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“The democratic dynamics of educational investment and income distribution”

by

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## 1. Introduction

Democracy is thought, by many, to be coextensive with justice. For instance, Adolfo Perez Esquivel, a Nobel Peace Prize laureate, recently said, “The vote does not define democracy. Democracy means justice and equality.” (*The Daily Journal* [Caracas], July 12, 2001) But for political scientists, democracy is a set of political institutions, and they can (or must) ask whether those institutions promote one or another kind of justice.

Of course, for those who define justice in a purely procedural fashion, it is conceivable that democracy, by its very definition, engenders justice. If, that is, justice is defined by certain procedures of citizen representation and competitive elections, then democracy might well *constitute* justice. But Esquivel clearly does not hold this procedural view, for he says that democracy promotes *equality*. Despite the fact that he says ‘justice and equality’, we might assume that he means that democracy is just in part because it promotes economic equality -- that is, equality in more than political representation.

Many recent views of justice are egalitarian: those that advocate equality of primary goods, or of resources, or of opportunities. In this paper, I pose the question: Does democracy, conceived of as competition between parties that represent the interests of different groups of citizens, promote, over time, equality of material condition among citizens?

There is an extremely simple and well-known model of this question that gives an unequivocal affirmative answer. Imagine that society consists of members characterized by a distribution of income. The income a citizen produces is, furthermore, independent

of the degree of taxation. We further suppose that median income,  $m$ , is less than mean income,  $\mu$ . Two parties form, each of which desires only to win the election – neither represents citizens in any conscious way. The set of policies these parties can propose are simple, affine tax rules: that is, to tax all income at a common rate  $t$ , and return a lump-sum payment of  $t\mu$  to everyone. Each citizen will vote for that tax rate, between the two proposed by the competing parties, that maximizes her after-tax income. The unique Nash equilibrium in the game between these two parties is for both to propose the ideal tax rate for the citizen with median income, which is unity, because  $m < \mu$ . Thus, all after-tax incomes are equal to the mean, and complete equality is achieved.

This model is too sparse, however, to answer our question convincingly. It presents too simplistic and incorrect a view of reality, in at least the following ways: (1) actual parties in democracies are partisan, they are not purely opportunistic, as in the Downsian model that I have described above; (2) surely one of the main, if not *the* main, institution through which equalization of incomes has and will take place is public education, which is ignored in this model; (3) the policy space for the parties is artificially restricted to the unidimensional space of ‘flat taxes,’ which we observe in virtually no democracy; (4) if democracy engenders equality, surely this will be a long process, that takes many years – it does not happen instantly; (5) it is assumed that the labor supply of citizens is entirely inelastic with respect to taxation.

My aim is to study the question whether democracy engenders economic equality in a model that corrects the first four problems just mentioned: that is, one where parties represent different coalitions of citizens in a partisan way, where a major political

concern is public investment in the education of children, and that education determines the future wages of those children, where the policy space is very large – indeed, infinite dimensional – and where equality, if it occurs, is the consequence of a long process. Unfortunately, I will not modify the assumption that labor is supplied inelastically by citizens with respect to taxation – to do that would introduce mathematical complexities which, together with the complexities engendered by the other four modifications, make the analysis (thus far) intractable.

Here is a preview of the model. Society consists of an infinite sequence of overlapping generations of parents and children. An individual lives for two periods: first, as a child, when he lives in his mother's household, and during which period he is educated, and second, as an adult, when she earns income, pays taxes, and raises her son. (I will refer to children as sons and parents as mothers; the cost of this convention is that little boys turn into adult women.) At the beginning, in the first period, mothers are characterized by a distribution of human capital,  $h$ . We take the human capital of an adult to be the wage she is capable of earning during a unit of time (say, a year). At each date, the living adults must decide four questions, through democratic competition: how much tax revenue to raise; how to partition the government budget into two sub-budgets, one for redistribution among adults, and one for the education of children; *how* to allocate the first sub-budget among existing families, and *how* to allocate the educational budget among children of different 'socio-economic status.'

