Pinning Down the Meanings of Quantifiers[1]

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1. Conciliationism about ontological questions

A useful thought experiment for any deadlocked debate: imagine isolated communities each of which behaves like an unreflective version of one of the sides in the real-world debate.

Three kinds of conciliationist about ontological debates (e.g. the Special Composition Question):

> *Predicate variantist:* the communities just mean different things by all the relevant predicates ('table', 'trout-turkey', 'part'…)
>
> *Quantifier variantist*: the communities mean different things by 'there is', 'something', etc.
>
> *Radical variantist:* No words, or almost no words, have the same meanings for any two of the communities.

I'll be focusing on the quantifier variantist who is not also a predicate variantist.

2. How logical role might pin down meaning: the case of 'or'

$\vee$-intro rule: $\phi \vdash \phi\vee\psi$, $\psi \vdash \phi\vee\psi$
$\vee$-elim rule: If $\phi \vdash \chi$ and $\psi \vdash \chi$, then $\phi\vee\psi \vdash \chi$

> $\vee$-intro property: $\forall P \forall Q(P \leq [\vee](P, Q) \wedge Q \leq [\vee](P, Q))$
> $\vee$-elim property: $\forall P \forall Q \forall R((R \leq P \wedge R \leq Q) \rightarrow R \leq ([\vee](P,Q))$

3. A dialectically ineffective collapse argument for quantifiers

Closed $\exists$-intro rule: $\phi \vdash \exists v \phi[v/a]$ for any variable v and name a
Closed $\exists$-elim rule: if $\phi \vdash \psi$ and the name a does not occur in ψ, $\exists v \phi[v/a] \vdash \psi$

> Closed $\exists$-intro property: $\forall x \forall F(Fx \leq [\exists](F))$
> Closed $\exists$-elim property: $\forall x \forall F \forall P((Fx \leq P \wedge P$ is not about x$) \rightarrow [\exists](F) \leq P))$

- Supplementary claim: $\forall x \forall F($F is not about x $\rightarrow [\exists](F)$ is not about x$)$

4. Consequence relations on open sentences

Open $\exists$-intro rule: $\phi \vdash \exists v \phi$

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[1] See 'Quantifier Variantism and the Collapse Theorems', *The Monist* 97 (2014): 503–570.

Open ∃-elim rule: if $\phi \vdash \psi$ and v is not free in ψ, $\exists v\phi \vdash \psi$

What could $\phi \vdash \psi$ usefully mean when ϕ and/or ψ contains free variables?

Idea: take open sentences to express properties and relations, and interpret $\vdash$ in terms of a notion of entailment for properties and relations which is not explained using object-quantifiers.

- 'x is square' $\vdash$ 'x is rectangular' will be true iff *squareness* ≤ *rectangularity*.
- 'x loves y' $\vdash$ 'x values y' will be true iff *loving* ≤ *valuing*; or (equivalently) if *being loved by* ≤ *being valued by*.
- 'x is square' $\vdash$ 'there are squares' will be true if *squareness* ≤ *being such that there are squares*. The latter property is the *expansion* of the proposition *there are squares*.

Denote the *expansion* operation (mapping propositions to properties, properties to binary relations, binary relations to ternary relations…) by +. To be +P is to be such that P.

5. Non-quantificational conceptions of entailment for properties

Some possible glosses on F=G not involving object quantification:

> To be F is to be G
> $\forall X(XF \leftrightarrow XG)$ (X a third-order variable)
> (When F and G are qualitative:) $\forall X$(X is qualitative$\rightarrow(XF \leftrightarrow XG)$)

Some possible glosses on 'F≤G':

> $F \leq G$ iff $F = F \wedge G$
> $F \leq G$ iff $G = F \vee G$
> $F \leq G$ iff $F \wedge G = F \vee (F \wedge G)$
> $F \leq G$ iff $(F \vee G) \wedge G = F \vee G$

6. A collapse argument that isn't hopeless

Open ∃-intro property: $\forall F(F \leq +[\exists]F)$
Open ∃-elim property: $\forall F \forall P(F \leq +P \rightarrow [\exists]F \leq P)$

Theorem: if two operations both have the open ∃-intro and the open ∃-elim property, then for any F, the results of applying them to F are equivalent (mutually entailing).

- Also: if two operations both have the *qualitative* restrictions of these properties, then the results of applying them to any qualitative F are equivalent.
- This is significant since it is not *obvious* that the quantifier variantist has any motivation for positing any variation in higher-order quantification restricted to qualitative properties and relations.

