Linguistic Applications of Mereology

Script to the ESSLLI 2012 course

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Expressions like ‘John and Mary’ or ‘the water in my cup’ intuitively involve reference to collections of individuals or substances. The parthood relation between these collections and their components is not modeled in standard formal semantics of natural language (Montague 1974; Heim and Kratzer 1998), but it plays central stage in what is known as mereological or algebraic semantics (Link 1998; Krifka 1998; Landman 2000). In this mini-lecture series, I present introductions into algebraic semantics and selected applications involving plural, mass reference, measurement, aspect, and distributivity. I discuss issues involving ontology and philosophy of language, and how these issues interact with semantic theory depending on how they are resolved.

This script is subject to change. Comments welcome.

Lucas Champollion, Tübingen, June 19th, 2012
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Lecture 1

Mereology: Concepts and axioms

1.1 Introduction

- Mereology: the study of parthood in philosophy and mathematical logic
- Mereology can be axiomatized in a way that gives rise to algebraic structures (sets with binary operations defined on them)

Figure 1.1: An algebraic structure

- Algebraic semantics: the branch of formal semantics that uses algebraic structures and parthood relations to model various phenomena
1.2 Mereology

1.2.1 Parthood

• Basic motivation (Link 1998): entailment relation between collections and their members

(1) a. John and Mary sleep. \( \Rightarrow \) John sleeps and Mary sleeps.
b. The water in my cup evaporated. \( \Rightarrow \) The water at the bottom of my cup evaporated.

• Basic relation \( \leq \) (parthood) – no consensus on what exactly it expresses

• Table 1.1 gives a few interpretations of the relation \( \leq \) in algebraic semantics

<table>
<thead>
<tr>
<th>Whole</th>
<th>Part</th>
</tr>
</thead>
<tbody>
<tr>
<td>some horses</td>
<td>a subset of them</td>
</tr>
<tr>
<td>a quantity of water</td>
<td>a portion of it</td>
</tr>
<tr>
<td>John, Mary and Bill</td>
<td>John</td>
</tr>
<tr>
<td>some jumping events</td>
<td>a subset of them</td>
</tr>
<tr>
<td>a running event from A to B</td>
<td>its part from A halfway towards B</td>
</tr>
<tr>
<td>a temporal interval</td>
<td>its initial half</td>
</tr>
<tr>
<td>a spatial interval</td>
<td>its northern half</td>
</tr>
</tbody>
</table>

• All these are instances of unstructured parthood (arbitrary slices).

• Compare this with structured parthood (Simons 1987; Fine 1999; Varzi 2010) in Table 1.2 (cognitively salient parts)

• In algebraic semantics one usually models only unstructured parthood.

• This contrasts with lexical semantics, which concerns itself with structured parthood (e.g. Cruse 1986).

• Mereology started as an alternative to set theory; instead of \( \in \) and \( \subseteq \) there is only \( \leq \).

• In algebraic semantics, mereology and set theory coexist.
Table 1.2: Examples of structured parthood from Simons (1987)

<table>
<thead>
<tr>
<th>Whole</th>
<th>Part</th>
</tr>
</thead>
<tbody>
<tr>
<td>a (certain) man</td>
<td>his head</td>
</tr>
<tr>
<td>a (certain) tree</td>
<td>its trunk</td>
</tr>
<tr>
<td>a house</td>
<td>its roof</td>
</tr>
<tr>
<td>a mountain</td>
<td>its summit</td>
</tr>
<tr>
<td>a battle</td>
<td>its opening shot</td>
</tr>
<tr>
<td>an insect’s life</td>
<td>its larval stage</td>
</tr>
<tr>
<td>a novel</td>
<td>its first chapter</td>
</tr>
</tbody>
</table>

- The most common axiom system is classical extensional mereology (CEM).
- The order-theoretic axiomatization of CEM starts with $\leq$ as a partial order:

  (2) **Axiom of reflexivity**
  \[ \forall x [x \leq x] \]
  (Everything is part of itself.)

  (3) **Axiom of transitivity**
  \[ \forall x \forall y \forall z [x \leq y \land y \leq z \rightarrow x \leq z] \]
  (Any part of any part of a thing is itself part of that thing.)

  (4) **Axiom of antisymmetry**
  \[ \forall x \forall y [x \leq y \land y \leq x \rightarrow x = y] \]
  (Two distinct things cannot both be part of each other.)

- The proper-part relation restricts parthood to nonequal pairs:

  (5) **Definition: Proper part**
  \[ x < y \overset{\text{def}}{=} x \leq y \land x \neq y \]
  (A proper part of a thing is a part of it that is distinct from it.)

- To talk about objects which share parts, we define overlap:
(6) **Definition: Overlap**
\[ x \circ y \overset{\text{def}}{=} \exists z [z \leq x \land z \leq y] \]
(Two things overlap if and only if they have a part in common.)

### 1.2.2 Sums

- Pretheoretically, sums are that which you get when you put several parts together.
- The classical definition of sum in (7) is due to Tarski (1929). There are others.

(7) **Definition: Sum**
\[ \text{sum}(x, P) \overset{\text{def}}{=} \forall y[P(y) \rightarrow y \leq x] \land \forall z[z \leq x \rightarrow \exists z'[P(z') \land z \circ z']] \]
(A sum of a set \( P \) is a thing that consists of everything in \( P \) and whose parts each overlap with something in \( P \). "\( \text{sum}(x, P) \)" means "\( x \) is a sum of (the things in) \( P \)."

**Exercise 1.1** Prove the following facts!

(8) **Fact**
\[ \forall x \forall y[x \leq y \rightarrow x \circ y] \]
(Parthood is a special case of overlap.)

(9) **Fact**
\[ \forall x[\text{sum}(x, \{x\})] \]
(A singleton set sums up to its only member.)

The answers to this and all following exercises are in the Appendix. □

- In CEM, two things composed of the same parts are identical:

(10) **Axiom of uniqueness of sums**
\[ \forall P[P \neq \emptyset \rightarrow \exists! z \text{sum}(z, P)] \]
(\( P \) has a unique sum.)

- In CEM, every nonempty set \( P \) has a unique sum \( \bigoplus P \).

(11) **Definition: Generalized sum**
For any nonempty set \( P \), its sum \( \bigoplus P \) is defined as \( \forall z \text{sum}(z, P) \).
(The sum of a set \( P \) is the thing which contains every element of \( P \) and whose parts each overlap with an element of \( P \).)
• As a shorthand for binary sum, we write $\bigoplus \{x, y\}$ as $x \oplus y$.

(12) **Definition: Binary sum**

$x \oplus y$ is defined as $\bigoplus \{x, y\}$.

(13) **Definition: Generalized pointwise sum**

For any nonempty $n$-place relation $R_n$, its sum $\bigoplus R_n$ is defined as the tuple $\langle z_1, \ldots, z_n \rangle$ such that each $z_i$ is equal to $\bigoplus \{x_i | \exists x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n[R(x_1, \ldots, x_n)]\}$.

(The sum of a relation $R$ is the pointwise sum of its positions.)

• Two applications of sum in linguistics are conjoined terms and definite descriptions.

  – For Sharvy (1980), [the water] = $\bigoplus$ water
  – For Link (1983), [John and Mary] = $j \oplus m$

• Another application: natural kinds as sums; e.g. the kind potato is $\bigoplus$ potato.

• But this needs to be refined for uninstantiated kinds such as dodo and phlogiston. One answer: kinds are individual concepts of sums (Chierchia 1998b). See Carlson (1977) and Pearson (2009) on kinds more generally.

### 1.3 Mereology and set theory

• Models of CEM (or “mereologies”) are essentially isomorphic to complete Boolean algebras with the bottom element removed, or equivalently complete semilattices with their bottom element removed (Tarski 1935; Pontow and Schubert 2006).

• CEM parthood is very similar to the subset relation (Table 1.3).

• Example: the powerset of a given set, with the empty set removed, and with the partial order given by the subset relation.

