Preface

Expressions like ‘John and Mary’ or ‘the water in my cup’ intuitively involve reference to collections of individuals or substances. The parthood relation between these collections and their components is not modeled in standard formal semantics of natural language (Montague 1974; Heim and Kratzer 1998), but it plays central stage in what is known as mereological or algebraic semantics (Link 1998; Krifka 1998; Landman 2000). In the first half of this course, I present introductions into algebraic semantics and selected applications involving plural, mass reference, measurement, aspect, and distributivity. I discuss issues involving ontology and philosophy of language, and how these issues interact with semantic theory depending on how they are resolved. The second half of this course develops strata theory, originally published in my dissertation (Champollion 2010) and essentially unchanged from that source. The last lecture is based on Champollion (2012).

This script is subject to change. Comments welcome (champollion@nyu.edu).

Lucas Champollion, New York, May 8th, 2013
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### 8 For, each, all

#### 8.1 All vs. every/each

- **8.1.1 Numerous-type predicates**
- **8.1.2 Gather-type predicates**

#### 8.2 Similarities between for and all

- **8.2.1 For and all block cumulative readings**
- **8.2.2 For and all license dependent plurals**

#### 8.3 Explaining the similarities between for and all

- **8.3.1 For and all block cumulative readings**
- **8.3.2 For and all license dependent plurals**

#### 8.4 Explaining the behavior of numerous and gather

- **8.4.1 Collective predication**
  - **8.4.1.1 Thematic collectivity**
  - **8.4.1.2 Nonthematic collectivity**
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- **8.4.3 Gather distinguishes between each and all**

### 9 The scopal behavior of for-adverbials

#### 9.1 Nonatomic distributivity in for-adverbials

- **9.1.1 Modeling interveners**

#### 9.2 Raising the bar: Deo and Piñango (2011)

- **9.2.1 Availability and cost of iterative interpretations**
- **9.2.2 Availability of partitive interpretations**
- **9.2.3 Problems of D&P’s account**
- **9.2.4 What’s the status of lexical cumulativity?**

### 10 Distance distributivity

#### 10.1 Introduction

- **10.1.1 Questions.**

#### 10.2 Illustrating Zimmermann’s Generalization

- **10.2.1 Jeweils-type DD Items**
- **10.2.2 Each-type DD Items**
- **10.2.3 A Note on Reduplication**

#### 10.3 Capturing the Semantic Variation

#### 10.4 Explaining Zimmermann’s Generalization

#### 10.5 Formalization

#### 10.6 Summary
Lecture 1

Mereology: Concepts and axioms

1.1 Introduction

- **Mereology**: the study of parthood in philosophy and mathematical logic

- Mereology can be axiomatized in a way that gives rise to **algebraic structures** (sets with binary operations defined on them)

  ![Figure 1.1: An algebraic structure](image)

  - **Algebraic semantics**: the branch of formal semantics that uses algebraic structures and parthood relations to model various phenomena
1.2 Mereology

1.2.1 Parthood

• Basic motivation (Link 1998): entailment relation between collections and their members

(1) a. John and Mary sleep. ⇒
    John sleeps and Mary sleeps.

   b. The water in my cup evaporated. ⇒
    The water at the bottom of my cup evaporated.

• Basic relation \( \leq \) (parthood) – no consensus on what exactly it expresses

• Table 1.1 gives a few interpretations of the relation \( \leq \) in algebraic semantics

<table>
<thead>
<tr>
<th>Whole</th>
<th>Part</th>
</tr>
</thead>
<tbody>
<tr>
<td>some horses</td>
<td>a subset of them</td>
</tr>
<tr>
<td>a quantity of water</td>
<td>a portion of it</td>
</tr>
<tr>
<td>John, Mary and Bill</td>
<td>John</td>
</tr>
<tr>
<td>some jumping events</td>
<td>a subset of them</td>
</tr>
<tr>
<td>a running event from A to B</td>
<td>its part from A halfway towards B</td>
</tr>
<tr>
<td>a temporal interval</td>
<td>its initial half</td>
</tr>
<tr>
<td>a spatial interval</td>
<td>its northern half</td>
</tr>
</tbody>
</table>

• All these are instances of unstructured parthood (arbitrary slices).

• Compare this with structured parthood (Simons 1987; Fine 1999; Varzi 2010) in Table 1.2 (cognitively salient parts)

• In algebraic semantics one usually models only unstructured parthood.

• This contrasts with lexical semantics, which concerns itself with structured parthood (e.g. Cruse 1986).

• Mereology started as an alternative to set theory; instead of \( \in \) and \( \subseteq \) there is only \( \leq \).

• In algebraic semantics, mereology and set theory coexist.
Table 1.2: Examples of structured parthood from Simons (1987)

<table>
<thead>
<tr>
<th>Whole</th>
<th>Part</th>
</tr>
</thead>
<tbody>
<tr>
<td>a (certain) man</td>
<td>his head</td>
</tr>
<tr>
<td>a (certain) tree</td>
<td>its trunk</td>
</tr>
<tr>
<td>a house</td>
<td>its roof</td>
</tr>
<tr>
<td>a mountain</td>
<td>its summit</td>
</tr>
<tr>
<td>a battle</td>
<td>its opening shot</td>
</tr>
<tr>
<td>an insect’s life</td>
<td>its larval stage</td>
</tr>
<tr>
<td>a novel</td>
<td>its first chapter</td>
</tr>
</tbody>
</table>

- The most common axiom system is **classical extensional mereology** (CEM).
- The order-theoretic axiomatization of CEM starts with \( \leq \) as a partial order:

  (2) **Axiom of reflexivity**
  \[ \forall x [x \leq x] \]
  (Everything is part of itself.)

  (3) **Axiom of transitivity**
  \[ \forall x \forall y \forall z [x \leq y \land y \leq z \rightarrow x \leq z] \]
  (Any part of any part of a thing is itself part of that thing.)

  (4) **Axiom of antisymmetry**
  \[ \forall x \forall y [x \leq y \land y \leq x \rightarrow x = y] \]
  (Two distinct things cannot both be part of each other.)

- The **proper-part** relation restricts parthood to nonequal pairs:

  (5) **Definition: Proper part**
  \[ x < y \overset{\text{def}}{=} x \leq y \land x \neq y \]
  (A proper part of a thing is a part of it that is distinct from it.)

- To talk about objects which share parts, we define overlap:
Definition: Overlap
\[ x \circ y \overset{\text{def}}{=} \exists z [ z \leq x \land z \leq y ] \]
(Two things overlap if and only if they have a part in common.)

1.2.2 Sums

- Pretheoretically, sums are that which you get when you put several parts together.
- The classical definition of sum in (7) is due to Tarski (1929). There are others.

Definition: Sum
\[ \text{sum}(x, P) \overset{\text{def}}{=} \forall y [ P(y) \rightarrow y \leq x ] \land \forall z [ z \leq x \rightarrow \exists z' [ P(z') \land z \circ z' ] ] \]
(A sum of a set \( P \) is a thing that consists of everything in \( P \) and whose parts each overlap with something in \( P \). "sum(\( x, P \))" means "\( x \) is a sum of (the things in) \( P \).")

Exercise 1.1 Prove the following facts!

Fact
\[ \forall x \forall y [ x \leq y \rightarrow x \circ y ] \]
(Parthood is a special case of overlap.)

Fact
\[ \forall x [ \text{sum}(x, \{ x \}) ] \]
(A singleton set sums up to its only member.)

The answers to this and all following exercises are in the Appendix. □

- In CEM, two things composed of the same parts are identical:

Axiom of uniqueness of sums
\[ \forall P [ P \neq \emptyset \rightarrow \exists ! z \text{ sum}(z, P) ] \]
(Every nonempty set has a unique sum.)

- In CEM, every nonempty set \( P \) has a unique sum \( \bigoplus P \).

Definition: Generalized sum
For any nonempty set \( P \), its sum \( \bigoplus P \) is defined as \( \iota z \text{ sum}(z, P) \).
(The sum of a set \( P \) is the thing which contains every element of \( P \) and whose parts each overlap with an element of \( P \).)
• As a shorthand for binary sum, we write $\bigoplus \{x, y\}$ as $x \oplus y$.

(12) **Definition: Binary sum**

$x \oplus y$ is defined as $\bigoplus \{x, y\}$.

(13) **Definition: Generalized pointwise sum**

For any nonempty $n$-place relation $R_n$, its sum $\bigoplus R_n$ is defined as the tuple $\langle z_1, \ldots, z_n \rangle$ such that each $z_i$ is equal to

$\bigoplus \{x_i | \exists x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n [R(x_1, \ldots, x_n)]\}$.

(The sum of a relation $R$ is the pointwise sum of its positions.)

• Two applications of sum in linguistics are conjoined terms and definite descriptions.
  
  – For Sharvy (1980), $\llbracket$the water$\rrbracket = \bigoplus$ water
  
  – For Link (1983), $\llbracket$John and Mary$\rrbracket = j \oplus m$

• Another application: natural kinds as sums; e.g. the kind *potato* is $\bigoplus$ potato.

• But this needs to be refined for uninstantiated kinds such as *dodo* and *phlogiston*. One answer: kinds are individual concepts of sums (Chierchia 1998b). See Carlson (1977) and Pearson (2009) on kinds more generally.

### 1.3 Mereology and set theory

• Models of CEM (or “mereologies”) are essentially isomorphic to complete Boolean algebras with the bottom element removed, or equivalently complete semilattices with their bottom element removed (Tarski 1935; Pontow and R. Schubert 2006).

• CEM parthood is very similar to the subset relation (Table 1.3).

• Example: the powerset of a given set, with the empty set removed, and with the partial order given by the subset relation.

**Exercise 1.2** If the empty set was not removed, would we still have a mereology? Why (not)? □
Table 1.3: Correspondences between CEM and set theory

<table>
<thead>
<tr>
<th>Property</th>
<th>CEM</th>
<th>Set theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Reflexivity</td>
<td>$x \leq x$</td>
<td>$x \subseteq x$</td>
</tr>
<tr>
<td>2 Transitivity</td>
<td>$x \leq y \land y \leq z \rightarrow x \leq z$</td>
<td>$x \subseteq y \land y \subseteq z \rightarrow x \subseteq z$</td>
</tr>
<tr>
<td>3 Antisymmetry</td>
<td>$x \leq y \land y \leq x \rightarrow x = y$</td>
<td>$x \subseteq y \land y \subseteq x \rightarrow x = y$</td>
</tr>
<tr>
<td>4 Interdefinability</td>
<td>$x \leq y \leftrightarrow x \oplus y = y$</td>
<td>$x \subseteq y \leftrightarrow x \cup y = y$</td>
</tr>
<tr>
<td>5 Unique sum/union</td>
<td>$P \neq \emptyset \rightarrow \exists!z \text{ sum}(z, P)$</td>
<td>$\exists!z \ z = \bigcup P$</td>
</tr>
<tr>
<td>6 Associativity</td>
<td>$x \oplus (y \oplus z) = (x \oplus y) \oplus z$</td>
<td>$x \cup (y \cup z) = (x \cup y) \cup z$</td>
</tr>
<tr>
<td>7 Commutativity</td>
<td>$x \oplus y = y \oplus x$</td>
<td>$x \cup y = y \cup x$</td>
</tr>
<tr>
<td>8 Idempotence</td>
<td>$x \oplus x = x$</td>
<td>$x \cup x = x$</td>
</tr>
<tr>
<td>9 Unique separation</td>
<td>$x &lt; y \rightarrow \exists!z [x \oplus z = y \land \neg x \circ z]$</td>
<td>$x \subseteq y \rightarrow \exists!z [z = y - x]$</td>
</tr>
</tbody>
</table>

1.4 Selected literature

- Textbooks:
  * Mathematical foundations: Barbara Hall Partee, ter Meulen, and Wall (1990)
  * Algebraic semantics: Landman (1991)

- Books on algebraic semantics: Link (1998); Landman (2000)
- Seminal articles on algebraic semantics: Landman (1996); Krifka (1998)
- Mereology surveys: Simons (1987); Casati and Varzi (1999); Varzi (2010)
Lecture 2

Nouns

2.1 Algebraic closure and the plural

- Link (1983) has proposed algebraic closure as underlying the meaning of the plural.

(1) a. John is a boy.
   b. Bill is a boy.
   c. ⇒ John and Bill are boys.

- Algebraic closure closes any predicate (or set) $P$ under sum formation:

(2) **Definition: Algebraic closure (Link 1983)**

The algebraic closure $\ast P$ of a set $P$ is defined as \{ $x$ | $\exists P' \subseteq P [x = \bigoplus P']$ \}. (This is the set that contains any sum of things taken from $P$.)

- Link translates the argument in (1) as follows:

(3) $\text{boy}(j) \land \text{boy}(b) \Rightarrow \ast \text{boy}(j \oplus b)$

- This argument is valid. Proof: From $\text{boy}(j) \land \text{boy}(b)$ it follows that $\{j, b\} \subseteq \text{boy}$. Hence $\exists P' \subseteq \text{boy}[j \oplus b = \bigoplus P']$, from which we have $\ast \text{boy}(j \oplus b)$ by definition.

**Exercise 2.1** Prove the following fact!

(4) **Fact**

$\forall P [P \subseteq \ast P]$  
(The algebraic closure of a set always contains that set.)
Definition: Algebraic closure for relations
The algebraic closure $^\ast R$ of a non-functional relation $R$ is defined as
$$\{ \vec{x} \mid \exists R' \subseteq R [\vec{x} = \bigoplus R'] \}$$
(The algebraic closure of a relation $R$ is the relation that contains any sum of tuples each contained in $R$.)

Definition: Algebraic closure for partial functions
The algebraic closure $^\ast f$ of a partial function $f$ is defined as
$$\lambda x : x \in ^\ast \text{dom}(f). \bigoplus \{ y \mid \exists z [z \leq x \land y = f(z)] \}$$
(The algebraic closure of $f$ is the partial function that maps any sum of things each contained in the domain of $f$ to the sum of their values.)

- There are different views on the meaning of the plural:
  - Exclusive view: the plural form $N_{pl}$ essentially means the same as two or more $N$ (Link 1983; Chierchia 1998a).
    
    (7) $[N_{pl}] = [N_{sg}] - [N_{sg}]$

    (8) a. $[\text{boy}] = \{ a, b, c \}$
    b. $[\text{boys}] = [\text{boy}] - [\text{boy}] = \{ a \oplus b, b \oplus c, a \oplus c, a \oplus b \oplus c \}$

  - Inclusive view: the plural form essentially means the same as one or more $N$ (Krifka 1986; Sauerland 2003; Sauerland, Anderssen, and Yatsushiro 2005; Chierchia 2010); the singular form blocks the plural form via competition
    
    (9) $[N_{pl}] = [N_{sg}]$

    (10) a. $[\text{boy}] = \{ a, b, c \}$
    b. $[\text{boys}] = [\text{boy}] = \{ a, b, c, a \oplus b, b \oplus c, a \oplus c, a \oplus b \oplus c \}$

  - Mixed view: plural forms are ambiguous between one or more $N$ and two or more $N$ (Farkas and de Swart 2010).

- Problem for the exclusive view (Schwarzschild 1996, p. 5): downward entailing contexts

(11) a. No doctors are in the room. (false if there is exactly one doctor in the room)
    b. Are there doctors in the room? (the answer is yes if there is exactly one doctor in the room)
Figure 2.1: Different views on the plural.

- Problem for the inclusive view: needs to be complemented by a blocking story

(12) *John is doctors.

- Problem for both views: dependent plurals (de Mey 1981)

(13) Five boys flew kites.
    \neq Five boys flew one or more kites.
    (because the sentence requires two or more kites in total to be flown)

(14) No boys flew kites.
    \neq No boys flew two or more kites.
    (because one kite flown by a boy already falsifies the sentence)

- Possible solution (Zweig 2008, 2009): take the inclusive view and treat the inference that at least two kites are flown as a grammaticalized scalar implicature. Scalar implicatures only surface when they strengthen the meaning of the sentence.
• A sentence S1 is stronger than a sentence S2 iff S1 is true in less scenarios than S2.

(15) Five boys flew kites.
   a. \( S = \text{Five boys flew one or more kites} \) \hspace{1cm} \text{inclusive}
   b. \( S + \text{scalar} = \text{Five boys flew two or more kites} \) \hspace{1cm} \text{exclusive}
   c. In a scenario where five boys flew one kite, S is true and \( S + \text{scalar} \) is false.
   d. Therefore, \( S + \text{scalar} \) is stronger than S.
   e. The scalar implicature surfaces and the plural is interpreted inclusively.

(16) No boys flew kites.
   a. \( S = \text{No boys flew one or more kites} \) \hspace{1cm} \text{inclusive}
   b. \( S + \text{scalar} = \text{No boys flew two or more kites} \) \hspace{1cm} \text{exclusive}
   c. In a scenario where one boy flew one kite, S is false and \( S + \text{scalar} \) is true.
   d. Therefore, S is stronger than \( S + \text{scalar} \).
   e. The scalar implicature does not surface and the plural is interpreted exclusively.

Exercise 2.2 What does this solution predict? Do the judgments in (11) follow? □

• On both the inclusive and exclusive view, plural nouns are \textit{cumulative}.

(17) \textbf{Definition: Cumulative reference}
\[
\text{CUM}(P) \equiv \forall x [P(x) \rightarrow \forall y [P(y) \rightarrow P(x \oplus y)]]
\]
(A predicate \( P \) is cumulative if and only if whenever it holds of two things, it also holds of their sum.)

• The property of cumulativity is common to plural nouns and mass nouns (see below).

2.2 \textbf{Singular count nouns}

• Counting involves mapping to numbers. Let a “singular individual” be something which is mapped to the number 1, something to which we can refer by using a singular noun.

• One can assume that all singular individuals are \textit{atoms}: the cat’s leg is not a part of the cat.

(18) \textbf{Definition: Atom}
\[
\text{Atom}(x) \equiv \neg \exists y [y < x]
\]
(An atom is something which has no proper parts.)
Definition: Atomic part
\[ x \leq_{\text{Atom}} y \overset{\text{def}}{=} x \leq y \land \text{Atom}(x) \]
(Being an atomic part means being atomic and being a part.)

• Group nouns like committee, army, league have given rise to two theories.
  
  – Atomic theory: the entities in the denotation of singular group nouns are mereological atoms like other singular count nouns (Barker 1992; Schwarzschild 1996; Winter 2001)
  
  – Plurality theory: they are plural individuals (e.g. Bennett 1974)

• The question is whether the relation between a committee and its members is linguistically relevant, and if so whether it is mereological parthood.

• One can also assume that singular count nouns apply to “natural units” (Krifka 1989) that may nevertheless have parts, or that there are two kinds of parthood involved (Link 1983).

• If one allows for nonatomic singular individuals, one might still want to state that all singular count nouns have quantized reference (Krifka 1989): the cat’s leg is not itself a cat.

Definition: Quantized reference
\[ \text{QUA}(P) \overset{\text{def}}{=} \forall x[P(x) \rightarrow \forall y[y < x \rightarrow \neg P(y)]] \]
(A predicate \( P \) is quantized if and only if whenever it holds of something, it does not hold of any its proper parts.)

• But a twig may have a part that is again a twig, a rock may have a part that is again a rock, and so on (Zucchi and White 2001).

• For these cases, one can assume that context specifies an individuation scheme (Chierchia 2010; Rothstein 2010).

2.3 Mass nouns and atomicity

• Anything which can be referred to by a proper name, or denoted by using a common noun

• Objects form a mereology, so they include plural objects (sums)
  
  – Individuals: firemen, apples, chairs, opinions, committees
  
  – Substances: the water in my cup or the air which we breathe
• Possible exceptions:
  – Nominalizations (which arguably involve reference to events)
  – Measure nouns (which arguably involve reference to intervals or degrees).
• Mass nouns are compatible with the quantifiers *much* and *little* and reject quantifiers such as *each, every, several, a/an, some* and numerals (Bunt 2006; Chierchia 2010)
• Many nouns can be used as count nouns or as mass nouns:

  (21) a. Kim put an apple into the salad.
      b. Kim put apple into the salad.

