Robust Regression Analysis:
Some Popular Statistical Package Options

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Robust regression analysis provides an alternative to a least squares regression model when fundamental assumptions are unfulfilled by the nature of the data. When the analyst estimates his statistical regression models and tests his assumptions, he frequently finds that the assumptions are substantially violated. Sometimes the analyst can transform his variables to conform to those assumptions. Often, however, a transformation will not eliminate or attenuate the leverage of influential outliers that bias the prediction and distort the significance of parameter estimates. Under these circumstances, robust regression that is resistant to the influence of outliers may be the only reasonable recourse. In this article, some robust regression options within popular statistical packages –including SAS9.0, STATA7, S-PLUS6.1, E-Views, LIMDEP8 --are considered.

Ordinary Least Squares Model Assumptions

There are several assumptions that have to be fulfilled for the ordinary least squares regression model to be valid. When the regression model does not meet the fundamental assumptions, the prediction and estimation of the model may become biased. Residuals, differences between the values predicted by the model and the real data, that are very large can seriously distort the prediction. When these residuals are extremely large, they are called outliers. The outliers will inflate the error variance. They inflate the standard errors. The confidence interval becomes stretched. The estimation cannot become asymptotically consistent. Outliers that bias the parameter estimates are those with leverage. They are called bad leverage points, whereas outlier that lie along the predicted line are those called good leverage points. When outliers inflate the error variance, they sap the model of power to detect the outliers.

One of the basic assumptions of regression analysis is equality of the error variance along the predicted line, a condition called homoskedasticity. Homoskedasticity provides a modicum of uniformity to the confidence intervals. If the residual distribution is normally distributed, the analyst can determine where the level of significance or rejection regions begin. Even if the sample size is large, the influence of the outlier can increase the local and possibly even the global error variance. This inflation of error variance decreases the efficiency of estimation.

Another form of violation resides in the lack of independence of observations. This lack of independence of the errors can manifest itself in terms of residual autocorrelation, which can further bias the estimation of significance tests as the error variance becomes artificially compressed by residual correlation. When this happens, the R^2, F and t values become inflated. Failures of these assumptions can predispose output toward false statistical significance. However, failure of basic classical regression model assumptions can be detected with the proper tests.

Another assumption is a normality of the residuals. When there are violations of the assumption of normality of the residuals in ordinary least square regression analysis, the estimation of significance becomes impaired.

Model Assumption Tests

The researcher should not only know what assumptions have to be fulfilled for the model to be valid, he should also know how to test them for fulfillment and what statistical packages contain which tests. Frequently, fundamental assumptions--such as, independence of observations, linear functional form, no influential outliers, sufficiently large sample size, normality and homoskedasticity of the residual distribution--are not adequately fulfilled. To test for independence of observations, the analyst may apply a runs test, a Durbin-Watson d or h test, or an autocorrelation function. To test for linear functional form, the statistician may graph the dependent with each of the independent variables or he may run curve estimation regressions. For the detection and assessment of influential outliers, he may extract the standardized residuals and run a frequencies analysis upon them or obtain the leverage and Cook's d statistics for additional tests (Stata, S-Plus, SAS). Another way to assess the influence of the outliers is to use the dffit or dfbeta options. Where these are available, the dffit, which show how much the outlier influences the model fit while the dfbeta will reveal how much the outlier influences the parameter estimate (*PROC REG* in SAS, in STATA, S-Plus). To test for nonnormality of the residuals, he may examine a quantile-quantile plot or a Cramer-Von Mises, Anderson-Darling, Kolmogorov-Smirnov or a Shapiro-Wilk test (available in SAS *PROC UNIVARIATE or* STATA *sktest*). He may graph the residuals on with normal quantile-quantile plot (SAS, STATA, LIMDEP, or S-PLUS). Should he wish to test the residuals for heteroskedasticity, he can run a Cook-Weisberg test (see *hettest* in STATA) or a White's general test (in SAS *PROC REG* or a different version in PROC MODEL) on the residuals. When tests for these assumptions do not pass muster or outliers plague the model, robust regression or nonparametric regression analysis may provide a reliable alternative. Before attempting a regression analysis, the researcher should run a sample size test to be sure that he will have enough statistical power to test his hypotheses. His sample size will have to be sufficiently larger than the number of independent variables in his model. Indeed, it should be large enough so that his standard errors will be small enough to test his hypotheses. If one, some, or all of the assumptions are not adequately fulfilled, he should know what fixes are available to deal with residual problems. In this article, we explain the applications of different types of robust regression analysis to situations when the tests show that the assumptions are not adequately met.

