Field on Nominalism

1. What is Field's fictionalism?

Realist (or mathematical idealist): someone who *literally believes* mathematical theories *taken at face value*
Fictionalist: someone who doesn't

(although the fictionalist may *in some nonliteral sense* believe mathematical theories taken at face value, and *literally* believe mathematical theories on some devious reinterpretation)

Questions for fictionalist (p.2): (i) are mathematical theories true on some non-face-value interpretation? (semantic fictionalism) (ii) does this interpretation give the 'real meaning' of the theories? (hermeneutic fictionalism). Field: (ii) is an uninteresting verbal question; a fictionalist does not need to be committed to (i).

What does a fictionalist who 'accepts' a mathematical claim C literally believe? That C *follows from standard mathematics*; or better (footnote 3) that it *follows from standard mathematics together with some purely nonmathematical facts*.

(p. 5) Is there any real difference between platonism and fictionalism?

2. Conservativeness

A mathematical theory S is *conservative* if, for any *purely nonmathematical* sentence A, and any body of such sentences N, A is a consequence of N+S only if A is a consequence of N.

3. Justification for belief in mathematical entities

(i) Claims like '2 + 2 = 4' are analytic truths (p. 4)
Reply: no they're not; existence-claims are never analytic.

(ii) The specifically mathematical methods we (mathematicians, elementary school kids,...) actually use for coming by mathematical beliefs are good methods: they confer justification on the beliefs we arrive at by using them.

Interpretation 1: the methods are things like the algorithms for addition and multiplication, deduction from the Zermelo-Fraenkel axioms for set theory, etc.

Interpretation 2: the methods are things like *believing whatever your elementary school teacher tells you, following your sense of intuitive plausibility, deducing consequences from things you already believe...*

- Epistemic conservativism / "general foundationalism" (Harman): whatever you already believe is automatically (prima facie) justified; what needs justification is change in view.

These interpretations differ as regards what they say about different imaginary cultures, e.g. a society of nominalists.

Field's response: radical shifts of practice can be rationally required, when the new practice 'serves our purposes' at least as well as the old one, while being less committal.

Q: what are these 'purposes'?

(iii) Mathematical beliefs are justified in the same way that beliefs in the other sciences are believed: the "indisensability argument".

4. Indispensability for explanation

What is a 'pretty good explanation' of some 'phenomena'?

- A relatively simple non-*ad hoc* body of principles from which they follow.
- 'As-if claims which ride piggyback on genuine explanations are not themselves to be construed as explanations (at least, non-*ad hoc* ones)'.

A purported general epistemological principle: if you have reason to believe that P_1, P_2..., and you have reason to think that that no [pretty good] explanation of these 'phenomena' which does not entail that S is possible, then you have reason to believe that S.

NB: no suggestion that the 'as-if' theories are unintelligible—merely that such theories are weaker than what we have reason to believe. They are 'bad' theories from a a scientific point of view.

Is this epistemological principle correct? Some competitors:

- Believe only the observable consequences of the best explanations.
- Believe only those consequences of the best explanations which are 'directly checkable in an experiment cheap enough for the government to fund' (footnote 10)
- Believe only those consequences of the best explanations which concern non-mathematical entities
- Believe only those consequences of the best explanations which concern causally relevant entities

These are epistemological theories according to which certain 'as-if' theories are perfectly fine.

How do we argue about this issue?

- Arbitrariness
- Our 'ordinary inductive methodology'.

5. Field's "nominalization program" in physics

Find theories which are at least as good (simple...) as current mathematical physics, but don't entail the existence of mathematicalia

(i) Find a vocabulary for describing the physical structure of the world 'intrinsically', without dragging in mathematical entities. Set up definitions, and prove *representation theorems*, which show how to understand mathematical physics as a consequence of the intrinsic description of world, together with straight mathematics.

(ii) State some simple laws using the intrinsic vocabulary which have all the same intrinsic consequences as mathematical physics, but don't entail the existence of mathematical entities.

(Of course, if the only point was to undermine the indispensability argument for *current mathematics*, we wouldn't have to find an 'intrinsic' theory: it would be enough if the entities we 'dragged in' were sufficiently unlike those described by current mathematics. This is pretty easy to do: so if the indispensability argument gives us any reason to believe "mathematical" theories, they are *very weak* ones.)

6. Against Platonism: explaining the reliability of mathematical belief

A Platonist seems forced to conclude that it's an *inexplicable coincidence* that his mathematical beliefs are true. But this conclusion should undermine any putative justification conferred on those beliefs by our mathematical methods.

It's not enough to have an explanation of the beliefs on the one hand, and an explanation of the mathematical facts on the other hand. What's needed for it not to be a coincidence is an explanation of the *correlation*.

A genuine explanation of a correlation would presumably be a *counterfactual-supporting* one.

Response: if certain mathematical facts had been different, the basic laws of physics would also have been different, and so things in general would have been very different.

Field's rejoinder: this is undermined even by the partial success of the nominalization program.

> This would be denied by a Platonist who denied that the notions of mathematical physics ('is the mass of', etc.) can be defined in terms of intrinsic notions.

Does this rejoinder really work for a defender of methods like *relying on initial plausibility*? Analogy: suppose that for some reason we all had strong intuitions about the values of fundamental physical constants.

7. Primitive modality

Objection to nominalism: if there are no sets, then in all models in which P_1, ... P_n hold, C holds; so by the Tarskian definition of logical consequence, C is a logical consequence of P_1... P_n. But this is absurd.

More pointed version of the objection: moreover, you, Field, say that to accept a mathematical claim is to hold that it follows from standard mathematics together with non-mathematical facts. So you must say that every nominalist accepts every mathematical claim.

Response: the Tarskian definition of logical consequence is not literally true. Logical consequence (or consistency, or necessity) cannot be defined: it is a "primitive" notion that we understand by learning rules of use.

Even a Platonist should agree.

Definition of the 'according to the mathematical fiction' operator in terms of logical consequence:

> 'According to the mathematical fiction, P' means
> $\sum_{\text{non-mathematical}} Q(Q \,\&\, ((S \,\&\, Q) \rightarrow P))$