Expressivism about Chance

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Framework for talking about chances

Basic ideology: for each proposition A, there is a real number ch(A): the timeless chance of A being true.

Derived ideology:
- the chance of A given background assumptions B = ch(A|B) ≈ ch(AB)/ch(B).
- the time-dependent indeterministic chance of A at t = ch(A|complete history up to t).
- the statistical-mechanical chance of A at t = ch(A|macrohistory up to t).
- ‘A is nomically necessary’ ≈ ‘ch(A) = 1’

Framework for talking about rational belief

If x is ideally rational, then we can find
- a probability function C_x (x’s prior credence function) and
- a function E_x from times to propositions (x’s evidence function), such that
x is disposed, at each time t, to believe each proposition A to degree C_x(A|E_x(t)).

The Principal Principle

PP: C(A|P = ch) = P(A)

(if the LHS is defined)

- C is any ideally rational prior credence function.
- A is any proposition.
- P is any probability function on propositions.
- ‘P = ch’ abbreviates ‘∀B.ch(B) = P(B)’
Bracketing technical worries about infinity, PP is equivalent to the claim that rational priors are weighted averages of self-assured probability functions:

\[
C = a_1P_1 + a_2P_2 + a_3P_3 + \ldots
\]

where \( P \) is "self-assured" iff \( P(P=\text{ch}) = 1 \)

**Corollary:**

\[
P(P = \text{ch}) = C(P = \text{ch}|P = \text{ch}) = 1
\]

if \( C(P=\text{ch}|P=\text{ch}) \) is defined

**PP:** \( \mathbf{C(A|P = \text{ch}) = P(A)} \)

(if the LHS is defined)

**A priori reductionism**

Each proposition of the form ‘\( \text{ch}(A)=x \)’ is a priori equivalent to some proposition about the total history of the world.
The undermining problem

Premise: For some distinct $P_1, P_2$ and total history $H$:
1. $C(H|P_1=ch)$ and $C(H|P_2=ch)$ are positive (by PP).
2. So $C(H|P_1=ch)$ and $C(H|P_2=ch)$ are positive.
3. If a priori settles whether $P_1=ch$ or $P_2=ch$ or neither (by a priori reductionism)
4. $H$ a priori entails either $P_1\neq ch$ or $P_2\neq ch$ (since $P_1=ch$ and $P_2=ch$ are a priori inconsistent).
5. So either $C(H\land P_1=ch) = 0$ or $C(H\land P_2=ch) = 0$.

The ‘New Principle’ to the rescue?

PP: $C(A|P=ch) = P(A)$ (if the LHS is defined)

NP: $C(A|P=ch) = P(A|P=ch)$ (if the LHS is defined)
Problems for a priori reductionism + NP

1. Jaggedness.
   Implausible that ideal prior credence functions make sudden, unsmooth transitions at the boundaries between \(P_i=\text{ch}\) and \(P_j=\text{ch}\).

2. Forced agreement
   Implausible that all ideal prior credence functions draw these boundaries in exactly the same places.

3. Vagueness
   Surely it's vague where the boundaries are. But if so, NP entails that no prior credence function is definitely ideally rational, which is implausible.

A posteri or reductionism to the rescue?

\(P_1\): probability function in which the coin is biased 2-1 towards Heads.
\(P_2\): probability function in which the coin is biased 2-1 towards Tails.
\(H_1\): total history in which the coin lands Heads every time.
\(H_2\): total history in which the coin lands Tails every time.

Surely ideally rational believers can at least know this much:
if \(H_1 \lor H_2\) is consistent with \(P_1=\text{ch} \lor P_2=\text{ch}\), then \(H_1\) entails \(P_1=\text{ch}\) and \(H_2\) entails \(P_2=\text{ch}\).

A problem for reductionists of all stripes: interpreting aliens

- Suppose the Martians follow some weird inductive method. E.g.: for any given coin, they assign bizarrely low credence to the hypothesis that that coin lands heads exactly 50% of the time.
- It's tempting to attribute correspondingly bizarre beliefs about chances to these Martians—e.g. that the chance that the coin will land heads exactly 50% of the time is low.
- But for the reductionist, such ascriptions are hard to justify! Even if the Martians have a word 'chance' that they use to make these bizarre remarks, we should deny that it means chance.