Types of Robust Regression

Several popular statistical packages have procedures for robust regression analysis. Among them are SAS, STATA, S-PLUS, LIMDEP, and E-Views. They will need to know in which statistical package the type of robust regression appropriate for that particular application can be found. There is a family of robust regression analysis that replaces the sum of squared errors as the criterion to be minimized with one less influenced by outliers. Least absolute deviations, sometimes called L_1 regression, is another method that seeks to minimize the influenced of outliers. Least median of squares (Rouseseeuw, 1984) is one member of this family. Least trimmed means (Rousseeuw and Leroy, 1987) is another approach. Weighted least squares can be employed to serve this purpose. There is a method called iteratively reweighted least squares using robust corrections for influence of the outliers (Mächler and Chatterjee, 1995). Another resistant regression method uses White variance estimators that are consistent and efficient amidst heteroskedastic residuals of unknown form. There are some nonparametric regression techniques that handle nonnormal distribution of the errors, some of which are addressed later. We consider how robust regression analysis attempts to handle these violations as well as some of the statistical package procedures that can be applied to correct these problems.

Least Median of Squares

One of the earliest types of robust regression is called median regression, which has the advantage of diminishing the influence of the residuals. According to Venables and Ripley (1999), this algorithm minimizes the median of ordered squares of residuals to obtain the regression coefficient, b:

$$\begin{aligned} &Least\ median\ of\ squares\,(LMS) \\ &= \min_b \underset{i}{median} |\, y_i - x_i b\,|^2 \end{aligned} \qquad (1)$$

SAS8.2 users can call least median of squares with the *LMS* call in *PROC IML*. S-Plus users can execute this algorithm with *lmsreg*. According to Martin (Martin, 2002), the median squared residuals lacks a smooth squared residual function and takes a long time to converge

Median Regression

Least absolute deviation, sometimes called L_1 or least absolute value (LAV) regression, is also known as median regression. SAS users call this procedure with the *LAV* command within the IML library. In STATA, median regression is performed with the quantile regression (qreg) procedure. The procedure forms initial estimates from a weighted least squares of absolute residuals. Then qreg estimates the quantile of the dependent variable, or by default, the median, by taking the raw sum of absolute deviations around the unconditional median. It finds the regression coefficient, b, that minimizes this function. The procedure estimates a constant and parameter estimates that predicts the median. For this reason, *qreg* is sometimes called median regression.

median regression estimates by

$$minimizing \sum_{i=1}^{n} | y - xb | \qquad (2)$$

The computation of the standard errors comes from the estimation of the weighted variance-covariance matrix for the regression coefficients. The weights are a function of the quantile being estimated divided by the true density of the residual. Median regression with bootstrapped standard errors will be in LIMDEP 8.

If influential outliers are not a serious problem, though the data are skewed and nonnormal, then *qreg* may be the answer. For influential outliers may distort the results. If the errors are homoskedastic, *qreg* may be applied with the caveat (STATA 7 Reference Q-ST) that it is reported to underestimate the standard errors for heteroskedastic errors. If outliers do plague the data set, the STATA qreg procedure will be sensitive to their distortion, whereas the STATA *rreg* procedure may be preferred because it downweights their influence.

Least Trimmed Squares

One way to eliminate possible outliers is to run the analysis on trimmed or winsorized distributions. Distributions that have their outliers trimmed prior to the analysis are sometimes called trimmed means procedures. While trimming a distribution means truncating it at the t and 1- t quantile (Wilcox, 1997), winsorizing a distribution means setting the values at or more extreme than the t quantile to that of the t quantile on one tail and setting those values at or more extreme than the 1- t quantile to those of that quantile. The trimmed or winsorized distribution is then estimated by minimizing the sum of squared absolute residuals. By trimming the alpha rejection region, the distorting effects of influential outliers could be pruned from the variables prior to processing. SAS has provision in its *INSIGHT or ANALYST* module for trimming or winsorizing the distribution to eliminate the outliers before the estimating the means. Winsorizing outliers limits the values of outliers beyond a particular sigma limit away from a measure of central tendency to those found at that particular sigma limit. Rousseeuw advocated trimming by minimizing the sum of the absolute values of the squared residuals to obtain the regression coefficient. He called this procedure least trimmed squares, as cited in Venables and Ripley (1999).

$$Least\ trimmed\ squares\,(LTS)$$
$$= \min_b \sum_{i=1}^{q} |\ y_i - x_i b\ |_{(i)}{}^2 \qquad (3)$$

According to Rousseeuw, the LTS procedure is more efficient than the LMS or M procedure. It has a smoother objective function that is "less sensitive to local effects" and it has "the highest possible breakdown value (Rousseau and Van Driessen, 1998)."

