

Linguistic Applications of Mereology

Lecture notes to the introductory course at the
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Preface

Expressions like *John and Mary* or *the water in my cup* intuitively involve reference to collections of individuals or substances. The parthood relation between these collections and their components is not modeled in standard formal semantics of natural language (Montague 1974, Heim & Kratzer 1998), but it takes central stage in what is known as algebraic or mereological semantics (Link 1998, Krifka 1998, Landman 1996, 2000). This course provides a gentle introduction into the mathematical framework of classical extensional mereology, and is designed to help students understand important issues in the following problem domains, all of them active areas of research: plural, mass reference, measurement, aspect, and distributivity. In particular, the course shows how mereology sheds light on cross-categorial similarities between oppositions that pervade these domains, such as the count-mass, singular-plural, telic-atelic, and collective-distributive opposition.

Much of the important seminal work in the area has been done in the 1980s and 1990s (e.g. Link, Krifka, Landman). More recently, significant new applications have cropped up, ranging from the influence of verbal semantics on cumulativity (e.g. Kratzer 2007) to the grammar of measurement (Schwarzschild 2006) to dependent plurals (Zweig 2009). The mereological perspective has kept opening up important new avenues for research across the decades (Kratzer 2007, Williams 2009). It has proven particularly useful in drawing out cross-categorial generalizations. Examples include recent work on pluractionality in Kaqchikel and its relation to group nouns (Henderson 2012), and my own work on a unified theory of distributivity, aspect and measurement (Champollion 2010, 2017).

This booklet is based on a full-semester course, whose lecture notes are available at <http://ling.auf.net/lingbuzz/002174>. These lecture notes are slightly out of date; see <http://www.nyu.edu/projects/champollion/book/> for a book that discusses their contents and more and is up to date. Comments welcome (champollion@nyu.edu).

Lucas Champollion, New York, July 6th, 2017

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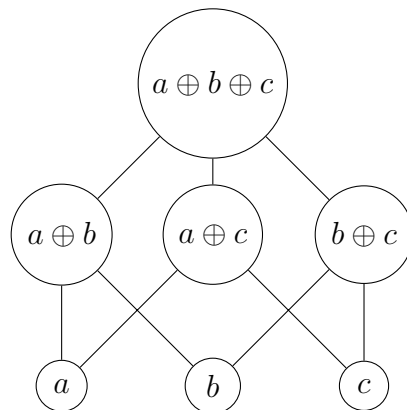
Lecture 1

Mereology: Concepts and axioms

1.1 Introduction

- **Mereology:** the study of parthood in philosophy and mathematical logic
- Mereology can be axiomatized in a way that gives rise to **algebraic structures** (sets with binary operations defined on them)

Figure 1.1: An algebraic structure



- **Algebraic semantics:** the branch of formal semantics that uses algebraic structures and parthood relations to model various phenomena

1.2 Mereology

1.2.1 Parthood

- Basic motivation (Link 1998): entailment relation between collections and their members
 - (1) a. John and Mary sleep. \Rightarrow
John sleeps and Mary sleeps.
 - b. The water in my cup evaporated. \Rightarrow
The water at the bottom of my cup evaporated.
- Basic relation \leq (parthood) – no consensus on what exactly it expresses
- Table 1.1 gives a few interpretations of the relation \leq in algebraic semantics

Table 1.1: Examples of unstructured parthood

Whole	Part
some horses	a subset of them
a quantity of water	a portion of it
John, Mary and Bill	John
some jumping events	a subset of them
a running event from A to B	its part from A halfway towards B
a temporal interval	its initial half
a spatial interval	its northern half

- All these are instances of unstructured parthood (arbitrary slices).
- Compare this with structured parthood (Simons 1987, Fine 1999, Varzi 2010) in Table 1.2 (cognitively salient parts)
- In algebraic semantics one usually models **only unstructured parthood**.
- This contrasts with lexical semantics, which concerns itself with structured parthood (e.g. Cruse 1986).
- Mereology started as an alternative to set theory; instead of \in and \subseteq there is only \leq .
- In algebraic semantics, mereology and set theory coexist.

Table 1.2: Examples of structured parthood from Simons (1987)

Whole	Part
a (certain) man	his head
a (certain) tree	its trunk
a house	its roof
a mountain	its summit
a battle	its opening shot
an insect's life	its larval stage
a novel	its first chapter

- The most common axiom system is **classical extensional mereology** (CEM).
- The order-theoretic axiomatization of CEM starts with \leq as a partial order:

(2) **Axiom of reflexivity**

$$\forall x[x \leq x]$$

(Everything is part of itself.)

(3) **Axiom of transitivity**

$$\forall x \forall y \forall z [x \leq y \wedge y \leq z \rightarrow x \leq z]$$

(Any part of any part of a thing is itself part of that thing.)

(4) **Axiom of antisymmetry**

$$\forall x \forall y [x \leq y \wedge y \leq x \rightarrow x = y]$$

(Two distinct things cannot both be part of each other.)

- The *proper-part* relation restricts parthood to nonequal pairs:

(5) **Definition: Proper part**

$$x < y \stackrel{\text{def}}{=} x \leq y \wedge x \neq y$$

(A proper part of a thing is a part of it that is distinct from it.)

- To talk about objects which share parts, we define overlap:

(6) **Definition: Overlap**

$$x \circ y \stackrel{\text{def}}{=} \exists z[z \leq x \wedge z \leq y]$$

(Two things overlap if and only if they have a part in common.)

1.2.2 Sums

- Pretheoretically, sums are that which you get when you put several parts together.
- The classical definition of sum in (7) is due to Tarski (1929). There are others.

(7) **Definition: Sum**

$$\text{sum}(x, P) \stackrel{\text{def}}{=} \forall y[P(y) \rightarrow y \leq x] \wedge \forall z[z \leq x \rightarrow \exists z'[P(z') \wedge z \circ z']]$$

(A sum of a set P is a thing that consists of everything in P and whose parts each overlap with something in P . “ $\text{sum}(x, P)$ ” means “ x is a sum of (the things in) P ”.)

Exercise 1.1 Prove the following facts!

(8) **Fact**

$$\forall x \forall y[x \leq y \rightarrow x \circ y]$$

(Parthood is a special case of overlap.)

(9) **Fact**

$$\forall x[\text{sum}(x, \{x\})]$$

(A singleton set sums up to its only member.)

The answers to this and all following exercises are in the Appendix. \square

- In CEM, two things composed of the same parts are identical:

(10) **Axiom of uniqueness of sums**

$$\forall P[P \neq \emptyset \rightarrow \exists! z \text{sum}(z, P)]$$

(Every nonempty set has a unique sum.)

- In CEM, every nonempty set P has a unique sum $\bigoplus P$.

(11) **Definition: Generalized sum**

For any nonempty set P , its sum $\bigoplus P$ is defined as $\iota z \text{sum}(z, P)$.

(The sum of a set P is the thing which contains every element of P and whose parts each overlap with an element of P .)

- As a shorthand for binary sum, we write $\oplus\{x, y\}$ as $x \oplus y$.

(12) **Definition: Binary sum**

$x \oplus y$ is defined as $\oplus\{x, y\}$.

(13) **Definition: Generalized pointwise sum**

For any nonempty n -place relation R_n , its sum $\oplus R_n$ is defined as the tuple $\langle z_1, \dots, z_n \rangle$ such that each z_i is equal to

$\oplus\{x_i | \exists x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n [R(x_1, \dots, x_n)]\}$.

(The sum of a relation R is the pointwise sum of its positions.)

- Two applications of sum in linguistics are conjoined terms and definite descriptions.

- For Sharvy (1980), $\llbracket \text{the water} \rrbracket = \oplus \text{water}$

- For Link (1983), $\llbracket \text{John and Mary} \rrbracket = j \oplus m$

- Another application: natural kinds as sums; e.g. the kind *potato* is \oplus potato.
- But this needs to be refined for uninstantiated kinds such as *dodo* and *phlogiston*. One answer: kinds are individual concepts of sums (Chierchia 1998b). See Carlson (1977) and Pearson (2009) on kinds more generally.