The key technological fact is that it costs more to educate children from poorer families, in the sense that the wage a child will eventually earn, when he becomes an adult, is a deterministic, increasing function of two inputs: how much is invested in his

education, and his mother's level of human capital. Thus, if society were to invest the same amount in the education of all children, then the children would, as adults, occupy exactly the same rank in the distribution of human capital as their mothers did. The model is silent about the source of this technological fact: it could be due either to nature or nurture. I prefer to think of the mother's level of human capital as an indicator of the quality of knowledge she transmits to her son, which affects how easily he absorbs education in the public schools – thus, the influence of mother's human capital on son's human capital is through nurture, what might be called the culture of the household.

Thus, how much society invests in a child from a particular social background will determine his future wage, and we suppose that the educational authorities can target children from different social backgrounds with different levels of educational investment. The precise way this targeting is done is not specified: we know, for instance, that in the United States, more public funds are spent on the education of children from wealthier backgrounds through the mechanism of property-tax financing of education, combined with residential segregation on the basis of income, and neighborhood schools.

For most of the discussion, I will assume that children do not differ in talent or effort: thus, the educational production function is entirely deterministic in the sense that two inputs -- educational finance and mother's wage-- determine the wage of the son. I will comment in the conclusion upon what happens if we assume that there is a stochastic factor in the educational production function, which can be thought of as a random child-talent factor.

It is assumed that parents care about two things: the consumption of their household, which is to say, their after-tax income, and the wage their child will come to have as a consequence of the educational process.

At each date, adults form two political parties, and there is a competition between these parties over the four fiscal questions. Each party represents the interests of a coalition of citizens, where those interests are the ones just mentioned: household consumption and future wages of their children. A concept of political equilibrium is defined; this equilibrium gives us the interests of two parties that form (that is, whom each party represents), and the policy proposals that each party puts forth in political equilibrium. There is a stochastic element in the election, so that each policy wins with a probability. After the election, it is assumed that the victorious party implements its announced policy. In particular, that policy includes a specification of how much is invested in children from every social background, and so the distribution of wages of next period's adults is determined.

In that next period, adults (who are last period's children) engage in the same kind of political competition; there is again a political equilibrium, and the outcome of the election determines the distribution of wages of adults in the third generation. And so on, forever.

Our question is: Is the asymptotic distribution of wages of this stochastic dynamic process one of equality? Roughly speaking, the political parties at each date represent the poor and the rich, and so we shall therefore call them Left and Right. Does democratic political competition between the poor and the rich, over educational policy, eventually annihilate the differences in human capital in society? More finely, we wish

to understand the effect that democracy has on the distribution of human capital, as compared to some benchmark. The benchmark we will use is an imaginary counterfactual in which there is no redistribution at all, and education is financed privately by each household. We call this *laissez-faire*.

Let it be noted that nothing like a median voter theorem can be invoked to analyze this model, for, as I have said, there are four political issues, and indeed, we shall be even more demanding, and propose a policy space that is infinite dimensional. The infinite dimensionality of the policy space will model the idea that political competition is *ruthless*, that no holds -- in the sense of artificial restrictions on the form that policies must take -- are barred. (Why, indeed, would any democratic polity limit the set of admissible tax regimes to affine income taxes? That restriction is a purely artificial one, made in order to be able to apply an equilibrium concept which only ‘works’ when the policy space is unidimensional.) Thus, the technical innovation in the analysis involves the proposal of a model of party competition in which equilibrium will *exist* even when the policy space is very large.

## 2. The economy: preferences and technology

The utility function of a mother is given by

$$u(x, h') = \text{Log}x + \gamma \text{Log}h', \quad (2.1)$$

where  $x$  is the consumption (after-tax income) of the household and  $h'$  is the future wage of her son.  $\gamma$  is a positive constant. The utility function  $u$  is a Cobb-Douglas function in household consumption and the child’s future wage.

