Propositions and counterpart theory

Cian Dorr

1. The problem

David Lewis was a leading advocate of the view that propositions are sets of possible worlds. His advocacy was not unqualified: he was willing to recognize alternative versions of the ‘proposition role’, played not by sets of possible worlds but by more complicated set-theoretic constructions out of possibilia (1986a: 55–59). But for at least one philosophically important version of the proposition role, sets of possible worlds were supposed to be ‘just right’.

Lewis was also the leading advocate of a counterpart-theoretic interpretation of de re modal discourse. According to counterpart theory, for it to be the case that an object could have been a certain way is for that
object to have a counterpart which is that way. For example, Humphrey could have won a presidential election iff Humphrey has a counterpart (in some possible world) who does win a presidential election. Importantly, Lewis (1968: 29) allows that an object can have more than one counterpart in some possible worlds. For example, at a world where the counterpart of the zygote that developed into Humphrey developed into twins, it would be natural to take both twins to be counterparts of Humphrey’s. Adding a ‘one counterpart per world’ requirement would diminish a counterpart theorist’s ability to accommodate our pre-theoretic modal beliefs, such as the belief that Humphrey could have either lost or won an election in which the other candidate was his twin brother.

A tension between these views emerges when we consider how we might go about interpreting de re modal claims about propositions. Take, for example, the claim that

(1) There is a false proposition which could have been true.

We could treat this counterpart-theoretically, translating it as

(1’) There is, in the actual world, a false proposition which has a true counterpart in some world

just as we would translate

(2) There is a short person who could have been tall

as

(2’) There is, in the actual world, a short person who has a tall counterpart in some world.

But there is no good way to make sense of (1’) as it stands, if propositions are supposed to be sets of possible worlds. It is not clear what it would be for a set of possible worlds to be ‘in’ a given possible world, or to be a counterpart of some other set of possible worlds. And, while a set of possible worlds can be taken to be ‘true at’ exactly those worlds that are its members, there is no reasonable way to interpret the unrelativized predicate ‘true’ that figures in (1’). (Truth at the actual world obviously isn’t what’s needed.)

The most natural way for one who takes propositions to be sets of possible worlds to understand (1) does not involve counterpart theory at all:

(1*) There is a proposition which does not have the actual world as a member, and has some world as a member.

It is a straightforward exercise to modify the counterpart theory of Lewis 1968 in such a way as to restrict the application of counterpart theory
to variables ranging over individuals, while giving a standard non-counterpart-theoretic treatment to variables ranging over propositions. The interesting cases for the resulting hybrid theory are ‘mixed’ de re modal claims, like

\[(3) \text{ For each individual } x, \text{ there is a proposition } p \text{ such that possibly, } x \text{ is } F \text{ and } p \text{ is true,} \]

which will be translated as follows:

\[(3^*) \text{ For each individual } x \text{ in the actual world, there is a proposition } p \text{ such that there is some world } w \text{ such that } w \text{ contains a counterpart of } x \text{ that is } F, \text{ and } p \text{ is true at } w. \]

So far, so good. But now consider the following claim:

\[(4) \text{ For every individual } x, \text{ there is a proposition } p \text{ such that necessarily, } p \text{ is true iff } x \text{ is } F. \]

The hybrid theory translates this as

\[(4^*) \text{ For every individual } x \text{ in the actual world, there is a proposition } p \text{ such that, for every world } w \text{ and every counterpart } x' \text{ of } x \text{ in } w, \text{ } p \text{ is true at } w \text{ iff } x' \text{ is } F \]

which logically entails

\[(5) \text{ For every individual } x \text{ in the actual world, and any two counterparts } x' \text{ and } x'' \text{ of } x \text{ which are both in the same world } w, \text{ } x' \text{ is } F \text{ iff } x'' \text{ is } F. \]

There wouldn’t be much benefit to be gained from allowing objects to have multiple counterparts in a single world, if these counterparts could never have different qualitative properties. Thus (5) must be rejected. But we can’t very well reject (4). For it is central to any reasonable conception of the ‘proposition role’ that, for every individual \( x \), there is such a thing as the proposition that \( x \) is \( F \). And one thing we know about the proposition that \( x \) is \( F \) is that it is, necessarily, true iff \( x \) is \( F \).