1.3 Mereology and set theory

- Models of CEM (or “mereologies”) are essentially isomorphic to complete Boolean algebras with the bottom element removed, or equivalently complete semilattices with their bottom element removed (Tarski 1935, Pontow & Schubert 2006).
- CEM parthood is very similar to the subset relation (Table 1.3).
- Example: the powerset of a given set, with the empty set removed, and with the partial order given by the subset relation.

Exercise 1.2 If the empty set was not removed, would we still have a mereology? Why (not)? \square

Table 1.3: Correspondences between CEM and set theory

Property	CEM	Set theory
1 Reflexivity	$x \leq x$	$x \subseteq x$
2 Transitivity	$x \leq y \wedge y \leq z \rightarrow x \leq z$	$x \subseteq y \wedge y \subseteq z \rightarrow x \subseteq z$
3 Antisymmetry	$x \leq y \wedge y \leq x \rightarrow x = y$	$x \subseteq y \wedge y \subseteq x \rightarrow x = y$
4 Interdefinability	$x \leq y \leftrightarrow x \oplus y = y$	$x \subseteq y \leftrightarrow x \cup y = y$
5 Unique sum/union	$P \neq \emptyset \rightarrow \exists! z \text{ sum}(z, P)$	$\exists! z z = \bigcup P$
6 Associativity	$x \oplus (y \oplus z) = (x \oplus y) \oplus z$	$x \cup (y \cup z) = (x \cup y) \cup z$
7 Commutativity	$x \oplus y = y \oplus x$	$x \cup y = y \cup x$
8 Idempotence	$x \oplus x = x$	$x \cup x = x$
9 Unique separation	$x < y \rightarrow$ $\exists! z [x \oplus z = y \wedge \neg x \circ z]$	$x \subset y \rightarrow \exists! z [z = y - x]$

1.4 Selected literature

- Textbooks:
 - * Montague-style formal semantics: Heim & Kratzer (1998)
 - * Mathematical foundations: Partee, ter Meulen & Wall (1990)
 - * Algebraic semantics: Landman (1991)
- Books on algebraic semantics: Link (1998), Landman (2000)
- Seminal articles on algebraic semantics: Landman (1996), Krifka (1998)
- Mereology surveys: Simons (1987), Casati & Varzi (1999), Varzi (2010)
- Linguistic applications of mereology: Champollion & Krifka (2016)
- Distributivity, aspect, and measurement: Champollion (2017) (see <http://www.nyu.edu/projects/champollion/book>)
- Extended (but slightly outdated) version of these lecture notes: <http://ling.auf.net/lingbuzz/002174>

Lecture 2

Nouns and Measurement

2.1 Algebraic closure and the plural

- Link (1983) has proposed algebraic closure as underlying the meaning of the plural.

- (1)
- a. John is a boy.
 - b. Bill is a boy.
 - c. \Rightarrow John and Bill are boys.

- Algebraic closure closes any predicate (or set) P under sum formation:

- (2) **Definition: Algebraic closure (Link 1983)**

The algebraic closure $*P$ of a set P is defined as $\{x \mid \exists P' \subseteq P[x = \bigoplus P']\}$.
(This is the set that contains any sum of things taken from P .)

- Link translates the argument in (1) as follows:

- (3) $\text{boy}(j) \wedge \text{boy}(b) \Rightarrow * \text{boy}(j \oplus b)$

- This argument is valid. Proof: From $\text{boy}(j) \wedge \text{boy}(b)$ it follows that $\{j, b\} \subseteq \text{boy}$. Hence $\exists P' \subseteq \text{boy}[j \oplus b = \bigoplus P']$, from which we have $* \text{boy}(j \oplus b)$ by definition.

Exercise 2.1 Prove the following fact!

- (4) **Fact**
 $\forall P[P \subseteq *P]$
(The algebraic closure of a set always contains that set.)

□

(5) **Definition: Algebraic closure for relations**

The algebraic closure $*R$ of a non-functional relation R is defined as

$$\{\vec{x} \mid \exists R' \subseteq R[\vec{x} = \bigoplus R']\}$$

(The algebraic closure of a relation R is the relation that contains any sum of tuples each contained in R .)

(6) **Definition: Algebraic closure for partial functions**

The algebraic closure $*f$ of a partial function f is defined as

$$\lambda x : x \in * \text{dom}(f). \bigoplus \{y \mid \exists z [z \leq x \wedge y = f(z)]\}$$

(The algebraic closure of f is the partial function that maps any sum of things each contained in the domain of f to the sum of their values.)

- There are different views on the meaning of the plural:
 - *Exclusive view*: the plural form N_{pl} essentially means the same as *two or more N* (Link 1983, Chierchia 1998a).

$$(7) \quad \llbracket N_{pl} \rrbracket = * \llbracket N_{sg} \rrbracket - \llbracket N_{sg} \rrbracket$$

$$(8) \quad \text{a. } \llbracket \text{boy} \rrbracket = \{a, b, c\}$$

$$\text{b. } \llbracket \text{boys} \rrbracket = * \llbracket \text{boy} \rrbracket - \llbracket \text{boy} \rrbracket = \{a \oplus b, b \oplus c, a \oplus c, a \oplus b \oplus c\}$$

- *Inclusive view*: the plural form essentially means the same as *one or more N* (Krifka 1989b, Sauerland 2003, Sauerland, Anderssen & Yatsushiro 2005, Chierchia 2010); the singular form blocks the plural form via competition

$$(9) \quad \llbracket N_{pl} \rrbracket = * \llbracket N_{sg} \rrbracket$$

$$(10) \quad \text{a. } \llbracket \text{boy} \rrbracket = \{a, b, c\}$$

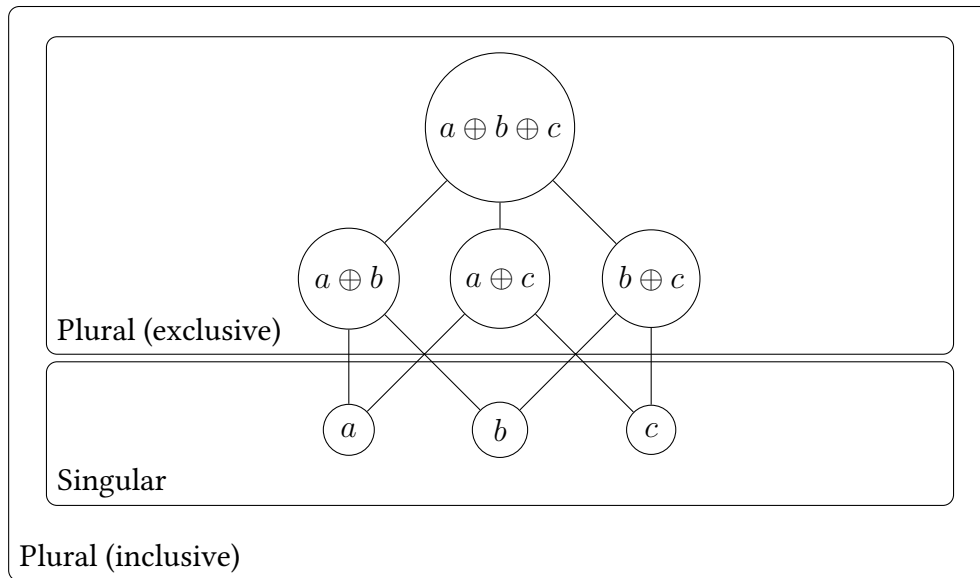
$$\text{b. } \llbracket \text{boys} \rrbracket = * \llbracket \text{boy} \rrbracket = \{a, b, c, a \oplus b, b \oplus c, a \oplus c, a \oplus b \oplus c\}$$

- *Mixed view*: plural forms are ambiguous between *one or more N* and *two or more N* (Farkas & de Swart 2010).

- Problem for the exclusive view (Schwarzschild 1996: p. 5): downward entailing contexts

(11) a. No doctors are in the room. (*false if there is exactly one doctor in the room*)

b. Are there (any) doctors in the room? (*the answer is yes if there is exactly one doctor in the room*)

Figure 2.1: Different views on the plural.