• Are these two different words or one word with two senses? Pelletier and L. K. Schubert (2002)
• Four traditional answers to what the denotation of a mass noun is (Bealer 1979; Krifka 1991):
  – *General term analysis*: a set of entities – e.g. *gold* denotes the set of all gold entities (like a count noun)
  – *Singular term analysis*: a sum – e.g. *gold* denotes the sum of all gold entities (like a proper name)
  – *Kind reference analysis*: a kind – e.g. *gold* denotes the kind \textsc{Gold}
  – *Dual analysis*: systematic ambiguity between sum/kind and set reading
• On the general term or dual analysis, we can apply higher-order properties to mass nouns
• Mass nouns have cumulative reference: add water to water and you get water
• In this, they parallel plural nouns (Link 1983)
• Mass nouns were proposed to have divisive reference (Cheng 1973); but this position is no longer popular (minimal-parts problem)

  (22) **Definition: Divisive reference**
  \[ \text{DIV}(P) \equiv \forall x[P(x) \rightarrow \forall y[y < x \rightarrow P(y)]] \]
  (A predicate \( P \) is divisive if and only if whenever it holds of something, it also holds of each of its proper parts.)
• **Question** Are the count and mass domains distinct?
• Link (1983): yes, they have distinct properties (the new ring consists of old gold)
  – But also within the mass domain: new snow consists of old water (Bach 1986)
  – We could also relativize the concepts new and old to concepts: the entity \( x \) is new \( qua \) ring, and old \( qua \) gold

• Chierchia (1998a, 2010): no, all nouns refer within the same domain
  – But then, what is the count-mass distinction? (Chierchia’s answer: vagueness)
  – What about pairs like letters/mail, furniture/Möbel etc.?

• What comes first: the count-mass distinction or the individual-substance distinction? Quine (1960) claims the former. Acquisition evidence suggests the latter (Spelke 1990).

• We can also talk about atoms in general, by adding one of the following axioms to CEM:

  \[ \forall y \exists x [x \leq_{\text{Atom}} y] \]
  (All things have atomic parts.)

  \[ \forall x \exists y [y < x] \]
  (All things have proper parts.)

• Of course, we don’t need to add either axiom. This is one of the advantages of mereology.

• Do count nouns always involve reference to atomic domains?
  – If yes: What about twig type nouns and group nouns?
  – If no: What determines if a concept is realized as a count noun?

• Do mass nouns ever involve reference to atomic domains? What about “fake mass nouns” (collective mass nouns like furniture, mail, offspring) (Barner and Snedeker 2005; Doetjes 1997; Chierchia 2010)?
  – If yes: why can’t you say *three furniture(s)?
  – If no: why do the parts of furniture not qualify as furniture?
2.4 Noun phrases

• Noun phrases can participate in cumulative, collective, and distributive readings.

• Distributive readings will be the topic of later lectures.

2.4.1 Cumulative readings

• Cumulative readings involve a “cross-product” interpretation:

(25) a. 600 Dutch firms use 5000 American computers. (Scha 1981)
    b. 600 Dutch firms each use at least one American computer and 5000 American computers are each used by at least one Dutch firm.

• Cumulative readings also occur with definite plurals:

(26) The men in the room are married to the girls across the hall. (Kroch 1974)

• Cumulative readings can be represented compactly given certain background assumptions we will learn about in later lectures (lexical cumulativity):

(27) \[ \exists e [ *\text{use}(e) \land *\text{dutch.firm}(\text{ag}(e)) \land |\text{ag}(e)| = 600 \land \]
    \[ *\text{american.computer}(\text{th}(e)) \land |\text{th}(e)| = 5000 ] \]

• This type of representation is easy to derive compositionally because the arguments are kept apart (Krifka 1986, 1999; Landman 2000).

• To derive the “cross-product” inference, one needs an additional meaning postulate that says that \textit{use} is distributive on both its arguments

(28) \textbf{Definition: Stratified reference} (we will see this again later)
    \[ \text{SR}_{f,e(K)}(P) \overset{\text{def}}{=} \forall x [P(x) \rightarrow x \in ^{*}\lambda y \left( \left( P(y) \land e(K)(f(y)) \right) \right) ] \]

(29) \textbf{Meaning postulates: use is distributive with respect to agents and themes}

a. \[ \text{SR}_{ag,\text{Atom}}(*\text{use}) \leftrightarrow \forall e [ *\text{use}(e) \rightarrow e \in ^{*}\lambda e' \left( *\text{use}(e') \land \text{Atom}(\text{ag}(e')) \right) ] \]
   (Every event in the denotation of use consists of one or more events in the denotation of use whose agents are atoms.)
b. $\text{SR}_{\text{th,Atom}}(\forall e \rightarrow e \in \text{use}(e') \land \text{Atom}(\text{th}(e'))$]

(Every event in the denotation of use consists of one or more events in the denotation of use whose themes are atoms.)

### 2.4.2 Collective readings

- In collective readings, some group bears collective responsibility for an event. There is no “cross product” style interpretation.

- Sometimes only the collective reading is available:

  (30) a. The cowboys sent an emissary to the Indians.
     
     (i) *Does not mean:* Each of the cowboys sent an emissary to one of the Indians, and each of the Indians was sent an emissary by one of the cowboys.

     (ii) *Means:* The cowboys as a group sent an emissary to the Indians as a group.

- Some authors do not consider cumulative and collective readings distinct from each other (Roberts 1987; Link 1998)

- But sometimes both readings are available (Landman 1996):

  (31) Three boys invited four girls.

     a. *Cumulative reading:* Three boys each invited a girl, and four girls each were invited by a boy.

     b. *Collective reading:* A group of three boys invited a group of four girls.

- Landman (1989, 1996) assumes that collective readings involve separate model-theoretic entities called groups, which are assumed to be related to their “underlying sums” via a group formation operator.

- The group formation operator $\uparrow$ introduces a distinction between the sum $a \oplus b$ whose proper parts are the individuals $a$ and $b$, and the impure atom $\uparrow (a \oplus b)$, which has no proper parts.

- Groups, understood as the output of a group formation operator, may or may not be involved in the denotations of group nouns – that is a separate question. Barker (1992) argues that they are not.
Here we only assume that they are involved in the denotations of collective readings, more specifically in the arguments that occur in these readings.

(32)  a. The cowboys sent an emissary to the Indians.
      b. \( \exists e[^\star \text{send}(e) \land ^* \text{ag}(e) = \uparrow (\bigoplus \text{cowboy}) \land \text{emissary}(* \text{th}(e)) \land ^* \text{recipient}(e) = \uparrow (\bigoplus \text{Indian})] \)

I call atoms which are not generated through the group formation operator pure atoms. I assume that only pure atoms occur in the denotations of singular count nouns.

(33)  \textbf{Definition: Impure atom}

\[
\text{ImpureAtom}(x) \overset{df}{=} \exists y[y \neq x \land x = \uparrow (y)]
\]
(An impure atom is an atom that is derived from a distinct entity through the group formation operation \( \uparrow \).)

(34)  \textbf{Definition: Pure atom}

\[
\text{PureAtom}(x) \overset{df}{=} \text{Atom}(x) \land \neg \text{ImpureAtom}(x)
\]
(A pure atom is an atom which is not impure.)

By contrast, example (26) only has a cumulative reading, not a collective reading:

(35)  a. The men in the room are married to the girls across the hall. \text{ (Kroch 1974)}
      b. \( \exists e[^* \text{married}(e) \land ^* \text{ag}(e) = \bigoplus \text{man.in.the.room} \land ^* \text{th}(e) = \bigoplus \text{girl.across.the.hall}] \)

Collective readings can also occur with only one noun phrase in the sentence:

(36)  a. Three boys (as a group) carried a piano upstairs.
      b. \( \exists e[^* \text{carry.upstairs}(e) \land \exists x[^* \text{boy}(x) \land |x| = 3 \land ^* \text{ag}(e) = \uparrow (x) \land \text{piano}(* \text{th}(e))] \)

In Landman’s account, noun phrases are ambiguous between sum and group interpretations.

(37)  a. \([a \text{ boy}] = [\text{boy}] = \lambda x[\text{boy}(x)]\)
      b. \([\text{boys}] = \lambda x[^* \text{boy}(x)]\)
      c. \([\text{three boys}_{\text{sum}}] = \lambda x[^* \text{boy}(x)] \land |x| = 3\)
      d. \([\text{three boys}_{\text{group}}] = \lambda y \exists x[y = \uparrow (x) \land ^* \text{boy}(x) \land |x| = 3]\)
      e. \([\text{the boys}_{\text{sum}}] = \bigoplus \text{boy}\)
      f. \([\text{the boys}_{\text{group}}] = \uparrow (\bigoplus \text{boy})\)

- Sum interpretations lead to cumulative readings
• Group interpretations lead to collective readings

• The sum/group distinction can model the collective/cumulative ambiguity.

(38) Three boys invited four girls.

(39) Cumulative reading:
$\exists e. \exists \text{boy}^{*}(\text{ag}(e)) \land |\text{ag}(e)| = 3 \land \exists \text{girl}^{*}(\text{th}(e)) \land |\text{th}(e)| = 4$
(+ meaning postulate on both thematic roles of invite)

(40) Collective reading:
$\exists e \exists x \exists y [\exists \text{invite}^{*}(e) \land \exists \text{boy}^{*}(x) \land \theta(x) = \exists \text{ag}^{*}(e) \land |x| = 3 \land \exists \text{girl}^{*}(y) \land \theta(y) = \exists \text{th}^{*}(e) \land |y| = 4]$

2.5 The compositional process

• I assume that verbs and verbal projections denote sets of events (Parsons 1990; Krifka 1992); this will be the topic of the next lectures.

• Applying a noun phrase amounts to intersecting two sets of events (Carlson 1984)

• At the end of the derivation, existential closure applies.

• Quantificational noun phrases like every boy are interpreted via quantifier raising.

• Implementing Carlson’s idea requires type shifters. No standard practice here.

• I assume that the following type shifters can apply freely:

(41) Predicative type shifters:
   a. VP, then NP: $\lambda (\theta_{(ve)} \lambda V_{(vt)} \lambda P_{(et)} \lambda e \text{[}V(e) \land \theta(e) = x\text{]}$  
   b. NP, then VP: $\lambda (\theta_{(ve)} \lambda P_{(et)} \lambda V_{(vt)} \lambda e \text{[}V(e) \land \theta(e) = x\text{]}$

(42) Referential type shifters:
   a. VP, then NP: $\lambda (\theta_{(ve)} \lambda V_{(vt)} \lambda x_{(e)} \lambda e \text{[}V(e) \land \theta(e) = x\text{]}$
   b. NP, then VP: $\lambda (\theta_{(ve)} \lambda x_{(e)} \lambda V_{(vt)} \lambda e \text{[}V(e) \land \theta(e) = x\text{]}$

• The compositional process is illustrated in Figure 2.2 for a cumulative reading.
Figure 2.2: The compositional process

John and Mary saw thirty zebras.

(i) Predicative type shifter, NP first: \( \lambda \theta \lambda P(\theta) \lambda V(e) \lambda e[V(e) \land P(\theta)] \)
Result: \( \lambda V(e) \lambda e[V(e) \land P(\theta)] \)

(ii) Function application
Result: \( \lambda e[\ast \text{zebra}(\ast \theta)] \land |\ast \theta| = 30 \)

(iii) Referential type shifter, VP first: \( \lambda \theta \lambda V(e) \lambda x(e) \lambda e[V(e) \land \theta(e) = x] \)
Result: \( \lambda x \lambda e[\ast \text{ag}(e) = x \land \ast \text{zebra}(\ast \theta)] \land |\ast \theta| = 30 \)

(iv) Function application
Result: \( \lambda e[\ast \text{see}(e) \land \ast \text{ag}(e) = j \oplus m \land \ast \text{zebra}(\ast \theta)] \land |\ast \theta| = 30 \)

(v) Existential closure
Result: \( \exists e[\ast \text{see}(e) \land \ast \text{ag}(e) = j \oplus m \land \ast \text{zebra}(\ast \theta)] \land |\ast \theta| = 30 \)
Lecture 3

Verbs

3.1 Introduction

- Early work represents the meaning of a verb with $n$ syntactic arguments as an $n$-ary relation
- Davidson (1967) argued that verbs denote relations between events and their arguments
- The neo-Davidsonian position (e.g. Carlson 1984; Parsons 1990; Schein 1993) relates the relationship between events and their arguments by thematic roles
- There are also intermediate positions, such as Kratzer (2000)

<table>
<thead>
<tr>
<th>Position</th>
<th>Verbal denotation</th>
<th>Example: Brutus stabbed Caesar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional</td>
<td>$\lambda y \lambda x \eta[stab(x, y)]$</td>
<td>$stab(b, c)$</td>
</tr>
<tr>
<td>Classical Davidsonian</td>
<td>$\lambda y \lambda x \lambda e[stab(e, x, y)]$</td>
<td>$\exists e[stab(e, b, c)]$</td>
</tr>
<tr>
<td>Neo-Davidsonian</td>
<td>$\lambda e[stab(e)]$</td>
<td>$\exists e[stab(e) \land ag(e, b) \land th(e, c)]$</td>
</tr>
<tr>
<td>Asymmetric</td>
<td>$\lambda y \lambda e[stab(e, y)]$</td>
<td>$\exists e[ag(e, b) \land stab(e, c)]$</td>
</tr>
</tbody>
</table>

- The Neo-Davidsonian position makes it easier to state generalizations across the categories of nouns and verbs, and to place constraints on thematic roles.
- Events are things like Jones’ buttering of the toast, Brutus’ stabbing of Caesar.
- Events form a mereology, so they include plural events (Bach 1986; Krifka 1998)
- Events can be both spatially and temporally extended, unlike intervals.
• Events are usually thought to have temporal parts (subevents which occupy less time). It is controversial whether individuals also do – this is the 3D/4D controversy (Markosian 2009). Most semanticists seem to be on the 3D side (individuals do not have temporal parts).

• Some authors treat events as built from atoms (Landman 2000), others distinguish between count and mass events (Mourelatos 1978). With mereology, we need not decide (Krifka 1998).

• Some authors also include states (e.g. John’s being asleep) as events. Others use event more narrowly as opposed to states.
  
  – Do even stative sentences have an underlying event (Parsons 1987, 1990, ch. 10)? Maybe individual-level predicates don’t (Kratzer 1995)?

### 3.2 Thematic roles

• Thematic roles represent ways entities take part in events (Parsons 1990; Dowty 1991)

• Two common views:

  – Traditional view: thematic roles encapsulate generalizations over shared entailments of argument positions in different predicates (Gruber 1965; Jackendoff 1972)
    * agent (initiates the event, or is responsible for the event)
    * theme (undergoes the event)
    * instrument (used to perform an event)
    * sometimes also location and time

  – Alternative view: thematic roles as verb-specific relations: Brutus is not the agent of the stabbing event but the stabber (Marantz 1984). But this misses generalizations. And what about subjects of coordinated sentences like *A girl sang and danced*?

• No consensus on the inventory of thematic roles, but see Levin (1993) and Kipper-Schuler (2005) for wide-coverage role lists of English verbs

**Questions:**

• Do thematic roles have syntactic counterparts, the theta roles (something like silent prepositions)? Generative syntax says yes at least for the agent role: the “little v” head (Chomsky 1995)
• Does each verbal argument correspond to exactly one role (Chomsky 1981) or is the subject of a verb like fall both its agent and its theme (Parsons 1990)?

• Thematic uniqueness / Unique Role Requirement: Does each event have at most one agent, at most one theme etc. (widely accepted in semantics, see Carlson (1984, 1998); Parsons (1990); Landman (2000)) or no (Krifka (1992): one can touch both a man and his shoulder in the same event)?

• Question: Let \( e \) is a talking event whose agent is John and \( e' \) is a talking event whose agent is Mary. What is the agent of \( e \oplus e' \)?

• More generally, are thematic roles their own algebraic closures (Krifka 1986, 1998; Landman 2000)?

(1) **Cumulativity assumption for thematic roles**

For any thematic role \( \theta \) it holds that \( \theta = *\theta \). This entails that

\[
\forall e, e', x, y \left[ \theta(e) = x \land \theta(e') = y \rightarrow \theta(e \oplus e') = x \oplus y \right]
\]

- I assume the answer is yes (makes things easier to formalize)
- To symbolize this, instead of writing \( \theta \), I will write \( \ast \theta \).

• As a consequence of (1), thematic roles are homomorphisms with respect to the \( \oplus \) operation:

(2) **Fact: Thematic roles are sum homomorphisms**

For any thematic role \( \theta \), it holds that \( \theta(e \oplus e') = \theta(e) \oplus \theta(e') \).

(The \( \theta \) of the sum of two events is the sum of their \( \theta \)s.)

• Potential challenge to this assumption: the rosebush story (Kratzer 2003). Suppose there are three events \( e_1, e_2, e_3 \) in which Al dug a hole, Bill inserted a rosebush in it, and Carl covered the rosebush with soil. Then there is also an event \( e_4 \) in which Al, Bill, and Carl planted a rosebush. Let \( e_4 \) be this event. If \( e_4 = e_1 \oplus e_2 \oplus e_3 \), we have a counterexample to lexical cumulativity.

**Exercise 3.1** Why is this a counterexample? How could one respond to this challenge? □

### 3.3 Lexical cumulativity

• Many authors assume **lexical cumulativity**: whenever two events are in the denotation of a verb, so is their sum (Scha 1981; Schein 1986, 1993; Lasersohn 1989; Krifka 1989, 1992; Landman 1996, 2000; Kratzer 2007).
a. John slept.
b. Mary slept.
c. ⇒ John and Mary slept.

(4) a. John saw Bill.
b. Mary saw Sue.
c. ⇒ John and Mary saw Bill and Sue.

- Verbs have plural denotations: they obey the same equation as plural count nouns on the inclusive view

\[ [V] = ^* [V] \]

\[ [N_{pl}] = ^* [N_{sg}] \]

- It is customary to indicate lexical cumulativity by writing \( \lambda e [^* \text{see}(e)] \) for the meaning of the verb see instead of \( \lambda e [\text{see}(e)] \).

**Exercise 3.2** Translate example (3) into logic using the following assumptions: the Neo-Davidsonian view; and is translated as \( \oplus \); verbs and thematic roles are each closed under sum. Show that these assumptions predict the entailment in (3).

This entailment is parallel to the entailment from singular to plural nouns:

(7) a. John is a boy.
b. Bill is a boy.
c. ⇒ John and Bill are boys.

- Lexical cumulativity does not entail that all verb phrases have cumulative reference. For example, the sum of two events in the denotation of the verb phrase carry exactly two pianos is not again in its denotation, because it involves four rather than two pianos.

**Exercise 3.3** Does the verb phrase see John have cumulative reference?

### 3.4 Aspectual composition

- Predicates can be telic or atelic.
  - Atelic predicates: walk, sleep, talk, eat apples, run, run towards the store
    \( \approx \text{as soon as you start X-ing, you have already X-ed} \)
Telic predicates: build a house, finish talking, eat ten apples, run to the store
($\approx$ you need to reach a set terminal point in order to have X-ed)

- Question: Do telic and atelic predicates form disjoint classes of events (Piñón 1995) or is this a difference of predicates (Krifka 1998)?

- Traditionally, atelicity is understood as the subinterval property or divisive reference. Telicity is understood as quantized reference. This brings out the parallel between the telic/atelic and count/mass oppositions (e.g. Bach 1986).

(8) a. telic : atelic :: count : mass
   b. quantized : (approximate) subinterval :: quantized : (approximate) divisive

- We will use the following definition of the subinterval property:

(9) $\text{SUBINTERVAL}(P) \overset{\text{def}}{=} \forall e[P(e) \rightarrow \forall i[i < \tau(e) \rightarrow \exists e'[P(e') \land e' < e \land i = \tau(e)]]]

(Whenever $P$ holds of an event $e$, then at every subinterval of the runtime of $e$, there is a subevent of which $P$ also holds.)

(10) *eat ten apples for three hours

Failing presupposition: $\text{SUBINTERVAL}([\text{eat ten apples}])$, i.e. every part of the runtime of an eating-ten-apples event $e$ is the runtime of another eating-ten-apples event that is a part of $e$.

- The “minimal-parts problem” (Taylor 1977; Dowty 1979): The subinterval property distributes $P$ literally over all subintervals. This is too strong.

(11) John and Mary waltzed for an hour

$\not\Rightarrow$ #John and Mary waltzed within every single moment of the hour

$\Rightarrow$ John and Mary waltzed within every short subinterval of the hour

- The length interval that counts as very small for the purpose of the for-adverbial varies relative to the length of the bigger interval:

(12) The Chinese people have created abundant folk arts … passed on from generation to generation for thousands of years.\(^1\)

• *Aspectual composition* is the problem of how complex constituents acquire the telic/atelic distinction from their parts. Verkuyl (1972); Krifka (1998)

• With “incremental theme” verbs like *eat*, the correspondence is clear:

(13) a. eat apples / applesauce for an hour
b. *eat an apple / two apples / the apple for an hour

(14) a. count : mass :: telic : atelic
b. apple : apples :: eat an apple : eat apples

(15) a. drink wine for an hour
b. *drink a glass of wine for an hour

• With “holistic theme” verbs like *push* and *see*, the pattern is different:

(16) a. push carts for an hour
b. push a cart for an hour

(17) a. look at apples / applesauce for an hour
b. look at an apple / two apples / the apple for an hour

• *Verkuyl’s Generalization* (Verkuyl 1972): When the direct object of an incremental-theme verb is a count expression, we have a telic predicate, otherwise an atelic one.