Analogous for universal quantification:

Open ∀-intro property: ∀F∀P(+P≤F then P≤[∀]F) Open ∀-elim property: ∀F(+[∀]F≤F)

7. Equivalently…

[∃] is a *lower adjoint* of expansion:	[∃]F≤P ↔ F≤+P for all F, P
[∀] is an *upper adjoint* of expansion:	P≤[∀]F ↔ +P≤F for all F, P

These follow from the open intro/elim properties, given

Expansion entailment: P≤Q iff +P≤+Q

Consequence: Monotonicity for [∃] and [∀]: F≤G → [∃]F≤[∃]G and [∀]F≤[∀]G

8. Or equivalently again…

Transparency for [∃] and [∀]: If F≈G (i.e. F≤G and G≤F), then [∃]F≈[∃]G and [∀]F≈[∀]G

[∃] and [∀] "respect conjunction": [∃](F∧G)≤[∃]F∧[∃]G and [∀]F∧[∀]G≤[∀](F∧G)

Open ∃-intro property F≤+[∃]F plus '∃-triviality property': [∃]+P≤P
Open ∀-elim property +[∀]F≤F plus '∀-triviality property': P≤[∀]+P

If you agree that our own quantifier-meanings have these properties but you're determined to be quantifier variantist, which of these should you say fails for the other communities' languages? In some cases, assuming that our language isn't the "biggest" one, I think transparency has to go!

- Suppose that there are no trout-turkeys, but 'There are trout-turkeys' is true in Universalese. To be a trout-turkey is to be a trout turkey such that there is a trout-turkey. Let ϕ be a sentence in Universalese that expresses the (false) proposition that there is a trout-turkey. Then 'There are trout-turkeys such that ϕ' is false in Universalese.
- 'To be F is to be G' must also vary. Even though 'trout-turkey' means the same thing in English and Universalese, and 'trout turkey such that ϕ' in Universalese means what 'trout-turkey such that there is a trout-turkey' means in English, 'To be a trout-turkey is to be a trout-turkey such that ϕ' is false in Universalese while 'To be a trout turkey is to be a trout-turkey such that there is a trout-turkey' is true in English.

9. Do the meanings of English quantifiers actually have these properties?

Contingentist objection: In English, [∃] doesn't have the open ∃-intro property, since *not being identical to anything* does not entail *being such that something is not identical to anything.*

10. Questions for those who answer 'no'

Are there upper and lower adjoints of expansion?

Which properties are "safe" (such that every proposition whose expansion they entail is true)? Which properties are "guaranteed" (entailed by the expansion of a truth)?

- Clearly every *instantiated* property is safe: if something is F and F≤+P, then something is +P, so P. Similarly, every guaranteed property is instantiated by everything.
- But the contingentist will think that some uninstantiated properties are also safe—for example *not being identical to anything*. And some universally instantiated properties fail to be guaranteed, for example *being identical to something*.

Natural answer: The safe properties are the properties for which it's possible for there to be something that *actually* instantiates them. The guaranteed properties are the properties for which it's necessary that everything *actually* instantiates them.

- More generally: we can introduce "possibilist" quantifiers $\exists_\diamond$ and $\forall_\diamond$ as follows:

$$\exists_\diamond x\phi =_{df} \forall W(W \to \diamond\exists x\diamond(W \wedge \phi))$$
$$\forall_\diamond x\phi =_{df} \exists W(W \wedge \Box\forall x\Box(W \to \phi))$$[2]

  Contingentists should, I suggest, say that the semantic values of $\exists_\diamond$ and $\forall_\diamond$ are lower and upper adjoints of expansion respectively.

- This follows from the claim that $\forall F\forall G(F\leq G \leftrightarrow \Box\forall x\Box(Fx\to Gx))$.

$$\Box(\forall W(W \to \diamond\exists x\diamond(W \wedge Fx)) \to P) \leftrightarrow \Box\forall x\Box(Fx \to P)$$

Could the quantifier variantist say that ∃ and ∀ vary in meaning between the different relevant communities while denying that $\exists_\diamond$ and $\forall_\diamond$ vary in a corresponding way?

- This would be a view on which, e.g., ∃x(x is a trout-turkey) is true in some of the relevant communities' languages and false in others, but $\exists_\diamond$x(x is a trout-turkey) is true in all of them. This is implausible.
- So the collapse argument still stands.

11. An objection to contingentism

- It seems like a weird coincidence that we can get the same operation O (up to equivalence) either by defining OF = $\exists_\diamond$xFx or by defining OF as the conjunction of all P for which F≤+P.
- Doesn't the operation that plays the simple algebraic role best deserve the label 'unrestricted quantification'?

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[2] These are non-standard: the standard versions would restrict W to 'maximal' propositions which necessitate every proposition with which they are consistent. But this restriction is actually redundant assuming that it is necessary that there is a maximal true proposition.