- Problem for the inclusive view: needs to be complemented by a blocking story

(12) *John is doctors.

- Problem for both views: dependent plurals (de Mey 1981)

(13) Five boys flew kites.
 \neq Five boys flew one or more kites.
 (because the sentence requires two or more kites in total to be flown)

(14) No boys flew kites.
 \neq No boys flew two or more kites.
 (because one kite flown by a boy already falsifies the sentence)

- On both the inclusive and exclusive view, plural nouns are *cumulative*.

(15) **Definition: Cumulative reference**
 $\text{CUM}(P) \stackrel{\text{def}}{=} \forall x[P(x) \rightarrow \forall y[P(y) \rightarrow P(x \oplus y)]]$

(A predicate P is cumulative if and only if whenever it holds of two things, it also holds of their sum.)

- The property of cumulativity is common to plural nouns and mass nouns (see below).

2.2 Singular count nouns

- Counting involves mapping to numbers. Let a “singular individual” be something which is mapped to the number 1, something to which we can refer by using a singular noun.
- One can assume that all singular individuals are *atoms*: the cat’s leg is not a part of the cat.

(16) **Definition: Atom**

$$\text{Atom}(x) \stackrel{\text{def}}{=} \neg \exists y [y < x]$$

(An atom is something which has no proper parts.)

(17) **Definition: Atomic part**

$$x \leq_{\text{Atom}} y \stackrel{\text{def}}{=} x \leq y \wedge \text{Atom}(x)$$

(Being an atomic part means being atomic and being a part.)

- Group nouns like *committee*, *army*, *league* have given rise to two theories.
 - Atomic theory: the entities in the denotation of singular group nouns are mereological atoms like other singular count nouns (Barker 1992, Schwarzschild 1996, Winter 2001)
 - Plurality theory: they are plural individuals (e.g. Bennett 1974)
- The question is whether the relation between a committee and its members is linguistically relevant, and if so whether it is mereological parthood.
- One can also assume that singular count nouns apply to “natural units” (Krifka 1989a) that may nevertheless have parts, or that there are two kinds of parthood involved (Link 1983).
- If one allows for nonatomic singular individuals, one might still want to state that all singular count nouns have quantized reference (Krifka 1989a): the cat’s leg is not itself a cat.

(18) **Definition: Quantized reference**

$$\text{QUA}(P) \stackrel{\text{def}}{=} \forall x [P(x) \rightarrow \forall y [y < x \rightarrow \neg P(y)]]$$

(A predicate P is quantized if and only if whenever it holds of something, it does not hold of any its proper parts.)

- But a twig may have a part that is again a twig, a rock may have a part that is again a rock, and so on (Zucchi & White 2001).
- For these cases, one can assume that context specifies an individuation scheme (Chierchia 2010, Rothstein 2010).

2.3 Mass nouns and atomicity

- Anything which can be referred to by a proper name, or denoted by using a common noun
- Objects form a mereology, so they include plural objects (sums)
 - Individuals: firemen, apples, chairs, opinions, committees
 - Substances: the water in my cup or the air which we breathe
- Possible exceptions:
 - Nominalizations (which arguably involve reference to events)
 - Measure nouns (which arguably involve reference to intervals or degrees).
- Mass nouns are compatible with the quantifiers *much* and *little* and reject quantifiers such as *each*, *every*, *several*, *a/an*, *some* and numerals (Bunt 2006, Chierchia 2010)
- Many nouns can be used as count nouns or as mass nouns:

(19) a. Kim put an apple into the salad.
b. Kim put apple into the salad.
- Are these two different words or one word with two senses? Pelletier & Schubert (2002)
- Four traditional answers to what the denotation of a mass noun is (Bealer 1979, Krifka 1991):
 - *General term analysis*: a set of entities – e.g. *gold* denotes the set of all gold entities (like a count noun)
 - *Singular term analysis*: a sum – e.g. *gold* denotes the sum of all gold entities (like a proper name)
 - *Kind reference analysis*: a kind – e.g. *gold* denotes the kind GOLD
 - *Dual analysis*: systematic ambiguity between sum/kind and set reading

- On the general term or dual analysis, we can apply higher-order properties to mass nouns
- Mass nouns have cumulative reference: add water to water and you get water
- In this, they parallel plural nouns (Link 1983)
- Mass nouns were proposed to have divisive reference (Cheng 1973); but this position is no longer popular (minimal-parts problem)

(20) **Definition: Divisive reference**

$$\text{DIV}(P) \stackrel{\text{def}}{=} \forall x[P(x) \rightarrow \forall y[y < x \rightarrow P(y)]]$$

(A predicate P is divisive if and only if whenever it holds of something, it also holds of each of its proper parts.)

- **Question** Are the count and mass domains distinct?
- Link (1983): yes, they have distinct properties (the new ring consists of old gold)
 - But also within the mass domain: new snow consists of old water (Bach 1986)
 - We could also relativize the concepts *new* and *old* to concepts: the entity x is new *qua* ring, and old *qua* gold
- Chierchia (1998a, 2010): no, all nouns refer within the same domain
 - But then, what *is* the count-mass distinction? (Chierchia's answer: vagueness)
 - What about pairs like *letters/mail*, *furniture/Möbel* etc.?
- What comes first: the count-mass distinction or the individual-substance distinction? Quine (1960) claims the former. Acquisition evidence suggests the latter (Spelke 1990).
- We can also talk about atoms in general, by adding one of the following axioms to CEM:

(21) **Optional axiom: Atomicity**

$$\forall y \exists x[x \leq_{\text{Atom}} y]$$

(All things have atomic parts.)

(22) **Optional axiom: Atomlessness**

$$\forall x \exists y[y < x]$$

(All things have proper parts.)

- Of course, we don't need to add either axiom. This is one of the advantages of mereology.

- Do count nouns always involve reference to atomic domains?
 - If yes: What about *twig* type nouns and group nouns?
 - If no: What determines if a concept is realized as a count noun?
- Do mass nouns ever involve reference to atomic domains? What about “fake mass nouns” (collective mass nouns like *furniture, mail, offspring*) (Barner & Snedeker 2005, Doetjes 1997, Chierchia 2010)?
 - If yes: why can't you say **three furniture(s)*?
 - If no: why do the parts of furniture not qualify as furniture?

2.4 Measure nouns and degrees

- Measure nouns (*liter, kilogram, year*) have a separate analysis from other count nouns
- Motivation: measure/individuating ambiguity of container pseudopartitives (Rothstein 2009: and references therein):

- (23) three glasses of wine
- a. *Measure reading*: a quantity of wine that corresponds to three glassfuls
 - b. *Individuating reading*: three actual glasses containing wine

- Only the individuating reading entails the existence of three glasses.

Exercise 2.2 How would you model the ambiguity? □

- Measure pseudopartitives like *three liters of water* only have the measure reading.
- These expressions are often analyzed as involving *degrees*.
- Degrees are totally ordered quantities assigned by measure functions.
 - Degrees of individuals: John's weight, the thickness of the ice at the South Pole
 - Degrees of events: the speed at which John is driving his car now
- Uses of degrees in semantics: gradable adjectives (*tall, beautiful*), measure nouns (*liter, hour*), measure phrases (*three liters*), comparatives (*taller, more beautiful, more water, more than three liters*), pseudopartitives (*three liters of water*)

- Degree scales are assumed to be totally ordered, while mereologies are typically only partially ordered.