• Krifka (1992): in incremental-theme verbs (also called “measuring-out” verbs, among other things), the parts of the event can be related to the parts of the theme (see Figure 3.1).

• Following Krifka, we can formalize the difference between holistic-theme and incremental-theme verbs by meaning postulates.

(18) **Definition: Incrementality**
Incremental$_\theta$(P) $\iff \forall e \forall e' \forall x [\theta(e) = x \land e' < e \rightarrow \theta(e') < x]$

(19) **Definition: Holism**
Holistic$_\theta$(P) $\iff \forall e \forall e' \forall x [\theta(e) = x \land e' < e \rightarrow \theta(e') = x]$

(20) **Meaning postulates**
a. Incremental$_{\text{theme}}$([eat])
b. Incremental$_{\text{theme}}$([drink])
c. Holistic$_{\text{theme}}$([see])
Then we apply these meaning postulates to prove or disprove that the various VPs above have divisive reference or the subinterval property.

**Claim:** [eat two apples] does not have the subinterval property.

**Proof:** Suppose it has, then let $e$ be an event in its denotation whose runtime is an hour. From the definition of the subinterval property, (9), at each subinterval of this hour there must be a proper subevent of $e$ whose theme is again two apples. Let $e'$ be any of these proper subevents. Let the theme of $e$ be $x$ and the theme of $e'$ be $y$. Then $x$ and $y$ are each a sum of two apples. From the “incremental theme” meaning postulate in (20a) we know that $y$ is a proper part of $x'$. Since two apples is quantized, $x$ and $y$ can not both be two apples. Contradiction.

**Exercise 3.4** Why does the proof not go through for see two apples? Why does it not go through for eat apples? □
Lecture 4

Distributivity

4.1 Introduction

- **What is distributivity?** In this lecture: a property of predicates
  
  - *Distributive*: e.g. walk, smile, take a breath (applies to a plurality just in case it applies to each of its members)
  
  - *Collective*: e.g. be numerous, gather, suffice to defeat the army (may apply to a plurality even if it does not apply to each of its members)

- Literature: Roberts (1987); Winter (2001), Section 6.2; Schwarzschild (1996), Chapter 6; Link (1997), Section 7.4.

4.2 Lexical and phrasal distributivity

(1) **Lexical distributivity/collectivity** involves lexical predicates

   a. The children smiled.  \textit{distributive}
   b. The children were numerous. \textit{collective}

(2) **Phrasal distributivity/collectivity** involves complex predicates

   a. The girls are wearing a dress. \textit{distributive}
   b. The girls are sharing a pizza. \textit{collective}
   c. The girls are building a raft. \textit{collective/distributive}

- The difference between lexical and phrasal distributivity corresponds to the difference between what can and what cannot be described using meaning postulates
(3) **Meaning postulate: smile is distributive**
\[ \forall e [\text{smile}(e) \rightarrow e \in \lambda e'(\text{smile}(e') \land \text{Atom}(\text{ag}(e'))) ] \]
(Every smiling event consists of one or more smiling events whose agents are atomic.)

- Meaning postulates can only apply to words. We cannot formulate a meaning postulate that says that *wear a dress* is distributive.

- Problems:
  - Meaning postulates are taken to be available only for lexical items
  - For mixed predicates like *build a raft*, we would need optional meaning postulates

- The classical solution is due to Link (1983): A covert distributive operator $D$ adjusts the meaning of a verb phrase like *wear a dress* into *be a sum of people who each wear a dress*.

- $D$ is in the lexicon, so it can apply to entire VPs (Dowty 1987; Roberts 1987; Lasersohn 1995).

- Link’s $D$ operator introduces a universal quantifier:

  \[(4) \quad [D^{\text{Link}}] = \lambda P(ε) \lambda x \forall y [y \leq_{\text{Atom}} x \rightarrow P(y)] \]

  (Takes a predicate $P$ over individuals and returns a predicate that applies to any individual whose atomic parts each satisfy $P$.)

(5) a. The girls built a raft.
  \[ \approx \text{The girls built a raft together.} \quad \text{collective} \]
b. The girls $D^{\text{Link}}$(built a raft).
  \[ \approx \text{The girls each built a raft.} \quad \text{distributive} \]

- This allows us to model the distributive meaning of (2a):

  \[(6) \quad \forall y [y \leq_{\text{Atom}} \bigoplus \text{girl} \rightarrow \exists z [\text{dress}(z) \land \text{wear}(y, z)]] \]

  (Every atomic part of the sum of all girls wears a dress.)

- Based on earlier work by Eddy Ruys, Winter (2001) observes that the existential and the distributivity imports of numeral indefinites can have two distinct scopes.

(7) If three workers in our staff have a baby soon we will have to face some hard organizational problems.

  a. If any three workers have a baby, there will be problems. \[ \text{if } > 3 > D > 1 \]
b. There are three workers such that if each of them has a baby, there will be problems.

3 > if > D > 1

- Unlike the indefinite, the distributive operator cannot take scope outside of the if-island:

(8) a. *There are three workers such that for each \( x \) of them, if \( x \) has a baby, there will be problems.

3 > D > if > 1

4.2.1 Reformulating the D operator

- Link’s formulation of the D operator needs to be adjusted for several reasons:

  - If we assume with Landman (1996) that groups are atoms too (“impure” atoms) and that the girls can introduce a group, then we need to specify that D distributes over “pure” atoms (singular individuals) only.
  - If VPs are of type \( \langle vt \rangle \) instead of \( \langle et \rangle \), we need to repair the type mismatch.
  - We also need to be able to coindex D with different thematic roles (Lasersohn 1995).

(9) a. The first-year students D(\( \text{took an exam} \)). \hspace{1cm} \text{Target: agent}

b. John D(\( \text{gave a pumpkin pie} \)) to two girls. \hspace{1cm} \text{Target: recipient}

c. John D(summarized) the articles. \hspace{1cm} \text{Target: theme}

- The D operator can be understood as shifting arbitrary predicates to a distributive interpretation with granularity Atom (i.e. singular individual):

(10) **Definition: Atomic event-based D operator**

\[
[D_a] \overset{\text{def}}{=} \lambda P_{\langle et \rangle} \lambda e[e \in *e' \left( P(e') \land \text{Atom}(\theta(e')) \right)]
\]

(Takes an event predicate \( P \) and returns a predicate that holds of any event \( e \) which consists entirely of events that are in \( P \) and whose thematic roles \( \theta \) are atoms.)

- Example:

(11) The girls are wearing a dress.

\[ \exists e[\text{ag}(e) = \bigoplus \text{girl} \land *\text{wear}(e) \land \text{dress}(\text{th}(e))] \]

(There is a potentially plural wearing event whose agents sum up to the girls, and whose theme is a dress.)
(12) The girls D(are wearing a dress.)
\[ \exists e [^*_{ag}(e) = \bigoplus \text{girl} \land \\
\quad e \in ^*_{e'}(\text{wear}(e') \land \text{dress}(\text{th}(e')) \land \text{Atom}(^*_{ag}(e'))) ] \]
(There is an event whose agents sum up to the girls, and this event consists of wearing events for each of which the agent is a atom and the theme is a dress.)

- The star operator \(^*_{\lambda e'}\) is introduced through the D operator and takes scope over the predicate dress introduced by the theme.

**Exercise 4.1** Which background assumptions ensure that (12) entails that each girl wears a dress?

4.2.2 The leakage problem

- There are various other proposals on how to reformulate the D operator.
- Lasersohn (1998) proposes the following entry (among others):

  (13) **Distributivity operator over events (Lasersohn)**

  \[
  [D^{Lasersohn}] = \lambda P(e,vt) \lambda x \lambda e \forall y[y \leq_{\text{Atom}} x \rightarrow \exists e'[e' \leq e \land P(y)(e')]]
  \]

  - This applies to a predicate of type \( \langle e, vt \rangle \), e.g. \([\text{smile}] = \lambda x \lambda e[\text{smile}(e) \land ag(e) = x]\).

  - Inserting a D operator into *The girls smiled* before existential closure applies:

    (14) a. Lasersohn’s representation:

    \[
    \lambda e \forall y[y \leq_{\text{Atom}} \bigoplus \text{girl} \land \rightarrow \exists e'[e' \leq e \land \text{smile}(e') \land ^*_{ag}(e') = y]
    \]

    b. My representation:

    \[
    \lambda e[^*_{ag}(e) = \bigoplus \text{girl} \land e \in ^*_{\lambda e'}[\text{smile}(e') \land \text{Atom}(^*_{ag}(e'))]]
    \]

    - (14a) applies to all events that contain a smiling subevent for each girl, even if they also contain extraneous material. It suffers from what Bayer (1997) calls leakage. Whenever it (14a) applies to an event \( e \), it also applies to any event of which \( e \) is a part.

    - (14b) applies to all events that contain a smiling subevent for each girl and nothing else.

    - Leakage causes problems in connection with event predicates such as surprisingly, unharmo-

    - These predicates do not have divisive reference: they can hold of an event even if they do not hold of its parts (Schein 1993).
(15) Unharmoniously, every organ student sustained a note on the Wurlitzer.

- This says that the ensemble event was unharmonious and not any one student’s note.
- Let Lasersohn stand for Lasersohn’s (14a) and let Mine stand for my (14b).
- Imagine an event G that satisfies both Lasersohn and Mine, that is, the girls smiled in it.
- Let B be an event in which the boys cry.
- Now G ⊕ B does not satisfy Mine, but it does satisfy Lasersohn.
- Suppose that G is not surprising by itself, but that G ⊕ B is surprising. Then we have these judgments:

(16)  

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>The girls smiled.</td>
<td>true</td>
</tr>
<tr>
<td>b</td>
<td>The girls smiled and the boys cried.</td>
<td>true</td>
</tr>
<tr>
<td>c</td>
<td>Surprisingly, the girls smiled.</td>
<td>false</td>
</tr>
<tr>
<td>d</td>
<td>Surprisingly, the girls smiled and the boys cried.</td>
<td>true</td>
</tr>
</tbody>
</table>

- If one of the D operators is applied to smile, then (17) is translated as (17a) or (17b).

(17) Surprisingly, the girls smiled.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>∃e[surprising(e) ∧ Lasersohn(e)]</td>
</tr>
<tr>
<td>b</td>
<td>∃e[surprising(e) ∧ Mine(e)]</td>
</tr>
</tbody>
</table>

- The problem is that G ⊕ B satisfies both Lasersohn (by leakage) and the predicate surprising (by assumption). So Lasersohn’s D operator wrongly predicts that (17) is judged true.
- The above implementation avoids this kind of leakage.

4.3 Atomic and nonatomic distributivity

- So far we have implemented the view called atomic distributivity: the D operator distributes over atoms, that is, over singular individuals (Lasersohn 1998, 1995; Link 1997; Winter 2001)
- Nonatomic view: phrasal distributivity may also quantify over nonatomic parts (Gillon 1987, 1990; van der Does and Verkuyl 1995; Verkuyl and van der Does 1996; Schwarzschild 1996; Brisson 1998, 2003; Malamud 2006a,b)
Traditional argument is based on sentences like this, adapted from Gillon (1987):

Rodgers, Hammerstein and Hart never wrote any musical together, nor did any of them ever write one all by himself. But Rodgers and Hammerstein wrote the musical *Oklahoma* together, and Rodgers and Hart wrote the musical *On your toes* together.

On the basis of these facts, (18a) and (18b) are judged as true in the actual world, although it is neither true on the collective interpretation nor on an “atomic distributive” interpretation.

(18)  

a. Rodgers, Hammerstein, and Hart wrote *Oklahoma* and *On Your Toes*.

b. Rodgers, Hammerstein, and Hart wrote musicals.

The traditional nonatomic argument: in order to generate the reading on which (18b) is true, the predicates *wrote musicals* and *wrote Oklahoma and On Your Toes* must be interpreted as applying to nonatomic parts of the sum individual to which the subject refers.

Generally implemented with covers (Gillon 1987): partitions of a set (19) or sum (20) whose cells/parts can overlap

(19)  

**Definition: Cover (set-theoretic)**

\[
\text{Cov}(C, P) \overset{\text{df}}{=} \bigcup C = P \land \varnothing \not\in C
\]

(C is a cover of a set P if and only if C is a set of subsets of P whose union is P.)

(20)  

**Definition: Cover (mereological)**

\[
\text{Cov}(C, x) \overset{\text{df}}{=} \bigoplus C = x
\]

(C is a cover of a mereological object x is a set of parts of x whose sum is x.)

Cover-based approaches modify the D operator to quantify over nonatomic parts of a cover of the plural individual.

The first cover-based approaches assumed that the cover can be existentially quantified by the operator that introduces it:

(21)  

**Nonatomic distributivity operator, existentially bound cover**

\[
[D_3] = \lambda P_{(x)} \lambda x \exists C [\text{Cov}(C, x) \land \forall y [C(y) \land y \leq x \rightarrow P(y)]]
\]

On this view, sentences (18a) and (18b) are translated as follows:

(22)  

\[
\exists C [\text{Cov}(C, \text{rodgers } \oplus \text{hammerstein } \oplus \text{hart}) \land \\
\forall y [C(y) \land y \leq x \rightarrow y \in [\text{wrote Oklahoma and On Your Toes}]]]
\]
\[(23) \exists C[\text{Cov}(C, \text{rogers} \oplus \text{hammerstein} \oplus \text{hart}) \land \forall y[\text{C}(y) \land y \leq x \rightarrow y \in \text{[wrote musicals]}]]
\]

**Exercise 4.2** For which value of C are these formulas true in the actual world?

- Existentially bound covers are now generally considered untenable because they overgenerate nonatomic distributive readings
- Lasersohn (1989)'s problem: Suppose John, Mary, and Bill are the teaching assistants and each of them was paid exactly $7,000 last year. (24a) and (24b) are true, but (24c) is false.

\[(24) \begin{align*}
a. & \quad \text{True: The TAs were paid exactly$7,000 last year.} \\
b. & \quad \text{True: The TAs were paid exactly $21,000 last year.} \\
c. & \quad \text{False: The TAs were paid exactly$14,000 last year.}
\end{align*}
\]

- Giving up the existential cover-based operator D_3 in (21) explains why (24c) is false, because without this operator, there is no way to generate a true reading for this sentence.
- But now why are (18a) and (18b) true?
- As it turns out, the lexical cumulativity assumption is already enough (Lasersohn 1989):

\[(25) \forall w, x, y, z[\text{write}(w, x) \land \text{write}(y, z) \rightarrow \text{write}(w \oplus y, x \oplus z)]
\]

**Exercise 4.3** What does this assumption translate to in a Neo-Davidsonian framework?

- Further support: (28) is false in the actual world (Link 1997):

\[(28) \text{Rodgers, Hammerstein and Hart wrote a musical.}
\]

\[
\begin{align*}
a. & \quad \text{True if the three of them wrote a musical together – not the case.} \quad \checkmark \text{collective} \\
b. & \quad \text{True if each of them wrote a musical by himself – not the case.}\ \checkmark \text{atomic distributive} \\
c. & \quad \text{False even though Rodgers and Hammerstein wrote a musical together, and Rodgers and Hart wrote another musical together.} \quad \ast \text{nonatomic distributive}
\end{align*}
\]

- The absence of the nonatomic distributive reading of (28) is predicted if we give up the existential cover-based operator D_3.
- Lexical cumulativity derives the (available) nonatomic distributive reading of (18a) and (18b) but not the (unavailable) nonatomic distributive reading of (28):
Rodgers, Hammerstein and Hart wrote *Oklahoma* and *On Your Toes*.

\[ \exists e [\text{write}(e) \land \text{ag}(e) = \text{rodgers} \oplus \text{hammerstein} \oplus \text{hart} \land \text{th}(e) = \text{okl} \oplus \text{oyt}] \]

(Allows for several writing events and for teamwork, as long as these two musicals are written.)

Rodgers, Hammerstein and Hart wrote musicals.

\[ \exists e [\text{write}(e) \land \text{ag}(e) = \text{rodgers} \oplus \text{hammerstein} \oplus \text{hart} \land \text{musical}(\text{th}(e))] \]

(Allows for several writing events and for teamwork, and there can be several musicals in total.)

Rodgers, Hammerstein and Hart wrote a musical.

\[ \exists e [\text{write}(e) \land \text{ag}(e) = \text{rodgers} \oplus \text{hammerstein} \oplus \text{hart} \land \text{musical}(\text{th}(e))] \]

(Allows for several writing events and for teamwork, but there has to be only one musical in total.)

- Lasersohn, as well as Winter (2001) and others, conclude from this and similar examples that the atomic approach to phrasal distributivity is superior to covers.

- However, Gillon (1990) and Schwarzschild (1996) identify a residue of cases in which a cover-based operator does seem necessary.

**Scenario** Two pairs of shoes are on display, each pair with a $50 price tag.

a. The shoes cost $100. \(\checkmark\) collective (together)

b. The shoes cost $25. \(?\) atomic distributive (per shoe)

c. The shoes cost $50. (Lasersohn 1995) \(\checkmark\) nonatomic distributive (per pair)

- Evidence that individual shoes, and not shoe pairs, are atoms in this context:

**Schwarzschild’s nonatomic distributivity operator, free cover**

\[ [D_C] = \lambda P_{(et)} \lambda x \forall y [C(y) \land y \leq x \rightarrow P(y)] \]

- See Malamud (2006a,b) for a decision-theoretic elaboration of this proposal.

- Nonatomic distributivity is always available for verbs, but for verb phrases it only occurs when context supplies a pragmatically salient cover. Atomic distributivity is available in both cases.
Figure 4.1: V level versus verb phrase level distributivity in atomic domains

<table>
<thead>
<tr>
<th>(a) Empirical generalization</th>
<th>(b) Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>lexical (V level)</td>
<td>lexical (V level)</td>
</tr>
<tr>
<td>phrasal (VP level)</td>
<td>phrasal (VP level)</td>
</tr>
<tr>
<td>atomic available</td>
<td>meaning post. Atomic D op.</td>
</tr>
<tr>
<td>nonatomic available only w. context</td>
<td>meaning post. Cover-based D op.</td>
</tr>
</tbody>
</table>

- To model nonatomic distributivity, I change the event-based atomic D operator repeated below as (35), by replacing the predicate \( \text{Atom} \) by a free predicate \( C \). It plays the same role as the \( C \) predicate in Schwarzschild’s operator:

\[ [D_{\theta}] \overset{\text{def}}{=} \lambda P_{(\ell e)} \lambda e[e \in ^{*} \lambda e' \left( P(e') \land \text{Atom}(\theta(e')) \right)] = (10) \]

(Takes an event predicate \( P \) and returns a predicate that holds of any event \( e \) which consists entirely of events that are in \( P \) and whose \( \theta s \) are \textbf{atoms}.)

(36) **Definition: Generalized event-based D operator**

\[ [D_{\theta,C}] \overset{\text{def}}{=} \lambda P_{(\ell e)} \lambda e[e \in ^{*} \lambda e' \left( P(e') \land C(\theta(e')) \right)] \]

(Takes an event predicate \( P \) and returns a predicate that holds of any event \( e \) which consists entirely of events that are in \( P \) and whose \( \theta s \) \textbf{satisfy the predicate} \( C \).)

- The generalized D operator has two parameters: dimension (thematic role) and granularity (C).

- Unlike Schwarzschild, I do not rely on pragmatics to ensure that \( C \) actually covers the \( \theta s \) of the event to which the output of the D operator is applied.

- Following Schwarzschild (1996), I assume that the \( C \) parameter of the D operator in (36) can only be set in one of two ways: either it is set to the predicate \( \text{Atom} \) or to an anaphorically salient level of granularity.
Lecture 5

Measurement

5.1 Introduction

- Partial functions (or just functions for brevity) formalize the relationships between the domains of the ontological zoo, as shown in Figure 5.1

Figure 5.1: The world (some details omitted)
5.2 Trace functions and intervals

• Trace functions map events to intervals which represent their temporal and spatial locations
  – \( \tau \), the temporal trace or runtime
  – \( \sigma \), the spatial trace

• Occur in phrases like three hours and three miles, from 3pm to 6pm, to the store and in tense semantics

• Trace functions also indicate the precise location in space and time

• Example: John sings from 1pm to 2pm and Mary sings from 2pm to 3pm. Although each event takes the same time, their runtimes are different.

• While two distinct events may happen at the same place and/or time, this is not possible for intervals.

• Axioms for temporal intervals and other temporal structures are found in van Benthem (1983). The integration into mereology is from Krifka (1998).

