggregate the observed sample of data at the daily level, and estimate the following bivariate regression equation:

\[ M_t = \beta_0 + \beta_1 \cdot t + \varepsilon, \]  

(1)

where \( M_t \) is the maximum serial observed (e.g., \( \{5, 8, 16\} \)) on day \( t \), \( t \) is the numeric calendar day (e.g., \( \{1, 2, 3\} \)), and \( \varepsilon \) is the error. At the embassy level, we estimate the total number of cables written over a 365 day period as

\[ \hat{N} = \hat{M}_{365} = \hat{\beta}_0 + \hat{\beta}_1 \cdot 365. \]  

(2)

Straightforwardly, the quantity \( \hat{\beta}_1 \) is an estimate of \( \frac{\partial E[M_t]}{\partial t} \). In leap years, the fitted value for day 366 would be used.

When there exists an association between the expected change in \( p_t \) and \( t \), however, this estimator may be biased. If \( \frac{\partial m}{\partial t} > 0 \), the estimator will tend to produce estimates that are somewhat larger than the true population size, and when \( \frac{\partial m}{\partial t} < 0 \) the estimates will tend to undershoot the true population size. So too, if there are associations between changes in rate of cable generation and changes in the probability with which cables are included in the sample, the estimator may be biased in expectation. The magnitude of this bias will depend precisely on magnitude of the unobserved shifts in cable generation and \( p_t \).

There may be cases in which sharp shifts in \( p_t \) do not threaten the validity of linear extrapolation, however. Consider the case when \( p_t = 0.1 \) for the first half of a calendar year, and \( p_t \approx 0 \) in the second half. (This scenario is approximated in the “Second Half Censored” study condition mentioned in the next section.) In this extreme case, linear extrapolation
given the observed data may be reasonable: even though the range of the observed data is weighted exclusively to the first half of a calendar year, if the true rate of cable generation in the first half of the year is close to the rate of cable generation in the second half of the year, the estimates $\frac{\partial E[M_t]}{\partial t}$ obtained from the first half of the calendar year should appropriately map to the second half of the year, even if no data are observed in sample from that period.

### C.2.2 Study Conditions and Outcomes of Interest

To assess how various estimators perform across various hypothetical data generation processes, we vary both the distribution from which daily cable counts are drawn, in addition to the probability that any cable written on day $t$ is to be included in the Manning sample. As before, we denote the probability that a cable is included in the Manning sample, given that it is written on day $t$, as $p_t$.

We report simulation results for eight different manipulations of $p_t$. The names of these study conditions are presented along with their formal definitions in Table 4.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed Probabilities</td>
<td>$p_t = p = 0.1$</td>
</tr>
<tr>
<td>First Half Censored</td>
<td>$p_t = 0.001$ if $t &lt; 183$; $p_t = 0.1$ if $t \geq 183$</td>
</tr>
<tr>
<td>Second Half Censored</td>
<td>$p_t = 0.1$ if $t &lt; 183$; $p_t = 0.001$ if $t \geq 183$</td>
</tr>
<tr>
<td>Random Uniform</td>
<td>$p_t \sim \text{Unif}(0, 0.1)$</td>
</tr>
<tr>
<td>Inverted U-Shape</td>
<td>$p_t = \sin(t \cdot \pi/365)/10$</td>
</tr>
<tr>
<td>U-Shape</td>
<td>$p_t = (1 - \sin(t \cdot \pi/365))/10$</td>
</tr>
<tr>
<td>Linear Increase</td>
<td>$p_t = (t/365)/10$</td>
</tr>
<tr>
<td>Linear Decrease</td>
<td>$p_t = (1 - t/365)/10$</td>
</tr>
</tbody>
</table>

In addition to varying the probability with which written cables appear in the Manning sample, we vary whether daily cables counts arise as a result of a Poisson process or a Hawkes process. For both the Poisson and the Hawkes study conditions, we have “Large $N$” and a
“Small $N$” variants. In the “Large $N$” conditions, the Poisson parameter is $\lambda = 5$, while the respective Hawkes parameters are defined as $\mu = 10/3$ (the baseline rate), $\alpha = 1$ (the excitation parameter), and $\beta = 3$ (the exponential decay). In the “Small $N$” study conditions, the Poisson parameter $\lambda = 1$, and the Hawkes parameters are $\mu = 0.2$, $\alpha = 0.8$, and $\beta = 1$.

Using Poisson and Hawkes data generation process, across both the “Large $N$” and “Small $N$” study conditions, we perform 2,500 random simulations of each of the study conditions listed in Table 4. In each of these simulations we record the “true” number of cables generated by either the Poisson or Hawkes processes, in addition to the estimates of each of the MLE, Goodman, and regression-based estimators. In each iteration of the simulation, we divide each estimator’s estimate of the total population size by the true number, yielding $\hat{N}/N$, and we store this value. If across multiple simulations a particular estimator systematically yields values of $\hat{N}/N > 1$, this provides evidence that an estimator tends to overestimate the true number of cables. Similarly, if a particular estimator on average yields values of $\hat{N}/N < 1$, this provides evidence that an estimator, given the study conditions, tends to underestimate the true number of cables in a given embassy year.

C.3 Results

Figures 3 through 6 present the results of this simulation study. In each subplot, the mean value of $\hat{N}/N$ across simulations is presented beneath each estimator’s name. The upper and lower boundaries of each boxplot denote the interquartile range of simulation results for each estimator. The median result is presented as a solid, horizontal line. The upper and lower whiskers denote values 1.5 above or below the interquartile range of the plot.

Overall, the regression-based estimator performs consistently well. When the discrete uniformity assumption is satisfied, however, the Goodman estimator is unbiased and exhibits
the lowest variance. The bias and variance of each estimator appears to be larger in the “Small N” study conditions. In our applied example, the Goodman estimator is most biased cases in which $p_t$ is decreasing over time. Relative to the “Inverted U-Shape”, the ”U-Shape” study conditions have distributions of $\tilde{N}/N$ closer to 1.
Figure 3: Simulation results for “Poisson, Large N” study. The results of 2500 random simulations reflected in each subplot. In this condition, $\lambda = 5$, such that the expected number of cables per year is 1825.
Figure 4: Simulation results for “Poisson, Small \( N \)” study. The results of 2500 random simulations reflected in each subplot. In this condition, \( \lambda = 1 \), such that the expected number of cables per year is 365.
Figure 5: Simulation results for “Hawkes, Large N” study. The results of 2500 random simulations reflected in each subplot. In this condition, $\mu = 10/3$, $\alpha = 1$, and $\beta = 3$, such that the expected number of cables per year is 1825.
Figure 6: Simulation results for “Hawkes, Small N” study. The results of 2500 random simulations reflected in each subplot. In this condition, $\mu = 0.2, \alpha = 0.8$, and $\beta = 1$, such that the expected number of cables per year is 365.
References