The educational production function describes what the wage of a child from an  $h$ -family will be, if resources  $r$  are invested in his education. It will be:

$$h' = \alpha h^b r^c, \quad (2.2)$$

where  $\alpha$ ,  $b$ , and  $c$  are positive constants. Equation (2.2), written in logarithmic form is:

$$\log h' = \log \alpha + b \log h + c \log r,$$

from which we can deduce that  $b$  is the elasticity of the child's wage with respect to his mother's wage, and  $c$  is the elasticity of the child's wage with respect to educational investment. There is some weak econometric evidence that in reality,  $b+c < 1$ , but we will not rely on this factoid. (See Bénabou [2001].)

Suppose that all education is publicly financed, and that a policy entails that an  $h$ -family receive an after-tax income of  $y$  and an educational investment of  $r$  in the son.

Then the mother's utility written as a function of  $y$  and  $r$  will be:

$$\begin{aligned} v(y,r;h) &= \log y + \gamma \log \alpha h^b r^c = \\ &= \log y + \gamma c \log r + \gamma \log \alpha h^b \cong \\ &= \log y + \gamma c \log r \end{aligned} \quad (2.3)$$

In the last line, I have dropped the term  $\gamma \log \alpha h^b$ , because, from the decision-maker's (mother's) viewpoint, this is just a constant term which is never affected by policy choice, and so it can be ignored. Thus, the *indirect utility function* of a mother of human-capital  $h$ , defined on pairs  $(y,r)$ , is:

$$v(y,r;h) = \log y + \gamma c \log r. \quad (2.4)$$

Using this utility function, the mother evaluates her preferences over consumption-educational investment pairs  $(y,r)$ .

Consider, now, the following thought experiment. Suppose that a mother were to receive an income level  $M$ , and that she must privately educate her child: that is, she must choose an allocation  $(y,r)$  such that  $M = y + r$ . She would choose that partition of  $M$  into consumption and educational investment that maximizes  $v$ , as defined in (2.4). The result of this optimization problem is that she would choose to allocate some funds to both education and consumption: to wit, she would choose

$$r = \frac{\gamma c}{1 + \gamma c} M, \quad y = \frac{1}{1 + \gamma c} M. \quad (2.5)$$

Note that it is assumed that all parents have the same *direct* utility function  $u$ , given by (2.1). Parents differ only in their levels of human capital,  $h$ , which we assume is distributed, initially, according to a probability measure (or, if you prefer a cumulative distribution function)  $F$ . There are no assumptions on the nature of  $F$ . We will, however, assume that there is a very large population, which we model by assuming that there is a continuum of individuals. Thus, we will take  $F$  to be a probability measure (or CDF) on the non-negative real numbers. Indeed, we lose nothing by supposing that the support of  $F$  is the set of non-negative real numbers (which is to say that there are adults who earn any given positive wage, and zero). We assume that, regardless of what fiscal rules are adopted, an adult of human capital level  $h$  always produces a pre-tax income of  $h$ . Thus, labor is inelastically supplied.

If there is no redistribution through the tax system, then, since the income of an  $h$  mother is  $h$ , according to (2.5), she invests  $\frac{\gamma c}{1 + \gamma c} h$  in her son's education, and from (2.2), it follows that her son's wage will be  $\alpha \left( \frac{\gamma c}{1 + \gamma c} \right)^c h^{b+c}$ . Now suppose that  $b+c=1$ .

Then the wages of sons are exactly proportional to the wages of their mothers.

Consequently, the distribution of human capital of sons is exactly the distribution of human capital of mothers, multiplied by a constant. In particular, the *coefficient of variation*<sup>2</sup> of the distribution of wages is constant over time. We state this as:

*Fact 1* If  $b+c=1$ , and there is no redistributive state, then the coefficient of variation of human capital is constant over time.

This is our laissez-faire benchmark. Although the assumption that  $b+c=1$  may not be empirically true, it is a convenient assumption for our theoretical investigation, because it provides a sharp result in the case of the ‘laissez-faire’ economy. In that economy, there would be no tendency for equalization of wage rates whatsoever.