2.5 Unit functions

- For Lønning (1987), degrees occupy an intermediate layer between individuals and numbers (see also Schwarzschild (2006)).

$$(26) \quad \begin{array}{l} \text{a. } \llbracket \text{liter} \rrbracket = \llbracket \text{liters} \rrbracket = \lambda n \lambda d [\text{liters}(d) = n] \\ \text{b. } \llbracket \text{year} \rrbracket = \llbracket \text{years} \rrbracket = \lambda n \lambda t [\text{years}(t) = n] \end{array}$$

- Example: John weighs 150 pounds (68 kilograms) and measures six feet (183 centimeters). *Weight* and *height* are measure functions, *feet* and *centimeters* are unit functions

$$(27) \quad \begin{array}{l} \text{a. } \text{pounds}(\text{weight}(\text{john}))=150 \\ \text{b. } \text{kilograms}(\text{weight}(\text{john}))=68 \\ \text{c. } \text{feet}(\text{height}(\text{john}))=6 \\ \text{d. } \text{centimeters}(\text{height}(\text{john}))=183 \end{array}$$

- Individuals are related to degrees via *measure functions*.
- Typical measure functions: height, weight, speed, temperature
- Measure nouns like *liter*, *kilogram*, *year* denote functions from degrees to numbers: what I will call *unit functions*.
- Advantage of Lønning's split: underspecification in pseudopartitives

$$(28) \quad \begin{array}{l} \text{three inches of oil} \\ \text{a. } \lambda x [\text{oil}(x) \wedge \text{inches}(\text{height}(x)) = 3] \quad \text{(by height)} \\ \text{b. } \lambda x [\text{oil}(x) \wedge \text{inches}(\text{diameter}(x)) = 3] \quad \text{(by diameter)} \end{array}$$

- Alternative: measure functions directly relate entities to numbers (Quine, Krifka)

Lecture 3

Measurement and Verbs

3.1 The measurement puzzle

- Pseudopartitives reject some measure functions (Krifka 1998, Schwarzschild 2006)

(1)	a.	five pounds of rice	<i>weight</i>
	b.	five liters of water	<i>volume</i>
	c.	five hours of talks	<i>duration</i>
	d.	five miles of railroad tracks	<i>spatial extent</i>
	e.	*five miles per hour of driving	<i>*speed</i>
	f.	*five degrees Celsius of water	<i>*temperature</i>

- Several other constructions behave analogously:

(2)	more rope	<i>by length / by weight / *by temperature</i>
(3)	*five miles per hour of my driving	<i>*speed</i>

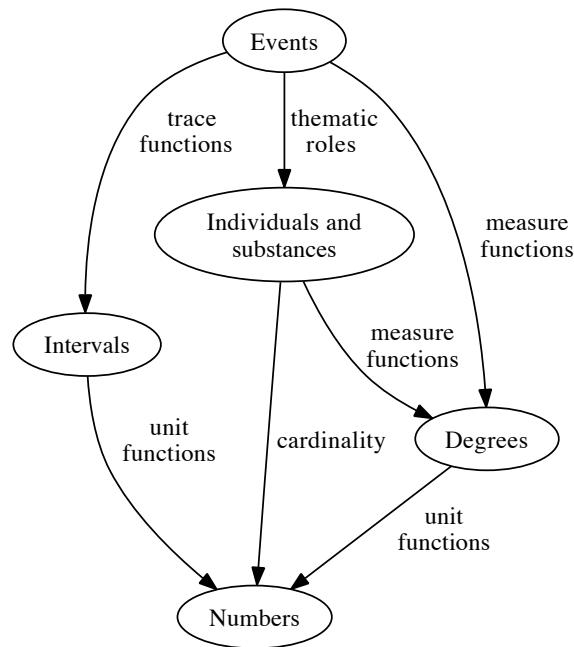
- Schwarzschild (2006): Only *monotonic* measure functions are admissible.
- A measure function μ is *monotonic* iff for any two entities a and b , if a is a proper part of b , then $\mu(a) < \mu(b)$. (See also Krifka (1998).)

3.2 Trace functions and intervals

- Let us zoom in on a special class of functions, the *trace functions*.

- Trace functions map events to intervals which represent their temporal and spatial locations
 - τ , the temporal trace or *runtime*
 - σ , the spatial trace
- Trace functions are pretty similar to measure functions but they relate events to intervals instead of events/individuals to degrees (Figure 3.1).

Figure 3.1: The world (some details omitted)



- Occur in phrases like *three hours* and *three miles*, *from 3pm to 6pm*, *to the store* and in tense semantics
- Trace functions also indicate the precise location in space and time
- Example: John sings from 1pm to 2pm and Mary sings from 2pm to 3pm. Although each event takes the same time, their runtimes are different.
- While two distinct events may happen at the same place and/or time, this is not possible for intervals.

- Axioms for temporal intervals and other temporal structures are found in van Benthem (1983). The integration into mereology is from Krifka (1998).
- Temporal inclusion (\leq) is like mereological parthood and subject to nonatomic CEM (most semanticists do not assume the existence of temporal atoms or instants)
- Temporal precedence (\ll) is irreflexive, asymmetric and transitive; holds between any two nonoverlapping intervals
- Example: if a is the interval from 2pm to 3pm today, b is the interval from 4pm to 5pm, and c is the interval from 1pm to 5pm, then we have $a \leq c$, $b \leq c$, and $a \ll b$
- Trace functions provide the bridge between interval logic and event logic:

(4) **Definition: Holding at an interval**

$$\text{AT}(V, i) \stackrel{\text{def}}{=} \exists e[V(e) \wedge \tau(e) = i]$$

(An event predicate V holds at an interval i if and only if it holds of some event whose temporal trace is i .)

- Trace functions are sum homomorphisms (Link 1998, Krifka 1998), like thematic roles.

(5) **Trace functions are sum homomorphisms**

σ is a sum homomorphism: $\sigma(e \oplus e') = \sigma(e) \oplus \sigma(e')$

τ is a sum homomorphism: $\tau(e \oplus e') = \tau(e) \oplus \tau(e')$

(The location/runtime of the sum of two events is the sum of their locations/runtimes.)

- Literature: Hinrichs (1985), Lasersohn (1988), Krifka (1998), Link (1998), Zwarts (2005, 2006)
- Prepositional phrases can be represented using trace functions

$$(6) \quad \llbracket \text{to the store} \rrbracket = \lambda V_{\langle vt \rangle} \lambda e[V(e) \wedge \text{end}(\sigma(e)) = \text{the.store}]$$

$$(7) \quad \llbracket \text{from 3pm to 4pm} \rrbracket = \lambda V_{\langle vt \rangle} \lambda e[V(e) \wedge \text{start}(\tau(e)) = 3\text{pm} \wedge \text{end}(\tau(e)) = 4\text{pm}]$$

- Literature on paths and aspect: Hinrichs (1985), Krifka (1998), Zwarts (2005, 2006)

3.3 Verbs

- Early work represents the meaning of a verb with n syntactic arguments as an n -ary relation

- Davidson (1967) argued that verbs denote relations between events and their arguments
- The neo-Davidsonian position (e.g. Carlson 1984, Parsons 1990, Schein 1993) relates the relationship between events and their arguments by thematic roles
- There are also intermediate positions, such as Kratzer (2000)

Table 3.1: Approaches to verbal denotations

Position	Verbal denotation	Example: Brutus stabbed Caesar
Traditional	$\lambda y \lambda x [\text{stab}(x, y)]$	$\text{stab}(b, c)$
Classical Davidsonian	$\lambda y \lambda x \lambda e [\text{stab}(e, x, y)]$	$\exists e [\text{stab}(e, b, c)]$
Neo-Davidsonian	$\lambda e [\text{stab}(e)]$	$\exists e [\text{stab}(e) \wedge \text{ag}(e, b) \wedge \text{th}(e, c)]$
Asymmetric	$\lambda y \lambda e [\text{stab}(e, y)]$	$\exists e [\text{ag}(e, b) \wedge \text{stab}(e, c)]$

- The Neo-Davidsonian position makes it easier to state generalizations across the categories of nouns and verbs, and to place constraints on thematic roles.
- Events are things like Jones' buttering of the toast, Brutus' stabbing of Caesar.
- Events form a mereology, so they include plural events (Bach 1986, Krifka 1998)
- Events can be both spatially and temporally extended, unlike intervals.
- Events are usually thought to have temporal parts (subevents which occupy less time). It is controversial whether individuals also do – this is the 3D/4D controversy (Markosian 2009). Most semanticists seem to be on the 3D side (individuals do not have temporal parts).
- Some authors treat events as built from atoms (Landman 2000), others distinguish between count and mass events (Mourelatos 1978). With mereology, we need not decide (Krifka 1998).
- Some authors also include states (e.g. John's being asleep) as events. Others use *event* more narrowly as opposed to states.
 - Do even stative sentences have an underlying event (Parsons 1987, 1990: ch. 10)? Maybe individual-level predicates don't (Kratzer 1995)?

