Probabilities of conditionals

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1. Descriptive generalisations about conditional probability

A reasonable reconstruction of what seems to be a standard approach to answering questions of the form 'How confident is A that if P, Q?': so long as we think A's degree of confidence that P is nonzero [and that A is not too irrational?], we go byA's conditional credence in Q given P.

- This also applies to a wide range of embedded confidence-ascriptions, e.g. in 'How confident should we be that if P, Q?'

'Adams's thesis': The *degree of assertability* of 'If P, Q' (by a speaker at a time) equals the speaker's conditional credence in Q given P.

- At least: a *necessary* condition for an assertion of 'If P, Q' to be acceptable is that the speaker's conditional credence in the proposition expressed by 'P', given the proposition expressed by 'Q', should be high.

2. Some exceptions

Cases where a conditional with a disjunctive antecedent is treated like a conjunction: 'If he ate the pizza or the week-old sausages, he was fine'.

'Compartmentalised' evaluation: 'It's starting to look like he died if he jumped'.

3. 'Stalnaker's Thesis'

When * is a binary operator on propositions (function from pairs of propositions to propositions), and P is a probability function, say that * **is a CCCP-function for P** iff for any propositions A and B such that $P(A)>0$ and $P(B)$ is defined, $P(A*B)=P(B \mid A)$.

ST: There is a binary operator → such that (i) → is a CCCP-function for any P that could be an ideally rational person's credence function, and (ii) for any sentences 'P', 'Q' that are naturally interpreted as expressing propositions A and B, 'If P, Q' is naturally interpreted as expressing A→B.

- The restriction to rational credence functions is obviously needed.
- If → satisfies MP and And-to-if, then → is a CCCP-function for P iff A→B is always *probabilistically independent* of A (in P).
- We could restricted ST in various ways, e.g. by requiring A and/or B to be *non-conditional* propositions.

- If $\rightarrow$ is a CCCP-function for P and $P(A)>0$, then $P(A\rightarrow B)+P(A\rightarrow\neg B)=1$, and $P(A\rightarrow(B\wedge\neg B))=0$. If $A\rightarrow(B\wedge\neg B)$ is logically equivalent to $(A\rightarrow B)\wedge(A\rightarrow\neg B)$, this means that $P((A\rightarrow B) \vee (A\rightarrow\neg B)) = P(A\rightarrow B)+P(A\rightarrow\neg B) - P((A\rightarrow B)\wedge(A\rightarrow\neg B)) = 1\text{-}0 = 1$.

4. Lewis's conditionalising result (strengthened version)

Premise: there exist a probability functions P and propositions A and B such that ¬B entails A, $0<P(B)<1$, and $P(\neg B)<P(A)<1$, and P is a rational credence function, and $P_B = P(\cdot \mid B)$ is a rational credence function.

Assumption for *reductio*: ST.

(i) $P(A\rightarrow\neg B) = P(\sim B \mid A) = P(\sim B\wedge A)/P(A) = P(\sim B)/P(A) > P(\sim B)$. So $P(B \wedge (A\rightarrow\neg B)) > 0$.

(ii) $P(B\wedge(A\rightarrow\neg B)) = P(A\rightarrow\neg B \mid B)P(B) = P_B(A\rightarrow\neg B)P(B) = P_B(\sim B \mid A)P(B)$

$= P(\sim B \mid A\wedge B)P(B) = 0$. Contradiction.

- This entails Lewis's first three triviality results.
- Fourth triviality result: given ST, no rational credence function is derived from any other by nondegenerate, two-celled Jeffrey conditioning.
- Hall's orthogonality result: given ST, for any rational credence functions P and P′, there exist some proposition A such that $P(A)=1$ and $P'(A)=0$.

5. Bullet-biting responses

6. The contextualist strategy

Contextualist explanation of the role of conditional probability in confidence attributions: when we are discussing what A believes at t, and A is ideally rational at t, we will find it natural to interpret indicative 'if' as expressing some operator that plays the CCCP-role for A's credence function at *t*.

- How is this to be generalised to other uses of 'if'?
- How is this context-sensitivity implemented?

Bad objections: Lewis 1976: 'Presumably our indicative conditional has a fixed interpretation, the same for speakers with different beliefs, and for one speaker before and after a change in his beliefs. Else how are disagreements about a conditional possible, or changes of mind?'

7. Contextualism and 'compartmentalized' evaluation.

8. Stalnaker's no-go result

Suppose $A\rightarrow B$ entails A, and is a CCCP-function for P where $0<P(A)<1$. Then $0 = P(A\rightarrow B \mid \neg A) = P(A\rightarrow B)$ (using the independence of the conditional from its antecedent) $= P(B \mid A)$. So $P(B)=0$.

Very Limited Antecedent Strengthening (CMon): $A \to B, A \to C \vdash (A \wedge B) \to C$

- Equivalently: we can strengthen the antecedent so long as the result is still weaker than the consequent.
- One corollary: $(A \vee C) \to (A \wedge B) \vdash A \to B$

Now we can just take $C = A \to B$ and we have an example of a conditional that entails its own antecedent.

9. van Fraassen's tenability results

First tenability result: any P can be extended to a P′ for which there is a CCCP-function → which obeys the logic CE.

Second tenability result: any P can be extended to a P′ for which there is an operator → which obeys the logic C2 (Stalnaker's logic), such that for any C and any nonconditional A and B, $P(A \to C) = P(C \mid A)$ and $P((A \to B) \to C) = P(C \mid A \to B)$.