**Exercise 1.2** If the empty set was not removed, would we still have a mereology? Why (not)? □
### Table 1.3: Correspondences between CEM and set theory

<table>
<thead>
<tr>
<th>Property</th>
<th>CEM</th>
<th>Set theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Reflexivity</td>
<td>$x \leq x$</td>
<td>$x \subseteq x$</td>
</tr>
<tr>
<td>2 Transitivity</td>
<td>$x \leq y \land y \leq z \rightarrow x \leq z$</td>
<td>$x \subseteq y \land y \subseteq z \rightarrow x \subseteq z$</td>
</tr>
<tr>
<td>3 Antisymmetry</td>
<td>$x \leq y \land y \leq x \rightarrow x = y$</td>
<td>$x \subseteq y \land y \subseteq x \rightarrow x = y$</td>
</tr>
<tr>
<td>4 Interdefinability</td>
<td>$x \leq y \leftrightarrow x \oplus y = y$</td>
<td>$x \subseteq y \leftrightarrow x \cup y = y$</td>
</tr>
<tr>
<td>5 Unique sum/union</td>
<td>$P \neq \emptyset \rightarrow \exists! z \text{ sum}(z, P)$</td>
<td>$\exists! z \bigcup P$</td>
</tr>
<tr>
<td>6 Associativity</td>
<td>$x \oplus (y \oplus z) = (x \oplus y) \oplus z$</td>
<td>$x \cup (y \cup z) = (x \cup y) \cup z$</td>
</tr>
<tr>
<td>7 Commutativity</td>
<td>$x \oplus y = y \oplus x$</td>
<td>$x \cup y = y \cup x$</td>
</tr>
<tr>
<td>8 Idempotence</td>
<td>$x \oplus x = x$</td>
<td>$x \cup x = x$</td>
</tr>
<tr>
<td>9 Unique separation</td>
<td>$x &lt; y \rightarrow \exists! z [x \oplus z = y \land \neg x \circ z]$</td>
<td>$x \subseteq y \rightarrow \exists! z [z = y - x]$</td>
</tr>
</tbody>
</table>

### 1.4 Selected literature

- Textbooks:
  * Mathematical foundations: Partee, ter Meulen, and Wall (1990)
  * Algebraic semantics: Landman (1991)

- Books on algebraic semantics: Link (1998); Landman (2000)

- Seminal articles on algebraic semantics: Landman (1996); Krifka (1998)

- Mereology surveys: Simons (1987); Casati and Varzi (1999); Varzi (2010)

- See my dissertation Champollion (2010) for a unified theory of distributivity, aspect and measurement based on the ideas presented in this course (available by email from champoll@gmail.com)

### 1.5 A preview of the course

- Lecture 1: done
- **Lecture 2 - Nouns**: semantic theories of singular and plural; cumulative, divisive, quantized reference; group nouns; mass nouns; atomicity
- **Lecture 3 - Measurement**: temporal and spatial trace; measure functions and degrees; measure nouns; pseudopartitives and the measurement puzzle
- **Lecture 4 - Verbs**: event semantics; thematic roles; aspectual composition
- **Lecture 5 - Distributivity**: lexical and phrasal distributivity; meaning postulates; D operator; leakage; nonatomic distributivity; covers
Lecture 2

Nouns

2.1 Algebraic closure and the plural

• Link (1983) has proposed algebraic closure as underlying the meaning of the plural.

(1) a. John is a boy.
   b. Bill is a boy.
   c. ⇒ John and Bill are boys.

• Algebraic closure closes any predicate (or set) \( P \) under sum formation:

(2) Definition: Algebraic closure (Link 1983)
    The algebraic closure \( *P \) of a set \( P \) is defined as \( \{ x \mid \exists P' \subseteq P \left[ x = \bigoplus P' \right] \} \).
    (This is the set that contains any sum of things taken from \( P \).)

• Link translates the argument in (i) as follows:

(3) \( \text{boy}(j) \land \text{boy}(b) \Rightarrow *\text{boy}(j \oplus b) \)

• This argument is valid. Proof: From \( \text{boy}(j) \land \text{boy}(b) \) it follows that \( \{ j, b \} \subseteq \text{boy} \). Hence \( \exists P' \subseteq \text{boy} [ j \oplus b = \bigoplus P' ] \), from which we have \( *\text{boy}(j \oplus b) \) by definition.

Exercise 2.1 Prove the following fact!

(4) Fact
    \( \forall P \left[ P \subseteq *P \right] \)
    (The algebraic closure of a set always contains that set.)
(5) **Definition: Algebraic closure for relations**
The algebraic closure \( \ast R \) of a non-functional relation \( R \) is defined as
\[
\{ \overline{x} \mid \exists R' \subseteq R[\overline{x} = \bigoplus R'] \}
\]
(The algebraic closure of a relation \( R \) is the relation that contains any sum of tuples each contained in \( R \).)

(6) **Definition: Algebraic closure for partial functions**
The algebraic closure \( \ast f \) of a partial function \( f \) is defined as
\[
\lambda x : x \in \ast \text{dom}(f). \bigoplus \{ y \mid \exists z \leq x \land y = f(z) \}
\]
(The algebraic closure of \( f \) is the partial function that maps any sum of things each contained in the domain of \( f \) to the sum of their values.)

- There are different views on the meaning of the plural:
  - **Exclusive view**: the plural form \( N_{pl} \) essentially means the same as *two or more N* (Link 1983; Chierchia 1998a).
  
  (7) \( [N_{pl}] = \ast [N_{sg}] - [N_{sg}] \)

  (8) a. \( [\text{boy}] = \{a, b, c\} \)
   
   b. \( [\text{boys}] = \ast [\text{boy}] - [\text{boy}] = \{a \oplus b, b \oplus c, a \oplus c, a \oplus b \oplus c\} \)

  - **Inclusive view**: the plural form essentially means the same as *one or more N* (Krivka 1986; Sauerland 2003; Sauerland, Anderssen, and Yatsushiro 2005; Chierchia 2010); the singular form blocks the plural form via competition

  (9) \( [N_{pl}] = \ast [N_{sg}] \)

  (10) a. \( [\text{boy}] = \{a, b, c\} \)
   
   b. \( [\text{boys}] = \ast [\text{boy}] = \{a, b, c, a \oplus b, b \oplus c, a \oplus c, a \oplus b \oplus c\} \)

  - **Mixed view**: plural forms are ambiguous between *one or more N* and *two or more N* (Farkas and de Swart 2010).

- Problem for the exclusive view (Schwarzschild 1996, p. 5): downward entailing contexts

  (11) a. No doctors are in the room.
   
   b. Are there doctors in the room?

- Problem for the inclusive view: needs to be complemented by a blocking story
Figure 2.1: Different views on the plural.

(12) *John is doctors.

- Problem for both views: dependent plurals (de Mey 1981)

(13) Five boys flew kites.
≠ Five boys flew one or more kites.
  (because the sentence requires two or more kites in total to be flown)

(14) No boys flew kites.
≠ No boys flew two or more kites.
  (because one kite flown by a boy already falsifies the sentence)

- Possible solution (Zweig 2008, 2009): take the inclusive view and treat the inference that at least two kites are flown as a grammaticalized scalar implicature. Scalar implicatures only surface when they strengthen the meaning of the sentence.

- A sentence $S_1$ is stronger than a sentence $S_2$ iff $S_1$ is true in less scenarios than $S_2$. 

Table:

<table>
<thead>
<tr>
<th></th>
<th>Singular</th>
<th>Plural (exclusive)</th>
<th>Plural (inclusive)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>a $\oplus$ b</td>
<td>a $\oplus$ c</td>
<td>b $\oplus$ c</td>
</tr>
<tr>
<td>b</td>
<td></td>
<td>a $\oplus$ c</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td></td>
<td></td>
<td>b $\oplus$ c</td>
</tr>
</tbody>
</table>

\(\text{Plural (exclusive)}\)
(15) Five boys flew kites.
a. \(S = \text{Five boys flew one or more kites}\) inclusive
b. \(S + \text{scalar} = \text{Five boys flew two or more kites}\) exclusive
c. In a scenario where five boys flew one kite, \(S\) is true and \(S + \text{scalar}\) is false.
d. Therefore, \(S + \text{scalar}\) is stronger than \(S\).
e. The scalar implicature surfaces and the plural is interpreted inclusively.

(16) No boys flew kites.
a. \(S = \text{No boys flew one or more kites}\) inclusive
b. \(S + \text{scalar} = \text{No boys flew two or more kites}\) exclusive
c. In a scenario where one boy flew one kite, \(S\) is false and \(S + \text{scalar}\) is true.
d. Therefore, \(S\) is stronger than \(S + \text{scalar}\).
e. The scalar implicature does not surface and the plural is interpreted exclusively.

- On both the inclusive and exclusive view, plural nouns are cumulative.

(17) **Definition:** Cumulative reference
\[\text{CUM}(P) \equiv \forall x [P(x) \rightarrow \forall y[P(y) \rightarrow P(x \oplus y)]]\]
(A predicate \(P\) is cumulative if and only if whenever it holds of two things, it also holds of their sum.)

- The property of cumulativity is common to plural nouns and mass nouns (see below).

### 2.2 Singular count nouns

- Counting involves mapping to numbers. Let a “singular individual” be something which is mapped to the number 1, something to which we can refer by using a singular noun.
- One can assume that all singular individuals are *atoms*: the cat’s leg is not a part of the cat.

(18) **Definition:** Atom
\[\text{Atom}(x) \equiv \neg \exists y [y < x]\]
(An atom is something which has no proper parts.)

(19) **Definition:** Atomic part
\[x \leq_{\text{Atom}} y \equiv x \leq y \land \text{Atom}(x)\]
(Being an atomic part means being atomic and being a part.)
• Group nouns like *committee, army, league* have given rise to two theories.
  
  – Atomic theory: the entities in the denotation of singular group nouns are mereological atoms like other singular count nouns (Barker 1992; Schwarzschild 1996; Winter 2001)
  
  – Plurality theory: they are plural individuals (e.g. Bennett 1974)

• The question is whether the relation between a committee and its members is linguistically relevant, and if so whether it is mereological parthood.

