# Data-Driven Incentive Alignment in Capitation Schemes

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**Abstract**

This paper explores whether Big Data, taking the form of extensive but high dimensional records, can reduce the cost of adverse selection in government-run capitation schemes. We argue that using data to improve the ex ante precision of capitation regressions is unlikely to be helpful. Even if types become essentially observable, the high dimensionality of covariates makes it infeasible to precisely estimate the cost of serving a given type. This gives an informed private provider scope to select types that are relatively cheap to serve. Instead, we argue that data can be used to align incentives by forming unbiased and non-manipulable ex post estimates of a private provider's gains from selection.

KEYWORDS: adverse selection, big data, capitation, observable but not interpretable, health-care regulation, detail-free mechanism design, model selection.

## 1 Introduction

This paper explores the value of Big Data in reducing the cost of adverse selection in government-run capitation or voucher schemes, with a particular emphasis on healthcare

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insurance.

Traditional capitation schemes pay private providers an estimate of expected public cost of service for each individual they enroll. Capitation payments can be conditioned on agreed upon user characteristics (then, they are said to be risk-adjusted). While capitation programs are a popular way to outsource government mandated services to the private sector, they are often plagued by adverse selection. Private service providers have strong incentives to select types that are cheaper to serve than their capitation payment, which increases the cost of serving the overall population. In the context of Medicare Advantage, a program which lets US Medicare recipients switch to private health insurance providers, Batata (2004) and Brown et al. (2014) report yearly overpayments in the thousands dollars for patients selected by private plans.

A natural strategy to reduce adverse selection is to build precise, risk-adjusted, ex ante capitation schemes, reimbursing private plans for the expected cost of taking care of the specific patients they select. This suggests that Big Data — i.e., the availability of high-dimensional patient records — which can be used to condition capitation payments on precise individual characteristics, may be of considerable help in reducing the effects of adverse selection. We take a different view and argue that under the correct Big Data limit, this naïve use of high-dimensional co-variates is likely to be of limited value. Instead, we suggest that data may be more successfully used to form unbiased ex post estimates of strategic selection by private providers. Correcting capitation formulas with these ex post estimates aligns the public and private providers' incentives.

Our model considers a single public provider seeking to outsource the provision of health-care services to a single private provider.[1] The private provider may have a comparative advantage in treating certain types so that some selection of patients may be welfare enhancing. However the private provider also has incentives to select patients whose cost of care is mispriced. This creates a distinction between legitimate selection characteristics,

[1]In the case of Medicare Advantage, the private provider would correspond to a PPO or HMO.

which predict comparative advantage, and illegitimate selection characteristics, which predict costs but not comparative advantage. Efficient selection need only depend on legitimate selection characteristics.

Our modeling choices reflect both the opportunities and limitations presented by Big Data. We assume that high-dimensional records isomorphic to patients' types — i.e. sufficient statistics for patients' cost of care — are observable. However, we also recognize that the number of such possible types need not be small relative to the sample size of available cost data, thereby limiting their use for prediction. This leads us to study mechanism design at a joint limit where both the sample size and number of relevant covariates are large.[2] At this Big Data limit, sufficient statistics of types are observable but not interpretable.

Our first set of results considers traditional capitation schemes, which, as emphasized by Brown et al. (2014), seek to reimburse private providers for the expected cost of treating patients given ex ante observables. We show that such schemes induce efficient selection when capitation fees conditional on types are precisely estimated, or when the private provider is constrained to select only on the basis of legitimate characteristics. However, we show that these conditions fail under our Big Data limit. Indeed, cost-estimates conditional on types remain noisy even for large samples. Hence, even though types are observable, it is possible for the private provider to maintain an informational advantage which induces inefficient selection and increases the average cost of care.

In spite of these limitations, we are able to show that an appropriate ex post use of data can achieve efficient selection at no excess cost for the public provider whenever legitimate selection characteristics are common knowledge. Instead of including a large number of covariates to obtain a more precise capitation formula, we argue that it is sufficient to augment the baseline capitation formula (based on legitimate characteristics) with a single additional term measuring ex post selection by the private provider. This additional term takes the

[2] This is the limit taken in the statistics literature concerned with Big Data. See Belloni et al. (2013, 2014) for recent examples in econometrics.

form of an appropriately weighted covariance between the distribution of types selected by the private provider, and the residuals from the basic capitation regression evaluated on out-of-sample costs. This "strategic capitation scheme" induces efficient selection, and, importantly, does not give the public provider any incentive to bias its report of out-of-sample costs. This last property allows us to extend our approach to health exchanges for which out-of-sample cost realizations would be reported by competing healthcare providers.

The basic idea behind strategic capitation can be extended to environments where legitimate selection characteristics are not common knowledge. In this case it is still possible to achieve a meaningful share of first-best efficiency by using generalized strategic capitation schemes that let private providers specify the characteristics they wish to select on. This flexibility comes at a cost related to the complexity of the class of models the private provider is allowed to select from. We show that the performance guarantees of this scheme are unimprovable by studying the exact direct mechanism design problem in specific environments.

The paper contributes to the theoretical literature on adverse selection in insurance markets.[3] Our work is particularly related to Glazer and McGuire (2000), who study optimal risk-adjustment in a Bayesian setting. They show that when selection is possible, optimal ex ante reimbursement schemes should deviate from simply reimbursing private providers for the expected cost of taking care of patients. In particular, capitation schemes should adjust reimbursement rates to dull the effect of cream-skimming by private providers. We show how to induce efficient selection by using information about patient types and ex post cost data.

Our mechanism is closely related to that of Mezzetti (2004), which also uses noisy ex post information to provide accurate ex ante incentives. Also related is the work of Riordan and Sappington (1988) who show how to exploit noisy ex post signals to screen agents at

[3]See for instance Rothschild and Stiglitz (1976), Bisin and Gottardi (1999, 2006), Dubey and Geanakoplos (2002).

no cost to the principal. As we clarify in greater detail later in the paper, our work differs for two main reasons. First, we are interested in prior-free mechanisms and do not make the identification assumptions required in Riordan and Sappington (1988). Second, ex post signals (here the public provider's hold-out cost data) need not be publicly observed and we must ensure that the relevant party has correct incentives for reporting. Third, unlike Mezzetti (2004), we require exact budget-balance.

Our work is motivated by a growing empirical literature which documents cream-skimming in health insurance and education markets, and studies the efficiency of various risk-adjustment schemes (Frank et al., 2000, Mello et al., 2003, Batata, 2004, Epple et al., 2004, Newhouse et al., 2012, Walters, 2012, Brown et al., 2014). Our analysis is largely inspired by Brown et al. (2014) which shows that increasing the number of covariates used in Medicare Advantage's capitation formulas has in fact led to an increase in the cost of adverse selection to the state.[4] We complement this result by showing that naïve uses of data are unlikely to resolve adverse selection, but suggest that progress can be made by using data to detect selection ex post.

The paper is structured as follows. Section 2 describes our framework, and in particular our approach to Big Data. Section 3 uses a simple example in which legitimate selection characteristics are common knowledge to delineate the mechanics of adverse selection under various capitation schemes. Section 4 generalizes the analysis to settings in which the private provider's comparative advantage is not common knowledge. Section 5 presents several extensions. In particular we show how to adapt our approach to address adverse selection in markets with multiple private providers and no public provider. Extensions allowing for heterogeneity in the quality of care across providers, as well as dynamic selection, are contained in Appendix A. Proofs are collected in Appendix B unless mentioned otherwise.

[4]Newhouse et al. (2012) argues that the cost of adverse selection may be overstated.

# 2 Framework

Our model seeks to capture three main features. The first is selection by private healthcare providers, such as HMOs or PPOs, which we model as a reduced form cost for attracting different populations.

The second is that public and private providers have heterogeneous comparative advantages in treating patients. Indeed, insurance providers serve a role beyond that of financial intermediaries. This role may include selecting, monitoring and generally resolving agency problems vis à vis doctors and hospitals, as well as encouraging preventive care and healthy habit formation. For instance, Bundorf et al. (2012) show that in their sample, HMOs have a comparative advantage over PPOs in treating high risk patients. As a result there is a reason for both public and private providers to be active and the question of efficient patient allocation becomes relevant.

Third, we seek to correctly capture the forces that make Big Data attractive but challenging: we assume that high dimensional records make patients' type observable, but that as a result, even with large samples of patients, it is not possible to form precise estimates of expected cost of treatment conditional on type. Types are observable but no interpretable.

The lead example for our work is Medicare Advantage, a program which lets US Medicare recipients switch to private insurance providers such as HMOs and PPOs. Medicare Advantage is a large and growing program. It covers a population of roughly 15 million, out of the roughly 50 million enrolled in Medicare, and its size was multiplied by three from 2005 to 2015. Selection by private providers is also an ongoing concern threatening the financial sustainability of the program (Batata, 2004, Brown et al., 2014).

## 2.1 Players, Actions, Payoffs

We study the relationship between a public health care provider $p_0$ responsible for the health expenses of a set $I = \{1, \cdots, N\}$ of patients and an independent private provider $p_1$.

**Treatment costs.** Each patient $i \in I$ has a type $\tau_i \in T \subset \mathbb{R}^n$ where the set of types $T$ is potentially very large, but finite. Type $\tau$ is a sufficient statistic for the patient's cost of care. For any sample $J$ of patients, we denote by $\mu_J \in \Delta(T)$ the sample distribution of types $\tau$ defined by $\mu_J(\tau) \equiv \frac{|J^\tau|}{|J|}$, where $J^\tau \equiv \{j \in J | \tau_j = \tau\}$, and $|J|$ denotes the cardinal of $J$.

Realized cost of care for a patient $i$ of type $\tau$, insured by provider $p$ are denoted by $\widehat{c}_i(p) \geq 0$, and the corresponding random variable conditional on $\tau$ and $p$ is denoted by $c(\tau, p) \in \Delta(\mathbb{R}_+)$. Treatment costs are exchangeable conditional on patient type $\tau$ and provider $p$.

We denote by $\mathbb{E}_c$ expectations under the realized distribution of costs $c$. Let $\kappa(\tau, p) \equiv \mathbb{E}_c[\widehat{c}|\tau, p]$ denote the expected realized cost of treatment for a patient of type $\tau$ by provider $p$, given distribution $c$, so that $\widehat{c}_i(p)$ can be written as

$$\widehat{c}_i(p) = \kappa(\tau_i, p) + e_{i,p}, \tag{1}$$

where $\mathbb{E}_c[e_{i,p}] = 0$.

To simplify welfare statements, we assume that the public and private provider share a common prior $\nu \in \Delta(\mathbb{R}^{T\times\{p_0,p_1\}})$ over costs $c$ (as well as over information structures to be described below). Note that the capitation mechanisms we study do not rely on the common prior assumption. Our performance bounds remain valid in a non-common prior setting, with expectations are taken under the private provider's prior.

**Selection.** Private provider $p_1$ can choose an expected selection policy $\lambda : T \to [0, 1]$ at a cost $K(\lambda) \geq 0$. Consistent with observations in Starc (2014), this reduced-form cost of selection may be thought of as a cost of advertisement.[5] Realized selection $\Lambda \subset I$ is a mean preserving spread of intended selection $\lambda$ defined by

$$\mathbf{1}_{i\in\Lambda} = \lambda(\tau_i) + \varphi_i$$

[5]Under a more standard model of selection along the lines of Rothschild and Stiglitz (1976), the private provider would screen patients through a menu of discounts and benefits specifically appealing to desirable types.

where error term $(\varphi_i)_{i \in I}$ has expectation equal to zero, and is independent of cost shocks $e_{i,p}$, but may be correlated across different types $\tau \in T$. For instance, recruitment ads may unexpectedly attract a population different from the targeted one.

**Realized payoffs.** Given a selection decision $\lambda$ by private provider $p_1$, a realized selection $\Lambda$, and a transfer $\Pi \in \mathbb{R}$ from $p_0$ to $p_1$, the realized surpluses $U_0$ and $U_1$ accruing to the public and private providers are

$$U_0 = -\Pi + \sum_{i \in \Lambda} \widehat{c}_i(p_0) \quad \text{and} \quad U_1 = \Pi - \sum_{i \in \Lambda} \widehat{c}_i(p_1) - K(\lambda).$$

## 2.2 Data

We model explicitly the role that data plays in the contracting problem. In particular we formalize a "Big Data limit" which captures the idea that although types are observable, when the type-space is large, the public provider may still have very imprecise estimates of expected treatment costs conditional on types. A consequence illustrated in Section 3 is that imprecise additional signals may give the private provider a significant advantage in selecting patients.

**Samples.** Both providers $p_0$ and $p_1$ observe a public dataset of types and cost realizations $D_0 = \{(i, \tau_i, \widehat{c}_i(p_0)) | i \in D_0\}$ for provider $p_0$, where $i \in D_0$ denotes a patient $i$ whose record is included in $D_0$. In addition, we denote by $D_0^\tau = \{(i, \tau_i, \widehat{c}_i(p_0)) | \tau_i = \tau, i \in D_0\}$ the cost data relating to patients of type $\tau$. We assume that for every $\tau \in T$, the set $D_0^\tau$ is non-empty, which implies $|T| \leq |D_0|$: the sample size of dataset $D_0$ is at least as large as the type space.

Provider $p_1$ privately observes a dataset $D_1 = \{(i, x_i, \widehat{c}_i(p_1)) | i \in D_1\}$ reporting both her own costs, and side-signals $x_i$ for a sample of patients $i \in D_1$. Side signal $x_i$ captures other signals beyond cost realizations that the provider may be able to use in order to select patients.

Finally, we assume that provider $p_0$ has access to a hold-out sample $H = \{(i, \tau_i, \widehat{c}_i(p_0)) | i \in H\}$ of her own costs, independent of data $D_1$ conditional on the realization of cost distribution $(c(\tau, p))_{\tau \in I, p \in \{p_0, p_1\}}$. Hold-out sample $H$ may consist of ex post cost realizations for the current set of patients enrolled by the public provider. Alternatively, $H$ may correspond to past cost data, securely encrypted, and verifiably released only after patient selection has occurred.[6] Contracts will be allowed to depend on hold-out sample $H$, but we will take seriously the public provider's incentive to reveal correct information.[7]

**Big Data.** Our model of Big Data consists of two assumptions (recall that $\mu_I \in \Delta(T)$ is the sample distribution of types $\tau$ in the patient population):

(i) types $\tau \in T$ are publicly observable;

(ii) sample data $D_0$, type space $T$ and sample $I$ grow large together, so that

$$\limsup_{|D_0| \to \infty} \frac{|I|}{|D_0|} < \infty \quad \text{and} \quad \liminf_{|D_0| \to \infty} \mathbb{E}_{\mu_I} \left[ \frac{1}{|D_0^\tau|} \right] > 0.$$

Points $(i)$ and $(ii)$ summarize what we think are the opportunities and limitations of Big Data. On the one hand, high dimensional records make types observable $(i)$. On the other hand, even though the aggregate sample size $D_0$ is large, the state space $T$ is not small compared to $D_0$. Under sample measure $\mu_I$, the size $|D_0^\tau|$ of sufficiently many subgroups $D_0^\tau$ remains bounded above, which implies that public provider $p_0$'s estimates of costs on the basis of data $D_0$ necessarily remain noisy. We note that for the results in this paper to hold, the first condition in $(ii)$ can be replaced with the weaker $\lim_{|I| \to \infty} |D_0| = \infty$, even though we believe the condition as stated to be realistic.