This problem arises no matter what method we adopt for identifying particular propositions with particular sets of worlds. But it can be seen as arising from the fact that, given the proposed semantics for de re modal claims about propositions, there is no acceptable answer to the question whether a world where \( x \) has both \( F \) and non-\( F \) counterparts is a member of the set that is the proposition that \( x \) is \( F \). Consider, for example, the proposition, concerning Hubert Humphrey, that he wins a presidential election. If we say that this proposition contains some world where

\[1 \text{ I am assuming that the predicate ‘} F \text{’ doesn’t contain any concealed references to individuals of the sort that would require a counterpart-theoretic treatment.} \]
Humphrey has both winning and losing counterparts – perhaps because we take it to be the set of worlds where Humphrey has some winning counterpart – the hybrid theory will yield the truth of

(6) The proposition that Humphrey wins is something which could be true even though Humphrey does not win.

If we say that it doesn’t contain all such worlds – perhaps because we take it to be the set of worlds where Humphrey has at least one counterpart and all his counterparts win – we will be committed to

(7) The proposition that Humphrey wins is something which could be false even though Humphrey wins

which is just as bad.

All of this generalizes straightforwardly to Lewis’s claim (1986a: 50ff.) that there are versions of the ‘property role’ and ‘relation role’ that are played by sets of (possible) individuals, and by sets of n-tuples of individuals. Consider the claim that

(8) For every individual x, there is a property p such that necessarily, for all y, y has p iff Rxy.

This should be part of the ‘property role’ on any reasonable conception. But if we adopt a hybrid semantics on which counterpart theory applies to individuals but not to properties, (8) will be translated as

(8*) For every individual x in the actual world, there is a property p such that whenever x’ is a counterpart of x in a world w and y is any individual at w, y has p iff Rx’y

which entails

(9) For every individual x in the actual world, any two counterparts x’ and x” of x which are both in the same world w, and any individual y in w, Rx’y iff Rx”y.

For example, if ‘R’ is ‘adjacent to’, this means that one cannot have two counterparts in a world such that something is adjacent to one but not the other.

2. Counterparts of propositions

The hybrid theory cannot account for the truth of sentences like (4) unless we give up the possibility of an object having several dissimilar counterparts at a world. So the hybrid theory must be rejected. No matter what sort of thing we take propositions to be, if we accept counterpart theory for individuals we must also accept some form of counterpart theory for propositions, properties and relations.
Extending counterpart theory to propositions gives us an appropriately symmetric thing to say about worlds where Humphrey has both winning and losing counterparts: these are worlds where the proposition that Humphrey wins also has two counterparts, one true and one false. But this isn’t enough on its own to solve the problem. According to the counterpart-theoretic translation scheme of Lewis 1968, the open sentence ‘Necessarily, \( p \) is true iff \( x \) is \( F \)’ will be satisfied by \( p \) and \( x \) if and only if, for any counterparts \( p' \) and \( x' \) of \( p \) and \( x \) in the same world, \( p' \) is true iff \( x' \) is \( F \). This entails that \( x \) cannot have both \( F \) and non-\( F \) counterparts in any world where \( p \) has at least one counterpart.\(^2\) This is a general deficiency in the ability of the 1968 theory to accommodate essential relations. In later work (1983: 44–45, 1986a: 232–33), Lewis suggests a way to avoid this difficulty. On the new approach, the translation of a sentence in which several terms occur in the scope of a modal operator will involve quantification over counterparts of the corresponding \( n \)-tuple of objects. Thus, (4) will be translated as follows:

\[
(4') \text{ For every individual } x \text{ in the actual world, there is some proposition } p \text{ in the actual world such that, for every world } w \text{ and for every ordered pair } \langle x', p' \rangle \text{ in } w \text{ which is a counterpart of } \langle x, p \rangle, \text{ } x' \text{ is } F \text{ iff } p' \text{ is true.}
\]

And this seems a reasonable claim. The property \textit{being such that one’s first member is } F \textit{ iff one’s second member is true} is one we would expect any ordered pair of the form \( \langle x, \text{ the proposition that } x \text{ is } F \rangle \) to have essentially.

This counterpart-theoretic treatment is most naturally combined with a conception of propositions as having the objects they are about as \textit{constituents}. In fact, Lewis grants that there is a legitimate version of the proposition role occupied by ‘singular propositions’: for example, the singular proposition that \( x \) is \( F \) might be identified with the ordered pair \( \langle x, \text{ the transworld set of all } F \text{ s} \rangle \). It is clear what unrelativized \textsl{truth} would amount to for such items: a proposition that is an ordered pair is true if its first member is a member of its second member.\(^3\) On this analysis, \( (4') \)

\(^2\) While the 1968 translation of (4) does not entail the unacceptable (5), this is so only for the bad reason that it does not require \( p \) to have a counterpart at every world where \( x \) has a counterpart. This is a separate problem for the 1968 theory: by making it too easy for claims of necessity to be true, the theory fails to validate sentences like ‘If it is necessary that \( p \) is true iff \( x \) is \( F \), and it is possible that \( x \) is \( F \), then it is possible that \( p \) is true’.