• One can also assume that singular count nouns apply to “natural units” (Krifka 1989) that may nevertheless have parts, or that there are two kinds of parthood involved (Link 1983).

• If one allows for nonatomic singular individuals, one might still want to state that all singular count nouns have quantized reference (Krifka 1989): the cat’s leg is not itself a cat.

(20) **Definition: Quantized reference**

\[
\text{QUA}(P) \equiv \forall x[P(x) \to \forall y[y < x \to \neg P(y)]]
\]

(A predicate \(P\) is quantized if and only if whenever it holds of something, it does not hold of any its proper parts.)

• But a twig may have a part that is again a twig, a rock may have a part that is again a rock, and so on (Zucchi and White 2001).

• For these cases, one can assume that context specifies an individuation scheme (Chierchia 2010; Rothstein 2010).

### 2.3 Mass nouns and atomicity

• Mass nouns are compatible with the quantifiers *much* and *little* and reject quantifiers such as *each, every, several, a/an, some* and numerals (Bunt 2006; Chierchia 2010)

• Many nouns can be used as count nouns or as mass nouns:

(21)  
  a. Kim put an apple into the salad.
  b. Kim put apple into the salad.

• Are these two different words or one word with two senses? Pelletier and Schubert (2002)

• Four traditional answers to what the denotation of a mass noun is (Bealer 1979; Krifka 1991):
- **General term analysis:** a set of entities – e.g. *gold* denotes the set of all gold entities (like a count noun)
- **Singular term analysis:** a sum – e.g. *gold* denotes the sum of all gold entities (like a proper name)
- **Kind reference analysis:** a kind – e.g. *gold* denotes the kind GOLD
- **Dual analysis:** systematic ambiguity between sum/kind and set reading

- On the general term or dual analysis, we can apply higher-order properties to mass nouns
- Mass nouns have cumulative reference: add water to water and you get water
- In this, they parallel plural nouns (Link 1983)
- Mass nouns were proposed to have divisive reference (Cheng 1973); but this position is no longer popular (minimal-parts problem)

\[(22) \textbf{Definition: Divisive reference} \]
\[\text{DIV}(P) \overset{\text{def}}{=} \forall x[P(x) \rightarrow \forall y[y < x \rightarrow P(y)]] \]
\[(A \text{ predicate } P \text{ is divisive if and only if whenever it holds of something, it also holds of each of its proper parts.)}\]

- **Question** Are the count and mass domains distinct?

- Link (1983): yes, they have distinct properties (the new ring consists of old gold)
  - But also within the mass domain: new snow consists of old water (Bach 1986)
  - We could also relativize the concepts *new* and *old* to concepts: the entity *x* is new *qua* ring, and old *qua* gold

- Chierchia (1998a, 2010): no, all nouns refer within the same domain
  - But then, what *is* the count-mass distinction? (Chierchia’s answer: vagueness)
  - What about pairs like *letters/mail, furniture/Möbel* etc.?

- What comes first: the count-mass distinction or the individual-substance distinction? Quine (1960) claims the former. Acquisition evidence suggests the latter (Spelke 1990).
- We can also talk about atoms in general, by adding one of the following axioms to CEM:
Optional axiom: Atomicity
\[ \forall y \exists x [x \leq_{\text{Atom}} y] \]
(All things have atomic parts.)

Optional axiom: Atomlessness
\[ \forall x \exists y [y < x] \]
(All things have proper parts.)

- Of course, we don’t need to add either axiom. This is one of the advantages of mereology.

- Do count nouns always involve reference to atomic domains?
  - If yes: What about twig type nouns and group nouns?
  - If no: What determines if a concept is realized as a count noun?

- Do mass nouns ever involve reference to atomic domains? What about “fake mass nouns” (collective mass nouns like furniture, mail, offspring) (Barner and Snedeker 2005; Doetjes 1997; Chierchia 2010)?
  - If yes: why can’t you say *three furniture(s)?
  - If no: why do the parts of furniture not qualify as furniture?
Lecture 3

Measurement

3.1 Introduction

- Partial functions (or just functions for brevity) formalize the relationships between the domains of the ontological zoo, as shown in Figure 3.1

Figure 3.1: The world (some details omitted)
3.2 Trace functions and intervals

- Trace functions map events to intervals which represent their temporal and spatial locations
  - $\tau$, the temporal trace or *runtime*
  - $\sigma$, the spatial trace

- Occur in phrases like *three hours* and *three miles*, *from 3pm to 6pm*, *to the store* and in tense semantics

- Trace functions also indicate the precise location in space and time

- Example: John sings from 1pm to 2pm and Mary sings from 2pm to 3pm. Although each event takes the same time, their runtimes are different.

- While two distinct events may happen at the same place and/or time, this is not possible for intervals.

- Axioms for temporal intervals and other temporal structures are found in van Benthem (1983). The integration into mereology is from Krifka (1998).

- Temporal inclusion ($\leq$) is like mereological parthood and subject to nonatomic CEM (most semanticists do not assume the existence of temporal atoms or instants)

- Temporal precedence ($\ll$) is irreflexive, asymmetric and transitive; holds between any two nonoverlapping intervals

- Example: if $a$ is the interval from 2pm to 3pm today, $b$ is the interval from 4pm to 5pm, and $c$ is the interval from 1pm to 5pm, then we have $a \leq c$, $b \leq c$, and $a \ll b$

- Trace functions provide the bridge between interval logic and event logic:

  (1) **Definition: Holding at an interval**
  
  $\text{AT}(V, i) \overset{df}{=} \exists e [V(e) \land \tau(e) = i]$ 
  
  (An event predicate $V$ holds at an interval $i$ if and only if it holds of some event whose temporal trace is $i$.)

- Trace functions are sum homomorphisms (Link 1998; Krifka 1998), like thematic roles.
(2) Trace functions are sum homomorphisms

\[ \sigma(e \oplus e') = \sigma(e) \oplus \sigma(e') \]

\[ \tau(e \oplus e') = \tau(e) \oplus \tau(e') \]

(The location/runtime of the sum of two events is the sum of their locations/runtimes.)


- Prepositional phrases can be represented using trace functions

\[ [\text{to the store}] = \lambda V (vt) \lambda e [V(e) \land \text{end}(\sigma(e)) = \text{the.store}] \]

\[ [\text{from 3pm to 4pm}] = \lambda V (vt) \lambda e [V(e) \land \text{start}(\tau(e)) = 3\text{pm} \land \text{end}(\tau(e)) = 4\text{pm}] \]


### 3.3 Measure functions and degrees

- While trace functions map entities to intervals, measure functions map entities to degrees (but some authors conflate them, e.g. Kratzer (2001))

- Typical measure functions: height, weight, speed, temperature

- Degrees are totally ordered quantities assigned by measure functions
  - Degrees of individuals: John’s weight, the thickness of the ice at the South Pole
  - Degrees of events: the speed at which John is driving his car now

- Uses of degrees in semantics: gradable adjectives (*tall, beautiful*), measure nouns (*liter, hour*), measure phrases (*three liters*), comparatives (*taller, more beautiful, more water, more than three liters*), pseudopartitives (*three liters of water*)

- Degree scales are assumed to be totally ordered, while mereologies are typically only partially ordered.

**Questions:**

- Are degrees ontological entities in their own right (Cresswell 1976) or contextual coordinates (Lewis 1972)? For comparisons of the two approaches, see Klein (1991), Kennedy (2007), and van Rooij (2008).
The contextual-coordinate analysis is mainly concerned with gradable adjectives and does not provide an obvious way to represent the meaning of measure phrases.

- If degrees are entities, what are they? Primitives (Parsons 1970; Cartwright 1975), numbers (Hellan 1981; Krifka 1998) or equivalence classes of individuals (Cresswell 1976; Ojeda 2003)?
  - Not sure if any linguistic facts motivate reductionism here; numbers aren’t sorted: six feet approximately equals 183cm, but 6 is not equal to 183

- Are degrees points or initial intervals (“extents”) on a scale (Krasikova 2009)? Are these scales always dense (Fox and Hackl 2006)?

- Should degree scales be special cases of mereologies (Szabolcsi and Zwarts 1993; Lassiter 2010a,b)?
  - Degrees are totally ordered, but mereological parthood is only a partial order.

### 3.4 Unit functions

- For Lønning (1987), degrees occupy an intermediate layer between individuals and numbers (see also Schwarzchild (2006)).