• Temporal inclusion (\( \leq \)) is like mereological parthood and subject to nonatomic CEM (most semanticists do not assume the existence of temporal atoms or instants)

• Temporal precedence (\( \ll \)) is irreflexive, asymmetric and transitive; holds between any two nonoverlapping intervals

• Example: if \( a \) is the interval from 2pm to 3pm today, \( b \) is the interval from 4pm to 5pm, and \( c \) is the interval from 1pm to 5pm, then we have \( a \leq c \), \( b \leq c \), and \( a \ll b \)

• Trace functions provide the bridge between interval logic and event logic:

\[(1) \textbf{ Definition: Holding at an interval} \]
\[
\text{AT}(V, i) \overset{df}{=} \exists e[V(e) \land \tau(e) = i]
\]
(An event predicate \( V \) holds at an interval \( i \) if and only if it holds of some event whose temporal trace is \( i \).)

• Trace functions are sum homomorphisms (Link 1998; Krifka 1998), like thematic roles.
(2) **Trace functions are sum homomorphisms**

\[ \sigma \] is a sum homomorphism: \( \sigma(e \oplus e') = \sigma(e) \oplus \sigma(e') \)

\[ \tau \] is a sum homomorphism: \( \tau(e \oplus e') = \tau(e) \oplus \tau(e') \)

(The location/runtime of the sum of two events is the sum of their locations/runtimes.)


- Prepositional phrases can be represented using trace functions

(3) \([\text{to the store}] = \lambda V(vt) \lambda e [V(e) \land \text{end}(\sigma(e))] = \text{the.store}\)

(4) \([\text{from 3pm to 4pm}] = \lambda V(vt) \lambda e [V(e) \land \text{start}(\tau(e))] = 3pm \land \text{end}(\tau(e)) = 4pm\)


### 5.3 Measure functions and degrees

- While trace functions map entities to intervals, measure functions map entities to degrees (but some authors conflate them, e.g. Kratzer (2001))

- Typical measure functions: height, weight, speed, temperature

- Degrees are totally ordered quantities assigned by measure functions
  - Degrees of individuals: John’s weight, the thickness of the ice at the South Pole
  - Degrees of events: the speed at which John is driving his car now

- Uses of degrees in semantics: gradable adjectives (*tall, beautiful*), measure nouns (*liter, hour*), measure phrases (*three liters*), comparatives (*taller, more beautiful, more water, more than three liters*), pseudopartitives (*three liters of water*)

- Degree scales are assumed to be totally ordered, while mereologies are typically only partially ordered.
Questions:

• Are degrees ontological entities in their own right (Cresswell 1976) or contextual coordinates (Lewis 1972)? For comparisons of the two approaches, see Klein (1991), Kennedy (2007), and van Rooij (2008).
  
  – The contextual-coordinate analysis is mainly concerned with gradable adjectives and does not provide an obvious way to represent the meaning of measure phrases.

• If degrees are entities, what are they? Primitives (Parsons 1970; Cartwright 1975), numbers (Hellan 1981; Krifka 1998) or equivalence classes of individuals (Cresswell 1976; Ojeda 2003)?
  
  – Not sure if any linguistic facts motivate reductionism here; numbers aren’t sorted: six feet approximately equals 183cm, but 6 is not equal to 183.

• Are degrees points or initial intervals (“extents”) on a scale (Krasikova 2009)? Are these scales always dense (Fox and Hackl 2006)?

• Should degree scales be special cases of mereologies (Szabolcsi and F. Zwarts 1993; Lassiter 2010a,b)?
  
  – Degrees are totally ordered, but mereological parthood is only a partial order.

5.4 Unit functions

• For Lønning (1987), degrees occupy an intermediate layer between individuals and numbers (see also Schwarzschild (2006)).

• Measure nouns like liter, kilogram, year denote functions from degrees to numbers: what I will call unit functions.

\[
\begin{align*}
&\text{a. } \text{[liter]} = \lambda n \lambda d [\text{liters}(d) = n] \\
&\text{b. } \text{[year]} = \lambda n \lambda t [\text{years}(t) = n]
\end{align*}
\]

• Example: John weighs 150 pounds (68 kilograms) and measures six feet (183 centimeters). Weight and height are measure functions, feet and centimeters are unit functions.

\[
\begin{align*}
&\text{a. } \text{pounds(weight(john))}=150 \\
&\text{b. } \text{kilograms(weight(john))}=68
\end{align*}
\]
c. feet(height(john))=6

d. centimeters(height(john))=183

- Advantage of Lønning’s split: underspecification in pseudopartitives

\[(7) \text{ three inches of oil} \]
\[\text{a. } \lambda x [\text{oil}(x) \land \text{inches}(\text{height}(x)) = 3] \quad \text{(by height)}\]
\[\text{b. } \lambda x [\text{oil}(x) \land \text{inches}(\text{diameter}(x)) = 3] \quad \text{(by diameter)}\]

- Ambiguity of container pseudopartitives (Rothstein 2009, and references therein):

\[(8) \text{ three glasses of wine} \]
\[\text{a. Measure reading: } \lambda x [\text{wine}(x) \land \text{glasses}(x) = 3] \quad \text{(a quantity of wine that corresponds to three glassfuls)}\]
\[\text{b. Individuating reading: } \lambda x [x = 3 \land \exists \text{glass}(x) \land \text{contains}(x, \text{wine})] \quad \text{(three actual glasses containing wine)}\]

- Alternative: measure functions directly relate entities to numbers (Quine, Krifka)

### 5.5 The cardinality function

- Cardinality maps sums which consist of singular individuals onto the number of singular individuals of which they consist

\[(9) \text{ Definition: Cardinality} \]

For any sum \(x\) such that \(\exists \text{Atom}(x)\), the cardinality of \(x\), written \(|x|\), is defined as \(|\{ y \mid y \leq \text{Atom}(x) \}|\).

- I let number words denote numbers, as in Krifka (1989), Hackl (2001) and Landman (2004): three denotes 3
  - Alternative 1: three denotes a generalized quantifier
  - Alternative 2: three denotes a predicate over pluralities with three atoms

- A silent head [many] introduces cardinality, as in Figure 5.2 (Hackl 2009)

- In measure phrases, the number word combines with the measure noun.
• Here I treat measure phrases like *three liters* as predicates of degrees (J. Zwarts 1997; Schwarzschild 2006), though there are other views (they might refer to degrees or quantify over them).

**Figure 5.2:** LF of the noun phrases *three boys* and *three liters*

\[\lambda x[|x| = 3 \land \neg \text{boy}(x)] \quad \lambda d[\text{liters}(d) = 3]
\]

| three | [many] | \[|x| = 3\] |
|-------|--------|-------------|
| 3     | \[\lambda n \lambda x[|x| = n]\] | \[\text{boys}\] |

**Figure 5.3:** Skeletal LF of an ordinary pseudopartitive

\[\lambda x[\text{water}(x) \land \text{liters(volume}(x)) = 3]
\]

| three | \[\langle n \rangle\] | \[\langle n, dt \rangle\] | \[|n, dt| = 3\] |
|-------|---------------------|------------------|----------------|
| \[\lambda n \lambda d[\text{liters}(d) = n]\] | \[\text{liters}\] | \[\text{volume}\] | \[\langle et\rangle\] |

**Questions:**

• Does *three boys* mean exactly three boys or at least three boys?

  – for the at least view, see Horn (1972, 1989), Barwise and Cooper (1981), Levinson (2000), van Rooij and Schulz (2006);
  – for the exactly view, see Harnish (1976), Sadock (1984), Barbara H. Partee (1987), Carston (1998), Geurts (2006), Breheny (2008);
for reviews of this literature, see Nouwen (2006) and Kennedy (2009).

• I assume an exactly interpretation: predicates like eat three apples do not apply to events in which more than three apples are eaten

• In event semantics, the maximality conditions involved in exactly readings have to be realized through other mechanisms anyway (Krifka 1999; Landman 2000; Robaldo 2010; Brasoveanu 2010).

• What do number words denote:
  • Alternative 1: three boys denotes the number 3, (Krifka 1989; Hackl 2001; Landman 2004):
  • Alternative 2: three denotes a predicate over pluralities with three atoms: \( \lambda x[|x| = 3] \)
  • Alternative 3: three denotes a generalized quantifier: \( \lambda A \lambda B[A \cap B = 3] \) – note that this is set-theoretic cardinality
Lecture 6

Aspect and measurement

6.1 The measurement puzzle

Pseudopartitives reject some measure functions (Krifka 1998; Schwarzchild 2006)

(1) a. five pounds of rice
b. five liters of water
c. five hours of talks
d. five miles of railroad tracks
e. *five miles per hour of driving
f. *five degrees Celsius of water

Several other constructions behave analogously:

(2) more rope
(3) *five miles per hour of my driving

6.1.1 Previous work

Schwarzchild (2006): Only monotonic measure functions are admissible.

- A measure function \( \mu \) is monotonic iff for any two entities \( a \) and \( b \), if \( a \) is a proper part of \( b \), then \( \mu(a) < \mu(b) \). (See also Krifka (1998).)

Examples:

- Volume is monotonic \( \rightsquigarrow \) thirty liters of water
- Temperature is not monotonic \( \rightsquigarrow \) *thirty degrees Celsius of water
• What about height? It had better be monotonic: $\rightsquigarrow$ five feet of snow

**Problem:** The snow that fell on West Berlin is a proper part of the snow that fell on Berlin. But, we don’t conclude that the height of the snow in West Berlin was less than the snow that fell on Berlin. So height is **not monotonic**.

### 6.1.2 Novel observation

Measure functions rejected by pseudopartitives are also rejected by *for*-adverbials.

(4) a. John waited for five hours. 
    b. The crack widens for five meters. 
    c. *John drove for thirty miles an hour. 
    d. *The soup boiled for 100 degrees Celsius.

This connection allows us to tap into the literature on aspect.

### 6.2 Answer strategy

We have seen in previous lectures that *For*-adverbials are most commonly associated with the telic/atelic opposition.

- **Atelic predicates:** walk, sleep, eat apples, run, run towards the store
  ($\approx$ *as soon as you start X-ing, you have already X-ed*)

- **Telic predicates:** build a house, eat ten apples, run to the store
  ($\approx$ *you need to reach a set terminal point in order to have X-ed*)

(5) a. John *ran* for five minutes. 
    b. *John ran to the store* for five minutes.
(6) a. John *ate apples* for an hour. 
    b. *John ate ten apples* for an hour.

**Plan of this lecture:**

- Introduce *stratified reference*, which generalizes the telic/atelic contrast.
- Derive the restriction on measure functions from this concept.
6.3 The aspect puzzle

As we’ve seen before, telicity is a property of predicates (Krifka 1998). But which one?

**Classical answer** To be atelic means to have the *subinterval property* (e.g. Bennett and Barbara H. Partee 1972; Dowty 1979). Here’s an event-based version of the subinterval property:

\[
\text{SUBINT}(P) = \text{def} \quad \forall e \left[ P(e) \rightarrow \forall i \left[ i < \tau(e) \rightarrow \exists e' \left[ P(e') \wedge \tau(e') < e \wedge i = \tau(e') \right] \right] \right]
\]

(Whenever \( P \) holds of an event \( e \), then at every subinterval of the runtime of \( e \), there is a subevent of which \( P \) also holds.)

In a previous lecture, we’ve assumed that *for*-adverbials presuppose the subinterval property.

6.3.1 Problems with the subinterval property

**First problem** The “minimal-parts problem” (Taylor 1977; Dowty 1979):

\( \text{8) John and Mary waltzed for an hour} \not\Rightarrow \text{John and Mary waltzed within every single moment of the hour} \Rightarrow \text{John and Mary waltzed within every short subinterval of the hour} \)

The minimal length varies relative to the length of the bigger interval:

\( \text{9) The Chinese people have created abundant folk arts … passed on from generation to generation for thousands of years.} \)

**Second problem** Spatial *for*-adverbials (Gawron 2005):

\( \text{10) a. The crack *widens* for 5 meters. b. *The crack *widens* 2cm for 5 meters.} \)

\( \text{spatially atelic} \)

\( \text{spatially telic} \)

\( \text{11) a. The road ends in a mile. b. *The road ends for a mile.} \)

\( \text{12) a. *The road meanders in a mile. b. The road meanders for a mile.} \)

\( \text{13) The police blocked streets for miles around [the museum].} \)

Spatial and temporal *for*-adverbials impose different constraints.

– see Figure 6.1.

---

(14) a. John pushed carts to the store for fifty minutes. \textit{temporally atelic}
    b. #John pushed carts to the store for fifty meters. \textit{ spatially telic}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6_1.png}
\caption{John pushed carts to the store is temporally atelic but spatially telic}
\end{figure}

(15) Snow fell throughout the area for two straight days.\textsuperscript{3}
    a. ⇒ Every part of “two straight days” is the runtime of an event in [Snow fell throughout the area]
    b. \(\not\Rightarrow\) Every part of “throughout the area” is the location of an event in [Snow fell throughout the area]

(16) Wine flowed from the jar to the floor for five minutes.\textsuperscript{4}
    a. ⇒ Every part of “five minutes” is the runtime of an event in [Wine fell from the jar to the floor]
    b. \(\not\Rightarrow\) Every part of “from the jar to the floor” is the location of an event in [Wine fell from the jar to the floor]

6.3.2 Generalizing the subinterval property

What the subinterval property says: An atelic predicate \(P\) distributes along the \textit{time} dimension down to intervals of \textit{ininitely short length}.

What it should say: An atelic predicate \(P\) distributes along the ____ dimension down to intervals of ____ length.

That is, we want to \textit{parametrize} the subinterval property.

\textsuperscript{3}Attested example (http://community.lawyers.com/forums/t/17235.aspx).
\textsuperscript{4}Beavers (2008)
We start with applying the subinterval property to \textit{waltz}:

\[(17) \quad \forall e [\text{waltz}(e) \rightarrow \forall i [i < \tau(e) \rightarrow \exists e' [\text{waltz}(e') \land e' < e \land i = \tau(e')]]] \]

(Whenever \textit{waltz} holds of an event \(e\), then at every subinterval of the runtime of \(e\), there is a subevent of which \textit{waltz} also holds.)

Let \(\varepsilon\) be a function that tells us what counts as very small: \(\varepsilon(\lambda t[\text{hours}(t) = 1])(t')\) is true just in case \(t'\) is very small with respect to one hour.

We want to be able to say:

\[(18) \quad \text{Whenever \textit{waltz} holds of an event, there is a way of dividing this event into subevents with very small runtimes such that \textit{waltz} also holds of each of these subevents.}\]

To express this formally, we use the star operator.

- Reminder: \(x \in \ast(\lambda y.B(y))\) means: \(x\) consists of one or more parts of which \(B\) holds

With the star operator, we can express (18) as follows:

\[(19) \quad \forall e [\text{waltz}(e) \rightarrow e \in \ast \lambda e' \left( \text{waltz}(e') \land \varepsilon(\lambda t[\text{hours}(t) = 1])(\tau(e')) \right) ] \]

Let us say that \textit{waltz} has \textbf{stratified reference} (SR) with respect to the dimension \textit{runtime} and the granularity \(\varepsilon(\lambda t[\text{hours}(t) = 1])\) ("very short time interval") just in case (19) above is true.

\[(20) \quad \textbf{Stratified reference (Example)}\]

Let \(\text{"SR}_{\text{runtime}, \varepsilon(\lambda t[\text{hours}(t) = 1])}(\lambda e[\text{waltz}(e)])\)" abbreviate (19).

By abstracting from this example, we arrive at the following definition:

\[(21) \quad \textbf{Stratified reference (Definition)}\]

\[
\text{SR}_{f, \varepsilon(K)}(P) \overset{\text{def}}{=} \forall x [P(x) \rightarrow x \in \ast \lambda y \left( P(y) \land \varepsilon(K)(f(y)) \right)]
\]

- A predicate \(P\) has stratified reference with respect to a function \(f\) and a threshold \(\varepsilon(K)\) if and only if there is a way of dividing every entity in its denotation exhaustively into parts which are each in \(P\) and which have a very small \(f\)-value. Very small \(f\)-values are those that satisfy \(\varepsilon(K)\).

The answer to the aspect puzzle.
Being atelic means having stratified reference with respect to time and a suitably instantiated granularity parameter.
For-adverbials presuppose stratified reference, not the subinterval property:

(22) waltz for an hour

**Satisfied presupposition:**
\[
\text{SR}_{τ,ε}(\text{[three hours]})(\text{[waltz]})
\]
\[
⇔ ∀e[\text{waltz}(e) \rightarrow e ∈ *λe' \left( \text{waltz}(e') ∧ \varepsilon(λt[\text{hours}(t) = 1])(τ(e')) \right)]
\]
(Every waltzing event consists of waltzing subevents whose runtimes are very small compared to an hour.)

(23) eat apples for three hours

**Satisfied presupposition:**
\[
\text{SR}_{τ,ε}(\text{[three hours]})(\text{[push carts]})
\]
\[
⇔ ∀e[\text{eat apples}(e) \rightarrow e ∈ *λe' \left( \text{eat apples}(e') ∧ \varepsilon(λt[\text{hours}(t) = 3])(τ(e')) \right)]
\]
(Every apple-eating event consists of apple-eating subevents whose runtimes are very small compared to three hours.)

(24) *eat ten apples for three hours

**Failing presupposition:**
\[
\text{SR}_{τ,ε}(\text{[three hours]})(\text{[eat ten apples]})
\]
\[
⇔ ∀e[\text{eat ten apples}(e) \rightarrow e ∈ *λe' \left( \text{eat ten apples}(e') ∧ \varepsilon(λt[\text{hours}(t) = 3])(τ(e')) \right)]
\]
(Every eating-ten-apples event consists of eating-ten-apples subevents whose runtimes are very small compared to three hours.)
(25) *widen 2cm for 5 meters
   Failing presupposition: SR_{location,\varepsilon(\{five\ \text{meters}\})}(\{\text{widen 2 cm}\})
   (Every widening-2-cm event consists of widening-2-cm subevents whose spatial locations
   are very small compared to five meters.)

6.4 Back to the measurement puzzle

Why can you not say *thirty degrees of water?

As we have seen, for-adverbials reject certain measure functions too:

(26) a. *John drove for thirty miles an hour.  
   b. *The soup boiled for 100 degrees Celsius.

Null assumption These sentences have parametrized presuppositions of the same kind as
temporal and spatial for-adverbials.

(27) *drive for thirty miles per hour
   Failing presupposition: SR_{speed,\varepsilon(\{thirty\ \text{mph}\})}(\{\text{drive}\})
   (Every driving event consists of driving subevents whose speeds are very small compared
to thirty mph.)

(28) *boil for 100 degrees Celsius
   Failing presupposition: SR_{temperature,\varepsilon(\{100\ \text{degrees}\})}(\{\text{boil}\})
   (Every boiling event consists of boiling subevents whose temperatures are very small
   compared to 100 degrees.)

Now we transfer this idea to pseudopartitives.

Intuition: run for three hours \approx three hours of running

(29) a. five pounds of books  
    b. thirty liters of water  
    c. *five pounds of book

“John walked for three hours.” three hours …is the runtime of … walk
“three hours of walking” three hours …is the runtime of … walk
“three liters of water” three liters …is the volume of … water
6.4.1 Baseline examples

Assumption: Same presuppositions for for-adverbials and pseudopartitives.

(30) run for three hours / three hours of running

Satisfied presupposition: \( SR_{\text{run}}([\text{three hours}])([\text{run}]) \)
(Every running event consists of running subevents whose runtimes are very small compared to three hours.)

The dimension parameter is the appropriate measure function.

(31) thirty liters of water

Satisfied presupposition: \( SR_{\text{volume}}([\text{thirty liters}])([\text{water}]) \)
(Every water amount consists of water parts whose volumes are very small compared to thirty liters.)

6.4.2 Temperature in pseudopartitives

No smaller temperatures as you go from bigger to smaller amounts of substance.

(32) *thirty degrees Celsius of water

Failing presupposition: \( SR_{\text{temperature}}([\text{thirty degrees Celsius}])([\text{water}]) \)
(Every water amount consists of water parts whose temperatures are very low compared to thirty degrees Celsius.)

6.4.3 The problematic snow example

Unlike Schwarzschild’s, this account has no monotonicity requirement.

(33) five feet of snow

Satisfied presupposition: \( SR_{\text{height}}([\text{five feet}])([\text{snow}]) \)
(Every snow amount consists of snow parts whose heights are very small compared to five feet.)

6.4.4 Ruling out singular count nouns

Singular count nouns are ruled out because they are quantized.

(34) *five pounds of book

Failing presupposition: \( SR_{\text{weight}}([\text{five pounds}])([\text{book}]) \)
Figure 6.3: Accepting five feet of snow

(Every book consists of parts which are themselves books (!) and whose weights are very small compared to five pounds.)

The answers to the measurement puzzle.

1. How can we characterize the class of admissible measure functions?
   - A pseudopartitive has to satisfy stratified reference, where the dimension parameter is specified by the measure function.

