

'If P, Q' and 'P \supset Q'

Cian Dorr

23 October 2012

1. The view

Example: For it to be the case that if Booth didn't shoot Lincoln, someone else did is for it to be the case that either Booth shot Lincoln or someone else did.

Metalinguistic generalisation: For some large class of sentences of the form 'If P, Q', the default interpretation is on on which they are equivalent to ' $\neg P \vee Q$ '.¹

What does 'equivalent' mean here? One might need to distinguish several varieties:

- Expressing the same proposition
- Expressing propositions such that necessarily, either both or neither are true
- Expressing propositions which *logically entail* one another
- Two-way logical entailment at the level of *sentences*.

2. Does 'If P, Q' entail 'P \supset Q'?

The people who don't like this are those (intuitionists, paracompletists) who accept 'If P, P' but not 'P \supset P' as valid. A huge territory, too big for us to explore.

3. Does 'P \supset Q' entail 'If P, Q'?

Note that this entailment stands or falls with the 'Or-to-if' inference from 'P or Q' to 'If not-P, Q'.

- Or-to-if inferences *just sound valid*. 'It's either a dog or a cat. So, if it isn't a dog, it's a cat'.
- Or-to-if can be derived using *disjunctive syllogism* and the metarule of *conditional proof*:

$P \vee Q, \neg P \vdash Q$	(disjunctive syllogism)
$P \vee Q \vdash P \rightarrow Q$	(conditional proof)

- Or-to-if can be derived using *Identity*, *Import-export* and *Modus Ponens*.

$\vdash ((P \vee Q) \wedge \neg P) \rightarrow ((P \vee Q) \wedge \neg P)$	(Identity)
$\vdash ((P \vee Q) \wedge \neg P) \rightarrow Q$	(RCK)
$\vdash (P \vee Q) \rightarrow (\neg P \rightarrow Q)$	(Import-Export)
$(P \vee Q) \vdash (\neg P \rightarrow Q)$	(Modus Ponens)

¹ NB: this is perfectly consistent with Kratzer's operator-restrictor-matrix analysis. 'Every actualised world at which P is a world at which Q' is equivalent to 'P \supset Q'.

An interestingly different argument: if I say ‘If the dice landed on an odd number it landed on 1 or 3, and if it landed on an even number it landed on 2’, and it turns out it landed on 1, that seems to be enough to settle that I spoke the truth.²

4. Challenge: why is ‘If P, Q’ often infelicitous even when ‘ $P \supset Q$ ’ is true?

First pragmatic tool: presupposition.

Second pragmatic tool: conversational implicature

- Gricean maxims: ‘Make your contribution as informative as required (for the current purposes of the exchange)’ (quantity); ‘Be brief (avoid unnecessary prolixity)’ (manner).
- ‘Assert the stronger when probabilities are close’

Third pragmatic tool: conventional implicature (‘but’, ‘even’, ‘nevertheless’, ‘anyway’)

- Jackson 1979: ‘If P, Q’ conventionally implicates that the speaker’s high confidence that $\neg P \vee Q$ would persist even in the face of evidence that P.
- ‘If Reagan worked for the KGB, I’ll never find out’.

Note: in general it is harder to use the tools of pragmatic to explain why an utterances is felicitous despite being true than to explain why an utterance is infelicitous despite being false. However it’s not so easy to find felicitous, natural-looking English sentences that come out false according to materialism—‘It is not the case that if P, Q’ is a bit stilted.

5. ‘Paradoxes of material implication’

$$\begin{aligned} Q \vdash P \rightarrow Q \\ \neg P \vdash P \rightarrow Q \\ \vdash P \rightarrow Q \vee Q \rightarrow R \\ (P \rightarrow Q) \rightarrow R \vdash P \rightarrow R \\ (P \rightarrow Q) \rightarrow R \vdash \neg Q \rightarrow R \end{aligned}$$

6. The confidence-theoretic challenge

Validity-Confidence Link: If the proposition that P logically entails the proposition that Q, then no ideally rational person could be less confident that Q than that P.