Note that since type space $T$ is changing, the limit described above considers sequences of models. It should be treated as a stylized approximation capturing the fact that in

[6] For instance, an encrypted version of the data can be released before selection occurs, with a decryption key publicized after patient enrollment has occurred.

[7] Specifically, we will address the public provider's incentives to bias its records in order to reduce payments to the private provider. For instance the public provider could down-code interventions happening to its own patients

the existing data, the number of a priori relevant characteristics (or columns) is not small compared to the number of data points (or rows). Throughout the paper, we provide bounds that depend explicitly on $\frac{|I|}{|D_0|}$ and $\mathbb{E}_{\mu_I}\left[\frac{1}{|D_0^\tau|}\right]$.

## 2.3 Contracts, Equilibrium and Welfare

**Contracts.** For any set $J \subset I$, let $\tau_J \equiv (\tau_i)_{i\in J}$, and $\widehat{c}_J(p) \equiv (\widehat{c}_i(p))_{i\in J}$. We denote by $H_R = \{(i, \tau_i, \widehat{c}_i^R(p_0)), i \in H\}$ the hold-out data reported ex post by $p_0$. We emphasize that these are reports of privately observed costs, and that the public provider must be given incentives to report truthfully. A capitation contract between the public and private provider is a mapping $\Pi(D_0, \Lambda, \tau_I, H_R) \in \mathbb{R}$, specifying the aggregate payments received by private provider $p_1$ as a function of public data $D_0$, realized selection $\Lambda$, the distribution of types $\tau_I$ in patient population $I$, and reported hold-out sample data $H_R$.

**Equilibrium.** We denote by $\beta$ the public provider's strategy, mapping hold-out data $H$ to reported hold-out data $H_R$. Given a capitation contract $\Pi$, a selection strategy $\lambda$, and a reporting strategy $\beta$, the public and private providers obtain expected payoffs,

$$\mathbb{E}_\nu U_0 = \mathbb{E}_\nu\left[-\Pi + \sum_{i\in\Lambda}\widehat{c}_i(p_0)\middle|\lambda,\beta\right],$$
$$\mathbb{E}_\nu U_1 = \mathbb{E}_\nu\left[\Pi - \sum_{i\in\Lambda}\widehat{c}_i(p_1)\middle|\lambda,\beta\right] - K(\lambda).$$

Given a contract $\Pi$, abstractly denoting by $\mathcal{I}_0$ and $\mathcal{I}_1$ the information available to providers $p_0$ and $p_1$, a strategy profile $(\beta, \lambda)$ is in equilibrium if and only if $\beta$ and $\lambda$ respectively solve

$$\max_\beta \mathbb{E}_\nu[-\Pi|\mathcal{I}_0,\beta,\lambda] \quad \text{and} \quad \max_\lambda \mathbb{E}_\nu\left[\Pi - \sum_{i\in\Lambda}\widehat{c}_i(p_1)\middle|\mathcal{I}_1,\beta,\lambda\right].$$

We denote by $\beta^*(H) \equiv H$ the truthful reporting strategy. We break indifferences in favor of truthful reporting, i.e. we assume that provider $p_0$ sends truthful reports whenever it is an optimal strategy, reflecting small costs in misreporting.

**Design objectives.** Conditional on selection rule $\lambda$ and expected costs $\kappa$, surplus takes the form

$$S(\lambda) = -K(\lambda) + \sum_{i \in I} \lambda(\tau_i) \left[ \kappa(p_0, \tau_i) - \kappa(p_1, \tau_i) \right].$$

We seek contracts $\Pi$ such that for all priors $\nu$, data $D_0, D_1$, and all equilibria $(\lambda, \beta)$:

$$\mathbb{E}_\nu[S|\lambda] = \mathbb{E}_{D_0, D_1 \sim \nu} \left[ \max_\lambda \mathbb{E}_\nu[S|\lambda, D_0, D_1] \right] - o(|I|) \tag{2}$$

$$\mathbb{E}_\nu \left[ U_0 \middle| \lambda, \beta, D_0 \right] \geq -o(|I|) \tag{3}$$

$$\mathbb{E}_\nu \left[ U_1 \middle| \lambda, \beta, D_1 \right] \geq 0. \tag{4}$$

In other terms, we seek ex post budget-balanced prior-free mechanisms that: maximize efficiency given available information up to a term negligible compared to $|I|$; satisfy at least approximate interim individual rationality for both providers. We highlight once again that the mechanisms we propose to attain these objectives do not exploit the common prior assumption, and would satisfy the same properties in a non-common prior setting, with expectations evaluated under the private provider's prior.[8]

# 3 An Example

To fix ideas, we delineate our main points using a simple instantiation of the model we introduced in Section 2.

[8] For recent work emphasizing prior-free approaches to mechanism design, see Segal (2003), Bergemann and Schlag (2008), Hartline and Roughgarden (2008), Chassang (2013), Carroll (2013), Madarász and Prat (2014), Brooks (2014), Antic (2014).

**Legitimate and illegitimate selection.** We assume in this example that there exists a common knowledge partition $E$ of type space $T$, with typical element $\eta \in E$ (so that $\eta \subset T$) such that treatment costs can be decomposed as

$$\widehat{c}_i(p) = \kappa(\eta_i, p) + e_{i,\tau_i} \tag{5}$$

where terms $e_{i,\tau}$ have mean zero conditional on $\eta$, and are distributed according to a log-normal distribution:

$$e_{i,\tau} = \underline{\kappa}\left[\exp\left(\varepsilon_\tau + \varepsilon_i - 1\right) - 1\right]$$

with $\varepsilon_\tau$ and $\varepsilon_i$ independent standard normal distributions $\mathcal{N}(0,1)$, and $\underline{\kappa} \in \left(0, \min_{\eta \in E, p \in \{p_0, p_1\}} \kappa(\eta, p)\right)$. By construction, $\mathbb{E}_\nu[e_{i,\tau}] = 0$ and $\widehat{c}_i(p) \geq 0$.[9]

Cost decomposition (5) is a special case of decomposition (1) in which the comparative advantages of providers $p_0$ and $p_1$, described by $\kappa(\eta, \cdot)$, depend only on characteristics $\eta \in E$. We think of $E$ as a small set compared to $T$, so that it is possible for each provider to form accurate estimates of their costs conditional on $\eta \in E$. For simplicity, we assume that the costs of the public provider $\kappa(\cdot, p_0)$ are known by both providers, and that private provider $p_1$ knows its own costs $\kappa(\cdot, p_1)$. Error term $e_{i,\tau}$ captures residuals in cost estimates that depend both on idiosyncratic shocks $\varepsilon_i$, and correlated type-specific shocks $\varepsilon_\tau$.

We assume in this example that the private provider is able to perfectly select the realized set $\Lambda$ of patients it treats at no cost. That is, for all $\lambda \in [0,1]^T$, $K(\lambda) = 0$. An immediate implication of costless selection and cost decomposition (5) is that surplus maximizing selection rules need only depend on characteristics $\eta \in E$.

**Remark 1.** *First-best surplus, defined by* $S_{\max} \equiv \max_\lambda \mathbb{E}_c\left[\sum_{i \in \Lambda} \widehat{c}_i(p_0) - \widehat{c}_i(p_1) \middle| \lambda\right]$ *is attained by a selection policy* $\lambda^*$ *that is measurable with respect to* $E$*:* $\lambda^*(\eta) = \mathbf{1}_{\kappa(\eta, p_0) > \kappa(\eta, p_1)}$.

Accordingly, a selection rule is said to be *legitimate* if and only if it is measurable with

[9] Throughout this example, we use the fact that a log-normal distribution $\ln \mathcal{N}(\mu, \sigma^2)$ has expectation $\exp\left(\mu + \frac{1}{2}\sigma^2\right)$.

respect to $E$. Selection rules that are not measurable with respect to $E$ depend on features of types $\tau$ that do not matter for efficiency. They are referred to as *illegitimate*. We denote by $\mathcal{M}(E)$ the set of selection rules measurable with respect to $E$.

**Private information.** For every $\tau \in T$, the private provider's data $D_1$ lets it observe a signal $x_\tau = \varepsilon_\tau + \varepsilon_x$ with $\varepsilon_x$ an independent error term distributed according to a standard normal $\mathcal{N}(0,1)$. Given that provider $p_1$ knows her cost mapping $\kappa(\eta, p_1)$ this is equivalent to observing a single conditionally independent realization of her own costs for each type $\tau \in T$.

**Bayesian updating.** The information structure defined above leads to tractable updated beliefs. Observing data $D_0^\tau$ is equivalent to observing signals $x_i = \varepsilon_\tau + \varepsilon_i$ for $i \in D_0^\tau$. Hence the public and private provider's beliefs over random cost parameter $\varepsilon_\tau$ follow normal distributions $\left(\mathcal{N}(\chi_{p,\tau}, \rho_{p,\tau}^{-1})\right)_{p\in\{p_0,p_1\}}$ where mean $\chi$ and precision $\rho$ satisfy

$$\chi_{p,\tau} = \frac{\mathbf{1}_{p=p_1} x_\tau + \sum_{i\in D_0^\tau} x_i}{1 + \mathbf{1}_{p=p_1} + |D_0^\tau|} \quad \text{and} \quad \rho_{p,\tau} = 1 + \mathbf{1}_{p=p_1} + |D_0^\tau|. \tag{6}$$

This implies conditional estimates of residual costs

$$\mathbb{E}_\nu[e_{i,\tau}|D_0^\tau, p] = \underline{\kappa}\left[\exp\left(\chi_{p,\tau} - \frac{1}{2\left(|D_0^\tau| + 1 + \mathbf{1}_{p=p_0}\right)}\right) - 1\right].$$

Note that conditional on sample size $|D_0^\tau|$, precision $\rho_{p,\tau}$ is deterministic, while mean $\chi_{p,\tau}$ has an ex ante distribution $\mathcal{N}\left(0, \frac{\left(\mathbf{1}_{p=p_1}+|D_0^\tau|\right)^2+\mathbf{1}_{p=p_1}+|D_0^\tau|}{\left(1+\mathbf{1}_{p=p_1}+|D_0^\tau|\right)^2}\right)$.

## 3.1 Why Ex Ante Capitation Schemes Fail

We begin by illustrating the limits of natural transfer schemes that attempt to align incentives through fixed capitation rates. Since payments are specified ex ante, such mechanisms remove concerns that the public provider may misreport its hold-out costs to reduce payments. We show that under restrictive strategic environments, these schemes can indeed attain efficiency and satisfy both providers' individual rationality constraints. However, whenever provider $p_1$ can engage in illegitimate selection, they are inefficient and generate losses for public provider $p_0$.

We consider sparse and rich capitation contracts that differ in the sophistication of the regressions used to predict treatment costs. Transfers take the following form:

$$\Pi^{\mathsf{sparse}}(\Lambda, \tau_I) = \sum_{i \in \Lambda} \mathbb{E}_\nu[c(\tau_i, p_0)|\eta_i, D_0] = \sum_{i \in \Lambda} \kappa(\eta_i, p_0) \tag{7}$$

$$\Pi^{\mathsf{rich}}(\Lambda, \tau_I) = \sum_{i \in \Lambda} \mathbb{E}_\nu[c(\tau_i, p_0)|\tau_i, D_0] = \sum_{i \in \Lambda} \kappa(\eta_i, p_0) + \mathbb{E}_\nu[e_{i,\tau_i}|\tau_i, D_0]. \tag{8}$$

Both schemes reimburse the private provider the public provider's expected cost of treating the patients it selects, conditional on some set of ex ante observables. Sparse capitation estimates patients' costs conditional on legitimate characteristics $\eta$ alone. Rich capitation estimates patients' costs conditional on the full set of observables $\tau$ — i.e. it exploits Big Data to form targeted estimates. We now show that neither scheme resolves the problem of adverse selection at the Big Data limit.

**Proposition 1** (sparse capitation)**.** *Consider capitation scheme $\Pi^{\mathsf{sparse}}$.*

*(i) Assume that the private provider is constrained to use legitimate selection, i.e. selection strategies $\lambda$ must be measurable with respect to $E$. Efficient selection and truthful reporting $(\lambda^*, \beta^*)$ is the unique equilibrium.*

*(ii) Assume that the private provider is not constrained to use legitimate selection rules. If $\mathbb{E}_{\mu_I}(|\kappa(\eta, p_0) - \kappa(\eta, p_1)|) > 0$, then there exists $h > 0$ such that for all sample size $|D_0|$, scheme $\Pi^{\mathsf{sparse}}$ induces an efficiency loss*

$$\mathbb{E}_\nu \left[S_{\max} - S^{\mathsf{sparse}}\right] \geq h|I|.$$

*If allocation does not matter for efficiency, i.e.* $\kappa(\eta, p_0) = \kappa(\eta, p_1)$*, the public provider makes expected losses*

$$\mathbb{E}_\nu[U_0|\Pi^{\textsf{sparse}}] \leq -h|I|.$$

If provider $p_1$ uses only legitimate selection strategies, sparse capitation induces efficient selection. Indeed the expected benefit that provider $p_1$ obtains from selecting in a patient with characteristic $\eta$ is equal to $\kappa(\eta, p_0) - \kappa(\eta, p_1)$. This gives the private provider incentives to engage in efficient selection.

However, the profit a private provider $p_1$ expects from selecting a patient of type $\tau \in \eta$ is in fact $\kappa(\eta, p_0) - \kappa(\eta, p_1) - \mathbb{E}_\nu[e_{i,\tau}|D_0, x]$. Whenever the private provider can select on the basis of non-legitimate characteristics $\tau$, term $\mathbb{E}_\nu[e_{i,\tau}|D_0, x]$ will induce deviations from efficient selection to avoid under-reimbursed patients and recruit over-reimbursed patients. This inefficiency arises because of bias in cost estimates, and does not vanish as the data gets large. Indeed, as we have observed, $\mathbb{E}_\nu[e_{i,\tau}|D_0, x] = \underline{\kappa}\left[\exp\left(\chi_\tau - \frac{1}{2}\frac{1}{|D_0^\tau|+2}\right) - 1\right]$ with $\chi_\tau$ following a Gaussian distribution $\mathcal{N}\left(0, \frac{(|D_0^\tau|+1)^2+|D_0^\tau|+1}{(|D_0^\tau|+2)^2}\right)$. Hence perturbation $\mathbb{E}_\nu[e_{i,\tau}|D_0, x]$ and the inefficiency loss it induces do not vanish as sample size $|D_0|$ grows large.

Since inefficiencies in sparse capitation schemes are driven by biased cost estimates, rich capitation schemes $\Pi^{\textsf{rich}}$, which condition capitation rates on the full set of observables $\tau$ emerge naturally as a candidate solution. The following holds.