Expressivism to the rescue?

Expressivism about whether \(P\): the psychological state we call “believing [to such-and-such degree] that \(P\)” is not strictly speaking the state of believing any proposition [to any degree].
Machinery for stating expressivist semantics

The semantic value of a sentence is a set of quasi-worlds — <world, probability function over sets of worlds> pairs.

Where $\phi$ is a sentence not about chance and $S$ is the set of worlds where $\phi$ is true:

$|\phi| = \{<w,P>|w \in S\}$

$|\text{ch}(\phi) = x| = \{<w,P>|P(S) = x\}.$

For arbitrary $\phi$:

$|\text{ch}(\phi) = x| = \{<w,P>|P^*(|\phi|) = x\}$

where $P^*$ is the self-assured extension of $P$:

$P^*(S) = _{df} P(\{w|<w,P> \in S\}.$

First steps to the goal

First: explain quasi-credences in terms of prior quasi-credences:

• $x$'s quasi-credence in $A$ is $x$'s prior quasi-credence in $A$, conditional on $\{<w,P>: E_x \text{ is true at } w\}$

Second: explain prior quasi-credences in terms of prior credences in genuine propositions.

The expressivist goal

Explain, in terms of one's attitudes towards genuine propositions, what it is to have a given "quasi-credence" in a given set of quasi-worlds.

The simplest possible strategy

Where $C$ is one's prior credence function, one's quasi-prior credence function is $C^*$ (the self-assured extension of $C$).

Problem: one never assigns positive credence to any two inconsistent (quasi)-propositions of the form $P = \text{ch}$. 
The “objectivisation” strategy (Skyrms, Jeffrey)

Takes as input a special partition \( \{H_i\} \).
Where one’s prior credence function is \( C \),
one’s quasi-prior credence function is the weighted sum
\[
C^+ = C(H_1|\cdot)C(H_1) + C(H_2|\cdot)C(H_2) + \ldots
\]
Corollary: where \( \phi \) is not about chance,
\[
C^+(\text{ch}(\phi) = x) = C(\lor \{H_i; C(\phi|H_i) = x\})
\]

Objections to the objectivisation strategy:
1. Where do we get \( \{H_i\} \)?
2. Even if you start with a \( C \) that is a weighted average of nice simple probability functions, typically \( C^+ \) will end up assigning zero credence to the chance function being any of these nice and simple functions.
3. One cannot rationally be uncertain what the chances are conditional on a completely detailed proposition about total history.

The “best decomposition” strategy

Suppose one’s prior credence distribution admits of a best decomposition as a weighted sum of relatively simple probability distributions:
\[
a_1P_1 + a_2P_2 + a_3P_3 + \ldots
\]
Then one’s prior quasi-credence distribution is the weighted sum
\[
a_1P_1^* + a_2P_2^* + a_3P_3^* + \ldots
\]
What makes a decomposition ‘best’?

- As far possible, give higher weight to simple probability functions.
- Give similar weight to similar probability functions.
- Maybe we should also look at the actual computational processes that underlie assigning the person those prior credences.
- To the extent that there’s no unique best way to do it, it’ll just be vague what one believes about chance.

How we avoided the Frege-Geach problem

- The semantic machinery applies in the same way to all sentences, including those where ‘ch(A)=x’ occurs embedded.
- The “best decomposition” strategy allows that even the ideally rational can have high credence in a disjunction of claims about chance without having high credence in any disjunct.

Challenges and Objections

1. What about agents whose degrees of belief aren’t probabilistically coherent, or whose inductive dispositions are too unstable to be encoded by a prior credence function?
2. Isn’t it possible, even without having incoherent credences, to acquire crazy beliefs about chance by picking them up from other language-users without “full understanding”?
3. Does expressivism about chance require expressivism about lots of other subject matters? Would that be bad? (What if one of the subject matters was belief itself?)