- Measure nouns like liter, kilogram, year denote functions from degrees to numbers: what I will call *unit functions*.

  \[
  \begin{align*}
  \text{a. } & \text{[liter]} = \lambda n. \lambda d. \text{litters}(d) = n \\
  \text{b. } & \text{[year]} = \lambda n. \lambda t. \text{years}(t) = n
  \end{align*}
  \]

- Example: John weighs 150 pounds (68 kilograms) and measures six feet (183 centimeters). *Weight* and *height* are measure functions, *feet* and *centimeters* are unit functions.

  \[
  \begin{align*}
  \text{a. } & \text{pounds(\text{weight}(\text{john})) = 150} \\
  \text{b. } & \text{kilograms(\text{weight}(\text{john})) = 68} \\
  \text{c. } & \text{feet(\text{height}(\text{john})) = 6} \\
  \text{d. } & \text{centimeters(\text{height}(\text{john})) = 183}
  \end{align*}
  \]

- Advantage of Lønning’s split: underspecification in pseudopartitives

  \[
  \begin{align*}
  \text{three inches of oil}
  \end{align*}
  \]
a. $\lambda x [\text{oil}(x) \land \text{inches(height}(x)) = 3]$ (by height)
b. $\lambda x [\text{oil}(x) \land \text{inches(diameter}(x)) = 3]$ (by diameter)

- Ambiguity of container pseudopartitives (Rothstein 2009, and references therein):

  (8) three glasses of wine
  a. Measure reading: $\lambda x [\text{wine}(x) \land \text{glasses}(x) = 3]$
      (a quantity of wine that corresponds to three glassfuls)
  b. Individuating reading: $\lambda x [\lvert x \rvert = 3 \land \text{*glass}(x) \land \text{contains}(x, \text{wine})]$
      (three actual glasses containing wine)

- Alternative: measure functions directly relate entities to numbers (Quine, Krifka)

### 3.5 The measurement puzzle

- Pseudopartitives and comparative determiners reject certain measure functions like speed and temperature (Krifka 1998; Schwarzschild 2006)

  (9) a. five pounds of rice weight
      b. five liters of water volume
      c. five hours of talks duration
      d. five miles of railroad tracks spatial extent
      e. *five miles per hour of driving *speed
      f. *five degrees Celsius of water *temperature

  (10) five carats of gold mass / *purity

- Several other constructions behave analogously, e.g. comparative determiners and true partitives:

  (11) more rope by length / by weight / *by temperature

  (12) *five miles per hour of my driving *speed

- Pseudopartitives have been claimed to only accept measure functions that are monotonic (Schwarzschild 2006).

- A measure function $\mu$ is monotonic iff for any two entities $a$ and $b$ in the model ($\approx$ physical world), if $a$ is a proper part of $b$, then $\mu(a) < \mu(b)$. 
• Problems: *five feet of snow is acceptable, but height is not monotonic.

• The same measure functions are also rejected by for-adverbials (Champollion 2010).

(13) a. John waited for five hours. \textit{duration}
b. The crack widens for five meters. \textit{spatial extent}
c. *John drove for thirty miles an hour. \textit{*speed}
d. *The soup boiled for 100 degrees Celsius. \textit{*temperature}

• Champollion (2010) proposes that these sentences have the same presuppositions as sentences with for-adverbials, except that time has been replaced by speed/temperature:

(14) SUBINTERVAL_{time}(P) =_{def} \\
∀e[P(e) → ∀i[i < τ(e) → ∃e'[P(e') ∧ e' < e ∧ i = τ(e)]]]

(Whenever P holds of an event e, then at every subinterval of the runtime of e, there is a subevent of which P also holds.)

(15) SUBINTERVAL_{speed}(P) =_{def} \\
∀e[P(e) → ∀i[i < \text{speed}(e) → ∃e'[P(e') ∧ e' < e ∧ i = \text{speed}(e)]]]

(Whenever P holds of an event e, then for every value lower than the speed of e, there is a subevent of e which has that speed and of which P also holds.)

• This assumption predicts presupposition failures for (15):

(16) *eat an apple for an hour

\textbf{Failing presupposition:} eat an apple has the subinterval property with respect to time (every eating-an-apple event consists of eating-an-apple subevents with shorter runtimes)

(17) *drive for thirty miles per hour

\textbf{Failing presupposition:} drive has the subinterval property with respect to speed (every driving event consists of driving subevents with smaller speeds)

(18) *boil for 100 degrees Celsius

\textbf{Failing presupposition:} boil has the subinterval property with respect to temperature (every boiling event consists of boiling subevents with smaller temperatures)

• To transfer this idea to pseudopartitives, we make use of a parallel between distinctions in the nominal and in the verbal domain (e.g. Bach 1986; Krifka 1998):

(19) \textit{atelic : telic :: mass/plural : singular count}
• Both telic predicates and singular count nouns are *quantized* (Krifka 1998).

• Pseudopartitives reject singular count nouns:

  (20)  a. five pounds of **books**
        b. thirty liters of **water**
        c. *five pounds of **book**

• Intuition: *run for three hours ≈ three hours of running*

• We assume that the same presupposition that is found in *for*-adverbials is also found in pseudopartitives, just with other parameters. To apply it, align *for*-adverbials and pseudopartitives:

  “John walked for three hours.”
  “three hours of walking”
  “three liters of water”

  \[
  \begin{array}{ccc}
  \text{three hours} & \ldots \text{is the} & \text{runtime} & \text{of} & \text{walk} \\
  \text{three hours} & \ldots \text{is the} & \text{runtime} & \text{of} & \text{walk} \\
  \text{three liters} & \ldots \text{is the} & \text{volume} & \text{of} & \text{water} \\
  \end{array}
  \]

• Event-denoting pseudopartitives work just like *for*-adverbials:

  (21) run for three hours / three hours of running

  \textbf{Satisfied presupposition:} run has the subinterval property with respect to time

• In substance-denoting pseudopartitives, we assume that the dimension parameter is the appropriate measure function

  (22) thirty liters of water

  \textbf{Satisfied presupposition:} water has the subinterval property with respect to volume

• Singular count nouns are ruled out because they are quantized:

  (23) *five pounds of book*

  \textbf{Failing presupposition:} book has the subinterval property with respect to weight

• A nonmonotonic measure function like *temperature* is ruled out because smaller values are not guaranteed as you go from bigger to smaller amounts of substance.

  (24) *thirty degrees Celsius of water*

  \textbf{Failing presupposition:} water has the subinterval property with respect to temperature
Lecture 4

Verbs

4.1 Introduction

- Early work represents the meaning of a verb with \( n \) syntactic arguments as an \( n \)-ary relation.
- Davidson (1967) argued that verbs denote relations between events and their arguments.
- The neo-Davidsonian position (e.g. Carlson 1984; Parsons 1990; Schein 1993) relates the relationship between events and their arguments by thematic roles.
- There are also intermediate positions, such as Kratzer (2000).

<table>
<thead>
<tr>
<th>Position</th>
<th>Verbal denotation</th>
<th>Example: Brutus stabbed Caesar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional</td>
<td>( \lambda y \lambda x [\text{stab}(x, y)] )</td>
<td>( \text{stab}(b, c) )</td>
</tr>
<tr>
<td>Classical Davidsonian</td>
<td>( \lambda y \lambda x \lambda e [\text{stab}(e, x, y)] )</td>
<td>( \exists e [\text{stab}(e, b, c)] )</td>
</tr>
<tr>
<td>Neo-Davidsonian</td>
<td>( \lambda e [\text{stab}(e)] )</td>
<td>( \exists e [\text{stab}(e) \land \text{ag}(e, b) \land \text{th}(e, c)] )</td>
</tr>
<tr>
<td>Asymmetric</td>
<td>( \lambda y \lambda e [\text{stab}(e, y)] )</td>
<td>( \exists e [\text{ag}(e, b) \land \text{stab}(e, c)] )</td>
</tr>
</tbody>
</table>

- The Neo-Davidsonian position makes it easier to state generalizations across the categories of nouns and verbs, and to place constraints on thematic roles.
- Events are things like Jones’ buttering of the toast, Brutus’ stabbing of Caesar.
- Events form a mereology, so they include plural events (Bach 1986; Krifka 1998).
- Events can be both spatially and temporally extended, unlike intervals.
• Events are usually thought to have temporal parts (subevents which occupy less time). It is controversial whether individuals also do – this is the 3D/4D controversy (Markosian 2009). Most semanticists seem to be on the 3D side (individuals do not have temporal parts).

• Some authors treat events as built from atoms (Landman 2000), others distinguish between count and mass events (Mourelatos 1978). With mereology, we need not decide (Krifka 1998).

• Some authors also include states (e.g. John’s being asleep) as events. Others use event more narrowly as opposed to states.
  
  – Do even stative sentences have an underlying event (Parsons 1987, 1990, ch. 10)? Maybe individual-level predicates don’t (Kratzer 1995)?
  
  – Do telic and atelic predicates form disjoint classes of events (Piñón 1995) or is this a difference of predicates (Krifka 1998)?

4.2 Thematic roles

• Thematic roles represent ways entities take part in events (Parsons 1990; Dowty 1991)

• Two common views:
  
  – Traditional view: thematic roles encapsulate generalizations over shared entailments of argument positions in different predicates (Gruber 1965; Jackendoff 1972)
    * agent (initiates the event, or is responsible for the event)
    * theme (undergoes the event)
    * instrument (used to perform an event)
    * sometimes also location and time
  
  – Alternative view: thematic roles as verb-specific relations: Brutus is not the agent of the stabbing event but the stabber (Marantz 1984). But this misses generalizations. And what about subjects of coordinated sentences like A girl sang and danced)?