**Proposition 2** (rich capitation)**.** *There exists continuous strictly increasing functions* $\underline{h}$ *and* $\overline{h}$ *such that* $\underline{h}(0) = \overline{h}(0) = 0$*, and for all sample size distributions* $(|D_0^\tau|)_{\tau\in T}$*, efficiency loss* $S_{\max} - S^{\textsf{rich}}$ *satisfies*

$$\mathbb{E}_\nu\left[S_{\max} - S^{\textsf{rich}}\right] \leq \underline{\kappa}\,|I|\,\overline{h}\left(\mathbb{E}_{\mu_I}\frac{1}{|D_0^\tau|}\right), \tag{9}$$

*and there exist* $\kappa(\eta, p_0)$, $\kappa(\eta, p_1)$ *such that*

$$\mathbb{E}_\nu \left[S_{\max} - S^{rich}\right] \geq \underline{\kappa}\,|I|\,\underline{h}\left(\mathbb{E}_{\mu_I}\frac{1}{|D_0^\tau|}\right). \tag{10}$$

*If* $\kappa(\eta, p_0) = \kappa(\eta, p_1)$ *for all* $\eta$*, the public provider makes expected losses*

$$\mathbb{E}_\nu[U_0|D_0, \Pi^{rich}] \leq -\underline{\kappa}\,|I|\,\underline{h}\left(\mathbb{E}_{\mu_I}\frac{1}{|D_0^\tau|}\right).$$

While sparse capitation schemes do not achieve efficiency, regardless of data $D_0$, rich capitation schemes may achieve efficiency provided that $\mathbb{E}_{\mu_I}\frac{1}{|D_0^\tau|}$ becomes arbitrarily small, i.e. for almost every type $\tau$, subsample $D_0^\tau$ becomes arbitrarily large. Of course, this is ruled out by definition at the Big Data limit. As a result, cost estimates $\mathbb{E}_\nu[e_{\tau,i}|D_0]$ are imprecise for a non-vanishing mass of types $\tau$ (under patient sample measure $\mu_I$) and signals $(x_\tau)_{\tau\in T}$ make it possible for private provider $p_1$ to profit from selecting mispriced types.

## 3.2 Strategic Capitation

We now describe a capitation scheme that correctly takes care of incentives for strategic selection by $p_1$ and strategic reporting by $p_0$. Payments can be expressed as

$$\Pi^{\text{strat}}(\Lambda, \tau_I, H_R) \equiv \sum_{i\in\Lambda} \pi(\eta_i) + \Delta\pi(\eta_i, H_R), \tag{11}$$

where $\pi(\eta) \equiv \kappa(\eta, p_0)$ is the baseline capitation rate conditional on legitimate characteristics used in sparse capitation, and $\Delta\pi(\eta, H_R)$ is a correction dependent on reported hold-out data $H^R$ and selected sample $\Lambda$ taking the form:

$$\Delta\pi(\eta_i, H_R, \Lambda) \equiv \text{cov}_I(s_i, r_i|\eta_i = \eta) = \frac{1}{|I^\eta|}\sum_{i\in I^\eta} s_i r_i, \quad \text{where}$$

- $s_i \equiv \frac{\mu_\Lambda(\tau_i|\eta_i)}{\mu_I(\tau_i|\eta_i)} - 1$ is a measure of sample $\Lambda$'s deviation from legitimate selection;[10]

- $r_i \equiv \frac{1}{|H_R^{\tau_i}|} \sum_{j \in H_R^{\tau_i}} \left[\widehat{c}_j^R(p_0) - \kappa(\eta, p_0)\right]$ is the average residual of costs for type $\tau_i$ in the reported hold-out sample $H_R^{\tau_i} \equiv \{(j, \tau_j, \widehat{c}_j^R) | j \in H_R, \tau_j = \tau_i\}$.

Strategic capitation satisfies the following key properties

$$\forall \lambda, \quad \mathbb{E}_\nu[\Delta\pi(\eta_i, H_R, \Lambda)|D_0, D_1, \beta^*, \lambda] = \mathbb{E}_\nu[(\lambda(\tau|\eta) - \mu_I(\tau|\eta))\mathbb{E}_\nu[e_{\tau,i}|D_1, D_0]] \tag{12}$$

$$\forall \lambda \in \mathcal{M}(E), \forall \beta, \quad \mathbb{E}_\nu[\Delta\pi(\eta_i, H_R, \Lambda)|D_0, D_1, \beta, \lambda] = 0. \tag{13}$$

Condition (12) implies that under truthful reporting $\beta^*$, the adjustment performed by strategic capitation is an unbiased estimate of the excess profits $p_1$ may have obtained through illegitimate selection. This noisy ex post estimate provides an accurate ex ante correction and dissuades inefficient selection. Condition (13) ensures that regardless of the public provider's reporting strategy $\beta$, the private provider can guarantee herself expected capitation payments $\pi(\eta) = \kappa(\eta, p_0)$, provided it uses a legitimate selection strategy $\lambda \in \mathcal{M}(E)$.

**Proposition 3.** *Strategic capitation contract $\Pi^{strat}$ induces a unique equilibrium $(\lambda^*, \beta^*)$ in which private provider $p_1$ selects patients efficiently, and the public provider $p_0$ truthfully reports hold-out sample $H$. Both providers get positive expected payoffs: $\mathbb{E}_\nu[U_0|D_0, D_1, \lambda^*, \beta^*] \geq 0$ and $\mathbb{E}_\nu[U_1|D_0, D_1, \lambda^*, \beta^*] \geq 0$.*

Note that the observability of types $\tau$ is needed to assemble the correct cost residuals from the hold-out data, as well as to measure the private provider's deviation from legitimate selection.

[10] Recall that for any sample $J$, $\mu_J(\tau|\eta) \equiv \frac{|J^\tau|}{|J^\eta|}$ denotes the distribution of types $\tau$ conditional on characteristic $\eta \subset T$ in sample $J$.

## 3.3 Alternative Mechanisms

To clarify the economic forces at work in our environment it is useful to delineate the mechanics of other relevant mechanisms.

**Mechanisms from the literature.** Other work has emphasized the value of ex post noisy signals in environments with quasi-linear preferences. Riordan and Sappington (1988) show that it is possible to efficiently regulate a monopoly with unknown costs by exploiting public signals correlated to the monopoly's type. Using a construction related to that of Cremer and McLean (1988), they show how to extract all the surplus by offering the monopoly appropriately chosen screening contracts. Strategic capitation also exploits the fact that noisy ex post signals (here, hold-out cost realizations) can be used to construct accurate ex ante incentives, but our environment differs in key ways. First, signals are not public, and we need to take care of the public provider's incentives to reveal its own cost. Second, the identification condition at the heart of Riordan and Sappington (1988) is not satisfied: neither the distribution of the public provider's cost, nor the private provider's beliefs thereover, are sufficient statistic of the private providers' costs.

Mezzetti (2004) shows that it's possible to obtain efficiency in common value environments using ex post reports of the players' realized payoffs. In our application the mechanism proposed by Mezzetti (2004) would proceed by making the private provider a negative ex post transfer equal to the public provider's realized cost, and making the private provider a positive ex ante transfer to cover expected costs. This mechanism does not satisfy budget balance and relies on priors to set ex ante transfers.

The differences between our environment and that of Mezzetti (2004) help clarify the role played by the Big Data assumption, i.e. the assumption that types are observable but not interpretable. We obtain budget balance by: forming a measure of the private provider's deviation from legitimate selection; interacting this measure with an unbiased estimate of the public provider's counterfactual costs. This ensures that in equilibrium, neither the private nor the public provider can affect their expected payoffs by deviating from legitimate selection and truthful reporting. The observability of types is used to compute the private provider's deviation from legitimate selection, as well as correctly reweight the distribution of types in the hold-out sample $H$ to obtain estimates of counterfactual costs in the sample $\Lambda$ of patients selected by the private provider.[11]

**Plausible alternative mechanisms.** A key step in strategic capitation is to use hold-out data to form estimates of counterfactual costs for the public provider. The assumption that types are observable is needed to reweight the distribution of types in the hold-out sample to match that of the selected sample. There may be other ways to form an unbiased estimate of counterfactuals. For instance, if it were possible to assign patients selected by the private provider back to the public provider with a fixed uniform probability, one could form an estimate of counterfactual costs without observing types. Beyond feasibility issues, a difficulty with this approach is that it does not take care of the public provider's incentives to bias its own cost reports.

Strategic capitation dissuades illegitimate selection by forming unbiased estimates of the private provider's excess profits. An alternative way to dissuade illegitimate selection is to impose sufficiently large penalties, say proportional to $\left|\frac{\mu_\Lambda(\tau_i|\eta_i)}{\mu_I(\tau_i|\eta_i)} - 1\right|$, when the sample selected by the private provider deviates from legitimate selection. This scheme requires the observability of types but does not require the availability of a hold-out sample. It induces efficient legitimate selection whenever the private provider can select patients precisely and at to cost. However this scheme carries an efficiency loss when ensuring the legitimacy of realized selection $\Lambda$ is costly. Strategic capitation avoids the issue by using hold-out data to form an unbiased estimate of the profits from selection.

[11] The distribution of types in $H$ and $\Lambda$ should typically be different. For instance, the hold-out sample may consist of types treated by the public provider and rejected by the private provider.

# 4 General Analysis

The strategic capitation scheme presented in Section 3 relies on strong assumptions. Chief among those, cost decomposition (5) ensures that the surplus maximizing policy depends on a small number of commonly known characteristics $\eta \in E$. This is not realistic: a private provider's comparative advantage is likely to be her private information, and it need not be the case that the optimal selection policy is measurable with respect to a small set of characteristics. Furthermore, private providers may be able to innovate and develop comparative advantages along new dimensions. We now extend strategic capitation to environments in which the relevant dimensions of comparative advantage are not common knowledge.

We assume for simplicity that realized costs are bounded, i.e. that there exists $c_{\max}$ such that $\widehat{c}_i(p) \in [0, c_{\max}]$. Recall that $\kappa(\tau, p) = \mathbb{E}_c[\widehat{c}|\tau, p]$ denotes expected costs of treatment given $\tau$, which yields decomposition $\widehat{c}_i(p) = \kappa(\tau_i, p) + e_i$, where $\mathbb{E}_\nu[e_i|\tau, p] = 0$. By construction, it must be that $e_i \in [-c_{\max}, c_{\max}]$. Finally, let

$$S(\lambda|D_0, D_1) \equiv \mathbb{E}_\nu \left[ \sum_{i \in I} \lambda(\tau_i) \left[ \kappa(p_0, \tau_i) - \kappa(p_1, \tau_i) \right] \middle| D_0, D_1 \right] - K(\lambda)$$

$$S_{E|D_0,D_1} \equiv \max_{\lambda \in \mathcal{M}(E)} S(\lambda|D_0, D_1)$$

respectively denote the surplus achieved by selection rule $\lambda$, and the maximum surplus achievable using selection rules measurable with respect to partition $E$.

## 4.1 Generalized Strategic Capitation

For any collection $\mathcal{E}$ of partitions $E \in \mathcal{E}$, our goal is to approach the maximum achievable efficiency $S_{E|D_0,D_1}$ with respect to partitions $E \in \mathcal{E}$. We define the generalized strategic capitation scheme $G^{\mathsf{strat}}_{\mathcal{E}}$ as follows:

1. data $D_0$ is shared with provider $p_1$;

2. provider $p_1$ picks a partition $E \in \mathcal{E}$ according to which it will be allowed to select patients; we continue to refer as characteristics $\eta \in E$ as legitimate selection characteristics;

3. provider $p_1$ is rewarded using the strategic capitation scheme $\Pi^{\mathsf{strat}}(\cdot|E)$ defined by

$$\Pi^{\mathsf{strat}}(\Lambda, \tau_I, H_R) \equiv \sum_{i \in \Lambda} \pi(\eta_i) + \Delta\pi(\eta_i, H_R)$$

   where $\pi(\eta) = \widehat{\kappa}(\eta, p_0) \equiv \sum_{\tau \in \eta} \mu_I(\tau|\eta) \frac{1}{|D_0^\tau|} \sum_{i \in D_0^\tau} \widehat{c}_i(p_0)$ is the sample estimate $\widehat{\kappa}(\eta, p_0)$ of expected treatment costs conditional on characteristic $\eta \in E$. As in Section 3, $\Delta\pi(\eta, H_R, \Lambda)$ takes the form:

$$\Delta\pi(\eta_i, H_R) \equiv \text{cov}_I(s_i, r_i|\eta_i = \eta) = \frac{1}{|I^\eta|} \sum_{i \in I^\eta} s_i r_i,$$

   with

$$s_i \equiv \frac{\mu_\Lambda(\tau_i|\eta_i)}{\mu_I(\tau_i|\eta_i)} - 1 \quad \text{and} \quad r_i \equiv \frac{1}{|H_R^{\tau_i}|} \sum_{j \in H_R^{\tau_i}} \left[\widehat{c}_j^R(p_0) - \widehat{\kappa}(\eta, p_0)\right].$$

An equilibrium of mechanism $G_{\mathcal{E}}^{\mathsf{strat}}$ is a triplet $(E, \lambda, \beta)$ where $E \in \mathcal{E}$ is $p_1$'s choice of characteristics it is allowed to select on.

Mechanism $G_{\mathcal{E}}^{\mathsf{strat}}$ expands on strategic capitation by letting the private provider specify the set of characteristics it wishes to select on. As we show below, this additional degree of freedom results in unavoidable losses related to the complexity of the class of models $\mathcal{E}$ the private provider is allowed to pick from. These losses are related to penalties encountered in the model selection literature (Vapnik, 1998, Massart and Picard, 2007), and indeed one can think of our problem as one of delegated model selection.

**Definition 1.** *For any $\mathcal{E}$ and $e = (e_i)_{i \in D_0}$, let $\Psi(\mathcal{E}, e)$ denote the random variable*

$$\Psi(\mathcal{E}, e) \equiv \max_{E \in \mathcal{E}} \left( \sum_{\eta \in E} |I^\eta| \left[ \sum_{\tau \in \eta} \mu_I(\tau|\eta) \frac{1}{|D_0^\tau|} \sum_{i \in D_0^\tau} e_i \right]^+ \right). \tag{14}$$

Variable $\Psi(\mathcal{E}, e)$ is an upper-bound to the gains a perfectly informed private provider could obtain from selecting the partition $E$ that lets her optimally target over-reimbursed types. The scope for selection comes from the fact that generalized capitation uses sample averages $\widehat{\kappa}(\eta, p_0)$ to estimate the public provider's cost of service $\mathbb{E}_\nu[c_i(p_0)|\eta, c]$ conditional on legitimate characteristics.

Generalized capitation extends the performance bounds described in Proposition 3 up to a penalty of order $\mathbb{E}_\nu[\Psi(\mathcal{E}, e)]$.