The LTS procedure propounded by Rosseeuw and Leroy (1987) can be called with the *LTS* function in SAS *PROC IML*. Although the original algorithm is time-consuming, a newer fast-LTS, which takes h random subsets of large data sets (Rosseuw and Van Driessen, 1998) has now become the default algorithm and is to become part of *PROC ROBUSTREG* in version 9 of SAS. This algorithm "minimizes the sum of the h smallest squared residual (SAS OnLineDoc):

Least Trimmed Squares (*SAS LTS*) minimizes

$$\sqrt{\left(\frac{1}{h}\ \sum_{i=1}^{h} r_{i:N}{}^2\right)}$$
$$where\ h\ is\ defined\ to\ be\ within$$
$$the\ range\ of\ \ \frac{N}{2}+1 \le h \le \frac{3N}{4}+\frac{p+1}{4} \qquad (4)$$
$$N = sample\ size$$
$$p = number\ of\ parameters$$

S-PLUS, however, contains this routine in its menu system so that the minimization of the sum of squared absolute residuals is run over a subset of the residuals. In this case q = [(n+p+1)/2], where p=number of parameters in the model and n=sample size. S-Plus contains this algorithm in its *ltsreg* procedure. Although this converges more smoothly and faster than the LMS algorithm, Martin (2002) maintains that it has low Gaussian efficiency that seems to roughen with the trimming fraction. Be that as it may, the slow convergence of the older procedure inclines developers toward use of the fast-LTS algorithm.

Heteroskedasticity and Weighted least squares

Weighted least squares (WLS) estimation is available in all of these packages. To compensate for heteroskedasticity, weights computed as the inverse of the variances of the dependent variable, can be applied. The weights have to be computed as a separate variable. When they are applied to the estimation, they compensate for the distorting effect of the heteroskedasticity. At the same time, they may be an inverse function of the leverage of the outlier to downweight the influence of outliers. Such fixed weights may be applied as an option in the SAS *PROC REG,* LIMDEP *Regress; wts=,* as an option in the lm function in S-PLUS, as well as in EVIEWS least squares and two-stage least squares procedures. In STATA, wls is used to find the initial estimates of qreg. WLS can also be applied in the variance weighted least squares procedure, called *vwls*. Once the weights are included, WLS estimation takes place.

Heteroskedasticity and Autocorrelation Consistent Variance Estimation

To perform regression analysis resistant to heteroskedasticity, there are other procedures. One of the ways to relax the requirement of homoskedasticity is to iteratively apply a robust filter to the heteroskedasticity. Harold White in 1980 showed that for asymptotic (large sample) estimation, the sample sum of squared error corrections approximated those of their population parameters under conditions of heteroskedasticity and yielded a heteroskedastically consistent sample variance estimate of the standard errors (STATA 7 Reference Manual, Q-ST).

$$
\begin{aligned}
&Robust\ Variance = \\
&\frac{N}{N-K}\widehat{V}\left(\sum_{k=1}^{N} u_k^{(c)\prime} u_k^{(c)}\right)\widehat{V} \qquad (5)\\
&c = clusters\\
&u_k = row\ \ vector\ \ of\ scores k = 1,\ldots,K\\
&\widehat{V} = variance\ matrix
\end{aligned}
$$

Thus, the robust White variance estimator rendered regression resistant to the heteroskedasticity problem. LIMDEP can generate the White estimators with the het option for the regress procedure, as can EVIEWS with its *LS and TSLS* options. Stata generates them with the *regress* y x1 x2, …, *robust* command.

In STATA, there are additional corrections. Regress, robust in STATA uses a degree of freedom correction of n/(n-k) times the error variance to improve the small sample estimates. The *hc2* option provides a leverage (h) correction for homoskedastic models by dividing the error variance by (1 – h) while the *hc3* option invokes a correction for heteroskedastic models that divides the error variance by $(1 - h)^2$.

White asymptotic covariance estimation can be performed with the ACOV option in SAS *PROC REG*. When tests are performed using this option, the heteroskedastic consistent standard errors are used (Meyer, 2002).