2. Why are not all measure functions admissible in the first place?
   - The constraint on measure functions is also instantiated in for-adverbials and other constructions.
Lecture 7

Distributivity and stratified reference

7.1 Introduction

- Today we will concentrate on cross-categorial parallels between atelic aspect, mass reference, plural reference, and distributivity.

- There is an intuitive parallelism between the telic-atelic, collective-distributive, singular-plural, and count-mass oppositions.

- Singular, telic, and collective predicates are delimited or bounded in ways that plural, mass, atelic, and distributive predicates are not.

- The boundedness question: How can the difference between boundedness and unboundedness be formally characterized?

- Let’s first look at distributivity in more detail.

7.2 What is distributivity?

- The use of the word *distributivity* generally indicates the application of a predicate to the members or subsets of a set, or to the parts of an entity.

- There are no standard definitions of distributivity. But it is diagnosed by the presence of *distributive entailments.*
7.2.1 Predicative distributivity

- We have already talked about predicative distributivity in Lecture 4.

(1) Distributive predicates
   a. The children smiled. ⇔ Every child smiled.

(2) Collective predicates

7.2.2 Quantificational distributivity

- This involves noun phrases headed by determiners like every or each.

- The truth conditions of these noun phrases involve application of the verbal predicate to each member of their witness set.

7.2.3 Relational distributivity

- Two constituents that contribute to the content of a distributive entailment stand in a distributive relation.

- A theory that relies heavily on this concept is developed in Choe (1987).

(3) a. Al and Bill each ate a pizza.
    b. Al and Bill ate a pizza.

- In (3a), the relation between the subject and the verb phrase is obligatorily distributive. In (3b), it is optionally distributive.

- A distributive relation can be indicated by distributive markers such as each in (3a) (see Gil (1982b) for a cross-linguistic survey).
7.3 Distributive constructions

- We can also see distributivity as a property of entire constructions.

- A distributive construction is a lexicosyntactic configuration that imposes an obligatory distributive relation between two of its constituents.

- Sentences with each are distributive constructions.

7.3.1 Pseudopartitives

- The pseudopartitive can also be classified as a distributive construction.

(4) Three liters of water are sufficient.

- Distributive entailment: among the parts of the water in question there exist one liter of water, two liters of water, and so on.

- It is required by the construction, because every pseudopartitive gives rise to similar entailments.

- Since pseudopartitives obligatorily involve a distributive relation, I classify them as distributive constructions.

7.3.2 For-adverbials

- For-adverbials license entailments of the following kind:

(5) a. John ran for five minutes.
    b. ⇒ John ran for four minutes.
    c. ⇒ John ran for three minutes.

- These entailments are distributive entailments because they are jointly determined by the for-adverbial and the predicate it modifies.

- So, for-adverbials are distributive constructions.
7.4 The components of a distributive relation

- There are many names for the components of a distributive relation. Table 7.1 lists a few.

Table 7.1: Terms for the components of a distributive relation

<table>
<thead>
<tr>
<th>Author</th>
<th>Name for “The boys”</th>
<th>Name for “(took) a breath”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Link (1986)</td>
<td>distributional domain</td>
<td>Distributive Share (DstrShr)</td>
</tr>
<tr>
<td>Choe (1987)</td>
<td>Sorting Key (SrtKy)</td>
<td>Distributed Share (DstrShr)</td>
</tr>
<tr>
<td>Safr and Stowell (1988)</td>
<td>Range NP</td>
<td>Distributing NP (DistNP)</td>
</tr>
<tr>
<td>Gil (1989), Choe (1991)</td>
<td>Key</td>
<td>Share</td>
</tr>
<tr>
<td>Zimmermann (2002)</td>
<td>DistKey</td>
<td>DistShare</td>
</tr>
<tr>
<td>Blaheta (2003)</td>
<td>Dist phrase</td>
<td>Range</td>
</tr>
<tr>
<td><strong>This work</strong></td>
<td><strong>Key</strong></td>
<td><strong>Share</strong></td>
</tr>
</tbody>
</table>

- I adopt the terms *Key* and *Share*.

- The term *Share* refers to the constituent whose denotation is distributed over the parts of the referent of the other constituent.

- This other constituent is called the *Key*.

- The property of reading a book is distributed over the individual boys, the property of being water is distributed over the liters, and the property of being an event of pushing a cart is distributed over the hours (see Table 7.2).

Table 7.2: A bridge from distributivity to aspect and measurement

<table>
<thead>
<tr>
<th>Construction</th>
<th>Example</th>
<th>Key</th>
<th>Share</th>
<th>Map</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adverbial <em>each</em></td>
<td>Three boys each laughed</td>
<td>three boys</td>
<td>laugh</td>
<td>agent</td>
</tr>
<tr>
<td><em>For</em>-adverbial</td>
<td>John ran for three hours</td>
<td>three hours</td>
<td>John run</td>
<td>runtime</td>
</tr>
<tr>
<td>Pseudopartitive</td>
<td>three liters of water</td>
<td>three liters</td>
<td>water</td>
<td>volume</td>
</tr>
</tbody>
</table>

- According to our background assumptions, constituents are related by certain covert functions: thematic roles, trace functions, and measure functions.

- These functions coincide with distributive relations in a particular way: they always map entities associated with the Share to entities associated with the Key. I call them *Maps*. 

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7.5 The constraints on distributive constructions

- Distributive constructions impose constraints on their Shares:

(6)  
  a. The boys each **walked**.  
  b. *The boys each **met**.  

(7)  
  a. John **ran** for five minutes.  
  b. *John **ran to the store** for five minutes.  

(8)  
  a. thirty pounds of **books**  
  b. thirty liters of **water**  
  c. *thirty pounds of **book**  

- I will formulate a constraint which subsumes the traditional accounts of these constraints.
- Along the way, we’ll motivate stratified reference in various ways.
- We start by modeling lexical distributivity:

(9)  
**Stratified distributive reference (preliminary definition 1)**

\[
\text{SDR}_{ag}(P) \equiv \forall e [P(e) \rightarrow \forall e' \leq e \left( P(e') \land \text{PureAtom}^{*}(ag(e')) \right)]
\]

- This preliminary definition does not work correctly (why)?
- Consider now the following definition.

(10)  
**Stratified distributive reference (preliminary definition 2)**

\[
\text{SDR}_{ag}(P) \equiv \forall e [P(e) \rightarrow e \in ^* \mu e' \left( P(e') \land \text{PureAtom}^{*}(ag(e')) \right)]
\]

- Now we generalize to arbitrary thematic roles \( \theta \).

(11)  
**Final definition: Stratified distributive reference**

\[
\text{SDR}_{\theta}(P) \equiv \forall e [P(e) \rightarrow e \in ^* \mu e' \left( P(e') \land \text{PureAtom}(\theta(e')) \right)]
\]

- This accounts for the inferential behavior of predicates like **kill** which are distributive on only one of their arguments, in this case the theme.
(12) a. Officers 1, 2, and 3 killed Bonnie and Clyde.
   \[ \exists e [ \text{kill}(e) \land \text{*ag}(e) = o_1 \oplus o_2 \oplus o_3 \land \text{*th}(e) = b \oplus c] \]
b. \( \Rightarrow \) Bonnie was killed.
c. \( \not\Rightarrow \) Officer 1 killed someone.

(13) **Meaning postulates: kill is distributive with respect to themes but not agents**
   a. \( \text{SDR}_{th}(\text{kill}) \)
   b. \( \neg \text{SDR}_{ag}(\text{kill}) \)

### 7.5.1 Adverbial *each*

- We can now formally express the fact that the Share of a sentence with adverbial *each* must be a distributive predicate.

(14) **Constraint on Shares of adverbial-*each* sentences**
A sentence with adverbial *each* whose Key is \( K \), whose Share is \( S \) and whose Map is \( M \) is acceptable only if \( S \) has stratified distributive reference with respect to \( M \) (formally: \( \text{SDR}_M(S) \)).

- Example of a distributive predicate:

(15) Three boys each laughed.
   a. Key: Three boys
   b. Map: agent
   c. Share: laugh

(16) \( \text{SDR}_{ag}(\text{laugh}) \)
\[ \iff \forall e [ \text{laugh}(e) \rightarrow e \in \lambda e' ( \text{laugh}(e') \land \text{PureAtom}(\text{*ag}(e'))) ] \]

- Example of a collective predicate:

(17) *Three boys each met.*
   a. Key: Three boys
   b. Map: agent
   c. Share: meet

(18) \( \text{SDR}_{ag}(\text{meet}) \)
\[ \iff \forall e [ \text{meet}(e) \rightarrow e \in \lambda e' ( \text{meet}(e') \land \text{PureAtom}(\text{*ag}(e'))) ] \]
7.5.2 Temporal for-adverbials

- The Share of a temporal for-adverbial must be an atelic predicate.

- We have seen in Lecture 6 that this is traditionally formalized in terms of the subinterval property:

\[(19) \text{ Definition: Subinterval property} \]

\[ \text{SUB}(P) \overset{\text{def}}{=} \forall i [\text{AT}(P, i) \rightarrow \forall j[j < i \rightarrow \text{AT}(P, j)]] \]

- I assume that what counts as very small is a vague notion. How can we formalize this? A cutoff point \( k \) is too crude but will do.

- Define \( \varepsilon \) as follows.

\[(20) \text{ Definition: } \varepsilon \]

For any predicate \( K \), if \( K \subseteq \star \text{PureAtom} \), then \( \varepsilon(K) = \text{PureAtom} \).

Otherwise, \( \varepsilon(K) = \lambda x[M(x) \leq k] \), where \( k \) is some constant that is very small with respect to the entities in \( K \).

- Definition (21) implements a weaker version of the subinterval property that avoids the minimal-parts problem:

\[(21) \text{ Definition: Stratified subinterval reference} \]

\[ \text{SSR}_{\varepsilon(K)}(P) \overset{\text{def}}{=} \forall e[P(e) \rightarrow e \in \star \lambda e'[P(e') \land \varepsilon(K)(\tau(e'))] \]

- The basic intuition here is the following:

  - Instead of testing whether the predicate holds at every subinterval of the interval \( i \), the definition only tests whether this interval can be divided into sufficiently small subintervals.
  
  - The definition of sufficiently small is specified through the threshold parameter \( \varepsilon(K) \).

- We can now formulate the constraint with the help of stratified subinterval reference.
(22) **Constraint on Shares of temporal for-adverbials**
A temporal for-adverbial whose Key is $K$ and whose Share is $S$ is acceptable only if $S$ has stratified subinterval reference (formally: $SSR_{\varepsilon(K)}(S)$), where the threshold $\varepsilon(K)$ holds of intervals whose runtime is very short with respect to $K$.

- Example:

(23) John and Mary waltzed for an hour.
   a. Key: an hour
   b. Threshold: e.g. $\lambda i.\text{seconds}(i) \leq 3$
   c. Share: waltz

- Applied to example (23), constraint (22) predicts that the sentence is acceptable only if the condition $SSR_{\lambda i.\text{seconds}(i)} \leq 3(\text{waltz})$ is fulfilled.

- This condition expands as follows:

(24) $SSR_{\lambda i.\text{seconds}(i)} \leq 3(\text{waltz})$
    $\Leftrightarrow \forall e[\text{waltz}(e) \rightarrow e \in ^{*}\lambda e' \left( \text{waltz}(e') \land \text{seconds}(\tau(e)) \leq 3 \right)]$

- Another example:

(25) *John built a sand castle for an hour.
   a. Key: an hour
   b. Threshold: e.g. $\lambda i.\text{seconds}(i) \leq 3$
   c. Share: build a sand castle

Applied to example (25), constraint (22) predicts that the sentence is acceptable only if the condition $SSR_{\lambda i.\text{seconds}(i)} \leq 3(\text{build.a.sand.castle})$ is fulfilled.

- This condition expands as follows:

(26) $SSR_{\lambda i.\text{seconds}(i)} \leq 3(\text{build.a.sand.castle})$
    $\Leftrightarrow \forall e[\text{build.a.sand.castle}(e) \rightarrow e \in ^{*}\lambda e' \left( \text{build.a.sand.castle}(e') \land \text{seconds}(\tau(e')) \leq 3 \right)]$

- Aspectual composition can be used to show that this condition is not fulfilled by the predicate $\text{build.a.sand.castle}$.
7.5.3 Pseudopartitives

- The Share (the substance noun) of a pseudopartitive must be a mass term or a plural count term.
- As a first approximation, this can be formalized using divisive reference:

\[(27) \text{ Definition: Divisive reference} \]
\[
\text{DIV}(P) \equiv \forall x [P(y) \rightarrow \forall y [y < x \rightarrow P(y)]]
\]

- But not all mass terms have divisive reference.
- Consider the word *cable* in its mass noun sense (e.g. *a spool of cable, a lot of cable*).
- Not every part of a cable segment qualifies itself as a cable segment. Linear segments of a cable segment do, but slices which are taken through the middle along the length of the cable segment do not. Schwarzschild (2002, 2006) observes that these facts correlate with the interpretation of pseudopartitives whose substance noun is *cable*:

\[(28) \text{three inches of cable} \quad \checkmark \text{length, *diameter}\]

- We can represent the two potential readings by varying the choice of Map:

\[(29) \lambda x [\text{cable}(x) \land \exists d [\text{length}(x) = d \land \text{inches}(d) = 3]]
\]
(available)

\[(30) \lambda x [\text{cable}(x) \land \exists d [\text{diameter}(x) = d \land \text{inches}(d) = 3]]
\]
(unavailable)

- What constraint rules out (30) but not (29)?
- Intuitively, the mass noun *cable* is divisive along its length, but not along its diameter.
- So let’s add a parameter \(\mu\) for measure functions to the concept of divisive reference.

\[(31) \text{Stratified measurement reference (preliminary definition)}
\]
\[
\text{SMR}_\mu(P) \equiv \forall x [P(x) \rightarrow \forall d [d < \mu(x) \rightarrow \exists y (y < x \land P(y) \land \mu(y) = d)]]
\]

- Example:
\( \text{SMR}_{\text{length}}(\text{cable}) \Leftrightarrow \)
\[ \forall x [\text{cable}(x) \to \forall d [d < \text{length}(x) \to \exists y (y < x \land \text{cable}(y) \land \text{length}(y) = d)]] \]

- Cable segments do not contain cable segments with a smaller diameter:

\( \neg \text{SMR}_{\text{diameter}}(\text{cable}) \Leftrightarrow \)
\[ \neg \forall x [\text{cable}(x) \to \forall d [d < \text{diameter}(x) \to \exists y (y < x \land \text{cable}(y) \land \text{diameter}(y) = d)]] \]

- We haven’t yet taken care of the minimal-parts problem for mass terms.
- Very short slices taken from a cable segment may not qualify as cable segments.
- The solution is analogous to the case of for-adverbials:

\( \text{Final definition: Stratified measurement reference} \)
\[ \text{SMR}_{\mu,\varepsilon(K)}(P) \overset{\text{df}}{=} \forall x [P(x) \to x \in \ast \lambda y \left( P(y) \land \varepsilon(K)(\mu(y)) \right)] \]

- We can now formulate the constraint on Shares in pseudopartitives:

\( \text{Constraint on Shares of pseudopartitives} \)

A pseudopartitive whose Key is \( K \), whose Share is \( S \) and whose Map is \( M \) is acceptable only if \( S \) has stratified measurement reference with respect to \( M \) (formally: \( \text{SMR}_{M,\varepsilon(K)}(S) \)), where the threshold \( \varepsilon(K) \) is very small with respect to \( K \).

- Why is length available as a Map?

\( \text{three inches of cable} \)

a. Key: three inches
b. Share: cable
c. Map: length
d. Threshold: e.g. \( \lambda d.\text{inches}(d) \leq 0.5 \)

- Is the condition \( \text{SMR}_{\text{length}, \lambda d.\text{inches}(d) \leq 0.5}(\text{cable}) \) fulfilled?

\[ \text{SMR}_{\text{length}, \lambda d.\text{inches}(d) \leq 0.5}(\text{cable}) \]
\[ \Leftrightarrow \forall x [\text{cable}(x) \to x \in \ast \lambda y \left( \text{cable}(y) \land \text{inches}(\text{length}(y)) \leq 0.5 \right)] \]
• We now no longer need to assume that cable has divisive reference relative to length.

• Why is diameter unavailable?

(38) three inches of cable
  a. Key: three inches
  b. Share: cable
  c. Map: diameter
  d. Threshold: e.g. \( \lambda d.\text{inches}(d) \leq 0.5 \)

• Is the condition \( \text{SMR}_{\text{diameter}, \lambda d.\text{inches}(d) \leq 0.5(\text{cable})} \) fulfilled?

(39) \( \text{SMR}_{\text{diameter}, \lambda d.\text{inches}(d) \leq 0.5(\text{cable})} \)
    \( \Leftrightarrow \forall x[ \text{cable}(x) \rightarrow x \in ^{*}\lambda y \left( \text{cable}(y) \land \text{inches}(\text{diameter}(y)) \leq 0.5 \right) ] \)

• This condition is not fulfilled (why not?)

7.6 Unifying the constraints

• Putting the properties side by side:

(40) **Definition: Stratified distributive reference**
    \[ \text{SDR}_{\theta}(P) \overset{\text{def}}{=} \forall e[P(e) \rightarrow e \in ^{*}\lambda e' \left( P(e') \land \text{PureAtom}(\theta(e')) \right) ] \]
    = (11)

(41) **Definition: Stratified subinterval reference**
    \[ \text{SSR}_{\epsilon(K)}(P) \overset{\text{def}}{=} \forall e[P(e) \rightarrow e \in ^{*}\lambda e' \left( P(e') \land \epsilon(K)(\tau(e')) \right) ] \]
    = (21)

(42) **Definition: Stratified measurement reference**
    \[ \text{SMR}_{\mu,\epsilon(K)}(P) \overset{\text{def}}{=} \forall x[P(x) \rightarrow x \in ^{*}\lambda y \left( P(y) \land \epsilon(K)(\mu(y)) \right) ] \]
    = (34)

• They can now be subsumed under one common property: stratified reference.

(43) **Definition: Stratified reference**
    \[ \text{SR}_{f,\epsilon(K)}(P) \overset{\text{def}}{=} \forall x[P(x) \rightarrow x \in ^{*}\lambda y \left( P(y) \land \epsilon(K)(f(y)) \right) ] \]
• Consider again the constraints on distributive constructions:

(44) **Constraint on Shares of adverbial-*each* sentences** = (14)
A sentence with adverbial *each* whose Key is $K$, whose Share is $S$ and whose Map is $M$ is acceptable only if $S$ has stratified distributive reference with respect to $M$ (formally: $\text{SDR}_M(S)$).

(45) **Constraint on Shares of temporal *for*-adverbials** = (22)
A temporal *for*-adverbial whose Key is $K$ and whose Share is $S$ is acceptable only if $S$ has stratified subinterval reference (formally: $\text{SSR}_{\varepsilon(K)}(S)$), where the threshold $\varepsilon(K)$ holds of intervals whose runtime is very short with respect to $K$.

(46) **Constraint on Shares of pseudopartitives** = (35)
A pseudopartitive whose Key is $K$, whose Share is $S$ and whose Map is $M$ is acceptable only if $S$ has stratified measurement reference with respect to $M$ (formally: $\text{SMR}_{M,\varepsilon(K)}(S)$), where the threshold $\varepsilon(K)$ is very small with respect to $K$.

• These constraints can now be subsumed by the following statement:

(47) **Distributivity Constraint**
A distributive construction whose Key is $K$, whose Share is $S$ and whose Map is $M$ is acceptable only if $S$ has stratified reference with respect to $M$ (formally: $\text{SR}_{M,\varepsilon(K)}(S)$).

### 7.7 Summary

• My answer to the boundedness question: Stratified reference characterizes what it means to be an unbounded predicate. Unboundedness can be understood in more than one way. These different ways correspond to the parameters of stratified reference.

  – One and the same verb phrase, *run to the store*, can be distributive (unbounded with respect to agents) and telic (bounded with respect to runtime).
  
  – One and the same verb, *kill*, can be collective (bounded) with respect to agents and distributive (unbounded) with respect to themes.
  