• No consensus on the inventory of thematic roles, but see Levin (1993) and Kipper-Schuler (2005) for wide-coverage role lists of English verbs
Questions:

- Do thematic roles have syntactic counterparts, the theta roles (something like silent prepositions)? Generative syntax says yes at least for the agent role: the “little v” head (Chomsky 1995)

- Does each verbal argument correspond to exactly one role (Chomsky 1981) or is the subject of a verb like fall both its agent and its theme (Parsons 1990)?

- Thematic uniqueness / Unique Role Requirement: Does each event have at most one agent, at most one theme etc. (widely accepted in semantics, see Carlson (1984, 1998); Parsons (1990); Landman (2000)) or no (Krifka (1992): one can touch both a man and his shoulder in the same event)?

- Question: Let e is a talking event whose agent is John and e’ is a talking event whose agent is Mary. What is the agent of e ⊕ e’?

- More generally, are thematic roles their own algebraic closures (Krifka 1986, 1998; Landman 2000)?

(1) **Cumulativity assumption for thematic roles**

For any thematic role θ it holds that \( \theta = \ast \theta \). This entails that

\[
\forall e, e', x, y [ \theta(e) = x \land \theta(e') = y \rightarrow \theta(e \oplus e') = x \oplus y ]
\]

- I assume the answer is yes (makes things easier to formalize)
- To symbolize this, instead of writing th, I will write \( \ast \)th.

- As a consequence of (1), thematic roles are homomorphisms with respect to the \( \oplus \) operation:

(2) **Fact: Thematic roles are sum homomorphisms**

For any thematic role θ, it holds that \( \theta(e \oplus e') = \theta(e) \oplus \theta(e') \).

(The \( \theta \) of the sum of two events is the sum of their \( \theta \)'s.)

- Potential challenge to this assumption: the rosebush story (Kratzer 2003). Suppose there are three events \( e_1, e_2, e_3 \) in which Al dug a hole, Bill inserted a rosebush in it, and Carl covered the rosebush with soil. Then there is also an event \( e_4 \) in which Al, Bill, and Carl planted a rosebush. Let \( e_4 \) be this event. If \( e_4 = e_1 \oplus e_2 \oplus e_3 \), we have a counterexample to lexical cumulativity.

**Exercise 4.1** Why is this a counterexample? How could one respond to this challenge? □
4.3 Lexical cumulativity

• Many authors assume *lexical cumulativity*: whenever two events are in the denotation of a verb, so is their sum (Scha 1981; Schein 1986, 1993; Lasersohn 1989; Krifka 1989, 1992; Landman 1996, 2000; Kratzer 2007).

\[
\text{(3) a. John slept.} \\
\text{b. Mary slept.} \\
\text{c. } \Rightarrow \text{John and Mary slept.}
\]

\[
\text{(4) a. John saw Bill.} \\
\text{b. Mary saw Sue.} \\
\text{c. } \Rightarrow \text{John and Mary saw Bill and Sue.}
\]

• Verbs have plural denotations: they obey the same equation as plural count nouns on the inclusive view

\[
\text{(5) } [V] = *[V] \\
\text{(6) } [N_{pl}] = *[N_{sg}]
\]

• It is customary to indicate lexical cumulativity by writing \(\lambda e[\ast \text{see(e)}]\) for the meaning of the verb *see* instead of \(\lambda e[\text{see(e)}]\).

**Exercise 4.2** Translate example (3) into logic using the following assumptions: anything to which the singular count noun *boy* applies is a mereological atom; *and* is translated as \(\oplus\); verbs and thematic roles are each closed under sum. Show that these assumptions predict the entailment in (3). □

This entailment is parallel to the entailment from singular to plural nouns:

\[
\text{(7) a. John is a boy.} \\
\text{b. Bill is a boy.} \\
\text{c. } \Rightarrow \text{John and Bill are boys.}
\]

• Lexical cumulativity does not entail that all verb phrases have cumulative reference. For example, the sum of two events in the denotation of the verb phrase *carry exactly two pianos* is not again in its denotation, because it involves four rather than two pianos.

**Exercise 4.3** Does the verb phrase *see John* have cumulative reference? □
4.4 Aspectual composition

• Predicates can be telic or atelic.

  − Atelic predicates: walk, sleep, talk, eat apples, run, run towards the store
    \((≈ \text{as soon as you start X-ing, you have already X-ed})\)
  − Telic predicates: build a house, finish talking, eat ten apples, run to the store
    \((≈ \text{you need to reach a set terminal point in order to have X-ed})\)

• Traditionally, atelicity is understood as the subinterval property or divisive reference. Telicity is understood as quantized reference. This brings out the parallel between the telic/atelic and count/mass oppositions (e.g. Bach 1986).

\[(E) \text{ a. telic : atelic :: count : mass} \]
\[(E) \text{ b. quantized : (approximate) subinterval :: quantized : (approximate) divisive} \]

• My dissertation (Champollion 2010) suggests a generalized notion that encompasses both the subinterval property and divisive reference.

• We will use the following definition of the subinterval property:

\[(F) \text{ SUBINTERVAL}(P) =_{df} \forall e[P(e) \rightarrow \forall i[i < \tau(e) \rightarrow \exists e'[P(e') \land e' < e \land i = \tau(e')]]]] \]
(Whenever \(P\) holds of an event \(e\), then at every subinterval of the runtime of \(e\), there is a subevent of which \(P\) also holds.)

\[(G) \text{ *eat ten apples for three hours} \]
\[(G) \text{ Failing presupposition: SUBINTERVAL(\[\text{eat ten apples}\]), i.e. every part of the runtime of an eating-ten-apples event } e \text{ is the runtime of another eating-ten-apples event that is a part of } e. \]

• The “minimal-parts problem” (Taylor 1977; Dowty 1979): The subinterval property distributes \(P\) literally over all subintervals. This is too strong.

\[(H) \text{ John and Mary waltzed for an hour} \]
\[(H) \text{ } \Leftrightarrow \# \text{John and Mary waltzed within every single moment of the hour} \]
\[(H) \text{ } \Rightarrow \text{John and Mary waltzed within every short subinterval of the hour} \]

• The length interval that counts as very small for the purpose of the for-adverbial varies relative to the length of the bigger interval:
(12) The Chinese people have created abundant folk arts ... passed on from generation to generation for thousands of years.¹

- *Aspectual composition* is the problem of how complex constituents acquire the telic/atelic distinction from their parts. Verkuyl (1972); Krifka (1998)

- With “incremental theme” verbs like *eat*, the correspondence is clear:

  (13)  
  a. eat apples / applesauce for an hour  
  b. *eat an apple / two apples / the apple for an hour

  (14)  
  a. count : mass :: telic : atelic  
  b. apple : apples :: eat an apple : eat apples

  (15)  
  a. drink wine for an hour  
  b. *drink a glass of wine for an hour

- With “holistic theme” verbs like *push* and *see*, the pattern is different:

  (16)  
  a. push carts for an hour  
  b. push a cart for an hour

  (17)  
  a. look at apples / applesauce for an hour  
  b. look at an apple / two apples / the apple for an hour

- *Verkuyl’s Generalization* (Verkuyl 1972): When the direct object of an incremental-theme verb is a count expression, we have a telic predicate, otherwise an atelic one.

- Krifka (1992): in incremental-theme verbs (also called “measuring-out” verbs, among other things), the parts of the event can be related to the parts of the theme (see Figure 4.1).

- Following Krifka, we can formalize the difference between holistic-theme and incremental-theme verbs by meaning postulates.

  (18) **Definition: Incrementality**
  \[
  \text{Incremental}_\theta(P) \iff \forall e \forall e' \forall x [\theta(e) = x \land e' < e \rightarrow \theta(e') < x]
  \]

  (19) **Definition: Holism**
  \[
  \text{Holistic}_\theta(P) \iff \forall e \forall e' \forall x [\theta(e) = x \land e' < e \rightarrow \theta(e') = x]
  \]

Figure 4.1: Incremental theme of *drink wine*, from Krifka (1992)

(20) **Meaning postulates**

a. Incremental\_theme([eat])

b. Incremental\_theme([drink])

c. Holistic\_theme([see])

- Then we apply these meaning postulates to prove or disprove that the various VPs above have divisive reference or the subinterval property.

- **Claim:** [eat two apples] does not have the subinterval property.

- **Proof:** Suppose it has, then let \( e \) be an event in its denotation whose runtime is an hour. From the definition of the subinterval property, (9), at each subinterval of this hour there must be a proper subevent of \( e \) whose theme is again two apples. Let \( e' \) be any of these proper subevents. Let the theme of \( e \) be \( x \) and the theme of \( e' \) be \( y \). Then \( x \) and \( y \) are each a sum of two apples. From the “incremental theme” meaning postulate in (20a) we know that \( y \) is a proper part of \( x' \). Since *two apples* is quantized, \( x \) and \( y \) can not both be two apples. Contradiction.