Argument: Since I am pretty confident that Romney won’t win the election, I am pretty confident that either Romney won’t win or Romney will introduce a carbon tax. But I am *far* from confident that if Romney wins, he will introduce a carbon tax. And given the evidence, it would be far from rationally ideal—indeed, crazy!—for me to be much

² This is due to Michael McDermott, ‘On the Truth-conditions of Certain “If”-sentences’. Note that McDermott is not himself a Materialist—he favours a kind of three-valued semantics where conditionals with false antecedents are gappy and the conjunction of something gappy with something true is true.

less confident that Romney won't win, or much more confident that he will introduce a carbon tax if he does. So, by the Validity-Confidence Link, the proposition that either Romney won't win or he will introduce a carbon tax does not logically entail the proposition that if Romney wins, he will introduce a carbon tax.

A rough stab at a principle about confidence in conditionals: Typically, 'A is [not] very confident that if P, Q' is naturally interpreted as expressing a proposition that is false in worlds where A is rational and [not] much less confident that P-and-Q than that P.

(A version of) 'Stalnaker's Thesis': If x is rational and has a nonzero degree of belief that P, then x 's degree of belief that if P, Q is the result of dividing x 's degree of belief that P and Q by x 's degree of belief that P.

Responses.

1. You are much less confident that if P, Q than that $P \supset Q$, but you are irrational! In the light of philosophical reflection, you should become much more confident that if Romney wins he will introduce a carbon tax.
2. Actually, you already *are* pretty confident that if P, Q
 - a. ...but you are bad at introspection, and thus falsely believe that you aren't.
 - b. ... and you know it! While your utterance of 'I am far from confident that if Romney wins, he will introduce a carbon tax' is literally false, you don't mean it literally. Given the way you mean it, it is acceptable.
3. Although *unembedded* occurrences of 'If Romney wins, he will introduce a carbon tax' are naturally interpreted as equivalent to material conditionals, whereas the occurrences embedded under 'confident' are not.³
 - a. This is something special about 'probability operators', like 'probably' and 'confident'.
 - This threatens to license weird speeches like 'One can believe/know that if P, Q even if one's degree of confidence that if P, Q is almost zero.'
 - b. This applies to all attitude verbs—'believe', 'know', 'assert', 'say'...
 - But surely the point of 'If P, Q' is to say that if P, Q, while the point of ' $P \supset Q$ ' is to say that $P \supset Q$. If these are very different kinds of sayings, what is the use of a notion of 'semantic value' under which 'if P, Q' and ' $P \supset Q$ ' are assigned the same 'semantic value'?
4. Reject the Validity-Confidence Link.

³ Kratzer might even claim that 'if Romney wins, he will institute a carbon tax' doesn't even form a syntactic constituent in 'I am confident that if Romney wins, he will introduce a carbon tax', so that the question what it expresses in that sentence doesn't even *arise*.

Inconsistency-Confidence Link: If the proposition that P and the proposition that Q are logically inconsistent, then any ideally rational person's degree of confidence that that P and Q must be zero.

7. If Or-to-If isn't valid, what's good about it?

Some different notions of 'quasivalidity' that one might explore:

- ' $P_1 \dots P_n \vdash Q$ ' is quasi-valid iff " $\text{''Must } P_1\text{''}, \dots, \text{''Must } P_n\text{''} \vdash Q$ " is valid.
- ' $P_1 \dots P_n \vdash Q$ ' is quasi-valid iff, whenever the speaker of a context c bears such-and-such epistemic relation to the propositions expressed relative to c by each of $P_1 \dots P_n$, the proposition expressed by Q relative to c is true.

Is 'It must be that either P or Q; therefore, if not-P then Q' valid?

- Note that if it is, one will (presumably) have to say that 'It must be that Q, therefore if P then Q' and 'It must be that not-P, therefore if P then Q' are valid.

8. Heterodox theories of validity

An increasingly popular view: 'If P, Q' and ' $P \supset Q$ ' are logically equivalent, but their negations aren't: 'Not (if P, Q)' does not logically entail 'P and not Q'.

- Advice for debating logical heresies: look for disagreements expressible using sentences that don't contain 'valid', 'true', etc.
- Does the heretic accept sentences like 'Everyone that is either non-rich or famous is famous if rich; but someone who is not famous if rich is non-rich or famous'?