  – One and the same noun, *cable*, can be divisive along its length (bounded with respect to length) and nondivisive along its diameter (unbounded with respect to diameter).
Figure 7.1: Skeletal LF of an ordinary pseudopartitive

\[
\langle et \rangle \\
\lambda x [\text{water}(x) \land \text{liters(volume}(x)) = 3]
\]
	hree liters \langle n, dt \rangle \lambda n \lambda d [\text{liters}(d) = n] \\
\text{volume} \langle ed \rangle \lambda x [\text{volume}(x)] \\
\text{of} \quad \text{water} \langle et \rangle \\
\lambda x [\text{water}(x)]

Figure 7.2: Skeletal LF of an event pseudopartitive

\[
\langle vt \rangle \\
\lambda e [\ast \text{walk}(e) \land \text{hours}(\tau(e)) = 3]
\]
	hree hours \langle n, it \rangle \lambda n \lambda t [\text{hours}(t) = n] \\
\text{runtime} \langle vi \rangle \lambda e [\tau(e)] \\
\text{of} \quad \text{walking} \langle vt \rangle \\
\lambda e [\ast \text{walk}(e)]

**Figure 7.3:** Skeletal LF of a *for*-adverbial

\[
\lambda e[\text{*walk}(e) \wedge \text{hours}(\tau(e)) = 3]
\]

**Figure 7.4:** Skeletal LF of an adverbial-\textit{each} construction

\[
\lambda e[\text{*walk}(e) \wedge \text{boy}(\text{*ag}(e)) \wedge |\text{ag}(e)| = 3]
\]
Lecture 8

For, each, all

8.1 All vs. every/each

8.1.1 Numerous-type predicates

• First observed by Kroch (1974) and Dowty (1987) independently.

(1) a. The students who came to the rally are numerous.
   b. The men who run this country are politically homogeneous.
   c. The people on this boat are a motley crew.
   d. The soldiers in this bataillon sufficed to defeat the army.

(2) a. *All the students who came to the rally are numerous.
   b. *All the men who run this country are politically homogeneous.
   c. *All the people on this boat are a motley crew.
   d. *All the soldiers in this bataillon sufficed to defeat the army.

(3) a. The ants in the colony were numerous. *distributive, ✓ collective
   b. The enemy armies were numerous. ✓ distributive, ✓ collective

(4) a. *Each ant in the colony was numerous. *distributive, *collective
   b. Each enemy army was numerous. ✓ distributive, *collective

(5) a. *All the ants in the colony were numerous. *distributive, *collective
   b. All the enemy armies were numerous. ✓ distributive, *collective

• These predicates motivate treating all in analogous terms to each and every.
8.1.2 Gather-type predicates

- *Gather-type* predicates can lead to distributive and collective interpretations, even in the presence of the word *all*.

(6)  a. The students gathered in the hallway.
    b. The professors met in the garden.
    c. The soldiers dispersed.

(7)  a. All the students gathered in the hallway.
    b. All the professors met in the garden.
    c. All the soldiers dispersed.

(8)  a. All the students gathered in the hall. *distributive, ✓ collective
    b. *Each student gathered in the hall. *distributive, *collective

(9)  a. All the committees gathered in the hall. ✓ distributive, ✓ collective
    b. Each committee gathered in the hall. ✓ distributive, *collective

- These predicates are a challenge for any account that treats *all* analogously to *each* and *every*.

8.2 Similarities between *for* and *all*

8.2.1 *For* and *all* block cumulative readings

- Zweig (2008) notes that the presence of *all* can block otherwise available cumulative readings.

(10)  a. All the safari participants saw thirty zebras.  

    *Unavailable cumulative reading*: Each safari participant saw at least one zebra, and thirty zebras were seen overall.

    b. Three safari participants saw thirty zebras.  

    *Available cumulative reading*: Three safari participants each saw at least one zebra and thirty zebras were seen overall.

- Similarly, a *for*-adverbial cannot enter a scopeless relation with an indefinite:
(11)  
   a. John saw thirty zebras for three hours.  
      *Unavailable reading:* John saw a total of thirty zebras over the course of a three-hour timespan.  
   b. John saw thirty zebras in three hours.  
      *Available reading:* John saw a total of thirty zebras over the course of a three-hour timespan.

8.2.2  *For* and *all* license dependent plurals

- *For*-adverbials and *all* both give rise to dependent plurals.

(12) All the boys flew kites.
   a.  *Distributivity component:* Each boy flew at least one kite.
   b.  *Multiplicity component:* At least two kites were flown in total.

(13) John flew kites for five hours.
   a.  *Distributivity component:* At each time, John flew at least one kite.
   b.  *Multiplicity component:* John flew at least two kites in total.

- Other examples:

(14) a.  At the party, five boys wore yellow neckties.
    b.  At the party, each boy wore yellow neckties.
    c.  At the party, all the boys wore yellow neckties.
    d.  At the party, John wore yellow neckties for five hours.

8.3  Explaining the similarities between *for* and *all*

(15) **Definition: Stratified reference**

\[
\text{SR}_{f,\varepsilon(K)}(P) \triangleq \forall x [P(x) \rightarrow x \in \overset{\star}{\lambda} y \left( P(y) \land \varepsilon(K)(f(y)) \right)]
\]

(A predicate \(P\) has stratified reference with respect to a function \(f\) and a threshold \(\varepsilon(K)\) if and only if there is a way of dividing every entity in its denotation exhaustively into parts which are each in \(P\) and which have a very small \(f\)-value. Very small \(f\)-values are those that satisfy \(\varepsilon(K)\).)

- I assume that *all* presupposes SR and lexically sets the granularity parameter to the predicate *Atom.*
\[ \text{all}_a = \lambda x \lambda P \langle vt \rangle \lambda e : \text{SR}_{e,\text{Atom}}(P). [P(e) \wedge *ag(e) = x] \]

\[ \text{all}_a \text{ the boys} = \lambda P \langle vt \rangle \lambda e : \text{SR}_{e,\text{Atom}}(P). [P(e) \wedge *ag(e) = \bigoplus \text{boy}] \]

### 8.3.1 For and all block cumulative readings

- For-adverbials and all both block cumulative readings. The first case is familiar from theories of aspectual composition.

(18) John saw thirty zebras for three hours.
\[ \exists e [*\text{see}(e) \wedge *ag(e) = j \wedge \text{hours}(\tau(e)) = 3 \wedge *\text{zebra}(x \text{th}(e)) \wedge |*\text{th}(e)| = 30] \]
Presupposition: \( \text{SR}_{e,\lambda t \langle \text{hours}(t) = 3 \rangle}(\lambda e [*\text{see}(e) \wedge *\text{zebra}(x \text{th}(e)) \wedge |*\text{th}(e)| = 30]) \)
(Every event in which thirty zebras are seen consists of one or more seeing events whose runtimes are very short compared with three hours and whose themes are sums of thirty zebras.)

(19) Three safari participants saw thirty zebras.
\[ \exists e [*\text{safari.participant}(x \text{ag}(e)) \wedge |*\text{ag}(e)| = 3 \wedge *\text{see}(e) \wedge *\text{zebra}(x \text{th}(e)) \wedge |*\text{th}(e)| = 30] \]

(20) John saw thirty zebras for three hours.
\[ \exists x [*\text{zebra}(x) \wedge |x| = 30 \wedge \forall y \leq \text{Atom} x \exists e [*\text{see}(e) \wedge *ag(e) = j \wedge \text{hours}(\tau(e)) = 3 \wedge *\text{th}(e) = y]] \]
(There are thirty zebras each of which John saw for three hours.)
Presupposition: \( \forall y \leq \text{Atom} x [\text{SR}_{e,\lambda t \langle \text{hours}(t) = 3 \rangle}(\lambda e [*\text{see}(e) \wedge *\text{th}(e) = y])] \)
(For each \( y \) among the thirty zebras, every seeing event whose theme is \( y \) consists of one or more seeing events whose runtimes are very short with respect to three hours, and whose theme is again \( y \).)

- Let all the safari participants combine directly with see thirty zebras.

(21) All the safari participants saw thirty zebras.
\[ \exists e [*\text{see}(e) \wedge *ag(e) = \bigoplus \text{safari.participant} \wedge *\text{zebra}(x \text{th}(e)) \wedge |*\text{th}(e)| = 30] \]
Presupposition: \( \text{SR}_{e,\text{Atom}}(\lambda e [*\text{see}(e) \wedge *\text{zebra}(x \text{th}(e)) \wedge |*\text{th}(e)| = 30]) \)
(Every event in which thirty zebras are seen consists of one or more seeing events whose agents are atomic and whose themes are sums of thirty zebras.)

- Sentence (21) does have available interpretations, just not the cumulative one. For example, it has a distributive reading. This can be derived by applying the D operator.
8.3.2 For and all license dependent plurals

- In Zweig’s system, only in-situ subjects can license dependent plurals. So what blocks the cumulative reading of all? Zweig proposes this and comments as follows (Zweig 2008, Section 6.2.6).

\[
\text{[all the boys]} \text{(Zweig)} = \lambda P_{(et)} [\text{\text{\begin{array}{l} P(\bigoplus \text{ boy}) \end{array}}}]
\]

(True of predicates whose algebraic closure holds of the sum of all boys.)

(23) “[T]here is no difference between saying “there is a sum of boys, such that each of its atomic parts is the agent of an event of flying kites”, and “there is a sum of boys that is the agent of an event of flying kites”. This stands in contrast to the case with a numerical object; “there is a sum of boys, such that each of its atomic parts is the agent of an event of flying two kites” is not equivalent to “there is a sum of boys that is the agent of a sum of events of flying two kites”.”

- This doesn’t describe (22) but rather it describes the effect that stratified reference has on all on my account. (22) won’t work.

8.4 Explaining the behavior of numerous and gather

- Idea: gather-type and numerous-type collective predicates instantiate two different notions that have both been called collective predication

8.4.1 Collective predication

- General idea: a predicate that applies to a plural entity as a whole, as opposed to applying to the individuals that form this entity

- Two distinct views of collectivity (Verkuyl 1994)

8.4.1.1 Thematic collectivity

- Defined by the presence of noninductive entailments, e.g. collective responsibility or collective action (Landman 2000)

(24) a. The Marines invaded Grenada. (Roberts 1987, p. 147)
b. The boys touch the ceiling. (Landman 2000)
c. The boys carried the piano upstairs. (Landman 2000)

• Sentence (24a) is about the Marines as an institution.
• Following Landman (1989), I model thematic collectivity by using groups ("impure" atoms)
• For any set of individuals there is also a group, and this group is an atom

(25)  
\[ \exists e [ \ast \text{carry.the.piano.upstairs}(e) \land \ast \text{ag}(e) = j \oplus b ] \]

\[ \exists e [ \ast \text{carry.the.piano.upstairs}(e) \land \ast \text{ag}(e) = \uparrow (j \oplus b) ] \]

8.4.1.2 Nonthematic collectivity

• Negative definition: a predicate that does not distribute down to singular individuals
• E.g. a plurality of people may be numerous (that is, if it has many members), but it does not even make sense to apply the predicate numerous to a single person.
• My conjecture:

(26)  
\[ \text{a. Thematic collectivity} = \text{gather-type collective predicates} \]
\[ \text{b. Nonthematic collectivity} = \text{numerous-type collective predicates} \]

(27)  
The boys are numerous.
_The sum of all boys is numerous, i.e. is large in number._
\[ \exists e [ \ast \text{numerous}(e) \land \ast \text{ag}(e) = \bigoplus \text{boy} ] \]

(28)  
The boys gathered.
_The group of all boys gathered._
\[ \exists e [ \ast \text{gather}(e) \land \ast \text{ag}(e) = \uparrow (\bigoplus \text{boy}) ] \]

• I assume that being numerous is a property that a sum of individuals has _qua_ sum, and that the agents of nonthematic collective predicates can never be impure atoms.

8.4.2 All distinguishes between be numerous and gather

(29)  
\[ \text{a. *All the boys were numerous.} \quad \text{nonthematic collectivity} \]
\[ \text{b. All the boys gathered.} \quad \text{thematic collectivity} \]
• Presupposition of (29a):

\[ \text{SR}_{ag, \text{Atom}}(\lambda e[\text{numerous}(e)]) \iff \\
\forall e[\text{numerous}(e) \to e \in ^* (\lambda e'. \text{Atom}(ag(e')) \wedge \text{numerous}(e'))] \]

(Every event \(e\) in the denotation of \textit{be numerous} can be divided into one or more parts each of which is in the denotation of \textit{be numerous} and has an atomic agent.)

• This fails because \textit{be numerous} is nonthematic collective

• What about sentences with group nouns?

\begin{align*}
(31) & \text{The enemy armies were numerous.} & \checkmark \text{distributive, \checkmark collective} \\
(32) & \text{All the enemy armies were numerous.} & \checkmark \text{distributive, } *\text{collective}
\end{align*}

• Questions:
  
  – Why can \textit{all} and \textit{be numerous} cooccur?
  
  – Why does only (31) but not (32) have a distributive reading?

• Answer: because a collective predicate like \textit{be numerous} can be shifted into a distributive reading by the D operator

• In (31), the D operator leads to a distributive reading and its absence to a collective reading.

\begin{align*}
(33) & \text{Definition: Generalized event-based D operator} \\
& [D_{\theta, C}] \equiv \lambda P_{(vt)} \lambda e[e \in ^* \lambda e' (P(e') \wedge C(\theta(e')))] \\
& \text{(Takes an event predicate } P \text{ and returns a predicate that holds of any event } e \text{ which consists entirely of events that are in } P \text{ and whose } \theta \text{s satisfy the predicate } C.)
\end{align*}

\begin{align*}
(34) & [D_{ag, \text{PureAtom}}] = \lambda P_{(vt)} \lambda e[e \in ^* \lambda e' (P(e') \wedge \text{PureAtom}(ag(e')))] \\
& \text{(Takes an event predicate } P \text{ and returns a predicate that holds of any event } e \text{ which consists entirely of events that are in } P \text{ and whose agent is a pure atom.)}
\end{align*}

• \([\text{be numerous}] = \text{true of any event } e \text{ whose agent is numerous (many in number)}

• \([D(\text{be numerous})] = \text{true of any event } e \text{ which consists of one or more events that are in } \textit{be numerous} \text{ and whose agent is a pure atom (a singular individual).}

• This predicate applies to less events than \textit{be numerous} does.
Example:

(35)  
   a. The boys are numerous. \hspace{1cm} true only if D is absent  
   b. The army is numerous. \hspace{1cm} true whether D is present or not  

   - Remember that the boys refers to a sum but the army refers to a pure atom!

- \( D([\text{be numerous}]) \) satisfies the presupposition of all, even though be numerous does not.

- Answers to the previous questions:
  - Sentences like (31) are acceptable because they contain the distributive shift.
  - They only have a distributive interpretation because this shift introduces distributivity.

- Consider now the case of gather

- We need to explain why the presupposition of all is not violated by gather

- Since gather is thematic collective, All the boys gathered involves a group agent.

- Its presupposition is as follows:

\[
\text{SR}_{ag,\text{Atom}} (\lambda e [\text{gather}(e)]) \leftrightarrow \\
\forall e [\text{gather}(e) \rightarrow e \in (\lambda e' \cdot \text{Atom}(\text{ag}(e')) \land \text{gather}(e'))]
\]

(Every event e in the denotation of gather can be divided into one or more parts each of which is in the denotation of gather and has an atomic agent.)

- This presupposition is vacuously satisfied.

### 8.4.3 Gather distinguishes between each and all

- Why are gather-type collective predicates incompatible with all but compatible with each?

(37)  
   a. All the students gathered.  
   b. *Each student gathered.

- Idea:
  
  - Every and each distribute over pure atoms
– *All* distributes over atoms no matter whether they are pure or impure

- On the group interpretation, *the students* refers to an impure atom, so it is compatible with *all* but not with *each*.

- The presupposition of *each* is as follows:

\[(38) \text{SR}_\text{ag, PureAtom}(\lambda e[P(e)])\]
\[\Leftrightarrow \forall e[^*\text{walk}(e) \rightarrow e \in *\lambda e' (P(e') \land \text{PureAtom}(ag(e')))]\]

(Every event in P can be divided into parts which are in P and whose agents are pure atoms.)

- The presupposition of *all* is as follows:

\[(39) \text{SR}_\text{ag, Atom}(\lambda e[P(e)])\]
\[\Leftrightarrow \forall e[^*\text{walk}(e) \rightarrow e \in *\lambda e' (P(e') \land \text{Atom}(ag(e')))]\]

(Every event in P can be divided into parts which are in P and whose agents are atoms.)

- *Gather* satisfies the presupposition of *all* but not of *each*. It satisfies the presupposition of *all* because it only applies to events whose agents are atoms. It fails to satisfy the one of *each* because these atoms may be impure.

- Note that for group nouns like *committee*, *gather* shows distributive/collective ambiguity:

\[(40) \text{The committees all gathered.}\]

✓ distributive, ✓ collective

- This suggests that (40) optionally contains the distributive operator.

- The output of the D operator always satisfies the presupposition of *each* and of *all*. (Exercise: Prove this!)

\[(41) \text{The committees all gathered.}\]

a. **Collective reading:**
\[\exists e[^*\text{ag}(e) = \uparrow (\bigoplus \text{committee}) \land *\text{gather}(e)]\]
(There is a gathering event whose agent is the group of all committees.)

b. **Distributive reading:**
\[\exists e[^*\text{ag}(e) = \bigoplus \text{committee} \land e \in *\lambda e' \left( *\text{gather}(e') \land \text{PureAtom}(ag(e')) \right)]\]
(There is a plural gathering event whose agent is the sum of all committees and which consists of gathering events whose agents are pure atoms.)
Lecture 9

The scopal behavior of for-adverbials

- Central innovation of this lecture: a temporal version of the D operator
- Assume as a baseline analysis that for is a universal quantifier over instants, as if it was the temporal counterpart of every:

\[(1) \quad \text{[for an hour] (baseline)} = \lambda P_{(vt)} \exists t \left[ \text{hours}(t) = 1 \land \forall t' [t' <_{\text{Atom}} t \rightarrow \exists e [P(e) \land \tau(e) = t']] \right]\]

- Given this assumption, the scopal behavior of for-adverbials is surprising (Carlson 1977; Zucchi and White 2001):

\[(2) \quad \begin{align*}
\text{a. } & \text{John pushed a cart for an hour.} & \exists > \forall; *\forall > \exists \\
\text{b. } & \text{I dialed a wrong phone number for five minutes.} & \exists > \forall; *\forall > \exists \\
\text{c. } & \text{She bounced a ball for 20 minutes.} & \exists > \forall; *\forall > \exists \\
\text{d. } & \text{He kicked a wall for a couple of hours.} & \exists > \forall; *\forall > \exists \\
\text{e. } & \text{She opened and closed a drawer for half an hour.} & \exists > \forall; *\forall > \exists \\
\text{f. } & \text{I petted a rabbit for two hours.} & \exists > \forall; *\forall > \exists
\end{align*}\]

- Same pattern in German (Kratzer 2007). This is remarkable since quantifier scope in German normally follows surface order:

\[(3) \quad \begin{align*}
\text{a. } & \text{Ich hab’ fünf Minuten lang eine falsche Telefonnummer gewählt.} & \exists > \forall; *\forall > \exists \\
\text{b. } & \text{Ich hab’ eine falsche Telefonnummer fünf Minuten lang gewählt.} & \exists > \forall; *\forall > \exists
\end{align*}\]

- Same pattern even when the missing reading would be more plausible:
(4) John found a flea on his dog for a month. (Zucchi and White 2001)

- Plural indefinites pattern like singular indefinites:

(5) John saw thirty zebras for three hours.

- Bare plurals and bare mass nouns are an exception (Verkuyl 1972):

(6) a. John discovered fleas on his dog for six weeks.
   b. John discovered crabgrass in his yard for six weeks.

(7) a. Tourists discovered that quaint little village for years.
   b. Water leaked through John’s ceiling for six months.

- Another exception is the quantifier no (Rooth 1995)

(8) John ate no cookies for thirty days.

- Overt temporal adverbials, call them interveners, can “trap” the indefinite:

  - Context-dependent temporal definites (Deo and Piñango 2011)

(9) Jane ate an egg/two eggs at breakfast for a month.

  - Temporal universal quantifiers (Zucchi and White 2001)

(10) John found a flea on his dog every day for a month.

  - Pluractional adverbials (see also Beck and von Stechow 2007)

(11) John found a flea on his dog day after day for a month.

- Compare the baseline analysis with the one in (13):

(12) [for an hour] (baseline)
    \[
    = \lambda P_{(\langle t \rangle \neq \text{hours}(t) = 1 \land \forall t' [t' < \text{end} \rightarrow \exists e (P(e) \land \tau(e) = t')])}
    \]

(13) [for an hour] (my proposal)
    \[
    = \lambda P_{(\langle t \rangle \neq \text{hours}(t) = 1} (P).P(e) \land \text{hours}(\tau(e)) = 1
    \]

- This immediately predicts that the indefinite in (2a) must take wide scope.
(14) John pushed a cart for an hour. = (2a)

(15) \[\text{[push a cart]} = \lambda e[^*\text{push}(e) \land \text{cart}(*\text{th}(e))]\] (True of any pushing event or sum of pushing events whose theme is one and the same cart.)