**Exercise 4.4** Why does the proof not go through for *see two apples*? Why does it not go through for *eat apples*? □
Lecture 5

Distributivity

5.1 Introduction

- What is distributivity? In this lecture: a property of predicates
  - Distributive: e.g. walk, smile, take a breath (applies to a plurality just in case it applies to each of its members)
  - Collective: e.g. be numerous, gather, suffice to defeat the army (may apply to a plurality even if it does not apply to each of its members)

- Literature: Roberts (1987); Winter (2001), Section 6.2; Schwarzschild (1996), Chapter 6; Link (1997), Section 7.4.

5.2 Lexical and phrasal distributivity

(1) Lexical distributivity/collectivity involves lexical predicates
  a. The children smiled. distributive
  b. The children were numerous. collective

(2) Phrasal distributivity/collectivity involves complex predicates
  a. The girls are wearing a dress. distributive
  b. The girls are sharing a pizza. collective
  c. The girls are building a raft. collective/distributive

- The difference between lexical and phrasal distributivity corresponds to the difference between what can and what cannot be described using meaning postulates
Meaning postulate: \textit{smile} is distributive

\[SR_{ag,Atom}(\langle \text{smile} \rangle)\]
\[\iff \forall e [\text{smile}(e) \rightarrow e \in * \lambda e' (\text{smile}(e') \land \text{Atom}(ag(e')))]\]

(\text{Every smiling event consists of one or more smiling events whose agents are atomic.})

- Meaning postulates can only apply to words. We cannot formulate a meaning postulate that says that \textit{wear a dress} is distributive.

- Problems:

  - Meaning postulates are taken to be available only for lexical items
  - For mixed predicates like \textit{build a raft}, we would need optional meaning postulates

- The classical solution is due to Link (1983): A covert distributive operator D adjusts the meaning of a verb phrase like \textit{wear a dress} into \textit{be a sum of people who each wear a dress}.

- D is in the lexicon, so it can apply to entire VPs (Dowty 1987; Roberts 1987; Lasersohn 1995).

- Link’s D operator introduces a universal quantifier:

\[\left[ D^{Link} \right] = \lambda P(x) \forall y [y \leq_{Atom} x \rightarrow P(y)]\]

(Takes a predicate \(P\) over individuals and returns a predicate that applies to any individual whose atomic parts each satisfy \(P\).)

(3) \[\left[ D^{Link} \right] = \lambda P(x) \forall y [y \leq_{Atom} x \rightarrow P(y)]\]

(4) \[\left[ D^{Link} \right] = \lambda P(x) \forall y [y \leq_{Atom} x \rightarrow P(y)]\]

\(\forall y [y \leq_{Atom} \bigoplus_{\text{girl}} \rightarrow \exists z [\text{dress}(z) \land \text{wear}(y, z)]\)

(5) a. The girls built a raft.
\(\approx\) The girls built a raft together.

b. The girls \(D^{Link}\)(built a raft).
\(\approx\) The girls each built a raft.

- This allows us to model the distributive meaning of (2a):

(6) \[\forall y \big[ y \leq_{Atom} \bigoplus_{\text{girl}} \rightarrow \exists z [\text{dress}(z) \land \text{wear}(y, z)]\]

(7) Based on earlier work by Eddy Ruys, Winter (2001) observes that the existential and the distributivity imports of numeral indefinites can have two distinct scopes.

(8) If three workers in our staff have a baby soon we will have to face some hard organizational problems.
a. If any three workers have a baby, there will be problems. \( 3 > D > 1 \)
b. There are three workers such that if each of them has a baby, there will be problems. \( 3 > if > D > 1 \)

- Unlike the indefinite, the distributive operator cannot take scope outside of the if-island:

\( 8 \) a. "There are three workers such that for each \( x \) of them, if \( x \) has a baby, there will be problems. \( 3 > D > if > 1 \)

### 5.2.1 Reformulating the D operator

- Link’s formulation of the D operator needs to be adjusted for several reasons:

  - If we assume with Landman (1996) that groups are atoms too ("impure" atoms) and that the girls can introduce a group, then we need to specify that D distributes over "pure" atoms (singular individuals) only.
  
  - If VPs are of type \( \langle vt \rangle \) instead of \( \langle et \rangle \), we need to repair the type mismatch.
  
  - We also need to be able to coindex D with different thematic roles (Lasersohn 1995).

(9) a. The first-year students D(took an exam).
   Target: agent
b. John D(gave a pumpkin pie) to two girls.
   Target: recipient
c. John D(summarized) the articles.
   Target: theme

- The D operator can be understood as shifting arbitrary predicates to a distributive interpretation with granularity Atom (i.e. singular individual):

\( 10 \) **Definition: Atomic event-based D operator**

\[
[D_\theta] \equiv \lambda P_{\langle vt \rangle} \lambda e \left[ e \in \ast \lambda e' \left( P(e') \land \text{Atom}(\theta(e')) \right) \right]
\]

(Takes an event predicate \( P \) and returns a predicate that holds of any event \( e \) which consists entirely of events that are in \( P \) and whose thematic roles \( \theta \) are atoms.)

- Example:

\( 11 \) The girls are wearing a dress.

\[
\exists e \left[ \ast \text{ag}(e) = \bigoplus \text{girl} \land \ast \text{wear}(e) \land \text{dress}(\text{th}(e)) \right]
\]

(There is a potentially plural wearing event whose agents sum up to the girls, and whose theme is a dress.)
(12) The girls D(are wearing a dress.)
\[ \exists e [ \ast ag(e) = \bigoplus \text{girl} \land \\ e \in \ast \lambda e' (\text{wear}(e') \land \text{dress}(\text{th}(e')) \land \text{Atom}(ag(e'))) ] \]
(The there is an event whose agents sum up to the girls, and this event consists of wearing events for each of which the agent is a atom and the theme is a dress.)

- The star operator \( \ast \lambda e' \) is introduced through the D operator and takes scope over the predicate \textit{dress} introduced by the theme.

**Exercise 5.1** Which background assumptions ensure that (12) entails that each girl wears a dress?

5.2.2 The leakage problem

- There are various other proposals on how to reformulate the D operator.
- Lasersohn (1998) proposes the following entry (among others):

(13) Distributivity operator over events (Lasersohn)

\[
[D^{\text{Lasersohn}}] = \lambda P(e,vt). \lambda x \lambda e \forall y [y \leq_{\text{Atom}} x \rightarrow \exists e'[e' \leq e \land P(y)(e')]]
\]

- This applies to a predicate of type \( \langle e, vt \rangle \), e.g. \([\text{smile}] = \lambda x \lambda e [\text{smile}(e) \land ag(e) = x] \).
- Inserting a D operator into \textit{The girls smiled} before existential closure applies:

(14) a. Lasersohn’s representation:

\[
\lambda e \forall y [y \leq_{\text{Atom}} \bigoplus \text{girl} \land e \in \ast \lambda e' [\text{smile}(e') \land \text{Atom}(\ast ag(e'))]]
\]

b. My representation:

\[
\lambda e [\ast ag(e) = \bigoplus \text{girl} \land e \in \ast \lambda e' [\text{smile}(e') \land \text{Atom}(\ast ag(e'))]]
\]

- (14a) applies to all events that contain a smiling subevent for each girl, even if they also contain extraneous material. It suffers from what Bayer (1997) calls leakage. Whenever it (14a) applies to an event \( e \), it also applies to any event of which \( e \) is a part.

- (14b) applies to all events that contain a smiling subevent for each girl and nothing else.

- Leakage causes problems in connection with event predicates such as \textit{surprisingly}, \textit{unharmonomiously} or \textit{in slow procession}.

- These predicates do not have divisive reference: they can hold of an event even if they do not hold of its parts (Schein 1993).
Unharmoniously, every organ student sustained a note on the Wurlitzer.

- This says that the ensemble event was unharmonious and not any one student’s note.
- Let Lasersohn stand for Lasersohn’s (14a) and let Mine stand for my (14b).
- Imagine an event G-SMILE that satisfies both Lasersohn and Mine, that is, the girls smiled in it.
- Let B-CRY be an event in which the boys cry.
- Now G-SMILE ⊕ B-CRY does not satisfy Mine, but it does satisfy Lasersohn.
- Suppose that G-SMILE is not surprising by itself, but that G-SMILE ⊕ B-CRY is surprising. Then we have these judgments:

  (16)  a. The girls smiled.  \( \text{true} \)
b. The girls smiled and the boys cried.  \( \text{true} \)
c. Surprisingly, the girls smiled.  \( \text{false} \)
d. Surprisingly, the girls smiled and the boys cried.  \( \text{true} \)

- If one of the D operators is applied to smile, then (17) is translated as (17a) or (17b).

  (17)  Surprisingly, the girls smiled.
         a. \( \exists e [\text{surprising}(e) \land \text{Lasersohn}(e)] \)
b. \( \exists e [\text{surprising}(e) \land \text{Mine}(e)] \)

- The problem is that G-SMILE ⊕ B-CRY satisfies both Lasersohn (by leakage) and the predicate surprising (by assumption). So Lasersohn’s D operator wrongly predicts that (17) is judged true.

- The above implementation avoids this kind of leakage.