3.4 Thematic roles

- Thematic roles represent ways entities take part in events (Parsons 1990, Dowty 1991)
- Two common views:
 - Traditional view: thematic roles encapsulate generalizations over shared entailments of argument positions in different predicates (Gruber 1965, Jackendoff 1972)
 - * *agent* (initiates the event, or is responsible for the event)
 - * *theme* (undergoes the event)
 - * *instrument* (used to perform an event)
 - * sometimes also *location* and *time*
 - Alternative view: thematic roles as verb-specific relations: Brutus is not the agent of the stabbing event but the stabber (Marantz 1984). But this misses generalizations. And what about subjects of coordinated sentences like *A girl sang and danced*?
- No consensus on the inventory of thematic roles, but see Levin (1993) and Kipper-Schuler (2005) for wide-coverage role lists of English verbs

Questions:

- Do thematic roles have syntactic counterparts, the theta roles (something like silent prepositions)? Generative syntax says yes at least for the agent role: the “little *v*” head (Chomsky 1995)
- Does each verbal argument correspond to exactly one role (Chomsky 1981) or is the subject of a verb like *fall* both its agent and its theme (Parsons 1990)?
- Thematic uniqueness / Unique Role Requirement: Does each event have at most one agent, at most one theme etc. (widely accepted in semantics, see Carlson (1984, 1998), Parsons (1990), Landman (2000)) or no (Krifka (1992): one can touch both a man and his shoulder in the same event)?
- Question: Let e is a talking event whose agent is John and e' is a talking event whose agent is Mary. What is the agent of $e \oplus e'$?
- More generally, are thematic roles their own algebraic closures (Krifka 1989b, 1998, Landman 2000)?

(8) **Cumulativity assumption for thematic roles**

For any thematic role θ it holds that $\theta = *\theta$. This entails that

$$\forall e, e', x, y [\theta(e) = x \wedge \theta(e') = y \rightarrow \theta(e \oplus e') = x \oplus y]$$

- I assume the answer is yes (makes things easier to formalize)
- To symbolize this, instead of writing θ , I will write $*\theta$.

- As a consequence of (8), thematic roles are homomorphisms with respect to the \oplus operation:

(9) **Fact: Thematic roles are sum homomorphisms**

For any thematic role θ , it holds that $\theta(e \oplus e') = \theta(e) \oplus \theta(e')$.

(The θ of the sum of two events is the sum of their θ s.)

- Potential challenge to this assumption: the rosebush story (Kratzer 2003). Suppose there are three events e_1, e_2, e_3 in which Al dug a hole, Bill inserted a rosebush in it, and Carl covered the rosebush with soil. Then there is also an event e_4 in which Al, Bill, and Carl planted a rosebush. Let e_4 be this event. If $e_4 = e_1 \oplus e_2 \oplus e_3$, we have a counterexample to lexical cumulativity.

Exercise 3.1 Why is this a counterexample? How could one respond to this challenge? \square

3.5 Lexical cumulativity

- Many authors assume *lexical cumulativity*: whenever two events are in the denotation of a verb, so is their sum (Scha 1981, Schein 1986, 1993, Lasersohn 1989, Krifka 1989a, 1992, Landman 1996, 2000, Kratzer 2007).

- (10) a. John slept.
 b. Mary slept.
 c. \Rightarrow John and Mary slept.

- (11) a. John saw Bill.
 b. Mary saw Sue.
 c. \Rightarrow John and Mary saw Bill and Sue.

- Verbs have plural denotations: they obey the same equation as plural count nouns on the inclusive view

$$(12) \quad \llbracket V \rrbracket = * \llbracket V \rrbracket$$

$$(13) \quad \llbracket N_{pl} \rrbracket = * \llbracket N_{sg} \rrbracket$$

- It is customary to indicate lexical cumulativity by writing $\lambda e[*\text{see}(e)]$ for the meaning of the verb *see* instead of $\lambda e[\text{see}(e)]$.

This entailment is parallel to the entailment from singular to plural nouns:

- (14) a. John is a boy.
 b. Bill is a boy.
 c. \Rightarrow John and Bill are boys.

- Lexical cumulativity does not entail that all verb phrases have cumulative reference. For example, the sum of two events in the denotation of the verb phrase *carry exactly two pianos* is not again in its denotation, because it involves four rather than two pianos.

Exercise 3.2 Does the verb phrase *see John* have cumulative reference? \square

3.6 Incremental vs. holistic themes

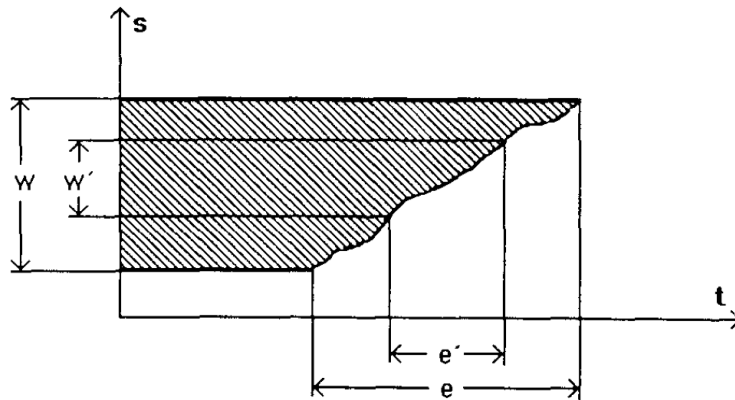
- Krifka (1992): in incremental-theme verbs (also called “measuring-out” verbs, among other things), the parts of the event can be related to the parts of the theme (see Figure 3.2).
- Tenny (1995) calls this “Krifka’s homomorphism”. Careful: this is different from the notion that all verbs are sum homomorphisms. Only some verbs are incremental-theme verbs.
- Following Krifka, we can formalize the difference between holistic-theme and incremental-theme verbs by meaning postulates.

(15) **Definition: Incrementality**
 $\text{Incremental}_{\theta}(P) \Leftrightarrow \forall e \forall e' \forall x [\theta(e) = x \wedge e' < e \rightarrow \theta(e') < x]$

(16) **Definition: Holism**
 $\text{Holistic}_{\theta}(P) \Leftrightarrow \forall e \forall e' \forall x [\theta(e) = x \wedge e' < e \rightarrow \theta(e') = x]$

- (17) **Meaning postulates**
- Incremental_{theme}($\llbracket \text{eat} \rrbracket$)
 - Incremental_{theme}($\llbracket \text{drink} \rrbracket$)
 - Holistic_{theme}($\llbracket \text{see} \rrbracket$)

Figure 3.2: Incremental theme of *drink wine*, from Krifka (1992)



Lecture 4

Verbs and Distributivity

4.1 Aspectual composition

- Predicates can be telic or atelic.
 - Atelic predicates: walk, sleep, talk, eat apples, run, run towards the store
(\approx *as soon as you start X-ing, you have already X-ed*)
 - Telic predicates: build a house, finish talking, eat ten apples, run to the store
(\approx *you need to reach a set terminal point in order to have X-ed*)
- Question: Do telic and atelic predicates form disjoint classes of events (Piñón 1995) or is this a difference of predicates (Krifka 1998)?
- Traditionally, atelicity is understood as the subinterval property or divisive reference. Telicity is understood as quantized reference. This brings out the parallel between the telic/atelic and count/mass oppositions (e.g. Bach 1986).
 - (1) a. telic : atelic :: count : mass
b. quantized : (approximate) subinterval :: quantized : (approximate) divisive
- We will use the following definition of the subinterval property:
 - (2) $\text{SUBINTERVAL}(P) =_{def} \forall e[P(e) \rightarrow \forall i[i < \tau(e) \rightarrow \exists e'[P(e') \wedge e' < e \wedge i = \tau(e')]]]$
(Whenever P holds of an event e , then at every subinterval of the runtime of e , there is a subevent of which P also holds.)