**Proposition 4** (efficiency bounds)**.** *Consider a collection of $\mathcal{E}$ of partitions. In any equilibrium $(E, \lambda, \beta)$ of mechanism $G_{\mathcal{E}}^{\mathsf{strat}}$ we have that*

$$S(\lambda) \geq \mathbb{E}_\nu \left[ \max_{E \in \mathcal{E}} S_{E|D_0, D_1} \right] - 2\mathbb{E}_\nu \left[ \Psi(\mathcal{E}, e) \right]; \tag{15}$$

$$\mathbb{E}_\nu \left[ -\Pi + \sum_{i \in \Lambda} \widehat{c}_i(p_0) \middle| D_0 \right] \geq -\mathbb{E}_\nu \left[ \Psi(\mathcal{E}, e) \right]; \tag{16}$$

$$\mathbb{E}_\nu \left[ \Pi - \sum_{i \in \Lambda} \widehat{c}_i(p_1) \middle| D_0, D_1 \right] \geq 0. \tag{17}$$

We do not endogenize the choice of the class of models $\mathcal{E}$. Still, if institutions are designed at a sufficiently ex ante period — specifically before data $D_0$ is realized — penalties $\Psi(\mathcal{E}, e)$ can be used to do so. The idea would be to let the private provider submit a class of models $\mathcal{E}$ ex ante that it will be able to pick from at the interim stage, and charge her complexity penalty $\mathbb{E}_\nu[\Psi(\mathcal{E}, e)]$. If data $D_0$ is renewed over time, the private provider may also be allowed to submit preferences over the class of models $\mathcal{E}$ to be used in the future.

Note however that $\mathbb{E}_\nu[\Psi(\mathcal{E},e)]$ depends on prior $\nu$ through error term $e$. The next lemma provides prior-free bounds for $\mathbb{E}_\nu\left[\Psi(\mathcal{E},e)\right]$. Denote by $\alpha \equiv \mathbb{E}_{\mu_I}\left[\frac{|I^\tau|}{|D_0^\tau|}\frac{|D_0|}{|I|}\right] \geq 1$ the average representativeness of data $D_0$ for patients in $I$.[12] Let $M \equiv \sum_{E\in\mathcal{E}}\left(2^{|E|}-1\right)$.

**Lemma 1** (selection bounds)**.** *(i) Let $(e'_i)_{i\in I}$ denote i.i.d. Rademacher random variables uniformly distributed over $\{-c_{max}, c_{max}\}$. For any class $\mathcal{E}$ and any centered error terms $(e_i)_{i\in I}$ arbitrarily distributed over $[-c_{\max}, c_{\max}]$, we have that*

$$\mathbb{E}_\nu\left[\Psi(\mathcal{E},e)\right] \leq \mathbb{E}_\nu\left[\Psi(\mathcal{E},e')\right].$$

*(ii) Regardless of the distribution of error terms $(e_i)_{i\in I}$,*

$$\mathbb{E}_\nu[\Psi(\mathcal{E},e)] \leq |I|c_{\max}\sqrt{\frac{2\alpha}{|D_0|}}\left(1+\sqrt{\log M}\right).$$

**Sparse linear classifiers.** It is informative to evaluate the bounds provided in Proposition 4 for a natural class of partitions $\mathcal{E}$: sparse linear classifiers. Specifically, we assume that type space $T$ is a subset of $\mathbb{R}^f$ (we will use the inequality $f \leq |T| \leq |D_0|$). For $d \in \{2,\cdots,f\}$, a $d$-sparse vector $v = (v_k)_{k\in\{1,\cdots,f\}} \in \mathbb{R}^f$ is a vector with at most $d$ non-zero coordinates. The family of partitions $\mathcal{E}$ induced by $d$-sparse classifiers is defined as

$$\begin{aligned}\mathcal{E} \equiv &\{E_v \equiv \{\eta_v^+, \eta_v^-\} | v \in \mathbb{R}^f,\ v\, d\text{-sparse}\}\\ &\text{where } \eta_v^+ = \{\tau \in T\ s.t.\ \langle \tau, v\rangle > 0\} \text{ and } \eta_v^- = \{\tau \in T\ s.t.\ \langle \tau, v\rangle < 0\}.\end{aligned}$$

The private provider is allowed to use any $d-$sparse linear classifier to decide whether or not to select a particular set of types or not.

**Corollary 1.** *When possible selection partitions $\mathcal{E}$ are induced by $d$-sparse classifiers, the*

[12] The fact that $\alpha \geq 1$ follows from the observation that $\alpha = \mathbb{E}_{\mu_I}\left[\mu_I(\tau)/\mu_{D_0}(\tau)\right] \geq 1/\mathbb{E}_{\mu_I}\left[\mu_{D_0}(\tau)/\mu_I(\tau)\right] = 1$.

*maximum expected loss* $\mathbb{E}_\nu[\Psi(\mathcal{E},e)]$ *from strategic capitation satisfies*

$$\mathbb{E}_\nu[\Psi(\mathcal{E},e)] \leq 4c_{\max}|I|\sqrt{\frac{\alpha d \log|D_0|}{|D_0|}}. \tag{18}$$

Indeed, the number of possible linear classifiers, which depends on $|T|$, is bounded by $\binom{f}{d}\cdot\binom{|T|}{d} \leq \frac{1}{4}|T|^{2d}$, where $\binom{m}{n} = \frac{m!}{(m-n)!n!}$. Since each $E \in \mathcal{E}$ contains two elements, we obtain that $M \leq K^{2d}$. Corollary 1 follows from a direct application of Lemma 1 and the fact that $|T| \leq |D_0|$.

Note that for all practical purposes, term $\sqrt{\log|D_0|}$ may be treated as a constant between 4 and 5. Indeed, for $|D_0| = 48\times 10^6$, approximately the size of the US Medicare population, $\sqrt{\log|D_0|} \simeq 4.2$, while for $|D_0| = 7\times 10^9$, roughly the current world population, $\sqrt{\log|D_0|} \simeq 4.8$.

## 4.2 Unimprovability of Strategic Capitation

In the spirit of Hartline and Roughgarden (2008), we now provide a lower-bound for the minimal efficiency losses that any mechanism can guarantee. Following the notation of Section 2, a state of the world is described by a tuple

$$\omega = (c(\tau,p), K(\cdot), D_0, D_1, H)_{\substack{p\in\{p_0,p_1\}\\ \tau\in T}} \in \Omega,$$

consisting of a distribution of treatment costs $c(\tau,p)$ conditional on types and provider, selection costs $K$ for the private provider, data sets $D_0$ and $D_1$ for the public and private provider, as well as hold-out data $H$ privately observed by the public provider.

State of the world $\omega$ is drawn according to common prior $\nu \in \Delta(\Omega)$. To provide lower bounds on worst case efficiency losses, it is sufficient forr us to consider the class of priors such that sample size $|D_0|$ and distributions of types $\mu_I \in \Delta(T)$ and $\mu_{D_0} \in \Delta(T)$ are known.

We consider the problem of Bayes-Nash implementation using budget-balanced direct mechanisms $g$ of the following form:

- data $D_0$ is publicly observable;
- provider $p_1$ sends a message $m_1 = (D_1^m, K^m(\cdot)) \in \nu_{|D_1,K(\cdot)}$;
- the mechanisms suggests a selection $\lambda_g(D_0, m_1) \in [0,1]^T$;
- provider $p_1$ makes a selection decision $\lambda \in [0,1]^T$, with realized selection $\Lambda \subset I$;
- provider $p_0$ sends a message $m_0 = H_R \in \operatorname{supp} \nu_{|H}$ corresponding to a reported hold-out sample;
- transfers $\Pi(D_0, m_1, m_0, \Lambda)$ from $p_0$ to $p_1$ are implemented.

We denote by $\mathcal{G}_\nu$ the set of incentive compatible direct revelation mechanisms under prior $\nu$. For any direct revelation mechanism $g \in \mathcal{G}_\nu$, the surplus $S(g,\nu)$ attained by mechanism $g$ under prior $\nu$ is

$$S(g,\nu) = \mathbb{E}_\nu \left[ \sum_{i \in \Lambda} \kappa(p_0, \tau_i) - \kappa(p_1, \tau_i) \,\middle|\, \lambda_g \right].$$

In turn, given a class $\mathcal{E}$ of partitions, the efficiency loss $L_{\mathcal{E}}(g,\nu)$ of mechanism $g$ relative to treatment allocations measurable with respect to $E \in \mathcal{E}$ is defined as:

$$L_{\mathcal{E}}(g,\nu) = \mathbb{E}_\nu \left[ \max_{E \in \mathcal{E}} S_{E|\mathcal{D}_0,\mathcal{D}_1} - S(g,\nu) \right].$$

**Proposition 5.** *There exists $k > 0$ such that for any class of partitions $\mathcal{E}$,*

$$\max_{\nu} \min_{g \in \mathcal{G}_\nu} L_{\mathcal{E}}(\nu, g) \geq k|I| c_{max} \max_{E \in \mathcal{E}} \mathbb{E}_{\mu_I} \left[ \frac{1}{\sqrt{|D_0^\eta|}} \right]. \tag{19}$$

In particular, the efficiency loss achieved by strategic capitation for linear classifiers (Corollary 1) is tight up to an order $\sqrt{\log |D_0|}$, which, for all plausible values of $|D_0|$, can be treated as a constant less than 5.

# 5 Discussion

This paper explores the value of Big Data in reducing the extent of adverse selection in government-run capitation schemes. We argue that at the correct Big Data limit, including an increasing number of covariates as part of an ex ante capitation formula is unlikely to succeed. Instead we suggest that Big Data may be used to align incentives by using ex post capitation adjustments interacting an unbiased estimate of counterfactual costs to the public provider with the private provider's deviation from legitimate selection.

This section discusses various extensions, including the use of strategic capitation in market settings, as well as dealing with heterogeneity in the quality of care, and dynamic selection.

## 5.1 Adverse Selection in Exchanges

Adverse selection is a significant concern in insurance markets such as the ones organized by American Healthcare Act. Indeed, if regulation constrains prices to depend only on a subset of observables (as is the case with community rating), providers will have incentives to select patients that are cheaper to serve given characteristics excluded from legal pricing formulas. This increases the cost of serving patients and can result in limited entry. A simple example suggests that strategic capitation may help improve market outcomes in such environments.

**A stylized model.** As in Section 2, a set $I$ of patients with types $\tau \in T$ has inelastic unit demand for insurance, where insurance corresponds to a single standardized insurance contract. Provider $p_0$ is now an incumbent private provider, while $p_1$ is a potential entrant. For simplicity, we assume that each provider's cost technology is the same: $\forall \tau \in T$, $c(p_0, \tau) \sim c(p_1, \tau)$. Here the objective is not to improve the allocation of patients to providers, but rather to increase competition so that insurance is priced at marginal cost. By law, plans are constrained to offer prices $\pi(\eta)$ that depend only on a coarse set of patient characteristics

$\eta \in E$, where $E$ is a partition of $T$. Prices are bounded above by $\overline{\pi}$.[13]

We assume that the private providers both know their common expected cost of treatment $\kappa(\tau)$ conditional on type $\tau$. Let $\kappa(\eta) \equiv \mathbb{E}_{\mu_I}[\kappa(\tau)|\eta]$. Each provider $p$ has access to a hold-out sample of its own cost $H_p$. We assume that both providers have lexicographic preferences over maximizing their own revenue and minimizing that of their competitor. The timing of decisions is as follows:

1. potential entrant $p_1$ decides to enter the market or not;
2. each provider $p$ active in the market submits a price formula $\pi_p : \eta \mapsto \pi_p(\eta)$;
3. each provider $p$ active in the market attempts to select a distribution $\lambda_p$ of patients;
4. if $\pi_{p_0}(\eta) \neq \pi_{p_1}(\eta)$, patients of type $\eta$ purchase insurance from the cheapest provider; if $\pi_{p_0}(\eta) = \pi_{p_1}(\eta)$, provider $p$ serves distribution of patients $\lambda_p + [\frac{\mu_I}{2} - \lambda_{\neg p}]$, where $\neg p$ denotes the other provider.[14]

The cross-price elasticity of patient demand is infinite, so that patients always go to the cheapest provider. As a result an entrant will at most make zero profit when entering. We assume that whenever the entrant can guarantee itself zero profits it enters.[15] The cost of engaging in selection $\lambda_p$ is denoted by $K(\lambda_p)$. We assume that $K$ is strictly convex, continuously differentiable, and minimized at $\lambda_p = \frac{\mu_I}{2}$. We denote by $\Lambda_p$ the realized selected sample of patients purchasing from provider $p$.

The following result holds.

**Proposition 6.** *The market entry game described above has a unique subgame perfect equilibrium in which the potential entrant does not enter, and the incumbent charges price* $\pi_{p_0}(\eta) = \overline{\pi}$.

*In the subgame following entry both the entrant and the incumbent make equilibrium losses* $-K(\lambda^*) < 0$ *where* $\lambda^*$ *solves* $\max_{\lambda \in [0,1]^T} \left[\sum_{\tau \in T} \lambda(\tau)\left(\kappa(\eta) - \kappa(\tau)\right) - K(\lambda)\right]$.

[13] Parameter $\overline{\pi}$ may be viewed as the patients' (common) value for insurance.

[14] We assume that the cost of selection $K(\lambda_p)$ is sufficiently steep around $\frac{\mu_I}{2}$ that $\lambda_p + \frac{\mu_I}{2} - \lambda_{\neg p} \in \Delta(T)$ for all individually rational selection policies.

[15] This could be due to small subsidies for entry, or high but finite cross-price elasticities.

Indeed, because cross-price elasticities are infinite, in equilibrium, both providers price at marginal cost conditional on $\eta$: $\pi_p(\eta) = \kappa(\eta)$. Furthermore, since the marginal cost of selection at $\lambda_p = \mu_I/2$ is zero, both players find it profitable to engage in non-zero selection. In aggregate however, selection efforts cancel one another and merely destroy surplus.

**Strategic capitation.** Consider now the following extension of the strategic capitation scheme introduced in Section 3. The game described above is modified in two ways:

- at stage 2, along with submitting pricing formulas $\pi_p(\cdot)$, each active plan submits a report $H_{R,p}$ of their hold-out sample.
- after selection has occurred, for each type $\eta$ it serves, plan $p$ receives capitation adjustment $\Delta\pi(\eta, H_{R,\neg p}, \Lambda_p)$ taking the form:

$$\Delta\pi(\eta_i, H_{R,p}, \Lambda_p) \equiv \text{cov}_I(s_{i,p}, r_{i,p}|\eta_i = \eta) = \frac{1}{|I^\eta|}\sum_{i \in I^\eta} s_{i,p} r_{i,p},$$

  with

$$s_{i,p} \equiv \frac{\mu_{\Lambda_p}(\tau_i|\eta_i)}{\mu_I(\tau_i|\eta_i)} - 1 \quad \text{and} \quad r_{i,p} \equiv \frac{1}{|H^{\tau_i}_{R,\neg p}|}\sum_{j \in H^{\tau_i}_{R,\neg p}} \left[\widehat{c}_j^{R,\neg p}(p_0) - \pi_p(\eta)\right].$$

**Proposition 7.** *The market game with strategic capitation described above has an efficient truthful equilibrium in which: the potential entrant enters; both providers submit prices* $\pi_p(\eta) = \kappa(\eta)$*; both providers select a representative population in expectation (*$\lambda_p = \mu_I/2$*); both providers submit their hold-out sample costs truthfully (*$H_{R,p} = H_p$*); expected ex post adjustments are equal to 0 (*$\mathbb{E}\Delta\pi_p = 0$*).*

## 5.2 Extensions and Implementation Concerns

**Quality.** Throughout the paper we assume that the quality of actual healthcare delivery is homogeneous across providers. In practice, insurance plans may differ in the quality of care they deliver to their enrollees. It is important to take into account the quality of health outcomes when designing capitation schemes. If not, costs may be kept low at the expense of quality. Appendix A describes an extension of strategic capitation that correctly reflects differences in the quality of care. An important limitation is that it requires that health outcomes (including death) be observable, and that they be assigned monetary values.