5.3 Atomic and nonatomic distributivity

- So far we have implemented the view called atomic distributivity: the D operator distributes over atoms, that is, over singular individuals (Lasersohn 1998, 1995; Link 1997; Winter 2001)
• Nonatomic view: phrasal distributivity may also quantify over nonatomic parts (Gillon 1987, 1990; van der Does and Verkuyl 1995; Verkuyl and van der Does 1996; Schwarzschild 1996; Brisson 1998, 2003; Malamud 2006a,b)

• Traditional argument is based on sentences like this, adapted from Gillon (1987):

• Rodgers, Hammerstein and Hart never wrote any musical together, nor did any of them ever write one all by himself. But Rodgers and Hammerstein wrote the musical Oklahoma together, and Rodgers and Hart wrote the musical On your toes together.

• On the basis of these facts, (18a) and (18b) are judged as true in the actual world, although it is neither true on the collective interpretation nor on an “atomic distributive” interpretation.

(18)  
  a. Rodgers, Hammerstein, and Hart wrote Oklahoma and On Your Toes.  
  b. Rodgers, Hammerstein, and Hart wrote musicals.

• The traditional nonatomic argument: in order to generate the reading on which (18b) is true, the predicates wrote musicals and wrote Oklahoma and On Your Toes must be interpreted as applying to nonatomic parts of the sum individual to which the subject refers.

• Generally implemented with covers (Gillon 1987): partitions of a set (19) or sum (20) whose cells/parts can overlap

(19)  
  **Definition:** Cover (set-theoretic)  
  \[\text{Cov}(C, P) \overset{\text{def}}{=} \bigcup C = P \land \emptyset \notin C\]  
  (C is a cover of a set P if and only if C is a set of subsets of P whose union is P.)

(20)  
  **Definition:** Cover (mereological)  
  \[\text{Cov}(C, x) \overset{\text{def}}{=} \bigoplus C = x\]  
  (C is a cover of a mereological object x is a set of parts of x whose sum is x.)

• Cover-based approaches modify the D operator to quantify over nonatomic parts of a cover of the plural individual.

• The first cover-based approaches assumed that the cover can be existentially quantified by the operator that introduces it:

(21)  
  **Nonatomic distributivity operator, existentially bound cover**  
  \[\lbrack D_\exists \rbrack = \lambda P_{(et)} \lambda x \exists C \{\text{Cov}(C, x) \land \forall y[C(y) \land y \leq x \rightarrow P(y)]\}\]

• On this view, sentences (18a) and (18b) are translated as follows:
Exercise 5.2 For which value of C are these formulas true in the actual world? □

- Existentially bound covers are now generally considered untenable because they overgenerate nonatomic distributive readings

- Lasersohn (1989)'s problem: Suppose John, Mary, and Bill are the teaching assistants and each of them was paid exactly $7,000 last year. (24a) and (24b) are true, but (24c) is false.

\[(24)\]

a. True: The TAs were paid exactly $7,000 last year. \textit{distributive}

b. True: The TAs were paid exactly $21,000 last year. \textit{collective}

c. False: The TAs were paid exactly $14,000 last year. \textit{nonatomic distributive}

- Giving up the existential cover-based operator $D_3$ in (21) explains why (24c) is false, because without this operator, there is no way to generate a true reading for this sentence.

- But now why are (18a) and (18b) true?

- As it turns out, the lexical cumulativity assumption is already enough (Lasersohn 1989):

\[(25)\]

$\forall w, x, y, z \left[ \text{write}(w, x) \land \text{write}(y, z) \rightarrow \text{write}(w \oplus y, x \oplus z) \right]$

Exercise 5.3 What does this assumption translate to in a Neo-Davidsonian framework? □

- Further support: (28) is false in the actual world (Link 1997):

\[(28)\]

Rodgers, Hammerstein and Hart wrote a musical.

a. True if the three of them wrote a musical together – not the case. \textit{collective}

b. True if each of them wrote a musical by himself – not the case. \textit{atomic distributive}

c. False even though Rodgers and Hammerstein wrote a musical together, and Rodgers and Hart wrote another musical together. \textit{nonatomic distributive}

- The absence of the nonatomic distributive reading of (28) is predicted if we give up the existential cover-based operator $D_3$. 

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• Lexical cumulativity derives the (available) nonatomic distributive reading of (18a) and (18b) but not the (unavailable) nonatomic distributive reading of (28):

\[(29) \quad [\text{Rodgers, Hammerstein and Hart wrote Oklahoma and On Your Toes.}] = \exists e [\text{write}(e) \land \text{ag}(e) = \text{rodgers} \oplus \text{hammerstein} \oplus \text{hart} \land \text{th}(e) = \text{okl} \oplus \text{oyt}]\]

(Allows for several writing events and for teamwork, as long as these two musicals are written.)

\[(30) \quad [\text{Rodgers, Hammerstein and Hart wrote musicals.}] = \exists e [\text{write}(e) \land \text{ag}(e) = \text{rodgers} \oplus \text{hammerstein} \oplus \text{hart} \land \text{musical}(\text{th}(e))]\]

(Allows for several writing events and for teamwork, and there can be several musicals in total.)

\[(31) \quad [\text{Rodgers, Hammerstein and Hart wrote a musical.}] = \exists e [\text{write}(e) \land \text{ag}(e) = \text{rodgers} \oplus \text{hammerstein} \oplus \text{hart} \land \text{musical}(\text{th}(e))]\]

(Allows for several writing events and for teamwork, but there has to be only one musical in total.)

• Lasersohn, as well as Winter (2001) and others, conclude from this and similar examples that the atomic approach to phrasal distributivity is superior to covers.

• However, Gillon (1990) and Schwarzschild (1996) identify a residue of cases in which a cover-based operator does seem necessary.

\[(32) \quad \text{Scenario} \quad \text{Two pairs of shoes are on display, each pair with a $50 price tag.} \]

\[\begin{align*}
\text{a.} & \quad \text{The shoes cost$100.} & \checkmark \text{collective (together)} \\
\text{b.} & \quad \text{The shoes cost$25.} & ? \text{atomic distributive (per shoe)} \\
\text{c.} & \quad \text{The shoes cost$50. (Lasersohn 1995)} & \checkmark \text{nonatomic distributive (per pair)}
\end{align*}\]

• Evidence that individual shoes, and not shoe pairs, are atoms in this context:

\[(33) \quad \text{How many shoes are on display?} \quad \text{– Four / #Two.}\]

• Schwarzschild (1996) proposes that the cover of D is anaphoric on context:

\[(34) \quad \text{Schwarzschild’s nonatomic distributivity operator, free cover} \]

\[\text{[D}_C\text{]} = \lambda P(e) \lambda x \forall y[C(y) \land y \leq x \rightarrow P(y)]\]

• See Malamud (2006a,b) for a decision-theoretic elaboration of this proposal.
• Nonatomic distributivity is always available for verbs, but for verb phrases it only occurs when context supplies a pragmatically salient cover. Atomic distributivity is available in both cases.

Figure 5.1: V level versus verb phrase level distributivity in atomic domains

<table>
<thead>
<tr>
<th></th>
<th>(a) Empirical generalization</th>
<th>(b) Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>lexical (V level)</td>
<td>phrasal (VP level)</td>
</tr>
<tr>
<td>atomic</td>
<td>available</td>
<td>available</td>
</tr>
<tr>
<td>nonatomic</td>
<td>available</td>
<td>only w. context</td>
</tr>
</tbody>
</table>

• To model nonatomic distributivity, I change the event-based atomic D operator repeated below as (35), by replacing the predicate Atom by a free predicate C. It plays the same role as the C predicate in Schwarzschild’s operator:

\[
[D_\theta] \overset{\text{def}}{=} \lambda P_{(\forall \theta)} \lambda e \left[ e \in * \lambda e' \left( P(e') \land \text{Atom}(\theta(e')) \right) \right] = (10)
\]

(Takes an event predicate \(P\) and returns a predicate that holds of any event \(e\) which consists entirely of events that are in \(P\) and whose \(\theta\)s are atoms.)

\[
[D_{\theta,C}] \overset{\text{def}}{=} \lambda P_{(\forall \theta)} \lambda e \left[ e \in * \lambda e' \left( P(e') \land \text{C}(\theta(e')) \right) \right]
\]

(Takes an event predicate \(P\) and returns a predicate that holds of any event \(e\) which consists entirely of events that are in \(P\) and whose \(\theta\)s satisfy the predicate \(C\).)

• The generalized D operator has two parameters: dimension (thematic role) and granularity (C).

• Unlike Schwarzschild, I do not rely on pragmatics to ensure that \(C\) actually covers the \(\theta\)s of the event to which the output of the D operator is applied.

• Following Schwarzschild (1996), I assume that the \(C\) parameter of the D operator in (36) can only be set in one of two ways: either it is set to the predicate Atom or to an anaphorically salient level of granularity.
Conclusion

Now that you have finished this course, you may be looking for topics to work on. Here are some:

- Distance-distributive items like *each* and *jeweils* occur remotely from the position in which they are compositionally interpreted (Zimmermann 2002; Champollion 2012). German and Japanese split quantifier constructions, in which a quantifier appears in adverbial position apart from the noun phrase over which it quantifies, are similar (Nakanishi 2004). But there is not yet a unified theory of both phenomena.