The Newey-West (1987) extended this heteroskedastically consistent variance estimator to handle residual autocorrelation as well. The White and Newey West estimators as well as their robust Generalized Methods of Moments analogs are available in LIMDEP nonlinear least squares *(NLSQ* with the *GMM* option), STATA (*regress* procedure with the *robust* option), LS, TSLS, ARCH, and GMM models in EVIEWS with the *HAC* option, and SAS (using *PROC MODEL* with the *GMM* option).

Iteratively Reweighted Least Squares

Another robust regression analysis is performed with M (Maximum likelihood-like) estimators. STATA begins regression analysis with computation of case weights from scaled residuals. The scale is a median absolute deviation about the median (MADAM) residual divided by a constant (Huber, 1981). If the residual is small the case weight is equal to one, but if it is larger the case weight is equal to tuning constant divided by the absolute value of the scale. Other weights, called biweights, are applied later. With biweights, all cases with residuals are downweighted and cases with large residuals are assigned zero weights, thereby eliminating their influence altogether. Because Huber weights sometimes have problems with extreme outliers and because bi-weights sometimes have trouble converging on one solution, Huber weights are first used in the early iterations and then biweights are used during later iterations of the regression until final convergence is reached. STATA applies with estimation with the *rreg* command (STATA7 Reference, Q-ST). SAS version 9 by default applies M estimation to minimize a robust function of the residuals in its experimental *ROBUSTREG* procedure. Although the Tukey weight is the default, one can employ other weights as an alternative. The SAS *ROBUSTREG* procedure has a number of appealing features including a nice menu of outlier diagnostics, which identify the outliers, Mahalanobis distances, robust MCD distances, and their leverages. These diagnostics make it easy to distinguish the good from the bad leverage points.

In **Robust MM Regression**, robust initial regression coefficients are used as starting values. The robust regression coefficients are found by minimizing a scale parameter, S, while c may be one of several bounded loss functions that serves the purpose of minimizing the empirical influence of troublesome residuals. c is an integral of c (u) in the formula

$$\begin{aligned}
&\sum_{i=1}^{n} c\left(\frac{y_i - x_i b}{c_0 S}\right) = (n-p)\,b \\
&c(u) = u^6 - 3u^4 + 3u^2, u \le 1 \qquad (6) \\
&c_0 = tuning\ constant = 1.548 \\
&b = .5
\end{aligned}$$

The M-estimate is derived from according to the loss function from the S estimate and the fixed scale estimate produced. The final M-estimate is computed and thus, the MM-estimator proposed by Yohai, Stahel, and Zammer in 1991 is generated. A test of bias that compares the M estimate to the least squares estimate is also produced. S-PLUS contains the *lmRobMM* function, which generates the above procedure, cited in Venables and Ripley (1999) and S-PLUS. 2000 Guide to Statistics, Vol. 1 (2000). The SAS version 9 *PROC ROBUSTREG* also includes both M and robust MM Regression.

Another method, utilizing iteratively reweighted least squares of a modified M-estimator weighting to deal with the outlier and heteroskedasticity problems, was developed by Prof. Samprit Chatterjee and Martin Mächler, respectively of NYU's Stern School and Swiss Federal Institute of Technology, have developed an iteratively reweighted least squares approach that downweights influential residuals to achieve a fit. Their case weights are updated at each iteration of the estimation process in accordance with the following formula:

$$w_i^j = \frac{(1-h_{ii})^2}{\max(|r_i^{j-1}|, med_i\,|r_i^{j-1}|)}\ ,\ i = 1,\ldots,n \qquad (7)$$

$$where\ \ med_i\ x_i = median(x_1, x_2, \ldots, x_n)$$

$$r_i = residual$$

$$h_{ii} = leverage\ \ of\ \ case\ \ i$$

This process estimates parameters by minimizing the weights least squares regression until convergence. Chatterjee and Mächler (1995) have tested their method on a variety of difficult data sets and found that it performed well on all of them. They have written SAS, S-Plus, and Minitab Macro code for their procedure (Chatterjee, 2002).

Bootstrapping

When distributional normality and homoskedasticity assumptions are violated, many researchers resort to nonparametric bootstrapping methods. Where the errors are independently distributed, bootstrapping can provide an empirical supplement to analytic means of assessing parameters, errors, standard errors, levels of significance and confidence intervals. Bootstrapping entails random resampling (according to Mooney and Duval, 1993) to obtain the desired empirical distribution. Based on asymptotic theory, bootstrapping estimation using one of several standard methods, one can determine with great accuracy the empirical standard errors for such samples. Forthcoming LIMDEP 8 (with linear regression via LAV with bootstrapped standard errors, as well as the current versions of STATA 7 (*bs, bstrap, or bsqreg*), S-PLUS 6.0 (with its bootstrap suite of functions), and EVIEWS 4.0 (with its bootstrapping function) all have procedures for resampling.