- (3) *eat ten apples for three hours
Failing presupposition: SUBINTERVAL(\llbracket eat ten apples \rrbracket), i.e. every part of the runtime of an eating-ten-apples event e is the runtime of another eating-ten-apples event that is a part of e .
- The “minimal-parts problem” (Taylor 1977, Dowty 1979): The subinterval property distributes P literally over all subintervals. This is too strong.
- (4) John and Mary waltzed for an hour
 \nrightarrow #John and Mary waltzed within every single moment of the hour
 \Rightarrow John and Mary waltzed within every short subinterval of the hour
- The length interval that counts as very small for the purpose of the *for*-adverbial varies relative to the length of the bigger interval:
- (5) The Chinese people have created abundant folk arts ... passed on from generation to generation for thousands of years.¹
- *Aspectual composition* is the problem of how complex constituents acquire the telic/atelic distinction from their parts. Verkuyl (1972), Krifka (1998)
 - With “incremental theme” verbs like *eat*, the correspondence is clear:
- (6) a. eat apples / applesauce for an hour
b. *eat an apple / two apples / the apple for an hour
- (7) a. count : mass :: telic : atelic
b. apple : apples :: eat an apple : eat apples
- (8) a. drink wine for an hour
b. *drink a glass of wine for an hour
- With “holistic theme” verbs like *push* and *see*, the pattern is different:
- (9) a. push carts for an hour
b. push a cart for an hour
- (10) a. look at apples / applesauce for an hour
b. look at an apple / two apples / the apple for an hour

¹Attested example (<http://www.twinbridge.com/detail.aspx?ID=315>). Nov 15, 2010.

- *Verkuyl's Generalization* (Verkuyl 1972): When the direct object of an incremental-theme verb is a count expression, we have a telic predicate, otherwise an atelic one.
- Krifka (1998): we use incremental-theme meaning postulates to prove or disprove that the various VPs above have divisive reference or the subinterval property.
- **Claim:** $\llbracket \text{eat two apples} \rrbracket$ does not have the subinterval property.
- **Proof:** Suppose it has, then let e be an event in its denotation whose runtime is an hour. From the definition of the subinterval property, (2), at each subinterval of this hour there must be a proper subevent of e whose theme is again two apples. Let e' be any of these proper subevents. Let the theme of e be x and the theme of e' be y . Then x and y are each a sum of two apples. From the “incremental theme” meaning postulate in (17a) we know that y is a proper part of x' . Since *two apples* is quantized, x and y can not both be two apples. Contradiction.

Exercise 4.1 Why does the proof not go through for *see two apples*? Why does it not go through for *eat apples*? \square

4.2 Distributivity

- **What is distributivity?** In this lecture: a property of predicates
 - *Distributive*: e.g. walk, smile, take a breath (applies to a plurality just in case it applies to each of its members)
 - *Collective*: e.g. be numerous, gather, suffice to defeat the army (may apply to a plurality even if it does not apply to each of its members)
- Literature: Roberts (1987); Winter (2001), Section 6.2; Schwarzschild (1996), Chapter 6; Link (1997), Section 7.4. Champollion (to appear) (handbook article). Champollion (2016a)

4.3 Lexical and phrasal distributivity

- (11) **Lexical distributivity/collectivity** involves lexical predicates
- | | |
|--------------------------------|---------------------|
| a. The children smiled. | <i>distributive</i> |
| b. The children were numerous. | <i>collective</i> |
- (12) **Phrasal distributivity/collectivity** involves complex predicates

- | | | |
|----|--------------------------------|--------------------------------|
| a. | The girls are wearing a dress. | <i>distributive</i> |
| b. | The girls are sharing a pizza. | <i>collective</i> |
| c. | The girls are building a raft. | <i>collective/distributive</i> |

- The difference between lexical and phrasal distributivity corresponds to the difference between what can and what cannot be described using meaning postulates

(13) **Meaning postulate: *smile* is distributive**

$$\forall e[\text{smile}(e) \rightarrow e \in * \lambda e'(\text{smile}(e') \wedge \text{Atom}(\text{ag}(e')))]$$

(Every smiling event consists of one or more smiling events whose agents are atomic.)

- Meaning postulates can only apply to words. We cannot formulate a meaning postulate that says that *wear a dress* is distributive.

- Problems:

- Meaning postulates are taken to be available only for lexical items
- For mixed predicates like *build a raft*, we would need optional meaning postulates

- The classical solution is due to Link (1983): A covert distributive operator D adjusts the meaning of a verb phrase like *wear a dress* into *be a sum of people who each wear a dress*.
- D is in the lexicon, so it can apply to entire VPs (Dowty 1987, Roberts 1987, Lasnik 1995).
- Link's D operator introduces a universal quantifier:

$$(14) \quad \llbracket D^{Link} \rrbracket = \lambda P_{\langle et \rangle} \lambda x \forall y [y \leq_{Atom} x \rightarrow P(y)]$$

(Takes a predicate P over individuals and returns a predicate that applies to any individual whose atomic parts each satisfy P .)

- | | | | |
|------|----|--|---------------------|
| (15) | a. | The girls built a raft. | |
| | | \approx The girls built a raft together. | <i>collective</i> |
| | b. | The girls D^{Link} (built a raft). | |
| | | \approx The girls each built a raft. | <i>distributive</i> |

- This allows us to model the distributive meaning of (12a):

(16) The girls D (are wearing a dress.)

$$\forall y [y \leq_{Atom} \bigoplus \text{girl} \rightarrow \exists z [\text{dress}(z) \wedge \text{wear}(y, z)]]$$

(Every atomic part of the sum of all girls wears a dress.)

4.4 Atomic and nonatomic distributivity

- So far we have implemented the view called *atomic distributivity*: the D operator distributes over atoms, that is, over singular individuals (Lasersohn 1998, 1995, Link 1997, Winter 2001)
- Nonatomic view: phrasal distributivity may also quantify over nonatomic parts (Gillon 1987, 1990, Verkuyl & van der Does 1991, van der Does & Verkuyl 1996, Schwarzschild 1996, Brisson 1998, 2003, Malamud 2006a,b, Champollion 2016a, 2017)
- Traditional argument is based on sentences like this, adapted from Gillon (1987):
- Rodgers, Hammerstein and Hart never wrote any musical together, nor did any of them ever write one all by himself. But Rodgers and Hammerstein wrote the musical *Oklahoma* together, and Rodgers and Hart wrote the musical *On your toes* together.
- On the basis of these facts, (17a) and (17b) are judged as true in the actual world, although it is neither true on the collective interpretation nor on an “atomic distributive” interpretation.

- (17) a. Rodgers, Hammerstein, and Hart wrote *Oklahoma* and *On Your Toes*.
 b. Rodgers, Hammerstein, and Hart wrote musicals.

- The traditional nonatomic argument: in order to generate the reading on which (17b) is true, the predicates *wrote musicals* and *wrote Oklahoma and On Your Toes* must be interpreted as applying to nonatomic parts of the sum individual to which the subject refers.
- Generally implemented with covers (Gillon 1987): partitions of a set (18) or sum (19) whose cells/parts can overlap

(18) **Definition: Cover (set-theoretic)**

$$\text{Cov}(C, P) \stackrel{\text{def}}{=} \bigcup C = P \wedge \emptyset \notin C$$

(C is a cover of a set P if and only if C is a set of nonempty subsets of P whose union is P .)

(19) **Definition: Cover (mereological)**

$$\text{Cov}(C, x) \stackrel{\text{def}}{=} \bigoplus C = x$$

(C is a cover of a mereological object x if and only if C is a set of parts of x whose sum is x .)

- Cover-based approaches modify the D operator to quantify over nonatomic parts of a cover of the plural individual.