- *For*-adverbials are not the only examples of aspectually sensitive constructions. As argued in Hitzeman (1991, 1997), *until* is also sensitive to the atelic-telic distinction (*He slept until midnight* vs. *He woke up until midnight*). The same appears to be true for English *since*, though the situation is more complicated here because *since* requires the Perfect, which under some analyses turns any predicate into an atelic predicate. The German equivalent *seit* does not require the Perfect and can only modify atelic predicates (*Dieter ist seit 1975 in Düsseldorf* vs. *Dieter ist seit 1975 achtmal in Düsseldorf*) (von Stechow 2002).

- Examples like *John found some fleas on his dog for an hour* pose a problem for current theories of *for*-adverbials, because it is difficult to explain why *some fleas* cannot have narrow scope. Indefinites like *some fleas* seem to behave for the purposes of theories of aspect as if they were quantized. The problem is also known as the quantization puzzle, and I know of no satisfying solution (though see Verkuyl (1972, 1993), Filip (2000, 2008), Zucchi and White (2001), Rothstein (2004) for attempts).

- The distribution of partitive and accusative case on direct objects in Finnish is sensitive to semantic properties of both the object and its verb (Krifka 1992; Kiparsky 1998). The direct object of a Finnish VP bears accusative case if and only if the VP is both quantized and nongradable; otherwise it bears partitive case. An account of the distribution of Finnish partitive could proceed along the lines of aspectual composition.

This concludes my lecture series. Do not hesitate to email me (champoll@gmail.com) with questions or comments, or to get a copy of my dissertation (Champollion 2010). Good luck!

Lucas Champollion, Tübingen, April 16th, 2012
Appendix

Answer to Exercise 1.1: The first claim is that parthood is a special case of overlap: $\forall x \forall y [x \leq y \to x \circ y]$. Using the definition of overlap in (6), this can be rewritten as $\forall x \forall y [x \leq y \to \exists z [z \leq x \land z \leq y]]$. We choose $z = x$ and rewrite this as $\forall x \forall y [x \leq y \to [x \leq x \land x \leq y]]$. Now $x \leq x$ follows from the axiom of reflexivity (2). The rest is trivial.

The second claim is that a singleton set sums up to its only member: $\forall x [\text{sum}(x, \{x\})]$. Here we can understand the singleton $\{x\}$ as standing for $\lambda z. z = x$, the predicate that applies to $x$ and to nothing else. Using the definition of sum in (7), we rewrite the claim as $\forall x \forall y [y = x \to y \leq x] \land \forall z [z \leq x \to \exists z' [z' = x \land z \circ z']]$. This simplifies to $\forall x [x \leq x] \land \forall z [z \leq x \to z \circ x]$. The first conjunct follows from the axiom of reflexivity (2), while the second conjunct follows from the proof of the first claim.

Answer to Exercise 1.2: If the empty set was not removed from the power set of any given set with at least two members, we would no longer have a mereology. The empty set is a subset of every set, so it would correspond to something which is a part of everything. If such a thing is included, any two things have a part in common, therefore any two things overlap. This contradicts unique separation, which states that whenever $x < y$, there is exactly one “remainder” $z$ that does not overlap with $x$ such that $x \oplus z = y$ (see line 9 in Table 1.3). Moreover, it contradicts the axiom of unique sum (10). A sum of a set $P$ is defined as a thing of which everything in $P$ is a part and whose parts each overlap with something in $P$ (see (7)). If any two things overlap, the second half of this definition becomes trivially true, so anything of which everything in $P$ is a part is a sum of $P$. From transitivity, it follow that if $x$ is a sum of $P$ and $x < y$, then $y$ is also a sum of $P$.

Answer to Exercise 2.1: The claim is that the algebraic closure of a set always contains that set: $\forall P [P \subseteq {}^*P]$. To prove this, we need to show that $\forall P [P \subseteq \{x \mid \exists P' \subseteq P [x = \bigoplus P']\}]$, or equivalently, $\forall P \forall x [x \in P \to \exists P' \subseteq P [\text{sum}(x, P')]]$. This follows for $P' = \{x\}$, given that a
singleton set sums up to its only member, as shown in the first exercise.

**Answer to Exercise 4.1:** If we consider \( e_4 = e_1 \oplus e_2 \oplus e_3 \), we have a counterexample to the lexical cumulativity assumption for the following reasons. The themes of \( e_1, e_2, e_3 \) are the hole, the rosebush, and the soil, while the theme of \( e_4 \) is just the rosebush. The theme of \( e_4 \) is not the sum of the themes of \( e_1, e_2, \) and \( e_3 \). This violates cumulativity.

One way to respond to this challenge is to reject the assumption that the mereological parthood relation should model all parthood relations that can be intuitively posited (see Section 1.2.1). In this case, we do not need to assume that \( e_4 \) is actually the sum of \( e_1, e_2, \) and \( e_3 \). Even though the existence of \( e_4 \) can be traced back to the occurrence of \( e_1, e_2, \) and \( e_3 \), nothing forces us to assume that these three events are actually parts of \( e_4 \), just like we do not consider a plume of smoke to be part of the fire from which it comes, even though its existence can be traced back to the fire. Without the assumption that \( e_4 \) contains \( e_1 \) through \( e_3 \) as parts, Kratzer’s objection against cumulativity vanishes. See also Williams (2009) and Piñón (2011) for more discussion.

**Answer to Exercise 4.2:** We only sketch the proof here. We translate *John slept* as \( \ast \text{sleep}(j) \) and similarly for *Mary slept*. The task is to show that this entails *John and Mary slept*, which we translate as \( \ast \text{sleep}(j \oplus m) \). Using the assumption that John is an atom, from \( \ast \text{sleep}(j) \) it can be shown that *sleep*(\( j \)) and analogously in the case of Mary. Hence the set \( \{ j, m \} \) is a subset of the set *sleep*. Given the definition of \( \ast \), we need to show that there is a subset \( S \) of the set *sleep* such that the sum of \( S \) is \( j \oplus m \). Now we already know that \( \{ j, m \} \) is a subset of the set *sleep*, so it remains to show that the sum of this set is \( j \oplus m \). This follows from the definition of binary sum in (12).

**Answer to Exercise 4.3:** Yes, on the assumption that cumulativity holds of *see* and of the theme relation, the verb phrase *see John* has cumulative reference. A seeing-John event is a seeing event whose theme is John. We therefore need to prove that the sum of any two seeing-John events is both a seeing event and an event whose theme is John. From cumulativity of *see*, we know that the sum of any two seeing events is a seeing event, so the sum of any two seeing-John events is a seeing event. From cumulativity of *theme*, the theme of the sum of any two events whose individual themes are John is the sum of their individual themes, then the theme of the sum of these events is the sum of John and John, which is John, given that the sum operation is idempotent.

**Answer to Exercise 4.4:** For *see two apples*, the proof does not go through because the theme of *see* is holistic and not incremental, that is, there is no meaning postulate like Incremental_theme(\( \{ \text{see} \} \)). For *eat apples*, the proof does not go through because *apples* is not quantized (the sum of any two things in the denotation of *apples* is again in the denotation of *apples*).
Answer to Exercise 5.1: The star operator $\lambda e'$ is introduced through the D operator and takes scope over the predicate $dress$ introduced by the theme. (12) does not directly require the theme of $e$ to be a dress, though it requires $e$ to consist of parts whose themes are dresses. This allows for the possibility that each girl wears a potentially different dress. The representation explicitly states that the dress-wearing events $e'$ have pure atoms as agents, but not that these pure atoms are girls. However, this fact is entailed by cumulativity of thematic roles together with the assumption that the entities in the denotation of singular count nouns are atoms. By cumulativity of thematic roles, any entity $x$ which is the agent of one of the dress-wearing events $e'$ is a part of the agent of $e$. This agent is the sum of all girls. By definition of sum, $x$ overlaps with a part of this agent. Being atomic, $x$ can only overlap with $y$ if it is a part of $y$. This means that $x$ is an atomic part of the girls. Given the background assumption that singular individuals like girls are mereological atoms, it follows that $x$ is a girl. In this way, the distributive interpretation of (12) is correctly captured.

Answer to Exercise 5.2: $C = \{\text{rodgers} \oplus \text{hammerstein}, \text{rodgers} \oplus \text{hart}\}$

Answer to Exercise 5.3: We assume that the verbal predicate is closed under sum:

$$\forall e, e' [\text{write}(e) \land \text{write}(e') \rightarrow \text{write}(e \oplus e')]$$

We also assume that the agent and theme relations are closed under sum:

$$\forall e, e', x, x'[\text{agent}(e) = x \land \text{agent}(e') = x' \rightarrow \text{agent}(e \oplus e') = x \oplus x']$$
$$\forall e, e', x, x'[\text{theme}(e) = x \land \text{theme}(e') = x' \rightarrow \text{theme}(e \oplus e') = x \oplus x']$$
Bibliography