**Dynamic selection.** The process of selection is dynamic. In the context of Medicare Advantage, patients have the opportunity to switch back and forth between public and private providers once a year. This implies that costs of care need to be evaluated over time. If a plan has low short-term cost of care, but skimps on quality (Ellis, 1998), it may end up generating greater longer term costs for the public provider if patients disenroll from the private plan once they get sick enough. Appendix A shows how to adjust strategic capitation to address this issue. It becomes important to keep track of the counterfactual distribution of types, should the patient have remained with the public provider.

**Surplus Extraction.** The paper focuses on the efficient allocation of patients across public and private providers. However, if there is a deadweight loss to public funds, it may be welfare improving for the public provider to extract some of the surplus. Since the private providers' has private information over her costs conditional on patient types, this is a difficult multidimensional screening problem. Two observations are helpful to make progress on this issue. First, given that we consider prior-free mechanisms, the argument of Carroll (2015) suggests there may not be much value in complex multidimensional screening. It may be near-optimal to focus on separable one-dimensional screening mechanisms that associate a discounted baseline capitation rate $\rho(\eta)\kappa(\eta, p_0)$ with $\rho(\eta) \in [0, 1]$ to each patient with characteristics $\eta$. A second useful observation is that strategic capitation adjustments used to prevent selection of mispriced types can be applied to any baseline repayment scheme. This suggests using capitation schemes of the form

$$\Pi(\Lambda, \tau_I, H_R) \equiv \sum_{i \in \Lambda} \rho(\eta)\kappa(\eta, p_0) + \Delta\pi(\eta_i, H_R)$$

where $\rho(\eta) \in [0,1]$ is a given discounting profile, and $\Delta\pi(\eta_i, H_R) = \frac{1}{|I^\eta|}\sum_{i \in I^\eta} s_i r_i$, with $s_i \equiv \frac{\mu_\Lambda(\tau_i|\eta_i)}{\mu_I(\tau_i|\eta_i)} - 1$ and $r_i \equiv \frac{1}{|H_R^{\tau_i}|}\sum_{j \in H_R^{\tau_i}}\left[\widehat{c}_j^R(p_0) - \kappa(\eta, p_0)\right]$. This separates the problem of extracting revenue, and that of preventing illegitimate selection.

**Ethics.** Regulators frequently ban indexing ex ante capitation rates on certain observables, such as ethnicity or income. One rationale for this is that the law has expressive content that affects social norms, and it is desirable to reinforce the norm that all citizens deserve equal treatment. This has subtle consequences on the social acceptability of contingent incentive schemes: having different equilibrium capitation payments for citizens of different ethnic background seems repugnant in a sense related to Roth (2007); but punishing discrimination against specific ethnic groups (which should mostly remain off of the equilibrium path) does not. The adjustments proposed by strategic capitation fall in this latter category: it punishes providers for non-representative selection of types.

**Volatility of revenues and profits.** One concern with strategic capitation is that the capitation payments that the private provider ultimately receives are uncertain at the interim stage: if noise in selection causes the private provider's to enroll types that are relatively cheap (resp. expensive) to treat for the public provider, it receives lower (resp. higher) payments than anticipated. While this increases the volatility of revenues, this may in fact reduce the volatility of profits. Indeed, types that are relatively cheap (resp. expensive) to treat to the public provider are also likely to be cheap (resp. expensive) to treat for the private provider. This means that departures from expected revenue are likely to be offset by departures from expected costs, resulting in lower volatility in profits.

# A Extensions

## A.1 Quality

If the private and public provider differ in the quality of health outcomes they deliver to patients, the value associated with different health outcomes needs to be reflected in capitation transfers. We assume that health outcomes (including death) for each patient $i \in I$ treated by provider $p$ are observable and associated with realized monetary values $\widehat{v}_i(p)$. By analogy to costs, we assume that the private provider's advantage function is measurable with respect to a relatively small set of types $\eta_i$. The strategic capitation scheme can then be extended to the scenario with outcome qualities. Given selection rule $\lambda$ and transfers $\Pi$, the surpluses accruing to the public and private providers take the form

$$\mathbb{E}_\nu U_0 = \mathbb{E}_\nu \left[ -\Pi + \sum_{i \in \Lambda} \widehat{c}_i(p_0) + \widehat{v}_i(p_1) - \widehat{v}_i(p_0) \middle| \lambda \right],$$

$$\mathbb{E}_\nu U_1 = \mathbb{E}_\nu \left[ \Pi - \sum_{i \in \Lambda} \widehat{c}_i(p_1) \middle| \lambda \right] - K(\lambda).$$

Differences in quality of care are isomorphic to a change in the public provider's cost of care. Since we assume that health outcomes are observable, data $D_0$ should now include values $\widehat{v}_i(p_0)$ to patients in $D_0$, and health outcomes $\widehat{v}_i(p_1)$ to patients in $D_1$ should be visible to the public provider. Strategic capitation can be extended by setting transfers:

$$\Pi(\Lambda, \tau_I, H_R) \equiv \sum_{i \in \Lambda} \widehat{v}_i(p_1) + \pi(\eta_i) + \Delta\pi(\eta_i, H_R)$$

where

$$\pi(\eta) \equiv \sum_{\tau \in \eta} \mu_I(\tau|\eta) \left[ \frac{1}{|D_0^\tau|} \sum_{i \in D_0^\tau} \widehat{c}_i(p_0) - \widehat{v}_i(p_0) \right]$$

and $\Delta\pi(\eta, H_R)$ takes the form:

$$\Delta\pi(\eta_i, H_R) \equiv \frac{1}{|I^\eta|} \sum_{i \in I^\eta} s_i r_i,$$

with $s_i \equiv \frac{\mu_\Lambda(\tau_i|\eta_i)}{\mu_I(\tau_i|\eta_i)} - 1$ and

$$r_i \equiv \frac{1}{|H_R^{\tau_i}|} \left[ \sum_{j \in H_R^{\tau_i}} \widehat{c}_j^R(p_0) - \widehat{v}_j(p_0) \right] - \pi(\eta).$$

## A.2 Dynamic Selection

In dynamic settings, capitation schemes need to control for differential transitions in health status across providers. For simplicity, as in Section 3, we assume that expected costs conditional on legitimate characteristics are known, and that at each time $t$, comparative advantage depends on a commonly known set of legitimate selection characteristics $\eta \in E$. We denote by $\tau_{i,t}$ the type of patient $i$ at date $t$, by $\eta_{i,t}$ her legitimate selection characteristic at date $t$, and by $\widehat{c}_i(t,p)$ her realized cost of care if treated by provider $p$ at time $t$. Types $(\tau_t)_{t \in \{0,\cdots,\overline{T}\}}$ and characteristics $(\eta_t)_{t \in \{0,\cdots,\overline{T}}$ follow separate Markov chains, summarized under notation $\Phi_p$, which depend on the provider $p$ that the patient is enrolled with. Future costs are discounted using discount factor $\delta \in (0,1]$, and $\overline{T}$ denotes an upper bound to the duration of patients' lives in the system.

For a patient $i$ of type $\tau_i$ enrolled with the public provider from time $t$ to time $\overline{T}$, we define

$$\widehat{C}_i(t, p_0) \equiv \sum_{s=t}^{\overline{T}} \delta^{s-t} \widehat{c}_i(s, p_0) \quad \text{and} \quad C(t, \eta, p_0) \equiv \mathbb{E}_\nu \left[ \widehat{C}_i(t, p_0) \middle| \eta_t = \eta \right].$$

In dynamic environments, strategic capitation must accommodate the possible reenrollment of patients with the public provider. As a result, transfers must occur at the reentry of patients into the public system. Let us denote by $\Lambda_t$ the selection of patients enrolled with the private provider at time $t$, and by $\Lambda_t^{re}$ the selection of patients disenrolling from the private provider and enrolling with the public provider at time $t$. The following scheme generalizes strategic capitation. At initial time of enrollment $t = 0$, the public provider commits to the following baseline payments conditional on legitimate characteristics $\eta \in E$:

- a capitation payment $\pi(t, \eta_0) = \mathbb{E}_\nu[\widehat{c}_i(t, p_0)|\eta_{i,0} = \eta_0]$ whenever patient $i$ with initial type $\eta_0$ is enrolled with the private provider at time $t$;
- a signed transfer $\pi_i^{re}$ (with positive transfers being made from the public provider to the private provider) at every time $T$ such that patient $i$ returns to the public provider: $\pi_i^{re} = \mathbb{E}_\nu[\widehat{C}(T, p_0)|\eta_{i,0}, \Phi_{p_0}] - C(T, \eta_{i,T}, p_0)$.

Provided that the private provider does not engage in illegitimate selection, this scheme induces efficient dynamic behavior by the private provider. To dissuade illegitimate selection, dynamic strategic capitation makes adjustments $\Delta\pi(t, \eta_0)$ and $\Delta\pi^{re}(T, \eta_0)$ using reported hold-out data $H_R$ as follows:

- $\Delta\pi(t, \eta_0) = \frac{1}{|I^{\eta_0}|} \sum_{i \in I^{\eta_0}} s_{i,t} r_{i,t}$, with

$$s_{i,t} \equiv \frac{\mu_{\Lambda_t}(\tau_{i,0}|\eta_{i,0} = \eta_0)}{\mu_I(\tau_{i,0}|\eta_{i,0} = \eta_0)} - 1, \quad \text{and} \quad r_{i,t} \equiv \frac{1}{|H_R^{\tau_{i,0}}|} \sum_{j \in H_R^{\tau_{i,0}}} \widehat{c}_j^R(t, p_0) - \pi(t, \eta_0).$$

- $\Delta\pi^{re}(t, \eta_t) = \frac{1}{|I^{\eta_t}|} \sum_{i \in I^{\eta_t}} s_{i,t}^{re} r_{i,t}^{re}$, with

$$s_{i,t}^{re} \equiv \frac{\mu_{\Lambda_t^{re}}(\tau_{i,t}|\eta_{i,t} = \eta_t)}{\mu_I(\tau_{i,t}|\eta_{i,t} = \eta_t)} - 1, \quad \text{and} \quad r_{i,t}^{re} \equiv \frac{1}{|H_R^{\tau_{i,t}}|} \sum_{j \in H_R^{\tau_{i,t}}} \left[ C(T, \eta_{i,T}, p_0) - \widehat{C}_j^R(t, p_0) \right].$$

# B Proofs

## B.1 Proofs for Section 3

**Proof of Proposition 1:** We begin with point $(i)$. Reports from provider $p_0$ do not affect reimbursements so that truth-telling strategy $\beta^*$ is dominant. In turn, for any selection $\Lambda$ measurable with respect to characteristics $\eta \in E$, the private provider's expected payoffs from selection take the form

$$\mathbb{E}_c\left[\sum_{i\in I}\mathbf{1}_{i\in\Lambda}(\kappa(\eta_i,p_0)-\kappa(\eta_i,p_1)+e_{i,\tau_i})\right]=\mathbb{E}_c\left[\sum_{i\in I}\mathbf{1}_{i\in\Lambda}(\kappa(\eta_i,p_0)-\kappa(\eta_i,p_1))\right]$$

where we used the fact that $\mathbb{E}_c[e_{i,\tau_i}|\eta_i] = 0$. It follows that the optimal selection rule is indeed $\Lambda = \Lambda_{\max} \equiv \{i \mid \kappa(\eta_i,p_0)-\kappa(\eta_i,p_1) > 0\}$.

Let us turn to point $(ii)$. It is useful to define

$$\xi_\eta \equiv \log\left(1+\frac{\kappa(\eta,p_0)-\kappa(\eta,p_1)}{\underline{\kappa}}\right)+\frac{1}{2(|D_0^\tau|+2)},$$

where we use the convention that $\log(x) = -\infty$ for $x \leq 0$.

Given data $D_0$ and signal $x_\tau$ the private provider's conditional belief over random cost parameter $\varepsilon_\tau$ follows a normal distribution $\mathcal{N}(\chi_\tau, \sigma_\tau^2)$ with

$$\chi_\tau = \frac{x_\tau + \sum_{i\in D_0^\tau} x_i}{2+|D_0^\tau|} \quad \text{and} \quad \sigma_\tau^2 = \frac{1}{2+|D_0^\tau|}.$$

This implies that $\mathbb{E}_\nu[e_{i,\tau}|D_0^\tau, x_\tau] = \underline{\kappa}\left[\exp\left(\chi_\tau - \frac{1}{2(|D_0^\tau|+2)}\right)-1\right]$. Furthermore, conditional on getting a data set of cardinal $|D_0^\tau|$, posterior belief $\chi_\tau$ itself follows a Gaussian distribution $\mathcal{N}\left(0, \frac{(|D_0^\tau|+1)^2+|D_0^\tau|}{(|D_0^\tau|+2)^2}\right)$.