(16) \[\text{[John pushed a cart for an hour]}\]
\[= \exists e : \text{SR}_{\tau \epsilon \pi}(\lambda \text{hours}(t) = 1) (\lambda e[^*\text{push}(e') \land \text{cart}(*\text{th}(e')))].\]
\[\text{[^*ag}(e) = j \land ^*\text{push}(e) \land \text{cart}(\text{th}(e)) \land \text{hours}(\tau(e)) = 1]\]
(There is a pushing event or sum of pushing events whose theme is one cart, whose agent is John, and whose runtime measures one hour.)

- As Kratzer (2007) points out, lexical cumulativity extends this style of account even to achievement verbs like find.

(17) \[\text{[John found a flea for a month]}\]
\[= \exists e : \text{SR}_{\tau \epsilon \pi}(\lambda \text{months}(t) = 1) (\lambda e[^*\text{find}(e') \land \text{flea}(\text{th}(e')))].\]
\[\text{[^*ag}(e) = j \land ^*\text{find}(e) \land \text{flea}(\text{th}(e)) \land \text{months}(\tau(e)) = 1]\]
(There is a finding event or sum of finding events whose theme is one flea, whose agent is John, and whose runtime measures one month.)

- Plurals and mass terms behave differently because they have cumulative reference.

(18) \[\text{[John found fleas for a month]}\]
\[= \exists e : \text{SR}_{\tau \epsilon \pi}(\lambda \text{months}(t) = 1) (\lambda e[^*\text{find}(e') \land ^*\text{flea}(\text{th}(e')))].\]
\[\text{[^*ag}(e) = j \land ^*\text{find}(e) \land ^*\text{flea}(\text{th}(e)) \land \text{months}(\tau(e)) = 1]\]
(There is a finding event or sum of finding events whose theme is a set of fleas, whose agent is John, and whose runtime measures one month.)

- This presupposition is fulfilled in this case (why?).

9.1 Nonatomic distributivity in for-adverbials

- We have seen that indefinites can’t covary with the for-adverbial.

(19) ??John found two fleas on his dog for a month.

(20) a. ??John noticed a discrepancy/two discrepancies for a week.
b. ??John discovered a new proof/two new proofs for a week. (Deo and Piñango 2011)
• But several authors have noted that there are exceptions.

(21) **Context**: the patient’s daily intake is discussed.
The patient took two pills for a month and then went back to one pill. (Moltmann 1991)

(22) **Context**: the bicycle is designed to carry around three children at a time. 
This bicycle carried three children around Amsterdam for twenty years. (Rothstein 2004)

(23) We built a huge snowman in our front yard for several years. 
(Deo and Piñango 2011)

• This inference takes time (self-paced reading tests, Todorova et al. (2000)):

(24) a. Even though Howard sent a large check to his daughter for many years, she refused to accept his money. longer reading time

   b. Even though Howard sent large checks to his daughter for many years, she refused to accept his money. shorter reading time

• This looks like nonatomic phrasal distributivity!

• Nonatomic lexical distributivity is shown in examples like this:

(25) Five thousand people gathered near Amsterdam. (van der Does 1993)

• Here the predicate *gather near Amsterdam* can be applied distributively (i.e. several gatherings) to nonatomic entities (a single person cannot gather)

• Nonatomic phrasal distributivity is usually unavailable (e.g. Lasersohn 1989):

(26) Rogers, Hammerstein and Hart wrote a musical.

   a. **True** if the three of them wrote a musical together. ✓ collective

   b. **True** if each of them wrote a musical by himself. ✓ atomic distributive

   c. **False** if Rodgers and Hammerstein wrote a musical together, and Rodgers and Hart wrote another musical together. *nonatomic distributive

(27) **Scenario** Al, Bill, Jim and Ed each weigh 100kg.

   a. **True**: The men weigh 300kg. ✓ collective (together)
b. **True**: The men weigh 100kg. ✓ atomic distributive (per man)
c. **False**: The men weigh 200kg. * nonatomic distributive (per pair)

- Exception: a level of granularity is made salient through context or world knowledge (Lasersohn 1995; Schwarzschild 1991, 1996)

(28) **Scenario** Two pairs of shoes are on display, each pair with a $50 price tag.
   a. The shoes cost $100. ✓ collective (together)
   b. The shoes cost $25. ? atomic distributive (per shoe)
   c. The shoes cost $50. ✓ nonatomic distributive (per pair)

- Nonatomic distributivity is much easier with a bare plural (Link 1997):

(29) a. Rodgers, Hammerstein and Hart wrote a musical. *pairwise
   b. Rodgers, Hammerstein, and Hart wrote musicals. pairwise

**Table 9.1**: V level versus verb phrase level distributivity in atomic domains

<table>
<thead>
<tr>
<th>(a) Empirical generalization</th>
<th>(b) Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>lexical (V level)</td>
<td>lexical (V level)</td>
</tr>
<tr>
<td>phrasal (VP level)</td>
<td>phrasal (VP level)</td>
</tr>
<tr>
<td>atomic available</td>
<td>meaning post. Atomic D op.</td>
</tr>
<tr>
<td>nonatomic available only w. context</td>
<td>meaning post. Cover-based D op.</td>
</tr>
</tbody>
</table>

**Table 9.2**: Distributivity in the temporal domain

<table>
<thead>
<tr>
<th>(a) Empirical generalization</th>
<th>(b) Explanation</th>
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<tbody>
<tr>
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<td>phrasal (VP level)</td>
</tr>
<tr>
<td>atomic n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>nonatomic available only w. context</td>
<td>lexical cum. Cover-based D op.</td>
</tr>
</tbody>
</table>

- Following Schwarzschild (1996), we can model the context dependency of nonatomic distributivity by assuming that there is a VP-level D operator that contains an anaphoric cover over contextually salient entities (pairs of shoes, etc.).
\[ D_{\tau,C} = \lambda P(\tau) \lambda e[ e \in *\lambda e' (P(e') \land C(e')) ] \]

(Takes an event predicate \( P \) and returns a predicate that holds of any event \( e \) which consists entirely of events that are in \( P \) and whose runtimes satisfy the anaphorically given predicate \( C \).)

- In a domain like time, there are no atomic covers. So setting dimension to \( \tau \) should be incompatible with setting granularity to \( Atom \).

(31) ??John found two fleas on his dog for a month.

(32) The patient took two pills for a month.

- The predicate \textit{find two fleas} by itself does not satisfy the presupposition of the \textit{for}-adverbial (why not?). Same for \textit{take two pills}.

(33) \textbf{Fact: \( D_{\tau,C} \) is a repair strategy}
\[
\forall P \forall C \forall C'[ C \subseteq C' \rightarrow SR_{\tau,C'}(D_{\tau,C}(P))]
\]

(When the granularity parameter of the temporal D operator is set to a given predicate \( C \), then for any \( C' \) such that \( C \subseteq C' \), the output of \( D \) satisfies stratified reference with respect to dimension \( \tau \) and granularity \( C' \).)

- Assume \( \lambda t[ \text{days}(t) \leq 1 ] \) ("once a day") is salient.

- Assume that \( \lambda t[ \text{days}(t) \leq 1 ] \subseteq \varepsilon(\lambda t[ \text{months}(t) = 1 ]) \). That is, any interval that qualifies as "once a day" must also qualify as very short with respect to one month.

- It follows that (32) can be interpreted by applying \( D_{\tau,\text{days}(t)\leq 1} \) to its verb phrase:

\[
D_{\tau,\text{days}(t)\leq 1}(\lambda e[ *\text{take}(e) \land *\text{pill}(\text{th}(e)) \land |\text{th}(e)| = 2 ])
\]

(True of any plural event that consists of one or more events of taking two pills which each take place within a day.)

\[
\exists e[ *\text{ag}(e) = \text{the.patient} \land \text{months}(\tau(e)) = 1 \land e \in *\lambda e'(*\text{take}(e') \land *\text{pill}(\text{th}(e')) \land |\text{th}(e')| = 2 ] \land \text{days}(\tau(e')) \leq 1 ]
\]

(There is a plural event that consists of one or more events of taking two pills which each take place within a day. Its agent is the patient, and its runtime measures a month.)

- For plural indefinites, there is an extra wrinkle:
John saw thirty zebras for three hours.  

- Predicates like *see thirty zebras* do not have the subinterval property: picture a safari in which the thirty zebras are seen in succession.

\[
\text{[see thirty zebras]} = \lambda e[\ast \text{see}(e) \land \ast \text{zebra}(\ast \text{th}(e)) \land |\text{th}(e)| = 30]
\]

(True of any possibly plural event in which a total of thirty zebras are seen)

- However, applying distributive QR (called SQI in Landman (1996)) to *thirty zebras* leaves a trace behind whose value is an atomic individual \(x\). For any fixed atomic \(x\), the predicate \(\text{see } x\) has the subinterval property. In this case, QR is driven by the need to satisfy the presupposition of the for-adverbial.

\[
\text{[see } t_1] = \lambda e[\ast \text{see}(e) \land \ast \text{th}(e) = g(1)]
\]

(True of any possibly plural event in which the atomic individual \(g(1)\) is seen)

- One can then nondistributively QR *three hours* above *thirty zebras* to ensure that the three-hour timespan does not covary with the zebras.

\[
\text{[[three hours] } \lambda 2 \text{ [[thirty zebras (each)] } \lambda 1 \text{ [[John saw } t_1 \text{ ] [for } t_2\text{]]]]}
\]

- In the LF in (39), the silent *each* stands for the effect of distributive QR.

### 9.1.1 Modeling interveners

- We have seen that temporal de/finitives, quantifiers, and pluractionals can lead to different-object effects:

\[
\text{(40) a. Jane ate an egg at breakfast for a month.}
\]
\[
\text{b. John found a flea on his dog every day for a month.}
\]
\[
\text{c. John found a flea on his dog day after day for a month.}
\]

- Every day and day by day mean the same as \(D^{\text{Schwarzschild}}\), except that the granularity is hard-wired to “day” instead of being given by context:

\[
\text{[every day] = [day by day] } \overset{\text{def}}{=} \lambda P_{(\forall t)} \lambda e[\ast \lambda e' \left( P(e') \land \underbrace{\text{days}(\tau(e')) \leq 1}_{\text{days}} \right)]
\]

(Takes an event predicate \(P\) and returns a predicate that holds of any event \(e\) which
consists entirely of events that are in \( P \) and whose runtimes are at most a day long.

- This leads to an interpretation like the following (ignoring the presupposition of for a month for convenience):

\[(42)\]

\[\text{a. } \text{John } [\text{found a flea every day } \text{for a month}]\]

\[\text{b. } \exists e[\text{ag}(e) = \text{john} \land \text{months}(\tau(e)) = 1 \land \]
\[e \in *\lambda e' (\text{find}(e') \land *\text{flea}(\text{th}(e')) \land |\text{th}(e')| = 1 \land \text{days}(\tau(e')) \leq 1)]\]

(There is a plural event that consists of one or more events of finding a flea which each take place within a day. Its agent is John, and its runtime measures a month.)

- At breakfast also means the same as \( D^{\text{Schwarzschild}} \), except that granularity is hard-wired to “breakfast”.

\[(43)\]

\[\text{[at breakfast]} \overset{\text{def}}{=} \lambda P_{(it)} \lambda e[e \in *\lambda e' (P(e') \land \exists i[\text{breakfast}(i) \land \tau(e') \leq i)])]\]

(Takes an event predicate \( P \) and returns a predicate that holds of any event \( e \) which consists entirely of events that are in \( P \) and whose runtimes are within a breakfast)

- These entries should be integrated into a more extensive account of temporal dependencies, see Pratt and Francez (2001), von Stechow (2002) and Champollion (2011). This is not trivial.

### 9.2 Raising the bar: Deo and Piñango (2011)

- Deo and Piñango (2011) attempt to account both for the scopal behavior and for the following facts:

#### 9.2.1 Availability and cost of iterative interpretations

- For-adverbials sometimes do combine successfully with telic predicates. This may lead to iterative interpretations (van Geenhoven 2004, 2005).

\[(44)\]

\[\text{a. Mary biked to the store for two months.}\]
\[\text{b. The girl dove into the pool for an hour.}\]
• *For*-adverbials also trigger iterative interpretations when they combine with atelic punctual predicates (i.e. semelfactives, Smith (1997)):

\[(45)\] a. The horse jumped for an hour.

• As reviewed by Deo and Piñango (2011), iterative interpretations engender cost by various psycho-/neurolinguistic measures:

  – increased centro-parietal activity (Downey 2006);
  – increased reading times and brain activity (Brennan and Pylkkänen 2008);
  – longer reaction time in cross-modal lexical decision (Piñango, Zurif, and Jackendoff 1999; Piñango, Winnick, et al. 2006);
  – comprehension difficulties in Wernicke’s Aphasics (Piñango and Zurif 2001).

### 9.2.2 Availability of partitive interpretations

• When there is not enough time for an iterative interpretation to make sense, *for*-adverbials can trigger a partitive interpretation instead:

\[(46)\] a. Mary read a book for an hour.
   b. Mary baked a cake for an hour.

• The availability of a partitive interpretation can be blocked by an explicit endpoint description (Smollett 2001):

\[(47)\] a. Mary polished the countertop for 15 minutes.
   b. *Mary polished the countertop **smooth** for 15 minutes.
   c. *Mary polished the countertop **to a shine** for 15 minutes.

• D&P argue for an analysis of *for*-adverbials which is based on regular partitions.

\[(48)\] **Definition.** A regular partition of an interval \( I \), written \( R_I \), is a set of disjoint intervals of equal length whose concatenation equals \( I \).

• They write \( R^{c}_I \) for a contextually determined regular partition of \( I \) and \( R^{inf}_I \) for a regular partition of \( I \) whose intervals have infinitesimal value (i.e. are moments).
• They write CoIN(P,I) to generalize over intervals and events – note that unlike At, it allows overlap if P is an event predicate:

\[
\text{CoIN}(P,I) = \begin{cases} 
P(I) & \text{if } P \text{ is an interval predicate} \\
\exists e[P(e) \land I \circ \tau(e)] & \text{if } P \text{ is an event predicate}
\end{cases}
\]

• D&P’s translation of a for-adverbial:

\[
\begin{align*}
\text{for an hour} &= \lambda P \lambda I \left[ \text{hours}(I) = 1 \land \forall J [J \in R^c_i \rightarrow \text{CoIN}(P,J)] \right] \\
\text{for an hour} &= \lambda P \lambda I \left[ \text{hours}(I) = 1 \land \forall J [J \in R^c_i \rightarrow \exists e[P(e) \land J \circ \tau(e)]] \right]
\end{align*}
\]

• In D&P’s system, predicates to which for-adverbials apply are always event predicates, except when they already contain another aspectual modifier. Since the latter case doesn’t occur in any of their examples, we can rewrite CoIN for presentation purposes.

\[
\begin{align*}
\text{for an hour} &= \lambda P \lambda I \left[ \text{hours}(I) = 1 \land \forall J [J \in R^c_i \rightarrow \exists e[P(e) \land J \circ \tau(e)]] \right] \\
\text{for an hour} &= \lambda P \lambda I \left[ \text{hours}(I) = 1 \land \forall J [J \in R^c_i \rightarrow \exists e[P(e) \land J \circ \tau(e)]] \right]
\end{align*}
\]

• D&P assume that to generate continuous (i.e. non-iterative) readings of for-adverbials, the partition cell length is set to infinitesimal. This amounts to universal quantification over moments. We can rewrite \( \circ \) as \( \sqsubseteq \) since we are dealing with moments (any moment that overlaps with an interval is contained in it).

\[
\begin{align*}
\text{for an hour} &= \lambda P \lambda I \left[ \text{hours}(I) = 1 \land \forall i [i \sqsubseteq I \rightarrow \exists e[P(e) \land i \sqsubseteq \tau(e)]] \right] \\
\text{for an hour} &= \lambda P \lambda I \left[ \text{hours}(I) = 1 \land \forall i [i \sqsubseteq I \rightarrow \exists e[P(e) \land i \sqsubseteq \tau(e)]] \right]
\end{align*}
\]

• Example:

\[
\begin{align*}
\text{John walk for an hour} &= \lambda I \left[ \text{hours}(I) = 1 \land \forall i [i \subseteq I \rightarrow \exists e[\text{john-walk}(e) \land i \subseteq \tau(e)]] \right] \\
\text{(True of any one-hour timespan of which every moment is temporally contained in an event of John walking.)}
\end{align*}
\]

• D&P assume that iterative readings arise when the interval is partitioned into subintervals which are relatively small-sized compared with the interval of the for-adverbial, but still larger than moments:

• Accomplishments:
(54) \[ \text{[Mary bike to the store for a month]_{iterative}^{D&P}} \]
\[ = \lambda I [\text{months}(I) = 1 \land \forall J [J \in \mathcal{R}_I \rightarrow \exists e \, [[\text{Mary bike to the store}]](e) \land J \circ \tau(e)]] \]
(True of any one-month timespan if each cell of its contextually given regular partition overlaps with an event of Mary biking to the store.)

- Semelfactives:

(55) \[ \text{[The horse jump for an hour]_{iterative}^{D&P}} \]
\[ = \lambda I [\text{months}(I) = 1 \land \forall J [J \in \mathcal{R}_I \rightarrow \exists e \, [[\text{The horse jump}]](e) \land J \circ \tau(e)]] \]
(True of any one-hour timespan if each cell of its contextually given regular partition overlaps with an event of the horse jumping.)

- The value of the variable \( c \) that determines the size of the subintervals is anaphoric on the context, following ideas in Moltmann (1991) and Champollion (2010).

- D&P attribute the higher processing costs for iterative interpretations to the process of retrieving a value for \( c \) from the context.

- They assume that this process is not necessary when \( c \) can be set to an infinitesimal value (this means no processing costs for continuous readings).

### 9.2.3 Problems of D&P’s account

- Indefinites which are interpreted in situ end up taking scope under the universal.

- By design, D&P do not model the aspectual sensitivity of for-adverbials.

- By making the infinitesimal partition length available at no cost, D&P predict partitive-like interpretations for all accomplishments.

- The contrast between indefinites and bare plurals is not predicted.

- Salience and world knowledge effects are unexplained.

### 9.2.4 What’s the status of lexical cumulativity?

- If we use lexical cumulativity for iterative readings of semelfactives, we don’t explain why they engender cost.
• Maybe restrict lexical cumulativity to count domains somehow, and introduce cumulativity using a silent verb-level ITER operator meaning “once or repeatedly” (see e.g. van Geenhoven (2004).

\[
\text{[The horse [jump ITER] for an hour]}_{step2} = \lambda P \lambda I [\exists e \ast \text{jump}(e, \text{the horse}) \land I = \tau(e) \land \text{hours}(I) = 1 \land \\
\forall J [J \in R_i \rightarrow \exists e' \ast \text{jump}(e', \text{the horse}) \land J = \tau(e')]]
\]

• Plausibility evidence for ITER is available from languages in which it is overt, e.g. West Greenlandic (van Geenhoven 2004). But the penalty remains a stipulation.

• D&P reject ITER because its processing cost of ITER shows up at the “wrong” place: about 300ms after the for-adverbial.

• It seems natural to assume that postulating silent operators should result in extra processing cost since if one seems them as a kind of garden path effect – ways to fix a sentence that would otherwise not make sense.

• D&P reject this because if people encounter these situations too often, they should conventionalize ITER and should not walk down the garden path anymore.

• But perhaps not all silent operators can be conventionalized?
Lecture 10

Distance distributivity

The content of this lecture has appeared as Champollion (2012) and is not contained in the dissertation. The main technical change is that the dissertation models each using stratified reference while this lecture uses the $D$ operator. This doesn’t make a difference empirically – the $D$ operator approach is formally a bit simpler, but breaks the parallel that the stratified-reference approach implements in the dissertation.

10.1 Introduction

- There are three uses of ‘each’ in English:

  (1)  
  a. **Adnominal each:** Two men have carried three suitcases each.  
  b. **Adverbial each:** Two men have each carried three suitcases.  
  c. **Determiner each:** Each man has carried three suitcases.

- This lecture focuses on data and observations by Zimmermann (2002).

- In German, adnominal and adverbial distance-distributive (DD) items are different from the distributive determiner:

  (2)  
  a. **Adnominal:** Die Männer haben jeweils drei Koffer getragen.  
  b. **Adverbial:** Die Männer haben jeweils drei Koffer getragen.  
  c. **Determiner:** Jeder/*Jeweils Mann hat drei Koffer getragen.

- Though adverbial and adnominal jeweils are similar, they can be teased apart syntactically. For more details, see Zimmermann.

- *Each and jeweils* generalize to two classes of DD items:
- *Each*-type DD items can also be used as determiners.
- *Jeweils*-type DD items cannot double as determiners.

• **Zimmermann’s Generalization** (illustrated below): All *each*-type DD items can only distribute over individuals. This contrasts with many *jeweils*-type DD items, which can also distribute over occasions (= salient chunks of time or space).