\(^3\) What about propositions that aren’t about any specific individuals, like the proposition that there is at least one cube? The easiest way to fit these into the counterpart-theoretic scheme is to take them to have the actual world itself as a constituent: the proposition that there is at least one cube could be taken to be the ordered pair \( \langle \text{the actual world, the set of all worlds that contain at least one cube} \rangle \).
will be derivable from set theory, the definitions of ‘proposition’ and ‘true’, and the independently plausible claim that ordered \( n \)-tuples have their elements essentially. However, there is nothing to stop us from combining this counterpart-theoretic account of \( de \) \( re \) modal claims about propositions with the identification of propositions with sets of possible worlds. In §1 I pointed out some obstacles to doing this: namely, that there is no way to make appropriate sense of the claim that a set of possible worlds is ‘in’ a given possible world, or true simpliciter. But these obstacles can be overcome fairly straightforwardly. The trick is to add extra argument places for worlds to the counterpart relation, enabling us to say things like ‘\( x \) at \( w_1 \) is a counterpart of \( y \) at \( w_2 \).’\(^4\) Given this new notion of counterparthood, we can translate

\[
(1) \quad \text{There is a false proposition which could have been true as}
\]

\[
(1\text{'}) \quad \text{There is a proposition } p \text{ such that } p \text{ is false at the actual world, and for some world } w, \text{ there is a proposition } p' \text{ such that } p' \text{ at } w \text{ is a counterpart of } p \text{ at the actual world, and } p' \text{ is true at } w.
\]

Likewise,

\[
(4) \quad \text{For every individual } x, \text{ there is a proposition } p \text{ such that necessarily, } p \text{ is true iff } x \text{ is } F
\]

can be translated as

\[
(4\text{'}) \quad \text{For every individual } x \text{ at the actual world, there is some proposition } p \text{ such that, for every world } w \text{ and every ordered pair } (x', p') \text{ which is a counterpart at } w \text{ of } (x, p) \text{ at the actual world, } x' \text{ is } F \text{ iff } p' \text{ is true at } w.
\]

This is consistent with the possibility of objects having dissimilar counterparts at the same world. However, if propositions are taken to be sets of possible worlds, there will be no hope of deriving (4) from antecedently plausible principles about the essences of sets and ordered pairs. For example, no one who was not already committed to the claim that the set of worlds where \( x \) has a counterpart that is \( F \) is the proposition that \( x \) is \( F \) would find it plausible that the property being such that one’s second member is the set of worlds containing at least one \( F \) counterpart of one’s first member is essential to the ordered pairs that have it. This does seem like an advantage of ‘singular propositions’ over sets of possible worlds.

\(^4\) Forbes (1985: 57ff.) recommends adding a single extra argument place for worlds to the counterpart relation, for unrelated reasons having to do with the treatment of contingent existence.
But this isn’t a decisive consideration: those who take set-theoretic reductions as seriously as Lewis will be prepared to revise their antecedent convictions about the essential characteristics of sets in the light of the reductions they end up endorsing.

Nevertheless, there is something pointless about the proposal to save the identification of propositions with sets of possible worlds by applying counterpart theory to propositions. On this approach, if we want to know whether a given proposition could have been true, we don’t look to see whether it has any worlds as members: rather, we have to ask ourselves whether it has any counterpart, with respect to some world, which has that world as a member. The question which non-actual worlds are members of a given proposition is relevant to our modal judgments only in so far as it plays a role in determining the extension of the counterpart relation. In effect, sets of possible worlds are functioning as mere codes, from which we recover a qualitative relation and a list of individuals. But this is a peculiarly arbitrary and indirect form of coding. Moreover, as we will see in the next section, it is an inadequate form of coding, since it sometimes maps non-equivalent singular propositions to the same set of possible worlds.

3. Indiscernible objects

Qualitatively indiscernible objects, like Max Black’s famous spheres (1952), are alike in all their qualitative properties. Let’s say that two things are strongly indiscernible if they are also alike in their qualitative relations to all objects other than their worldmates. (Given Lewis’s thesis (1986a: 69–71) that spatio-temporally related objects are always worldmates, this is not a very significant strengthening. Any qualitatively indiscernible objects in worlds where the only natural external relations are spatio-temporal ones – thus, any qualitatively indiscernible objects in worlds where Humean Supervenience (Lewis 1986b) is true – will automatically be strongly indiscernible.) Suppose, then, that $a$ and $b$ are strongly indiscernible objects at the same world, and let ‘$F$’ stand for a qualitative property. If propositions are sets of possible worlds, it seems to follow that the proposition that $a$ is $F$ is identical to the proposition that $b$ is $F$. For since all worlds are alike in their qualitative relations to $a$ and $b$, any reason to count a world as a member of the proposition that $a$ is $F$ is just as good a reason to count it as a member of the proposition that $b$ is $F$.