In the laissez-faire economy, the total resources available to an  $h$ -family are simply  $h$ . Thus, trivially, the *rate of increase* of total resources with respect to  $h$  is just unity. We state this fact to motivate what comes next.

## 2. Political competition

### (a) The policy space

Each political party must choose a policy in the *policy space*. Let  $\mu$  be the average wage in the society (at a particular date). We assume that the policy space consists of all pairs of continuous functions, denoted  $(\psi, r)$ , defined on the non-negative real line that satisfy three constraints:

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<sup>2</sup> The coefficient of variation of a distribution is its variance divided by its mean.

$$(3.1) \quad \int_0^{\infty} (\psi(h) + r(h)) dF(h) = \mu,$$

$$(3.2) \quad 0 \leq \psi'(h) + r'(h), \text{ for all } h,$$

$$(3.3) \quad \psi'(h) + r'(h) \leq 1, \text{ for all } h.^3$$

The function  $\psi$  is the after-tax income function: that is,  $\psi(h)$  is the after-tax income of a household with a parent who earns wage  $h$ ; and  $r$  is the educational investment function: that is,  $r(h)$  is the educational investment in a child from an  $h$ -family. We can think of the sum  $\psi(h) + r(h)$  as the *total resource* allocated to a type  $h$  family. Equation (3.1) is the budget constraint, which says that the average total resource in the society must be equal to the average income. Inequality (3.2) says that average total resources must weakly increase with pre-tax income, and inequality (3.3) says that average total resources cannot increase *too fast* with pre-tax income: in particular that rate of increase is bounded above by 1, the rate at which total resource increases in our benchmark laissez-faire economy.

If you wish to think of parties' proposing tax policies, instead of after-tax incomes, then simply imagine a party as proposing a tax schedule  $t(h) = h - \psi(h)$ . If  $t(h)$  is positive, then  $h$  pays net taxes, and if it is negative, then  $h$  receives a net transfer payment.

Equation (3.1) is a constraint of economic feasibility, but we have no such motivation for constraints (3.2) and (3.3): thus we will say they are *social norms*. For some reason, not here modeled, it would be unacceptable, in this society, were a party to

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<sup>3</sup> Actually, we do not assume these two functions are differentiable. Thus, (3.2) and (3.3), precisely, mean that *when the derivatives exist*, these inequalities should hold.

propose a policy  $(\psi, r)$  that violated (3.2) or (3.3). Perhaps there would be a revolution (overthrowing the democracy) if this were to happen – for instance, the rich would revolt if (3.1) were violated, and the poor, if (3.2) were violated. We do, however, note that in the laissez-faire policy, the rate of increase of total resources is exactly one, and so (3.3) simply says that a party cannot propose a policy in which total resources increase more rapidly than they do in laissez-faire. (In a sense, then, we restrict democracy to not being more regressive than laissez-faire.)

This policy space is very large: it is infinite dimensional. We put no restrictions on the functions  $\psi$  and  $r$  except those displayed above – they need not be linear, or even polynomial, or even everywhere differentiable. As I said earlier, the fact that parties can choose from this large space models the idea that political competition is ‘no holds barred’—well, no holds, except those precluded by the two social norms, are barred. This is not simply a mathematical nicety: it is, I contend, an important feature of the *argument*. For, when we derive a result in political economy where competition is restricted to a policy space of small dimension – such as the median-voter result discussed in the introduction—we never know if the result is robust, or whether it is an artifact of the (artificial) restriction on the flexibility of policy. Actual tax systems and educational financing policies are extremely complex – they are surely not drawn from a space of small dimension – and we attempt to capture this, in the model, with an infinite dimensional policy space.