- The first cover-based approaches assumed that the cover can be existentially quantified by the operator that introduces it:

(20) **Nonatomic distributivity operator, existentially bound cover**

$$\llbracket D_{\exists} \rrbracket = \lambda P_{\langle et \rangle} \lambda x \exists C [\text{Cov}(C, x) \wedge \forall y [C(y) \wedge y \leq x \rightarrow P(y)]]$$

- On this view, sentences (17a) and (17b) are translated as follows:

(21)
$$\exists C [\text{Cov}(C, \text{rodgers} \oplus \text{hammerstein} \oplus \text{hart}) \wedge \forall y [C(y) \wedge y \leq x \rightarrow y \in \llbracket \text{wrote Oklahoma and On Your Toes} \rrbracket]]$$

(22)
$$\exists C [\text{Cov}(C, \text{rodgers} \oplus \text{hammerstein} \oplus \text{hart}) \wedge \forall y [C(y) \wedge y \leq x \rightarrow y \in \llbracket \text{wrote musicals} \rrbracket]]$$

Exercise 4.2 For which value of C are these formulas true in the actual world? \square

- Existentially bound covers are now generally considered untenable because they overgenerate nonatomic distributive readings
- Lasersohn (1989)'s problem: Suppose John, Mary, and Bill are the teaching assistants and each of them was paid exactly \$7,000 last year. (23a) and (23b) are true, but (23c) is false.

(23) a. *True*: The TAs were paid exactly \$7,000 last year. *distributive*
 b. *True*: The TAs were paid exactly \$21,000 last year. *collective*
 c. *False*: The TAs were paid exactly \$14,000 last year. **nonatomic distributive*

- Giving up the existential cover-based operator D_{\exists} in (20) explains why (23c) is false, because without this operator, there is no way to generate a true reading for this sentence.
- But now why are (17a) and (17b) true?
- As it turns out, the lexical cumulativity assumption is already enough (Lasersohn 1989):

(24)
$$\forall w, x, y, z [\text{write}(w, x) \wedge \text{write}(y, z) \rightarrow \text{write}(w \oplus y, x \oplus z)]$$

Exercise 4.3 What does this assumption translate to in a Neo-Davidsonian framework? \square

- Further support: (27) is false in the actual world (Link 1997):

(27) Rodgers, Hammerstein and Hart wrote a musical.

- a. **True** if the three of them wrote a musical together – not the case. ✓ *collective*
- b. **True** if each of them wrote a musical by himself – not the case. ✓ *atomic distributive*
- c. **False** even though Rodgers and Hammerstein wrote a musical together, and Rodgers and Hart wrote another musical together. ✓ **nonatomic distributive*
- The absence of the nonatomic distributive reading of (27) is predicted if we give up the existential cover-based operator D_{\exists} .
 - Lexical cumulativity derives the (available) nonatomic distributive reading of (17a) and (17b) but not the (unavailable) nonatomic distributive reading of (27):
- (28) $\llbracket \text{Rodgers, Hammerstein and Hart wrote } \textit{Oklahoma} \text{ and } \textit{On Your Toes}. \rrbracket$
 $= \exists e [* \text{write}(e) \wedge * \text{ag}(e) = \text{rodgers} \oplus \text{hammerstein} \oplus \text{hart} \wedge * \text{th}(e) = \text{okl} \oplus \text{oyt}]$
 (Allows for several writing events and for teamwork, as long as these two musicals are written.)
- (29) $\llbracket \text{Rodgers, Hammerstein and Hart wrote musicals}. \rrbracket$
 $= \exists e [* \text{write}(e) \wedge * \text{ag}(e) = \text{rodgers} \oplus \text{hammerstein} \oplus \text{hart} \wedge * \text{musical}(* \text{th}(e))]$
 (Allows for several writing events and for teamwork, and there can be several musicals in total.)
- (30) $\llbracket \text{Rodgers, Hammerstein and Hart wrote a musical}. \rrbracket$
 $= \exists e [* \text{write}(e) \wedge * \text{ag}(e) = \text{rodgers} \oplus \text{hammerstein} \oplus \text{hart} \wedge \text{musical}(* \text{th}(e))]$
 (Allows for several writing events and for teamwork, but there has to be only one musical in total.)
- Lasersohn, as well as Winter (2001) and others, conclude from this and similar examples that the atomic approach to phrasal distributivity is superior to covers.
 - However, Gillon (1990) and Schwarzschild (1996) identify a residue of cases in which a cover-based operator does seem necessary.
- (31) **Scenario** Two pairs of shoes are on display, each pair with a \$50 price tag.
- a. The shoes cost \$100. ✓ *collective (together)*
- b. The shoes cost \$25. ? *atomic distributive (per shoe)*
- c. The shoes cost \$50. (Lasersohn 1995) ✓ **nonatomic distributive (per pair)*
- Evidence that individual shoes, and not shoe pairs, are atoms in this context:

(32) How many shoes are on display? – Four / #Two.

- Schwarzschild (1996) proposes that the cover of D is anaphoric on context:

(33) **Schwarzschild's nonatomic distributivity operator, free cover**
 $\llbracket D_C \rrbracket = \lambda P_{\langle et \rangle} \lambda x \forall y [C(y) \wedge y \leq x \rightarrow P(y)]$

- See Malamud (2006a,b) for a decision-theoretic elaboration of this proposal.
- Nonatomic distributivity is always available for verbs, but for verb phrases it only occurs when context supplies a pragmatically salient cover. Atomic distributivity is available in both cases. (Champollion 2016a)

Figure 4.1: V level versus verb phrase level distributivity in atomic domains

(a) Empirical generalization			(b) Explanation	
	lexical (V level)	phrasal (VP level)	lexical (V level)	phrasal (VP level)
atomic	available	available	meaning post.	Atomic D op.
nonatomic	available	only w. context	meaning post.	Cover-based D op.

4.5 Discussion: Aspectual composition and distributivity

- Aspect and distributivity have each been given a compositional treatment.
- Historically, both aspect and distributivity used to be seen as properties of verbs (Vendler 1957, Scha 1981)
- Verkuyl (1972) recognized that aspect is a property of larger constituents and that it must be derived compositionally from the properties of the verb and of its arguments
- Winter (2000) (among others) made the same argument for distributivity
- In each case the argument has served to move the relevant empirical phenomena from lexical semantics into compositional semantics
- Can you think of more parallels between the two?

Appendix A

Parts of a whole

If you have enjoyed this course, you might be interested in the following book (Champollion 2017).

Parts of a whole: Distributivity as a Bridge Between Aspect and Measurement

This book uses mathematical models of language to explain why there are certain gaps in language: things that we might expect to be able to say but can't. For instance, why can we say *I ran for five minutes* but not **I ran to the store for five minutes*? Why is *five pounds of books* acceptable, but **five pounds of book* not acceptable? What prevents us from saying **sixty degrees of water* to express the temperature of the water in a swimming pool when *sixty inches of water* can express its depth? And why can we not say **all the ants in my kitchen are numerous*? The constraints on these constructions involve concepts that are generally studied separately: aspect, plural and mass reference, measurement, and distributivity. This book provides a unified perspective on these domains, connects them formally within the framework of algebraic semantics and mereology, and uses this connection to transfer insights across unrelated bodies of literature and formulate a single constraint that explains each of the judgments above.

This book is a substantially reworked and extended revision of my dissertation (Champollion 2010), which it supersedes. I first introduced the framework of strata theory and the concept of stratified reference in that dissertation, and I have used it in my research ever since. The book collects in one place all the work I have carried out on algebraic semantics and mereology from about 2009 to 2016, and presents it in a unified and self-contained way. It is planned and written as a coherent whole and not merely a collection of papers. I have published some parts as self-contained articles over the last few years. The revisions to the theory that resulted from peer review have been propagated through the book.

I published an overview of strata theory as a target article in the open peer-review journal *Theoretical Linguistics* (Champollion 2015b). This article mostly centers on applications to aspect and measurement but also discusses distributivity. The responses to the target article prompted me to develop strata theory further, in particular by refining the definition of stratified reference. This is described in detail in a reply article (Champollion 2015a). I have folded these refinements back into the text of the dissertation as I prepared it for publication as a book.