Since counterparthood is supposed to be a qualitative relation, any principle for identifying propositions with sets of worlds which depends on the counterpart relation – for example, the principle that identifies the proposition that $x$ is $F$ with the set of worlds where some counterpart of $x$ is $F$ – will evidently entail that the propositions that $a$ is $F$ and proposition that $b$ is $F$ are identical. But my argument doesn’t depend on any
particular principle of this kind: all I need is the premiss that the relation
being an \( x \) and \( w \) such that \( w \) is a member of the proposition that \( x \) is \( F \)
is a qualitative one.

Suppose we accept that the proposition that \( a \) is \( F \) is the proposition
that \( b \) is \( F \); what follows? Let’s assume to begin with that we’re in one of
the contexts where the principle of the necessity of identity is true: that
is, the counterparts of identity pairs are themselves identity pairs. Then
we can infer that the propositions are necessarily identical, and hence
necessarily equivalent:

\[
(10) \text{Necessarily, the proposition that } a \text{ is } F \text{ is true iff the proposition}
\text{that } b \text{ is } F \text{ is true.}
\]

This entails that

\[
(11) \text{Necessarily, } a \text{ is } F \text{ iff } b \text{ is } F.
\]

Since \( a, b \) and ‘\( F \)’ were arbitrary, we can generalize to the conclusion that
necessarily, whenever two objects are strongly indiscernible, they neces-
sarily have exactly the same qualitative properties. But this is very implau-
sible. Qualitative indiscernibility, surely, is a rather precarious relation
even at worlds where Humean Supervenience is true. It seems obvious,
for example, that one of Black’s spheres could have been a little bit smaller
while the other one remained the same size.

Granted, Lewis holds that in some contexts identity pairs can have non-
identity pairs as counterparts. He also maintains that modal idioms are
highly context-dependent, with different counterpart relations being
selected even within the same sentence depending on how we refer to the
objects whose modal properties we are discussing. Lewis uses this
machinery to explain how one could express a truth in asserting a sen-
tence like ‘Although this bit of plastic is identical to that dishpan, the
former could have been made a day before the latter’ (1986a: 253). We
could do the same for ‘Although the proposition that \( a \) is \( F \) is identical to
the proposition that \( b \) is \( F \), the former could have been true while the
latter was false’.

But even if we make this move, we still have to say counterintuitive
things in contexts where our means of referring to the proposition don’t
privilege either of the two ways of thinking of it. For example, suppose
that Joe in fact asserted that \( a \) is \( F \) and that \( b \) is \( F \), and nothing else.
Intuitively, anyone who uttered the following sentence would be saying
something false:

\[
(12) \text{If any proposition asserted by Joe is necessarily true iff } a \text{ is } F,
\text{no proposition asserted by Joe is necessarily true iff } b \text{ is } F.
\]

This seems beyond the power of the context-switching story to explain.
In a sincere utterance of (12), there is nothing to induce the switch in
context from the antecedent to the consequent which would be required for the utterance to express a falsehood.\(^5\)

If there were some deep theoretical or intuitive reason to identify propositions with sets of possible worlds, perhaps it would be worth trying to learn to live with such aberrations. But I can see no such reason, even from Lewis’s point of view. The impression that sets of possible worlds are ‘just right’ for a version of the proposition role seems to be due to the thought that, by encapsulating the information about what has to be the case for them to be true, they allow for an especially simple and natural way to analyse modal claims about truth and falsity. Once we realize that these claims will have to be given a counterpart-theoretic treatment in any case (given counterpart theory for individuals), this apparent advantage of sets of possible worlds over ‘singular propositions’ disappears. I conclude that, for the counterpart theorist, singular propositions outperform sets of possible worlds across the board as candidates to be cast in the proposition role, even on the narrowest conception of what that role might amount to.

University of Pittsburgh
Pittsburgh, PA 15260, USA
csd6@pitt.edu

References

\(^5\) For more arguments showing the limitations of the context-switching idea as a means of reconciling apparent inconsistencies between claims of identity and intuitions about possibility, see Fine 2003.