One might ask: If voters have the power to redistribute income, why care about the level of human capital of one’s offspring? The main answer (there are others) is that I assume that human ‘capital’ is a misnomer: people derive welfare directly from their

human capital, as well as from the (future) income it provides. This obvious point – that one’s self-esteem increases with one’s level of human capital, that one derives welfare from understanding how the world works and from performing highly skilled work – has been obscured by the Chicago tradition, in which education is only an investment good. Thus, mothers care about their sons’ level of human capital. Mothers also care about their own level of human capital, but by the time one is an adult, that level is fixed, and so it is pointless to include it in the adult’s utility function.

(b) Parties

The problem is that the almost ubiquitous Downsian model of political competition – that of two candidates (or parties) each trying to maximize his (its) probability of victory – will induce a game with no Nash equilibrium on this policy space. How, then, can we conceptualize political competition so that an equilibrium will exist? I have recently proposed that this can be accomplished by conceiving of parties as consisting of factions that bargain internally over policy. Thus, parties compete *against* each other, in the sense of Nash equilibrium, but these parties are not ‘simple’: each is comprised of internal factions that bargain with each other, when facing the opposition party’s proposal. It turns out that, when *inter*-party competition is combined with *intra*-party bargaining, Nash equilibria exist in the competitive struggle, even when the policy space is infinite dimensional.

The details of this conception of political competition are laid out in my recent book (Roemer (2001)), to which the interested reader is referred. Here, I will give only a verbal description of the model of party competition. A party is conceived of as

consisting of three factions, called opportunists, reformists, and militants. The *opportunists* are the Downsian dramatis personae: they desire only to maximize the probability of the party's victory, and have no interest in policy per se. The *reformists* are the dramatis personae of Donald Wittman's (1973) model: facing the policy proposed by the opposition, they want to propose that policy that maximizes the expected utility of their constituents; the utility is 'expected' because the outcome of the election is uncertain, and constituent utility is taken to be the average utility of the party's *members*. The *militants* are party activists who do not care about winning this particular election: they would like to propose a policy as close as possible to the ideal policy of the party's (average) constituent. They are uncompromising, and use electoral competition as a forum to advertise, as it were, their constituents' interests. In a word, the three factions are interested, respectively, in *winning*, *constituent welfare*, and *publicity*.

Denote the policy space by  $T$ , and let  $t$  be the generic policy. Suppose there are two parties, denoted  $L$  and  $R$ , and suppose  $R$  has proposed a policy  $t^R$ . What happens inside party  $L$ ? Its three factions bargain to some proposal,  $t$ . All that we shall assume about the outcome of this bargaining process is that it is *Pareto efficient* with respect to the three factions, which means that, at the bargain  $t$ , there is no policy that would increase the payoff of any faction in  $L$ , without reducing the payoff of at least one other faction, given that the opposition is playing  $t^R$ .

What, then, constitutes a political equilibrium in the Nash competition between the two parties? It is a pair of policies  $(t^L, t^R)$  such that:

(a) Given  $t^L$ , policy  $t^R$  is Pareto-efficient for the three factions of party  $R$ ; that is, there is no policy  $t$  that increases the payoff of any one of  $R$ 's factions, and leaves the other two factions at least as satisfied as they are at  $t^R$ ;

(b) Given  $t^R$ , policy  $t^L$  is Pareto-efficient for the three factions of party  $L$ .

Thus, each policy is a best response to the other, where 'best response' means that no policy could be proposed that dominates the policy proposed from the viewpoint of the *collectivity* of the three factions. I call this concept *party unanimity Nash equilibrium* (PUNE), because there is no policy that either party's factions would *unanimously* prefer to the one their party is proposing.

I have yet to explain how the party memberships are determined *endogenously*. Consider any partition of the set of citizens (voters) into two elements –  $A$  and  $B$ . Now construct the parties associated with  $A$  and  $B$ : this means, we define the payoff functions of the three factions associated with each coalition of citizens. For example, the militants in the party representing  $A$  want to maximize the average utility of  $A$ 's members. Since we know the utility functions of all citizens, we can compute this function. Similarly, the reformists want to maximize the average *expected* utility of  $A$ 's members, and so on. Now let  $(t^A, t^B)$  be a PUNE associated with this party structure. We ask: does every member of  $A$  prefer the policy  $t^A$  to the policy  $t^B$ ? Most likely not! But sometimes, this will be the case!