We prove the first part of $(ii)$ by showing that

$$S_{\max} - S^{\mathsf{sparse}} = \sum_{i \in I} \text{prob}\left(\chi_{\tau_i} \geq \xi_{\eta_i}\right) \left[\kappa(\eta_i, p_0) - \kappa(\eta_i, p_1)\right]^+ + \sum_{i \in I} \text{prob}\left(\chi_{\tau_i} \leq \xi_{\eta_i}\right) \left[\kappa(\eta_i, p_1) - \kappa(\eta_i, p_0)\right]^+.$$

We prove the second part of $(ii)$ by showing that if the private provider has no comparative advantage, i.e. $\kappa(\eta, p_0) = \kappa(\eta, p_1)$, the public provider makes losses

$$\mathbb{E}_\nu[U_0 | D_0, x, \Pi^{\mathsf{sparse}}] = -\underline{\kappa} \sum_{i \in I} \left[\exp\left(\chi_{\tau_i} - \frac{1}{2(|D_0^{\tau_i}| + 2)}\right) - 1\right]^- .$$

Indeed, conditional on her information $(x_\tau, D_0^\tau)$, provider $p_1$'s expected payoff from selecting a patient of type $\tau$ is

$$\kappa(\eta, p_0) - \kappa(\eta, p_1) - \mathbb{E}_\nu[e_{i,\tau} | D_0^\tau, x_\tau].$$

Since $\mathbb{E}_\nu[e_{i,\tau} | D_0^\tau, x_\tau] = \underline{\kappa}\left[\exp\left(\chi_\tau - \frac{1}{2(|D_0^\tau|+2)}\right) - 1\right]$, provider $p_1$ will select type $\tau$ if and only if

$$\kappa(\eta, p_0) - \kappa(\eta, p_1) - \underline{\kappa}\left[\exp\left(\chi_\tau - \frac{1}{2(|D_0^\tau| + 2)}\right) - 1\right] > 0 \iff \chi_\tau < \xi_\eta.$$

This implies that efficiency losses indeed take the form

$$L^{\mathsf{sparse}} = \sum_{i \in I} \text{prob}\left(\chi_{\tau_i} \geq \xi_{\eta_i}\right) \left[\kappa(\eta_i, p_0) - \kappa(\eta_i, p_1)\right]^+ + \sum_{i \in I} \text{prob}\left(\chi_{\tau_i} \leq \xi_{\eta_i}\right) \left[\kappa(\eta_i, p_1) - \kappa(\eta_i, p_0)\right]^+.$$

When provider $p_1$ has no comparative advantage, it selects all types $\tau$ such that $\mathbb{E}_\nu[e_{i,\tau} | D_0^\tau, x_\tau] <$

0, and $p_0$'s expected payoffs are equal to

$$\begin{aligned}\mathbb{E}_\nu[U_0|D_0, D_1] &= \sum_{i\in I} \mathbb{E}_\nu[e_{i,\tau_i}|D_0, D_1]\mathbf{1}_{\mathbb{E}_\nu[e_{i,\tau}|D_0,x]<0} \\ &= -\underline{\kappa} \sum_{i\in I} \left[\exp\left(\chi_{\tau_i} - \frac{1}{2(|D_0^{\tau_i}|+2)}\right) - 1\right]^- .\end{aligned}$$

□

**Proof of Proposition 2:** It is useful to define

$$\forall \tau \in T, \quad \zeta_\tau \equiv \log\left(1 + \frac{\kappa(\eta, p_0) - \kappa(\eta, p_1)}{\underline{\kappa}} \exp\left(-\chi_{p_0,\tau} + \frac{1}{2(|D_0^\tau|+2)}\right)\right).$$

Provider $p_1$'s expected profit from selecting a patient of type $\tau$ is

$$\begin{aligned}&\kappa(\eta, p_0) - \kappa(\eta, p_1) + \mathbb{E}_\nu[e_{i,\tau}|D_0^\tau] - \mathbb{E}_\nu[e_{i,\tau}|D_0^\tau, x_\tau] \\ &\qquad = \kappa(\eta, p_0) - \kappa(\eta, p_1) + \underline{\kappa}\left[\exp\left(\chi_{p_0,\tau} - \frac{1}{2(|D_0^\tau|+1)}\right) - \exp\left(\chi_{p_1,\tau} - \frac{1}{2(|D_0^\tau|+2)}\right)\right].\end{aligned}$$

This implies that provider $p_1$ will select patients of type $\tau$ if and only if[16]

$$\begin{aligned}&\zeta_\tau > \chi_{p_1,\tau} - \chi_{p_0,\tau} + \frac{1}{2(|D_0^\tau|+1)(|D_0^\tau|+2)} \\ \iff &\zeta_\tau > \frac{1}{|D_0^\tau|+2}\left(x_\tau - \chi_{p_0,\tau} + \frac{1}{2(|D_0^\tau|+1)}\right).\end{aligned}$$

Observing that $\zeta_\tau$ has the same sign as $\kappa(\eta, p_0) - \kappa(\eta, p_1)$, this implies that the efficiency

[16]Selection will not occur when $\zeta_\tau$ is not defined.

loss $L^{\mathsf{rich}}$ can be written as

$$L^{\mathsf{rich}} = \sum_{i \in I} \text{prob}_{x_{\tau_i}} \left( x_{\tau_i} - \chi_{p_0,\tau} + \frac{1}{2(|D_0^\tau| + 1)} < -(|D_0^\tau| + 2)\zeta_{\tau_i}^- \right) [\kappa(\eta_i, p_0) - \kappa(\eta_i, p_1)]^-$$
$$+ \sum_{i \in I} \text{prob}_{x_{\tau_i}} \left( x_{\tau_i} - \chi_{p_0,\tau} + \frac{1}{2(|D_0^\tau| + 1)} > (|D_0^\tau| + 2)\zeta_{\tau_i}^+ \right) [\kappa(\eta_i, p_0) - \kappa(\eta_i, p_1)]^+$$

where we use the convention $z^- = \max\{0, -z\}$. The first term corresponds to the inefficiency loss from types that are more efficiently treated by $p_0$ but end up selected by $p_1$. The second term corresponds to the inefficiency loss from types that are more efficiently treated by $p_1$, but end up being treated by $p_0$.

Recall that $\chi_{p_0,\tau} \sim \mathcal{N}\left(0, \frac{|D_0^\tau|^2 + |D_0^\tau|}{(1+|D_0^\tau|)^2}\right)$, and therefore there are constants $c_1, c_2, c_3 > 0$ such that with probability greater than 1/2,

$$c_1 \cdot \frac{|\kappa(\eta_i, p_0) - \kappa(\eta_i, p_1)|}{\underline{\kappa}} \leq |\zeta_{\tau_i}| \leq c_2 \cdot \frac{|\kappa(\eta_i, p_0) - \kappa(\eta_i, p_1)|}{\underline{\kappa}}, \tag{20}$$

and for all $t > 0$, the probability that $|\zeta_{\tau_i}| < \exp(-t) \cdot |\kappa(\eta_i, p_0) - \kappa(\eta_i, p_1)|/\underline{\kappa}$ is at most $\exp(-c_3 t^2)$.

For the upper bound (9), suppose that $|\kappa(\eta_i, p_0) - \kappa(\eta_i, p_1)|/\underline{\kappa} = s/|D_0^\tau|$ for some $s > 1$. We have,

$$\begin{aligned} \text{prob}_{x_{\tau_i}, \chi_{p_0,\tau}} \left( x_{\tau_i} - \chi_{p_0,\tau} + \frac{1}{2(|D_0^\tau| + 1)} > (|D_0^\tau| + 2)\zeta_{\tau_i}^+ \right) < \\ \text{prob}_{x_{\tau_i}, \chi_{p_0,\tau}} \left( x_{\tau_i} - \chi_{p_0,\tau} > \sqrt{s} \right) + \text{prob}_{\chi_{p_0,\tau}} \left( \zeta_{\tau_i}^+ < \frac{\sqrt{s}}{|D_0^\tau|} \right) < \\ \exp(-c_4 \cdot s^2) + \text{prob}_{\chi_{p_0,\tau}} \left( |\zeta_{\tau_i}| < s^{-1/2} \cdot |\kappa(\eta_i, p_0) - \kappa(\eta_i, p_1)|/\underline{\kappa} \right) \leq \\ \exp(-c_4 \cdot s^2) + \exp(-c_3 (\log s)^2) < \\ \frac{c_5}{s}, \end{aligned}$$

for some constants $c_4, c_5 > 0$. Therefore, the expected contribution of patient $i \in I$ to efficiency loss $S_{\max} - S^{\mathsf{rich}}$ is bounded above by

$$\frac{c_5}{s} \cdot \frac{|\kappa(\eta_i, p_0) - \kappa(\eta_i, p_1)|}{\underline{\kappa}} \cdot \underline{\kappa} = \frac{c_5}{|D_0^\tau|} \cdot \underline{\kappa}.$$

When $s \leq 1$, the contributions of $i$ to efficiency loss is bounded above by $|\kappa(\eta_i, p_0) - \kappa(\eta_i, p_1)| \leq \underline{\kappa}/|D_0^\tau|$, thus completing the proof of the upper bound.

We now prove the lower bound (10). For concision, we use the notation $\delta \equiv \mathbb{E}_{\mu_I} \frac{1}{|D_0^\tau|} \in (0, 1]$. We will prove an efficiency loss of $c_6 \delta^2 \underline{\kappa}$ for some $c_6 > 0$. We first claim that there exists $k > 0$ such that $\text{prob}_{\mu_I}(|D_0^\tau| \leq k) \geq \delta^2 k/10$. Suppose this is not the case. Then

$$\begin{aligned} \delta = \mathbb{E}_{\mu_I} \frac{1}{|D_0^\tau|} &= \sum_{k=1}^{\infty} \frac{1}{k \cdot (k+1)} \cdot \text{prob}_{\mu_I}(|D_0^\tau| \leq k) \\ &\leq \sum_{k=1}^{10/\delta^2} \frac{1}{k \cdot (k+1)} \cdot \text{prob}_{\mu_I}(|D_0^\tau| \leq k) + \sum_{k \geq 10/\delta^2} \frac{1}{k \cdot (k+1)} \\ &< \frac{\delta^2}{10} + \sum_{k=1}^{10/\delta^2} \frac{\delta^2}{10(k+1)} < \frac{\delta^2}{10} + \int_1^{10/\delta^2} \frac{\delta^2}{10x} \mathrm{d}x. \end{aligned}$$

Using the fact that $\int_1^{10/\delta^2} \frac{\delta^2}{10x} \mathrm{d}x = \frac{\delta^2}{10} \log\left(\frac{10}{\delta^2}\right) < \frac{\delta^2}{5} \log(4/\delta) < \frac{\delta^2}{5}(4/\delta - 1)$, we obtain a contradiction.

Set $|\kappa(\eta_i, p_0) - \kappa(\eta_i, p_1)| = \underline{\kappa}/k$. By our choice of $k$, for a fraction of patients greater than $\delta^2 k/10$, $|D_0^\tau| \leq k$. By (20), with probability greater than $1/2$, $|\zeta_{\tau_i}| \leq c_2/k$. Thus $|\zeta_{\tau_i}| \cdot (|D_0^\tau| + 2) < c_2 + 2$, and for some $\epsilon > 0$, $i$ contributes at least $\epsilon$ efficiency loss $S_{\max} - S^{\mathsf{rich}}$ with probability greater than $\epsilon$. This leads to a per-patient expected efficiency loss of order

$$c_7 \cdot \frac{\delta^2 k}{10} \cdot \frac{\underline{\kappa}}{k} = (c_7/10) \cdot \delta^2 \underline{\kappa}.$$

When private provider $p_1$ has no comparative advantage, expected payoffs to the public provider take the form

$$\mathbb{E}_\nu[U_0|D_0,x,\Pi^{\mathsf{rich}}] = -\underline{\kappa}\sum_{i\in I}\left[\exp\left(\chi_{p_0,\tau_i}-\frac{1}{2(|D_0^{\tau_i}|+1)}\right)-\exp\left(\chi_{p_1,\tau_i}-\frac{1}{2(|D_0^{\tau_i}|+2)}\right)\right]^+$$
$$= -\underline{\kappa}\sum_{i\in I}\exp\left(\chi_{p_0,\tau}-\frac{1}{2(|D_0^{\tau_i}|+1)}\right)\left[1-\exp\left(\frac{1}{|D_0^{\tau_i}|+2}\left[x_{\tau_i}-\chi_{p_0,\tau_i}+\frac{1}{2(|D_0^{\tau_i}|+1)}\right]\right)\right]^+,$$

which is at least of the order of $\underline{\kappa}\,|I|\mathbb{E}_{\mu_I}\frac{1}{|D_0^{\tau_i}|}$ with probability bounded away from zero. □

**Proof of Proposition 3:** Provider $p_1$'s payoff takes the form

$$\mathbb{E}_\nu\left[\Pi(\Lambda)-\sum_{i\in\Lambda}\widehat{c}_i(p_1)\right] =\mathbb{E}_\nu\left[\sum_{i\in\Lambda}\kappa(\eta_i,p_0)-\kappa(\eta_i,p_1)\right]+\mathbb{E}_\nu\left[\sum_{i\in\Lambda}\Delta\pi(\eta_i,H_R,\Lambda)-e_{i,\tau_i}\right].$$

In any equilibrium $(\lambda,\beta)$, the expected cost of transfers to the public provider must be weakly lower under $\beta$ than under truthful reporting $\beta^*$. Recalling that $r_\tau\equiv\frac{1}{|H_R^\tau|}\sum_{j\in H_R^\tau}\left[\widehat{c}_j^R(p_0)-\widehat{\kappa}(\eta,p_0)\right]$ denotes reported residuals from the baseline capitation formula on hold-out sample costs, this implies that

$$\mathbb{E}_\nu\left[\sum_{i\in\Lambda}\Delta\pi(\eta_i,H_R,\Lambda)-e_{i,\tau_i}\Big|\lambda,\beta\right] = \mathbb{E}_\nu\left[\sum_{\eta\in E}|\Lambda^\eta|\left(\sum_{\tau\in T_\eta}[\mu_\Lambda(\tau|\eta)-\mu_I(\tau|\eta)]\widehat{r}_\tau-\sum_{i\in\Lambda}e_{i,\tau_i}\right)\Big|\lambda,\beta\right]$$
$$\leq -\mathbb{E}_\nu\left[\sum_{\eta\in E}|\Lambda^\eta|\sum_{\tau\in\eta}\mu_I(\tau|\eta)]\mathbb{E}_\nu[e_{i,\tau}|\tau]\right]=0.$$

Therefore it follows that provider $p_1$ gets a payoff at most equal to surplus

$$\mathbb{E}_\nu\left[\sum_{i\in\Lambda}\kappa(p_0,\eta_i)-\kappa(p_1,\eta_i)\right].$$

Since strategic-capitation adjustments have mean to zero when the private provider uses legitimate selection, the private provider can guarantee herself this payoff by using efficient selection strategy $\Lambda_{\max}$. Hence, in any equilibrium $\lambda=\Lambda_{\max}$. Since the private provider uses a legitimate selection rule, the public provider cannot reduce capitation payments by biasing reports, and uses truthful reporting strategy $\beta^*$. □

## B.2 Proofs for Section 4

**Proof of Proposition 4:** Let $\kappa(\eta, p) \equiv \mathbb{E}_{\mu_I}[\kappa(\tau, p)|\eta]$ denote the expected cost of service for provider $p$ conditional on legitimate selection characteristic $\eta$. Given a partition $E$ and a selection rule $\lambda$, provider $p_1$'s expected returns are

$$\begin{aligned}
\mathbb{E}_\nu[U_1|D_0, D_1] =& \mathbb{E}_\nu\left[\sum_{i\in I}\lambda(\tau_i)\left[\widehat{\kappa}(\eta_i, p_0) + \Delta\pi(\eta_i, H_R) - \kappa(\tau_i, p_1)\right]\Big| D_0, D_1\right] - K(\lambda)\\
=& \mathbb{E}_\nu\left[\sum_{i\in I}\lambda(\tau_i)\left[\kappa(\tau_i, p_0) - \kappa(\tau_i, p_1)\right]\Big| D_0, D_1\right] - K(\lambda)\\
+& \mathbb{E}_\nu\left[\sum_{i\in I}\lambda(\tau_i)\left[\kappa(\eta_i, p_0) + \Delta\pi(\eta_i, H_R) - \kappa(\tau_i, p_0)\right]\Big| D_0, D_1\right]\\
+& \mathbb{E}_\nu\left[\sum_{i\in I}\lambda(\tau_i)\left[\widehat{\kappa}(\eta_i, p_0) - \kappa(\eta_i, p_0)\right]\Big| D_0, D_1\right]\\
=& U_1^A + U_1^B + U_1^C.
\end{aligned}$$

where $U_1^A$, $U_1^B$ and $U_1^C$ are defined as the three respective terms in the expression above. Note that $U_1^A = S(\lambda|D_0, D_1)$. The key steps of the proof are the following,