---

### 10.1.1 Questions.
- How can we capture the synonymy of the determiner, adnominal and adverbial uses of *each* in English?
- How can we represent the fact that DD items across languages share some part of their meanings?
- How do DD items fit into distributivity theory more generally? How can we formally capture the semantic variation among DD items?
- How can we explain Zimmermann’s Generalization?

---

### 10.2 Illustrating Zimmermann’s Generalization

#### 10.2.1 *Jeweils*-type DD Items

<table>
<thead>
<tr>
<th>Occur in German, Czech, Bulgarian, Japanese, Korean (Zimmermann)</th>
</tr>
</thead>
</table>

- These languages have adnominal DD items that cannot double as determiners.
- All of these DD items can distribute over individuals, like English *each*.
- Except for Japanese *sorezore*, all of them can also (given supporting context) distribute over temporal/spatial intervals or ‘occasions’:

(3) *Die Kinder haben jeweils zwei Affen gesehen.*

The children have *dist* two monkeys seen.

a. *Always available*: ‘Each of the children has seen two monkeys’
b. *Available, though only with supporting context*: ‘The children have seen two monkeys on each occasion’
(4) *Hans hat jeweils zwei Affen gesehen.*

Hans has distr two monkeys seen.
‘Hans has seen two monkeys on each occasion’

- Many languages express adnominal distance distributivity by reduplicating a numeral (Gil 1982a). In this category, we both find cases where reduplication does not give rise to occasion readings, such as Hungarian (Farkas 1997; Szabolcsi 2010), and cases where it does, such as Telugu (Balusu 2005).

(5) *pilla-lu1 renDu renDu kootu-lu-ni cuus-ee-ru.*

Telugu
kid-Pl two two monkey-Pl-Acc see-Past-3PPl.

a. ‘Each of the children has seen two monkeys’
b. ‘The children have seen two monkeys on each occasion’

(6) *Raamu renDu renDu kootu-lu-ni cuus-ee-Du.*

Telugu
Ram two two monkey-Pl-Acc see-Past-3PSg.
‘Ram has seen two monkeys on each occasion’

- The import of these cases on Zimmermann’s generalization is unclear, as reduplication is not expected to be able to act as a determiner.

10.2.2 *Each*-type DD Items

Occur in English, French, Dutch, Norwegian, Icelandic, Italian, Russian (Zimmermann) and Turkish (Tuğba Çolak, p.c.)

- These languages have adnominal DD items that can also be used as distributive determiners.
- All of these DD items can distribute only over individuals, not over occasions:

(7) The children have seen two monkeys each.

a. *Available:* ‘Each of the children has seen two monkeys’
b. *Unavailable:* ‘The children have seen two monkeys on each occasion’

(8) *John has seen two monkeys each.*

*Intended:* John has seen two monkeys on each occasion.
10.2.3 A Note on Reduplication

- Many languages express adnominal distance distributivity by reduplicating a numeral (Gil 1982a). In this category, we both find cases where reduplication does not give rise to occasion readings, such as Hungarian (Farkas 1997; Szabolcsi 2010), and cases where it does, such as Telugu (Balusu 2005).

(9) \[ \text{pilla-}lu1 \text{ renDu kootu-}lu-\text{ni cuus-}ee-\text{ru.} \]

\[ \text{kid-Pl two two monkey-Pl-Acc see-Past-3PPl.} \]

a. ‘Each of the children has seen two monkeys’

b. ‘The children have seen two monkeys on each occasion’

(10) \[ \text{Raamu renDu kootu-}lu-\text{ni cuus-}ee-\text{Du.} \]

\[ \text{Ram two two monkey-Pl-Acc see-Past-3PSg.} \]

‘Ram has seen two monkeys on each occasion’

- The import of these cases on Zimmermann’s generalization is unclear, as reduplication is not expected to be able to act as a determiner.

10.3 Capturing the Semantic Variation

- I propose that adnominal each and jeweils include two versions of the distributivity operator (cf. Link (1986) for a similar claim for German je, a short form of jeweils).

- In Lecture 4, I have motivated a formulation of the D operator that equips it with a dimension parameter and a granularity parameter, just like stratified reference.

- In Lecture 8, I have used the distinction between impure atoms and pure atoms to model the distinction between each and all. In this lecture, this distinction doesn’t matter. I’ll instantiate the granularity parameter of each as Atom. But really it should be PureAtom, to distinguish it from all.

(11) **Definition: Atomic event-based D operator**

\[ [[D_0]] \overset{\text{def}}{=} \lambda P(v_e) \lambda e [e \in *e' \left( P(e') \land \text{Atom}(\theta(e')) \right)] \]

(Takes an event predicate \( P \) and returns a predicate that holds of any event \( e \) which consists entirely of events that are in \( P \) and whose thematic roles \( \theta \) are atoms.)
Definition: Event-based D operator with covers

\[ [D_{θ,C}] ≜ \lambda P_{(ut)} \lambda e [e \in *λe' \left( P(e') ∧ C(θ(e')) \right) ] \]

(Takes an event predicate \( P \) and returns a predicate that holds of any event \( e \) which consists entirely of events that are in \( P \) and whose \( θs \) satisfy the contextually salient ‘cover predicate’ \( C \).)

- The thematic role parameter allows us to capture the fact that DD items can also target different thematic roles (Zimmermann 2002):

  (13) The boys told the girls two stories each. \( \text{Target: agent} \)
  
  (14) The boys told the girls two stories each. \( \text{Target: recipient} \)

- When we instantiate \( C \) with \( \text{Atom} \), we get Link’s VP-level D operator.

- Schwarzschild’s Part operator is a generalization: instead of specifying the granularity parameter to be atomic, we leave it free.

- We can use these parameters to describe the difference between each and jeweils.
  - Each includes the atomic distributivity operator of Link (1987), which can only distribute over count domains (granularity=atom).
  - jeweils includes the cover-based distributivity operator of Schwarzschild (1996), which can also distribute over noncount domains like time (granularity=unspecified).

- The setting “granularity=atom” blocks “dimension=time” because time is continuous and noncount – there are no atoms to distribute over.

- I propose that adnominal each comes prespecified for “granularity=atom”. This blocks “dimension=time”, so distributivity over occasions is unavailable. jeweils does not come prespecified for anything.

### 10.4 Explaining Zimmermann’s Generalization

- What follows is similar to Zimmermann’s own account, but reconceptualized in the context of strata theory.
I propose that in English, adnominal, adverbial and *each* have identical meanings up to type-shifting.

Determiner *each* is only compatible with count domains (“granularity=atomic”) – *each mud, *each water* etc.

Adnominal *each* is formally identical, so it inherits this property.

The count domain restriction of adnominal *each* is incompatible with time because they do not contain atoms: “granularity=atom” blocks “dimension=time”.

*Jeweils*-type DD items are formally different from determiners. So it is unsurprising that they do not inherit “granularity=atom”.

### 10.5 Formalization

Recall that we use type shifters for composition like these (see Section 2.5):

\[
\begin{align*}
(15) & \quad \text{a. Type shifter for indefinites: } \lambda \theta_{\text{en}} \lambda P_{\text{et}} \lambda V_{\text{et}} \lambda e [V(e) \wedge P(\theta(e))] \\
& \quad \text{b. Type shifter for definites: } \lambda \theta_{\text{en}} \lambda x \lambda V_{\text{et}} \lambda e [V(e) \wedge \theta(e) = x]
\end{align*}
\]

Each of these type shifters combines a noun phrase with its theta role head to build an event predicate modifier of type ⟨vt, vt⟩.

Example:

\[
\begin{align*}
(16) & \quad [[\text{agent [the boys]]}] = \lambda V \lambda e [V(e) \wedge ^{*}\text{ag}(e) = \bigoplus \text{boy}] \\
(17) & \quad [[\text{theme [two monkeys]]}] = \lambda V \lambda e [V(e) \wedge |^{*}\text{th}(e)| = 2 \wedge ^{*}\text{monkey} (^{*}\text{th}(e))] 
\end{align*}
\]

The event variable is existentially bound if the sentence is uttered out of the blue (see Figure 10.1). If the sentence is understood as referring to a specific event, the event variable is instead resolved to that event.

Adverbial *each* is a VP modifier, and is synonymous to Link’s D operator:

\[
(18) \quad [\text{each}_\theta]_{\text{adverbial}} = [D_{\theta, \text{Atom}}] = (10)
\]

Adnominal and determiner *each* need to be type-shifted, but both are defined in terms of D:
The boys saw two monkeys each.
\[
\text{[\textit{each}]} \text{\textsubscript{adnominal}} = \lambda P_e \lambda \Theta \lambda V \lambda e \left[ [D_{\theta, \text{Atom}}] \left( \lambda e' [V(e') \land P(\Theta(e'))] \right) \right](e)
\]

- Adnominal \textit{each} combines with an indefinite NP and then with a theta head:

\[
\begin{align*}
\left[ \left[ \text{[two monkeys] each} \right] \text{\textsubscript{theme}} \right] & = \lambda V \lambda e \left[ [D_{\theta, \text{Atom}}] \left( \lambda e' [V(e') \land \text{\textquoteleft th(e')\textquoteright}] = 2 \land \text{\textquoteleft monkey\textquoteright}(\text{\textquoteleft th(e')\textquoteright}) \land \text{Atom}(ag(e'))] \right) \right] \\
& = \lambda V \lambda e \left[ [D_{\theta, \text{Atom}}] \left( \lambda e' [V(e') \land \text{\textquoteleft see\textquoteright}(e') \land \text{\textquoteleft th(e')\textquoteright}] = 2 \land \text{\textquoteleft monkey\textquoteright}(\text{\textquoteleft th(e')\textquoteright}) \land \text{Atom}(ag(e'))] \right) \right]
\end{align*}
\]

(21) \text{[\textit{each}]} \text{\textsubscript{determiner}} = \lambda P_e \lambda \theta \lambda V \lambda e \left[ \theta(e) = \bigoplus P \land [D_{\theta, \text{Atom}}](V)(e) \right]

Determiner \textit{each} combines first with a nominal and then with a theta head:

\[
\begin{align*}
\left[ \left[ \text{[Each child] agent} \right] \right] & = \lambda V \lambda e \left[ \text{\textquoteleft agent\textquoteright}(e) = \bigoplus \text{\textquoteleft child\textquoteright} \land [D_{\theta, \text{Atom}}](V)(e) \right] \\
& = \lambda V \lambda e \left[ \text{\textquoteleft agent\textquoteright}(e) = \bigoplus \text{\textquoteleft child\textquoteright} \land e \in \text{\textquoteleft \textit{monkeys}\textquoteright}(V(e') \land \text{Atom}(ag(e'))) \right]
\end{align*}
\]

The result of these derivations is always the same, which reflects their synonymy:

(23) \left[ \text{The children each} \text{\textsubscript{ag} saw two monkeys} \right] \\
= \left[ \text{The children saw two monkeys each} \text{\textsubscript{ag}} \right] \\
= \left[ \text{Each child saw two monkeys} \right] \\
= \left[ \text{The children} \text{\textsubscript{ag, Atom} saw two monkeys} \right]

- The same type shift as in (19) brings us from Schwarzschild’s Part operator to adnominal \textit{jeweils}:

(24) \text{[\textit{jeweils}]} \text{\textsubscript{adverbial}} = \left[ D_{\theta, C} \right] = (36)

(25) \text{[\textit{jeweils}]} \text{\textsubscript{adnominal}} = \lambda P \lambda \Theta \lambda V \lambda e \left[ [D_{\theta, C}] \left( \lambda e' [V(e') \land P(\Theta(e'))] \right) \right](e)

- Setting \( C \) to \text{\textit{Atom}} and \( \theta \) to \text{\textit{agent}} leads to distribution over individuals:

\[
\text{Die Kinder haben jeweils} \text{\textsubscript{ag, Atom} zwei Affen gesehen.} \\
\text{The children have each seen two monkeys.} \\
\text{“The children have each seen two monkeys.”}
\]

- With supporting context, the anaphoric predicate \( C \) can be set to a salient antecedent other than \text{\textit{Atom}}. Then \( \theta \) is free to adopt values like \text{\textquoteleft runtime\textquoteright}.

- Example: the children have been to the zoo to see animals last Monday, last Wednesday and last Friday. (27) is uttered with reference to that sum event:
Die Kinder haben jeweils \( \tau \), \textit{zoovisit} zwei Affen gesehen.

\[
\text{ag}(e_0) = \bigoplus \text{child} \land e_0 \in \lambda e'[\text{see}(e') \land |\text{th}(e')| = 2 \land \text{monkey}(\text{th}(e')) \land \text{zoovisit}(\tau(e'))]
\]

“The children have seen two monkeys on each occasion.”

- This sentence refers specifically to the sum \( e_0 \) of the three events in question.
- The salient predicate \textit{zoovisit} is an antecedent for \( C \). Then \( \theta \) can be set to \( \tau \).
- (27) asserts that \( e_0 \) has the children as its agents; that it can be divided into subevents, each of whose runtimes is the time of a zoo visit; and that each of these subevents is a seeing-two-monkeys event.
- Dividing \( e_0 \) results in parts whose runtimes sum up to \( \tau(e_0) \) (Krifka 1989).
- Assuming that \( \tau(e_0) \) is the (discontinuous) sum of the times of the three zoo visits in question, this entails that each of these zoo visits is the runtime of one of the seeing-two-monkeys events.

### 10.6 Summary

How can we capture the synonymy of the determiner, adnominal and adverbial uses of \textit{each} in English?
- They are all derived from Link’s D operator.

How can we represent the fact that DD items across languages share some part of their meanings?
- They are derived from related distributivity operators (Link’s or Schwarzschild’s) which differ only in their parameter settings.

How do DD items fit into distributivity theory more generally? How can we formally capture the semantic variation among DD items?
- They display the same parametric variation as other flavors of distributivity do.

How can we explain Zimmermann’s generalization?
- \textit{Each}-type DD items are formally identical to determiners and therefore inherit their “granularity=atomic” value. \textit{Jeweils}-type DD items may have any setting for the granularity parameter.
Appendix

Answer to Exercise 1.1: The first claim is that parthood is a special case of overlap: \( \forall x \forall y [x \leq y \rightarrow x \odot y] \). Using the definition of overlap in (6), this can be rewritten as \( \forall x \forall y [x \leq y \rightarrow \exists z [z \leq x \land z \leq y]] \). We choose \( z = x \) and rewrite this as \( \forall x \forall y [x \leq y \rightarrow [x \leq x \land x \leq y]] \). Now \( x \leq x \) follows from the axiom of reflexivity (2). The rest is trivial.

The second claim is that a singleton set sums up to its only member: \( \forall x [\text{sum}(x, \{x\})] \). Here we can understand the singleton \( \{x\} \) as standing for \( \lambda z. z = x \), the predicate that applies to \( x \) and to nothing else. Using the definition of sum in (7), we rewrite the claim as \( \forall x \forall y [y = x \rightarrow y \leq x] \land \forall z [z \leq x \rightarrow \exists z' [z' = x \land z \odot z']] \). This simplifies to \( \forall x [x \leq x] \land \forall z [z \leq x \rightarrow z \odot x] \). The first conjunct follows from the axiom of reflexivity (2), while the second conjunct follows from the proof of the first claim.

Answer to Exercise 1.2: If the empty set was not removed from the powerset of any given set with at least two members, we would no longer have a mereology. The empty set is a subset of every set, so it would correspond to something which is a part of everything. If such a thing is included, any two things have a part in common, therefore any two things overlap. This contradicts unique separation, which states that whenever \( x < y \), there is exactly one “remainder” \( z \) that does not overlap with \( x \) such that \( x \oplus z = y \) (see line 9 in Table 1.3). Moreover, it contradicts the axiom of unique sum (10). A sum of a set \( P \) is defined as a thing of which everything in \( P \) is a part and whose parts each overlap with something in \( P \) (see (7)). If any two things overlap, the second half of this definition becomes trivially true, so anything of which everything in \( P \) is a part is a sum of \( P \). From transitivity, it follow that if \( x \) is a sum of \( P \) and \( x < y \), then \( y \) is also a sum of \( P \).

Answer to Exercise 2.1: The claim is that the algebraic closure of a set always contains that set: \( \forall P [P \subseteq \ast P] \). To prove this, we need to show that \( \forall P [P \subseteq \{x \mid \exists P' \subseteq P [x = \bigoplus P']\}] \), or equivalently, \( \forall P \forall x [x \in P \rightarrow \exists P' \subseteq P [\text{sum}(x, P')]] \). This follows for \( P' = \{x\} \), given that a singleton set sums up to its only member, as shown in the first exercise.
Answer to Exercise 2.2: For *No doctors are in the room*, the prediction is that the literal meaning is *No doctor is in the room*. The scalar implicature is computed as *No set of two or more doctors is in the room*. But it doesn’t surface because it doesn’t make the sentence stronger. For *Are there doctors in the room?*, the answer will depend on how one extends the definition of strength to questions.

Answer to Exercise 3.1: If we consider \( e_4 = e_1 \oplus e_2 \oplus e_3 \), we have a counterexample to the lexical cumulativity assumption for the following reasons. The themes of \( e_1, e_2, e_3 \) are the hole, the rosebush, and the soil, while the theme of \( e_4 \) is just the rosebush. The theme of \( e_4 \) is not the sum of the themes of \( e_1, e_2, \) and \( e_3 \). This violates cumulativity.

One way to respond to this challenge is to reject the assumption that the mereological parthood relation should model all parthood relations that can be intuitively posited (see Section 1.2.1). In this case, we do not need to assume that \( e_4 \) is actually the sum of \( e_1, e_2, \) and \( e_3 \). Even though the existence of \( e_4 \) can be traced back to the occurrence of \( e_1, e_2, \) and \( e_3 \), nothing forces us to assume that these three events are actually parts of \( e_4 \), just like we do not consider a plume of smoke to be part of the fire from which it comes, even though its existence can be traced back to the fire. Without the assumption that \( e_4 \) contains \( e_1 \) through \( e_3 \) as parts, Kratzer’s objection against cumulativity vanishes. See also Williams (2009) and Piñón (2011) for more discussion.

Answer to Exercise 3.2: not given here.

Answer to Exercise 3.3: Yes, on the assumption that cumulativity holds of *see* and of the theme relation, the verb phrase *see John* has cumulative reference. A seeing-John event is a seeing event whose theme is John. We therefore need to prove that the sum of any two seeing-John event is both a seeing event and an event whose theme is John. From cumulativity of *see*, we know that the sum of any two seeing events is a seeing event, so the sum of any two seeing-John events is a seeing event. From cumulativity of theme, the theme of the sum of any two events whose individual themes are John is the sum of their individual themes, then the theme of the sum of these events is the sum of John and John, which is John, given that the sum operation is idempotent.

Answer to Exercise 3.4: For *see two apples*, the proof does not go through because the theme of *see* is holistic and not incremental, that is, there is no meaning postulate like Incremental\(_{\text{theme}}([\text{see}])\). For *eat apples*, the proof does not go through because *apples* is not quantized (the sum of any two things in the denotation of *apples* is again in the denotation of *apples*).

Answer to Exercise 4.1: The star operator \( ^* \lambda e' \) is introduced through the D operator and takes scope over the predicate *dress* introduced by the theme. (12) does not directly require the theme of \( e \) to be a dress, though it requires \( e \) to consist of parts whose themes are dresses. This allows for the possibility that each girl wears a potentially different dress. The representation explicitly states
that the dress-wearing events \( e' \) have pure atoms as agents, but not that these pure atoms are girls. However, this fact is entailed by cumulativity of thematic roles together with the assumption that the entities in the denotation of singular count nouns are atoms. By cumulativity of thematic roles, any entity \( x \) which is the agent of one of the dress-wearing events \( e' \) is a part of the agent of \( e \). This agent is the sum of all girls. By definition of sum, \( x \) overlaps with a part of this agent. Being atomic, \( x \) can only overlap with \( y \) if it is a part of \( y \). This means that \( x \) is an atomic part of the girls. Given the background assumption that singular individuals like girls are mereological atoms, it follows that \( x \) is a girl. In this way, the distributive interpretation of (12) is correctly captured.

**Answer to Exercise 4.2:** \( C = \{ \text{rodgers} \oplus \text{hammerstein}, \text{rodgers} \oplus \text{hart} \} \)

**Answer to Exercise 4.3:** We assume that the verbal predicate is closed under sum:

\[
\forall e, e' [\text{write}(e) \land \text{write}(e') \rightarrow \text{write}(e \oplus e')] \tag{26}
\]

We also assume that the agent and theme relations are closed under sum:

\[
\begin{align*}
(27) \quad & \forall e, e', x, x' [\text{agent}(e) = x \land \text{agent}(e') = x' \rightarrow \text{agent}(e \oplus e') = x \oplus x'] \\
& \text{b. } \forall e, e', x, x' [\text{theme}(e) = x \land \text{theme}(e') = x' \rightarrow \text{theme}(e \oplus e') = x \oplus x']
\end{align*}
\]
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