Thus, we say that a partition  $(A, B)$  defines a party structure endogenously (an *equilibrium* party structure) if, at the PUNE  $(t^A, t^B)$ , every member of  $A$  at least weakly prefers  $t^A$  to  $t^B$  and every member of  $B$  at least weakly prefers  $t^B$  to  $t^A$ . This is a condition

of *party stability*, for we can imagine that if a member of  $A$  preferred party  $B$ 's proposal to  $A$ 's proposal, he would switch his party membership.

Thus, to be complete, we define a quadruple  $(A, B, t^A, t^B)$  to be a *party unanimity equilibrium with endogenous parties* (PUNEED) if:

- (a)  $(A, B)$  is a partition of the set of voters;
- (b)  $(t^A, t^B)$  is a PUNE for the parties with factions that represent  $A$  and  $B$ ;
- (c) for all members  $a \in A$ , policy  $t^A$  is at least as good as policy  $t^B$ ;  
for all members  $b \in B$ , policy  $t^B$  is at least as good as policy  $t^A$ .

Thus, the *members* of a party and *those who vote for it* are identical sets of voters, in equilibrium.

The concept of PUNEED takes as *data* the distribution of preferences of the polity, and produces as *output* a division of the polity into two parties, and the policy that each party proposes in political competition. In this sense, it does, qualitatively, just what the classical Downsian model does – *but*, it turns out, that these equilibria exist for policy spaces of any dimension.

Qualitatively speaking, the concept of PUNEED articulates parties that are complex, in the sense that they care about many things – winning, constituent welfare, and publicity. Mathematically speaking, the reason that equilibria exist, even with policy spaces of high dimension, is that an agreement among three preference orders has to be achieved before a party can move from a given position. Thus, it is much harder to ‘deviate’ in this game than in a classical game where each player has only one preference order, and this means that more pairs will survive the deviation test required of Nash equilibrium than in the Downsian game.

In fact, it turns out that there are many PUNEES – typically, an infinite number of them (more precisely, a two dimensional manifold of them). We cannot specify *which* PUNEES will occur without specifying more about the internal party bargaining process – what, for instance, the relative bargaining strengths of the factions are. Indeed, our main result will be that the asymptotic nature of the distribution of human capital depends intimately on the relative strength of these factions in the intra-party bargaining process.

### 3. Equilibrium policies

It is difficult to characterize, analytically, what the set of equilibrium (PUNEES) policies is for our model, at any date. It turns out, however, to be possible to characterize precisely the policies associated with a *larger* set of allocations, which I call *quasi-PUNES*. I will not present the definition of quasi-PUNE here: all the reader needs to know is that every quasi-PUNE consists of an object that looks, formally, like a PUNEES – that is, a partition of the set of voters into two elements,  $L$  and  $R$ , and a pair of policies  $(t^L, t^R)$  proposed by the two parties that represent those coalitions. I have written the partition of voters as  $(L, R)$  because one element,  $L$ , always consists of the set of voters whose human capital is less than some number  $h^*$ , and the other element,  $R$ , consists of all voters whose human capital is greater or equal to  $h^*$ .<sup>4</sup> Of course, in our particular application, each policy consists of an after-tax income function and an educational investment function, so we write  $t^L = (\psi^L, r^L)$  and  $t^R = (\psi^R, r^R)$ . Because every

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<sup>4</sup> This is a restriction that I impose: there may be PUNES in which parties represent disconnected sets of voters, but I do not study them.

PUNEEP is indeed a quasi-PUNE (though not conversely), everything we say about quasi-PUNEs will, a fortiori, be true of ‘real’ equilibria, that is, of PUNEES.