(i) in any equilibrium $(E, \lambda, \beta)$, $U_1^B \leq 0$;

(ii) for any reporting strategy $\beta$, if $\lambda$ is measurable with respect to $E$, then $U_1^B = 0$;

(iii) for any $E$ and $\lambda$,

$$\left|\mathbb{E}_\nu[U_1^C|D_0]\right| \leq \mathbb{E}_\nu[\Psi(\mathcal{E}, e)].$$

Let us first show that points ($i$), ($ii$) and ($iii$) imply properties (15), (16) and (17). We have that under equilibrium strategies $(E, \lambda, \beta)$,

$$\begin{aligned}\mathbb{E}_\nu[U_1|D_0, D_1] \leq & S(\lambda|D_0, D_1) + \mathbb{E}_\nu[U_1^B|D_0, D_1, \lambda, \beta] + \mathbb{E}_\nu[U_1^C|D_0, D_1, \lambda, \beta] \\ \leq & S(\lambda|D_0, D_1) + \mathbb{E}_\nu[U_1^C|D_0, D_1, \lambda, \beta].\end{aligned}$$

In addition, from the fact that the private provider is weakly better off using $(E, \lambda)$ over any strategy $(E', \lambda')$ where $\lambda'$ is measurable with respect to $E'$, it follows that

$$\mathbb{E}_\nu[U_1] \geq \mathbb{E}_\nu\left[\max_{E' \in \mathcal{E}} S_{E'|D_0, D_1}\right] - \mathbb{E}_\nu[\Psi(\mathcal{E}, e)].$$

Overall this implies that $S(\lambda) \geq \mathbb{E}_\nu\left[\max_{E \in \mathcal{E}} S_{E|D_0,D_1}\right] - 2\mathbb{E}_\nu[\Psi(\mathcal{E}, e)]$. Condition (16) follows from the fact that truthful reporting $\beta^*(c, \tau)$ guarantees that

$$\begin{aligned}\mathbb{E}_\nu\left[\sum_{i \in I} \lambda(\tau_i)\left[\widehat{\kappa}(\eta_i, p_0) + \Delta\pi(\eta_i, H_R) - \kappa(\tau_i, p_0)\right]\right] \geq & \underbrace{\mathbb{E}_\nu\left[\sum_{i \in I} \lambda(\tau_i)\left[\kappa(\eta_i, p_0) + \Delta\pi(\eta_i, H_R) - \kappa(\tau_i, p_0)\right]\right]}_{=0} \\ & + \underbrace{\mathbb{E}_\nu\left[\sum_{i \in I} \lambda(\tau_i)\left[\widehat{\kappa}(\eta_i, p_0) - \kappa(\eta_i, p_0)\right]\right]}_{\geq -\mathbb{E}_\nu[\Psi(\mathcal{E}, e)]}.\end{aligned}$$

Finally, condition (17) follows from the fact that provider $p_1$ can choose a selection strategy measurable with respect to $E$, which guaranteed $p_1$ positive expected payoffs.

Let us return to the proofs of points ($i$), ($ii$) and ($iii$) above. Point ($i$) follows from the fact that in equilibrium the expected transfers of $p_0$ to provider $p_1$ under equilibrium reporting strategy $\beta$ must be weakly lower than under truthful reporting strategy $\beta^*$, i.e.

$\mathbb{E}_\nu\left[\Pi|\beta\right] \le \mathbb{E}_\nu\left[\Pi|\beta^*\right]$. This implies that

$$\mathbb{E}_\nu\left[\sum_{i\in I}\lambda(\tau_i)\left[\widehat{\kappa}(\eta_i,p_0)+\Delta\pi(\eta_i,H_R)\right]\Big|\beta\right] \le \mathbb{E}_\nu\left[\sum_{i\in I}\lambda(\tau_i)\left[\widehat{\kappa}(\eta_i,p_0)+\Delta\pi(\eta_i,H_R)\right]\Big|\beta^*\right], \text{ so that}$$

$$\mathbb{E}_\nu\left[\sum_{i\in I}\lambda(\tau_i)\left[\kappa(\eta_i,p_0)+\Delta\pi(\eta_i,H_R)-\kappa(\tau_i,p_0)\right]\Big|\beta\right] \le \mathbb{E}_\nu\left[\sum_{i\in I}\lambda(\tau_i)\left[\kappa(\eta_i,p_0)+\Delta\pi(\eta_i,H_R)-\kappa(\tau_i,p_0)\right]\Big|\beta^*\right].$$

Using the fact that

$$\begin{aligned}\mathbb{E}_\nu[\Delta\pi(\eta_i,H_R)|i\in\Lambda,\beta^*] &= \mathbb{E}_\nu\left[\sum_{\tau\in\eta_i}\left(\mu_\Lambda(\tau|\eta_i)-\mu_I(\tau|\eta_i)\right)\left(\kappa(\tau,p_0)-\widehat{\kappa}(\eta_i,p_0)\right)\Big|i\in\Lambda,\beta^*\right]\\ &= \mathbb{E}_\nu\left[\sum_{\tau\in\eta_i}\left(\mu_\Lambda(\tau|\eta_i)-\mu_I(\tau|\eta_i)\right)\left(\kappa(\tau,p_0)-\kappa(\eta_i,p_0)\right)\Big|i\in\Lambda,\beta^*\right]\\ &= \mathbb{E}_\nu\left[\sum_{\tau\in\eta_i}\mu_\Lambda(\tau|\eta_i)(\kappa(\tau,p_0)-\kappa(\eta_i,p_0))\Big|i\in\Lambda,\beta^*\right]\end{aligned}$$

and the fact that

$$\mathbb{E}_\nu\left[\sum_{i\in\Lambda}\kappa(\eta_i,p_0)-\kappa(\tau_i,p_0)+\Delta\pi(\eta_i,H_R)\right] = \mathbb{E}_\nu\left[\sum_{\eta\in E}|\Lambda^\eta|\left[\Delta\pi(\eta,H_R)+\sum_{\tau\in\eta}\mu_\Lambda(\tau|\eta)[\kappa(\eta,p_0)-\kappa(\tau,p_0)]\right]\right]$$

we obtain that indeed,

$$\mathbb{E}_\nu\left[\sum_{i\in I}\lambda(\tau_i)\left[\kappa(\eta_i,p_0)+\Delta\pi(\eta_i,H_R)-\kappa(\tau_i,p_0)\right]\Big|\beta^*\right] = 0,$$

and hence, for any reporting strategy $\beta$, $U_1^B \le 0$, which yields point $(i)$.

Point $(ii)$ follows from the fact that whenever $\lambda$ is measurable with respect to $E$, then for all reporting strategies $\beta$

$$\mathbb{E}_\nu\left[\sum_{i\in I}\lambda(\tau_i)(\kappa(\eta_i,p_0)-\kappa(\tau_i,p_0))\right] = 0$$

and

$$\mathbb{E}_\nu \left[\Delta\pi(\eta, H)\right] = \mathbb{E}_\nu \left[\sum_{\tau\in\eta}[\mu_\Lambda(\tau|\eta) - \mu_I(\tau|\eta)]r_\tau^H\right] = 0,$$

where $r_\tau^H = \frac{1}{|H^\tau|}\sum_{i\in H^\tau}\widehat{c}_i(p_0) - \widehat{\kappa}(\eta, p_0)$ denotes the mean residual of the baseline capitation formula computed in the hold-out sample.

Finally point $(iii)$ follows from the fact that

$$\begin{aligned}
U_1^C &\leq \max_{\lambda\in[0,1]^T, E\in\mathcal{E}} \mathbb{E}_\nu \left[\sum_{i\in I}\lambda(\tau_i)(\widehat{\kappa}(\eta_i, p_0) - \kappa(\eta_i, p_0))\right] \\
&\leq \max_{\lambda\in\mathcal{M}(E), E\in\mathcal{E}} \mathbb{E}_\nu \left[\sum_{i\in I}\lambda(\tau_i)(\widehat{\kappa}(\eta_i, p_0) - \kappa(\eta_i, p_0))\right] \\
&\leq \max_{\lambda\in\mathcal{M}(E), E\in\mathcal{E}} \mathbb{E}_\nu \left[\sum_{\eta\in E}\left[\sum_{i\in I^\eta}\widehat{\kappa}(\eta_i, p_0) - \kappa(\eta_i, p_0)\right]^+\right],
\end{aligned}$$

which yields point $(iii)$. □

**Proof of Lemma 1:** We begin with point $(i)$ and show that $\mathbb{E}_\nu\left[\Psi(\mathcal{E}, e)\right] \leq \mathbb{E}_\nu[\Psi(\mathcal{E}, e')]$ using a coupling argument, i.e. by carefully jointly sampling original errors $e$ and Rademacher errors $e'$.

Consider the following process for generating errors $e$ and $e'$. Errors $e$ are generated according to the original distribution of $e_i$ (where the different $e_i$'s are independent of one another). In turn, each error term $e'_i$ is generated from $e_i$ as follows: conditional on $e_i$, $e'_i \in \{-c_{max}, c_{max}\}$ is chosen so that $\mathbb{E}_\nu[e'_i|e_i] = e_i$. Note that this is possible since $e_i \in [-c_{max}, c_{max}]$, and there is a unique such distribution. Since error terms $(e_i)_{i\in D_0}$ are independent, so are error terms $(e'_i)_{i\in I}$. In addition,

$$\mathbb{E}_\nu[e'_i] = \mathbb{E}_{e_i}\mathbb{E}_\nu[e'_i|e_i] = \mathbb{E}_{e_i}e_i = 0,$$

which implies $e'_i \sim U\{-c_{max}, c_{max}\}$.

We now show that necessarily $\mathbb{E}_\nu [\Psi(\mathcal{E}, e)] \leq \mathbb{E}_\nu[\Psi(\mathcal{E}, e')]$. Note that $\Psi(\mathcal{E}, e)$ can be viewed as the maximum value for $S \subset E \in \mathcal{E}$ of

$$\Sigma_S \equiv \sum_{\eta \in S} |I^\eta| \left[ \sum_{\tau \in \eta} \mu_I(\tau|\eta) \frac{1}{|D_0^\tau|} \sum_{i \in D_0^\tau} e_i \right].^{17} \tag{21}$$

Fix $e$, and assume that $\Psi(\mathcal{E}, e)$ is realized by $\Sigma_S$ for some set $S$ of $\eta$'s. We have by linearity of expectation that

$$\begin{aligned}\Psi(\mathcal{E}, e) &= \sum_{\eta \in S} |I^\eta| \left[ \sum_{\tau \in \eta} \frac{|I^\tau|}{|I^\eta||D_0^\tau|} \sum_{i \in D_0^\tau} e_i \right] = \mathbb{E}_\nu \left[ \sum_{\eta \in S} |I^\eta| \left[ \sum_{\tau \in \eta} \frac{|I^\tau|}{|I^\eta||D_0^\tau|} \sum_{i \in D_0^\tau} e'_i \right] \Big| e \right] \\ &\leq \mathbb{E}_\nu \left[ \Psi(\mathcal{E}, e')|e \right].\end{aligned}$$

Using the law of iterated expectations, this completes the proof of point $(i)$.

We now turn to point $(ii)$. Let $E \in \mathcal{E}$ be any partition, and let $S \subset E$ be a selection of elements in partition $E$. We first show that for all $t$,

$$\text{prob}\left(\Sigma_S > t\right) \leq \exp\left( -\frac{t^2 |D_0|}{2c_{max}^2 \alpha^2 |I|^2} \right) \tag{22}$$

where $\Sigma_S$ is defined by (21). Using Hoeffding's inequality (see Hoeffding (1963) or Cesa-Bianchi and Lugosi (2006), Lemma 2.2) we have that

$$\begin{aligned}\text{prob}\left(\Sigma_S > t\right) &= \exp\left[ -\frac{2t^2}{\sum_{\eta \in S, \tau \in \eta} \sum_{i \in D_0^\tau} 4c_{\max}^2 \frac{|I^\tau|^2}{|D_0^\tau|^2}} \right] \\ &\leq \exp\left[ -\frac{t^2}{2c_{\max}^2 \sum_{\tau \in T} \frac{|I^\tau|^2}{|D_0^\tau|}} \right] = \exp\left[ -\frac{t^2}{2c_{\max}^2 \frac{|I|^2}{|D_0|} \sum_{\tau \in T} \frac{|I^\tau|}{|D_0^\tau|} \frac{|D_0|}{|I|} \frac{|I^\tau|}{|I|}} \right] \\ &\leq \exp\left[ -\frac{t^2}{2c_{\max}^2 \frac{|I|^2}{|D_0|} \alpha} \right].\end{aligned}$$

[17] Indeed, the corresponding set $S$ will only select $\eta$s such that $\sum_{\tau \in \eta} \mu_I(\tau|\eta) \frac{1}{|D_0^\tau|} \sum_{i \in D_0^\tau} e_i > 0$.

Since there are at most $M = \sum_{E \in \mathcal{E}} 2^{|E|} - 1$ possible non-empty sets $S$, this implies that

$$\text{prob}\left(\Psi(\mathcal{E}, e) > t\right) \leq M \exp\left[-\frac{t^2}{2c_{\max}^2 \frac{|I|^2}{|D_0|}\alpha}\right].$$

To complete the proof, we use the fact that $\mathbb{E}_\nu[\Psi(\mathcal{E}, e)] = \int_0^{+\infty} \text{prob}(\Psi(\mathcal{E}, e) > t)\mathrm{d}t$. Pick $t_0$ such that $M \exp\left[-\frac{t_0^2}{2c_{\max}^2 \frac{|I|^2}{|D_0|}\alpha}\right] = 1$, i.e. $t_0 = |I|c_{\max}\sqrt{\frac{2\alpha \log M}{|D_0|}}$. We have

$$\begin{aligned}
\mathbb{E}_\nu[\Psi(\mathcal{E}, e)] &\leq \int_0^{t_0} \text{prob}(\Psi(\mathcal{E}, e) > t)\mathrm{d}t + \int_{t_0}^{+\infty} \text{prob}(\Psi(\mathcal{E}, e) > t)\mathrm{d}t \\
&\leq t_0 + \int_{t_0}^{+\infty} M \exp\left[-\frac{t^2}{2c_{\max}^2 \frac{|I|^2}{|D_0|}\alpha}\right] \\
&\leq |I|c_{\max}\sqrt{\frac{2\alpha \log M}{|D_0|}} + \frac{\sqrt{2\pi}}{2}|I|c_{\max}\sqrt{\frac{\alpha}{|D_0|}} M \exp\left[-\frac{t^2}{2c_{\max}^2 \frac{|I|^2}{|D_0|}\alpha}\right] \\
&\leq |I|c_{\max}\sqrt{\frac{2\alpha}{|D_0|}}\left(\sqrt{\log M} + 1\right).
\end{aligned}$$

□

**Proof of Proposition 5:** Let $E$ be the partition maximizing $\sum_{\eta \in E} \frac{1}{1+\sqrt{|D_0^\eta|}}\mu_I(\eta)$. We start with the following simple claim:

**Claim 1** (hard to distinguish distributions)**.** *For each integer $d \geq 0$, there exists a pair of distributions $\phi_0$, $\phi_1$ with finite support over $[0, c_{max}]$ such that $\mathbb{E}_{\phi_0} c = c_l^d$, $\mathbb{E}_{\phi_1} c = c_h^d$, $c_h^d, c_l^d \in [c_{max}/4, 3c_{max}/4]$, $c_h^d - c_l^d \geq k' c_{max}/(1 + \sqrt{d})$, and $\phi_0^d$ is hard to distinguish from $\phi_1^d$, in the sense that*

$$\sup_{S \subset [0, c_{\max}]^d} \phi_0^d(S) - \phi_1^d(S) \leq 1/4,$$

*for some universal constant $k' > 0$, where $\phi_0^d$ and $\phi_1^d$ denote the $d$ product measures.*

We defer the proof of Claim 1 until after the proof of the proposition. We use the notation $d(\eta) \equiv |D_\eta^0|$. It is sufficient for our lower bound to consider the following class of environments $\nu$.