One advantage of bootstrapping is that some of the methods provide more accurate confidence intervals than the convention asymptotic distribution approaches do. As Mooney and Duval (1993) have it, the percentile bootstrap method empirically proves to be the most accurate. *Bsqreg* in STATA and a forthcoming bootstrap procedure in the new LIMDEP 8 will perform quantile regression with bootstrapped standard errors.

Those who would resort to the bootstrap must be aware that the bootstrapped sampling distribution will reflect bias already in the sample, unless bias correction is performed. Mooney and Durval (1993) warn that not much work has been done in that area as of 1993.

Other Techniques

William Cleveland proposed a robust scatterplot smoothing with a weighted least squares algorithm that locally weights the data. Once the bandwidth is set, a regression is performed that gives local weights the most influence. This locally weighted scatterplot smoothing (LOESS) is available in many statistical packages, including *PROC LOESS* in SAS and a "*loess*" procedure in S-Plus. These approaches lend themselves to nonnormal data. STATA also performs LOWESS with "*roblowes*." One needs only to search for the keyword, lowess, and then download and install the available files. Even SPSS can perform a LOWESS plot with a fit option to a scatterplot.

Robust spline smoothing can also be performed with S-PLUS, SAS, and STATA. With the nonparametric spline smoothing, a function is formed that sums the squares of the residuals plus some penalty function. The penalty function is a product of a smoothing parameter times a measure of roughness, such as the integral of the square of the second derivative of the function. The minimization of function yields the cubic spline. With SAS, the spline smoothing can be performed with *PROC LOESS or PROC TPSPLINE* for thin plate splines. SAS also has transformation regression in *PROC TRANSREG* with a spline option for transforming variables. With the "*spline*" command, or the "ksm" command coupled with the lowess option, STATA7SE can also perform these procedures. In S-Plus, one can use the spline command to interpolate the data points and when one selects the 2-D line and scatterplots, one can opt for spline smoothing.

The generalized additive model was proposed by Hastie and Tibshirani more than a decade ago. In these models, the mean of the dependent variable is connected to the predictors via a link function. The distribution of the response need not be normal; it can be any member of the exponential family of distributions. These can include binomial logistic functions. They can include poisson, inverse Gaussian, or quasi link functions. Other terms may be added as well. The model is semi-parametric. The parametric part iteratively smoothes the partial residuals as part of a process called backfitting. The nonparametric part is smoothed with loess or splines. The gam procedures in S-Plus, STATA, and *PROC GAM* in SAS are all general additive model procedures.

Kernel regression is a form of nonparametric regression. This procedure is planned for release in LIMDEP 8. The user can choose from 8 types of kernels to apply to his model. SAS has a procedure, *PROC KDE*, to do univariate or bivariate kernal density estimation with a normal kernel. There is a movement toward providing options to render ordinary statistical procedures more resistant against the effects of heteroskedasticity and outliers. As these options become more available, the teaching of the robust procedures as backups will naturally follow the teaching of the classical procedures in the near future. As S-Plus provides in its robust library, the analysis will be run with both procedures and when they agree, the classical procedures will suffice. When the results disagree substantially, the robust procedures will be preferred.

To recapitulate, when fundamental regression analysis assumptions are violated, the researcher may wish to consider what alternatives are available to him. If the data contain influential outliers, then he may wish to employ some form or robust regression that downweights the influence of the troublesome outliers. He could use least trimmed squares, weighted least squares, iteratively reweighted least squares, a form of M or MM-estimators or least median squares regression. If the data are heteroskedastic, the analyst may wish to employ one of the weighted least squares, iteratively reweighted least squares, MM-estimators, median regression or least median squares estimation options. If the data are nonnormal, he may wish to use least absolute deviation, iteratively reweighted least squares, median regression, or least median squares estimation. He may compare his robust results to his classical results. He may also wish to estimate his bias from his parametric estimate with bootstrapping. When there are substantial differences, Martin and others maintain that the robust statistics yield better estimation and prediction. These are some of the ways by which the analyst can fortify his model against unruly data.

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