Parts of chapter 2, which is about background assumptions, appeared in 2016 as a handbook article on linguistic applications of mereology coauthored with Manfred Krifka (Champollion & Krifka 2016).

Chapter 8, on covert distributivity, has been significantly expanded for publication in the open-access journal *Semantics and Pragmatics* (Champollion 2016a).

Chapter 9, on overt distributivity, was written after the dissertation was completed and published in *Semantics and Pragmatics*, back-to-back with the previous chapter (Champollion 2016c).

Chapter 10, on the word *all*, is based on chapter 9 of the dissertation and has undergone substantial revision. Setting aside a brief NELS proceedings paper (Champollion 2016b), most of this chapter has not appeared anywhere else.

Chapter 11, the conclusion of the book, has been rewritten from scratch and substantially expanded to include a detailed chapter-by-chapter summary and many suggestions for future work.

Compared with my dissertation (Champollion 2010), Chapters 1-4 and 6-7 have been only lightly changed; Chapter 5 has been partly rewritten.

For information on how to obtain the book, and for excerpts, see <http://www.nyu.edu/projects/champollion/book>.

Appendix B

Solutions to exercises

Answer to Exercise 1.1: The first claim is that parthood is a special case of overlap: $\forall x \forall y [x \leq y \rightarrow x \circ y]$. Using the definition of overlap in (6), this can be rewritten as $\forall x \forall y [x \leq y \rightarrow \exists z [z \leq x \wedge z \leq y]]$. We choose $z = x$ and rewrite this as $\forall x \forall y [x \leq y \rightarrow [x \leq x \wedge x \leq y]]$. Now $x \leq x$ follows from the axiom of reflexivity (2). The rest is trivial.

The second claim is that a singleton set sums up to its only member: $\forall x [\text{sum}(x, \{x\})]$. Here we can understand the singleton $\{x\}$ as standing for $\lambda z. z = x$, the predicate that applies to x and to nothing else. Using the definition of sum in (7), we rewrite the claim as $\forall x \forall y [y = x \rightarrow y \leq x] \wedge \forall z [z \leq x \rightarrow \exists z' [z' = x \wedge z \circ z']]$. This simplifies to $\forall x [x \leq x] \wedge \forall z [z \leq x \rightarrow z \circ x]$. The first conjunct follows from the axiom of reflexivity (2), while the second conjunct follows from the proof of the first claim.

Answer to Exercise 1.2: If the empty set was not removed from the powerset of any given set with at least two members, we would no longer have a mereology. The empty set is a subset of every set, so it would correspond to something which is a part of everything. If such a thing is included, any two things have a part in common, therefore any two things overlap. This contradicts unique separation, which states that whenever $x < y$, there is exactly one “remainder” z that does not overlap with x such that $x \oplus z = y$ (see line 9 in Table 1.3). Moreover, it contradicts the axiom of unique sum (10). A sum of a set P is defined as a thing of which everything in P is a part and whose parts each overlap with something in P (see (7)). If any two things overlap, the second half of this definition becomes trivially true, so anything of which everything in P is a part is a sum of P . From transitivity, it follows that if x is a sum of P and $x < y$, then y is also a sum of P .

Answer to Exercise 2.1: The claim is that the algebraic closure of a set always contains that set: $\forall P [P \subseteq *P]$. To prove this, we need to show that $\forall P [P \subseteq \{x \mid \exists P' \subseteq P [x = \bigoplus P']\}]$, or equivalently, $\forall P \forall x [x \in P \rightarrow \exists P' \subseteq P [\text{sum}(x, P')]]$. This follows for $P' = \{x\}$, given that a

singleton set sums up to its only member, as shown in the first exercise.

Answer to Exercise 2.2: One way to model the measure/individuating ambiguity of pseudopartitives is to assume the existence of functions like *liters* and *years* for measure nouns:

- (24) a. $\llbracket \text{liter} \rrbracket = \llbracket \text{liters} \rrbracket = \lambda n \lambda d [\text{liters}(d) = n]$
 b. $\llbracket \text{year} \rrbracket = \llbracket \text{years} \rrbracket = \lambda n \lambda t [\text{years}(t) = n]$

Assume that *glass* are taken to be ambiguous between an ordinary and a measure noun interpretation. The measure noun interpretation can be taken to give rise to the measure reading and the latter to the individuating reading:

- (25) three glasses of wine
 a. *Measure reading:* $\lambda x [\text{wine}(x) \wedge \text{glasses}(x) = 3]$
 (a quantity of wine that corresponds to three glassfuls)
 b. *Individuating reading:* $\lambda x [|x| = 3 \wedge * \text{glass}(x) \wedge \text{contains}(x, \text{wine})]$
 (three actual glasses containing wine)

In the individuating reading, *wine* appears as the argument of the relation *contains*. Rothstein treats *wine* as a kind in this case. Another possibility would be to treat it as the set of all wine entities or its characteristic function, corresponding to its occurrence in the measure reading.

Answer to Exercise 3.1: If we consider $e_4 = e_1 \oplus e_2 \oplus e_3$, we have a counterexample to the lexical cumulativity assumption for the following reasons. The themes of e_1, e_2, e_3 are the hole, the rosebush, and the soil, while the theme of e_4 is just the rosebush. The theme of e_4 is not the sum of the themes of e_1, e_2 , and e_3 . This violates cumulativity.

One way to respond to this challenge is to reject the assumption that the mereological parthood relation should model all parthood relations that can be intuitively posited (see Section 1.2.1). In this case, we do not need to assume that e_4 is actually the sum of e_1, e_2 , and e_3 . Even though the existence of e_4 can be traced back to the occurrence of e_1, e_2 , and e_3 , nothing forces us to assume that these three events are actually parts of e_4 , just like we do not consider a plume of smoke to be part of the fire from which it comes, even though its existence can be traced back to the fire. Without the assumption that e_4 contains e_1 through e_3 as parts, Kratzer's objection against cumulativity vanishes. See also Williams (2009) and Piñón (2011) for more discussion.

Answer to Exercise 3.2: Yes, on the assumption that cumulativity holds of *see* and of the theme relation, the verb phrase *see John* has cumulative reference. A seeing-John event is a seeing event whose theme is John. We therefore need to prove that the sum of any two seeing-John event is both

a seeing event and an event whose theme is John. From cumulativity of *see*, we know that the sum of any two seeing events is a seeing event, so the sum of any two seeing-John events is a seeing event. From cumulativity of *theme*, the theme of the sum of any two events whose individual themes are John is the sum of their individual themes, then the theme of the sum of these events is the sum of John and John, which is John, given that the sum operation is idempotent.

Answer to Exercise 4.1: For *see two apples*, the proof does not go through because the theme of *see* is holistic and not incremental, that is, there is no meaning postulate like $\text{Incremental}_{\text{theme}}(\llbracket \text{see} \rrbracket)$. For *eat apples*, the proof does not go through because *apples* is not quantized (the sum of any two things in the denotation of *apples* is again in the denotation of *apples*).

Answer to Exercise 4.2: $C = \{\text{rodgers} \oplus \text{hammerstein}, \text{rodgers} \oplus \text{hart}\}$

Answer to Exercise 4.3: We assume that the verbal predicate is closed under sum:

$$(25) \quad \forall e, e' [\text{write}(e) \wedge \text{write}(e') \rightarrow \text{write}(e \oplus e')]$$

We also assume that the agent and theme relations are closed under sum:

$$(26) \quad \begin{array}{l} \text{a. } \forall e, e', x, x' [\text{agent}(e) = x \wedge \text{agent}(e') = x' \rightarrow \text{agent}(e \oplus e') = x \oplus x'] \\ \text{b. } \forall e, e', x, x' [\text{agent}(e) = x \wedge \text{theme}(e') = x' \rightarrow \text{theme}(e \oplus e') = x \oplus x'] \end{array}$$

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