- Selection cost $K(\cdot)$ is identically equal 0.
- Cost distributions for the public and private providers are determined as follows. Let $(b_\eta)_{\eta\in E}$ be independent Bernoulli draws such that prob$(b_\eta = 1) = 1/2$. For all $\tau \in \eta$, cost distributions $c(p_0, \tau)$ are independent and identically distributed according to the distribution $\phi^{d(\eta)}_{b_\eta}$ described in Claim 1. Its expected value is $c^\eta_0 \in \{c^{d(\eta)}_h, c^{d(\eta)}_l\}$.
- For all $\tau \in \eta$, the private providers' cost $c(p_1, \tau_i)$ is distributed according to $\frac{1}{3}(\phi^{d(\eta)}_h + \phi^{d(\eta)}_l + c(p_0, \tau))$.
- Holdout set $H$ contains sufficient information to identify $(b_\eta)_{\eta\in E}$.
- Private provider $p_1$ knows $(b_\eta)_{\eta\in E}$.

For notational convenience, we denote by $\underline{c}(p_j, \eta)$ and $\overline{c}(p_j, \eta)$ the cost distributions for provider $j$ and characteristic $\eta$ when $b_\eta$ is respectively equal to 0 and 1. More generally, denote by $\underline{c}(p_j)$ the vector of expected per-patient cost functions for $p_j$ assuming $b_\eta = 0$. Note that $\underline{c}(p_j)$ and $\overline{c}(p_j)$ will agree on patients outside of $\eta$.

Let $g \in \mathcal{G}_\nu$ be an incentive compatible direct-revelation mechanism. Fix an $\eta \in E$, and a realization of $D^\eta_0$. We derive a lower bound for the efficiency loss incurred by $g$ over patients with characteristic $\eta$ (the number of such patients is $|I| \cdot \mu^E_I(\eta)$).

We exploit incentive compatibility conditions using the following set of messages. Message $\overline{m}_0$ is the message of public provider $p_0$ that correctly reports $(b_{\eta'})_{\eta'\neq\eta}$ but reports $b_\eta = 1$. Messages $\underline{m}_0$, $\overline{m}_1$, $\underline{m}_1$ are defined similarly. Note that message $m_1$ affects both transfers $\Pi(\mathcal{D}_0, m_0, m_1)$ and the selection of patients $\lambda(m_1)$.

For notational convenience, we will treat distribution $\lambda(\cdot)$ as a vector. Throughout, we take expectations over the realization of $b_{-\eta}$ and $\widehat{c}$ (cost indicators for groups other than $\eta$, and realized costs of care). Thus, for example, $\mathbb{E}_{b_{-\eta},\widehat{c}}\langle \overline{c}(p_1), \lambda(\overline{m}_1)\rangle$ is the expected cost of care for private provider $p_1$ assuming that $b_\eta = 1$; and $\mathbb{E}_{b_{-\eta},\widehat{c}}\langle \overline{c}(p_1), \lambda(\underline{m}_1)\rangle$ is the expected cost accrued to $p_1$ from treating its patients when $b_\eta = 1$, but $p_1$ reports that $b_\eta = 0$. We drop the $b_{-\eta}, \widehat{c}$ subscript from now on.

Incentive compatibility of provider $p_1$'s messages if $b_\eta = 1$ implies that

$$\mathbb{E}_\nu \Pi(\mathcal{D}_0, \overline{m}_0, \overline{m}_1) - \mathbb{E}_\nu \langle \overline{c}(p_1), \lambda(\overline{m}_1) \rangle \geq \mathbb{E}_\nu \Pi(\mathcal{D}_0, \overline{m}_0, \underline{m}_1) - \mathbb{E}_\nu \langle \overline{c}(p_1), \lambda(\underline{m}_1) \rangle. \tag{23}$$

Incentive compatibility of provider $p_0$'s message when $b_\eta = 0$ implies that

$$-\mathbb{E}_\nu \Pi(\mathcal{D}_0, \underline{m}_0, \underline{m}_1) + \mathbb{E}_\nu \langle \underline{c}(p_0), \lambda(\underline{m}_1) \rangle \geq -\mathbb{E}_\nu \Pi(\mathcal{D}_0, \overline{m}_0, \underline{m}_1) + \mathbb{E}_\nu \langle \underline{c}(p_0), \lambda(\underline{m}_1) \rangle,$$

which simplifies to

$$\mathbb{E}_\nu \Pi(\mathcal{D}_0, \underline{m}_0, \underline{m}_1) \leq \mathbb{E}_\nu \Pi(\mathcal{D}_0, \overline{m}_0, \underline{m}_1). \tag{24}$$

Combining (23) and (24) we obtain that

$$\mathbb{E}_\nu \Pi(\mathcal{D}_0, \overline{m}_0, \overline{m}_1) - \mathbb{E}_\nu \Pi(\mathcal{D}_0, \underline{m}_0, \underline{m}_1) \geq \mathbb{E}_\nu \langle \overline{c}(p_1), \lambda(\overline{m}_1) - \lambda(\underline{m}_1) \rangle. \tag{25}$$

A symmetric argument implies that

$$\mathbb{E}_\nu \Pi(\mathcal{D}_0, \overline{m}_0, \overline{m}_1) - \mathbb{E}_\nu \Pi(\mathcal{D}_0, \underline{m}_0, \underline{m}_1) \leq \mathbb{E}_\nu \langle \underline{c}(p_1), \lambda(\overline{m}_1) - \lambda(\underline{m}_1) \rangle. \tag{26}$$

Together, (25) and (26) imply

$$\mathbb{E} \langle \overline{c}(p_1) - \underline{c}(p_1), \lambda(\overline{m}_1) - \lambda(\underline{m}_1) \rangle \leq 0. \tag{27}$$

Since $\overline{c}(p_1) - \underline{c}(p_1)$ is a positive constant on $\eta$ and 0 elsewhere, (27) implies that in expectation at least as many patients from $\eta$ are treated by $p_1$ when $b_\eta = 0$ as when $b_\eta = 1$. Note that the efficiency loss that occurs when a patient $i \in \eta$ is treated by $p_0$ when $b_\eta = 1$ or is treated by $p_1$ when $b_\eta = 0$ is $(c_h^{d(\eta)} - c_l^{d(\eta)})/3$. Denote by $L_0$ the expected loss per patient in $\eta$ if

$b_\eta = 0$, and by $L_1$ the expected loss per patient if $b_\eta = 1$. We thus have

$$\text{prob}[p_1 \text{ treats}|b_\eta = 0] \geq \text{prob}[p_1 \text{ treats}|b_\eta = 1] = 1 - \text{prob}[p_0 \text{ treats}|b_\eta = 1],$$

and

$$L_0 + L_1 = ((c_h^{d(\eta)} - c_l^{d(\eta)})/3) \cdot (\text{prob}[p_1 \text{ treats}|b_\eta = 0] + \text{prob}[p_0 \text{ treats}|b_\eta = 1]) \geq (c_h^{d(\eta)} - c_l^{d(\eta)})/3. \tag{28}$$

Define $q_\eta \equiv \text{prob}[b_\eta = 1|\mathcal{D}_0]$. The expected efficiency loss accrued per patient in $\eta$ is greater than

$$\begin{aligned} &\min(L_0, L_1) \cdot \max(q_\eta, 1 - q_\eta) + \max(L_0, L_1) \cdot \min(q_\eta, 1 - q_\eta) \geq \\ &\qquad (L_0 + L_1) \cdot \min(q_\eta, 1 - q_\eta) \geq (c_h^{d(\eta)} - c_l^{d(\eta)}) \cdot \min(q_\eta, 1 - q_\eta)/3. \end{aligned} \tag{29}$$

Exploiting Claim 1 we will show that

$$\mathbb{E} \min(q_\eta, 1 - q_\eta) = \mathbb{E}_{D_0^\eta} \min\left(\frac{\phi_0(D_0^\eta)}{(\phi_0 + \phi_1)(D_0^\eta)}, \frac{\phi_1(D_0^\eta)}{(\phi_0 + \phi_1)(D_0^\eta)}\right) \geq \frac{1}{4}, \tag{30}$$

where we abuse notation and set $\phi_0(D_0^\eta) = \prod_{\widehat{c}_i \in D_\eta^0} \phi_0(\widehat{c}_i)$. Note that $D_0^\eta$ is distributed according to $(\phi_0^{d(\eta)} + \phi_1^{d(\eta)})/2$. The first equality of (30) holds by Bayes rule, and the fact that $b_\eta$ is a uniform Bernoulli. Furthermore, Claim 1 implies that

$$\begin{aligned} &\mathbb{E}_{D_0^\eta} \min\left(\frac{\phi_0(D_0^\eta)}{(\phi_0 + \phi_1)(D_0^\eta)}, \frac{\phi_1(D_0^\eta)}{(\phi_0 + \phi_1)(D_0^\eta)}\right) = \\ &\qquad \mathbb{E}_{D_0^\eta}\left[\frac{1}{2} - \frac{|\phi_0(D_0^\eta) - \phi_1(D_0^\eta)|}{2(\phi_0 + \phi_1)(D_0^\eta)}\right] = \frac{1}{2} - \sum_{D_0^\eta} \frac{|\phi_0(D_0^\eta) - \phi_1(D_0^\eta)|}{4} > \frac{1}{4}. \end{aligned}$$

Combining (29) and (30) it follows that per-patient efficiency loss in $\eta$ is at least

$$(c_h^{d(\eta)} - c_l^{d(\eta)}) \times \min(q_\eta, 1 - q_\eta)/3 \geq (c_h^{d(\eta)} - c_l^{d(\eta)})/12 = \frac{(k'/12)c_{max}}{1 + \sqrt{|D_0^\eta|}}.$$

Setting $k = k'/12$ completes the proof. □

We now prove Claim 1.

**Proof of Claim 1:** Given $d \geq 1$, let $\phi_0 \sim c_{\max} B_{1/2-\varepsilon}$, and $\phi_1 \sim c_{max} \cdot B_{1/2+\varepsilon}$, where $B_q$ denotes Bernoulli variables of parameter $q$, and $0 < \varepsilon < 1/4$ (with the relationship between $\varepsilon$ and $d$ to be specified below). Standard results from information theory (Cover and Thomas, 2012) imply that the statistical distance between $\phi_0^d$ and $\phi_1^d$ satisfies

$$2 \sup_{S \subset [0, c_{\max}]^d} \phi_0^d(S) - \phi_1^d(S) \leq \sqrt{d \cdot D(B_{1/2+\varepsilon} \| B_{1/2})/2} = \sqrt{d \cdot O(\varepsilon^2)} < k_1 \cdot \varepsilon \sqrt{d},$$

where $D(\cdot \| \cdot)$ is the Kullback-Leibler divergence, and $k_1 \geq 2$ is a constant. Choose $\varepsilon = 1/(2k_1\sqrt{d}) \leq 1/4$. Claim 1 holds with

$$c_h^d - c_l^d = 2\varepsilon = 1/(k_1\sqrt{d}).$$

Setting $k' \leq 1/k_1$ completes the proof. □

## B.3 Proofs for Section 5

**Proof of Proposition 6:** Consider the subgame following entry. For any continuation pricing equilibrium $(\pi_{p_0}, \pi_{p_1})$, the usual Bertrand competition argument implies that price formulas must satisfy

$$\forall \eta, \pi_0(\eta) = \pi_1(\eta) = \kappa(\eta).$$

Given these prices, profits are determined by the providers' selection behavior $(\lambda_{p_0}, \lambda_{p_1})$. Given the selection rule $\lambda_{\neg p}$ of her competitor, provider $p$ chooses

$$\begin{aligned}\lambda_p &\in \arg\max_{\lambda \in [0,1]^T} \sum_{\tau \in T} (\kappa(\eta) - \kappa(\tau)) \left( \frac{\mu_I}{2}(\tau) + \lambda(\tau) - \lambda_{\neg p}(\tau) \right) - K(\lambda) \\ &= \arg\max_{\lambda \in [0,1]^T} \sum_{\tau \in T} \lambda(\tau)(\kappa(\eta) - \kappa(\tau)) - K(\lambda).\end{aligned}$$

Since $K$ is strictly convex, minimized at $\mu_I/2$, and smooth, it follows that its gradient $\nabla K_{|\mu_I/2}$ at $\mu_I/2$ is equal to 0. As a result both providers engage in the same non-zero amount of selection $\lambda^*$, so that in aggregate, selection has no effect on each provider's treated sample. Strict convexity of $K$ implies that $K(\lambda^*) > 0$. This means that the entrant gets strictly negative expected profits following entry.

It follows that the unique equilibrium involves no entry, allowing the incumbent to charge prices equal to $\overline{\pi}$. □

**Proof of Proposition 7:** Consider the subgame following entry. For any continuation pricing equilibrium $(\pi_{p_0}, \pi_{p_1})$, the usual Bertrand competition argument implies that price formulas must satisfy

$$\forall \eta, \pi_0(\eta) = \pi_1(\eta) = \kappa(\eta).$$

Assuming truthful reporting by provider $\neg p$, strategic capitation ensures that provider $p$ does not benefit from selecting a non representative sample of types. Hence provider $p$'s payoffs boils down to

$$\sum_{\tau \in T} (\kappa(\eta) - \kappa(\tau)) (\mu_I(\tau) - \lambda_{\neg p}(\tau)) - K(\lambda).$$

It is therefore optimal for provider $p$ to set $\lambda_p = \mu_I/2$ and minimize selection cost. Given this choice, it is indeed optimal for provider $\neg p$ to report its hold-out sample truthfully. □

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