We can prove that, in any quasi-PUNE, and therefore, in any PUNEEP, the policies put forth by the *L*(eft) and *R*(ight) parties are as depicted in Figure 1. I have graphed the total resource that each party proposes to assign to citizens, by type. Now it is a fact that each party proposes to divide the total resource assigned to each household, into after-tax income and educational investment, exactly as the household would divide it, that is, according to equations (2.5). Thus, educational investment is just a proportion of the total resource. Consequently, Figure 1 is also a schematic representation of the educational investment functions proposed by the two parties, except that the slope of the increasing segments will not be one, but  $\frac{\gamma c}{1 + \gamma c}$ . The Left policy invests more in poor children and less in rich children than the Right policy; the two policies invest the same amount in a group of ‘middle income’ children; both policies are weakly increasing in the amount they invest in children as a function of family human capital. (Thus, it is interesting to note that the Left does not advocate investing more in children from poor families than rich families.)

As I wrote above, there are many possible PUNEES (and therefore many possible quasi-PUNEs), but their educational policies all look, qualitatively, like those pictured in Figure 1. In Figure 2, I present a graph which illustrates the entire set, or manifold, of quasi-PUNEs. On the horizontal axis is the wage level  $h^*$ , which partitions the population into the members of the Left and Right parties, and on the vertical axis is the total resource that type  $h^*$  receives from both parties at the equilibrium in question. Once we are given these two numbers, the picture of Figure 1 is completely

determined by the constraint that the total resource assigned to the population must exhaust society's total income. (That is, we can compute where the break points in the two policies occur.) The set of quasi-PUNEs is associated with the points inside the figure presented in Figure 2.

To what equilibria do the upper and lower curves of the set of quasi-PUNEs correspond? The lower curve contains equilibria where the militants of both parties are as powerful as possible<sup>5</sup>. The upper boundary contains equilibria where type  $h^*$ , who is the pivot type between the two parties, is receiving her ideal policy: the policy that maximizes her utility, in the set of feasible policies. It turns out that, at equilibria on this upper boundary, both Left and Right parties propose this policy. Thus, the upper boundary corresponds to 'the dictatorship of the pivotal voter' – shades of 'median voter' politics in the Downsian model. These equilibria result when the opportunists are extremely powerful in intra-party bargaining. Thus, as we move upward in the figure, holding  $h^*$  constant, from the lower boundary to the upper boundary, we trace out equilibria that reflect increasing power of the opportunist factions.

#### 4. Dynamics

Without further restrictions on the model, there are many possible equilibria at each date. To enable a dynamic analysis, we must fix a sequence of equilibria over time. I will examine two special sequences of equilibria.

The first sequence is gotten by fixing a type  $h_1^*$  and defining the equilibria as those in which, at every date, the pivot between the two parties is the descendent of this

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<sup>5</sup> Indeed, the militants of at least one party play their ideal point at these equilibria.

type, and the equilibrium is always on the lower boundary of the manifold of quasi-PUNEs – in other words, equilibria where the militants are very powerful. Call this intertemporal sequence of equilibria, *A*. The second fixes a given type (it need not be the same type as before) and defines the equilibria as those in which, at every date, the pivot between the two parties is the descendent of this type, and the equilibrium is always on the upper boundary of the manifold of quasi-PUNEs – that is, the opportunists are very powerful. Call this sequence of equilibria *B*. Looking at sequences of equilibria where the pivotal type remains in the same family is equivalent to saying that the probability of victory of each party remains constant over time. We might assume that, as an approximation, this probability should be about 0.5, although we need not assume that for the result to follow<sup>6</sup>.

Our main result is:

*Theorem* *If  $b+c=1$ , then in sequence A, the coefficient of variation of the distribution of wages converges to zero and the coefficient of variation in sequence B converges to a positive number.*

Recall the result of section 2, that in the laissez-faire benchmark, the coefficient of variation is unchanging over time when  $b+c=1$ . Our theorem says that, when militants dominate the intra-party bargaining process at each date, then convergence to equality of human capital occurs; but if opportunists in both parties dominate at each date, then it

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<sup>6</sup> If this probability were significantly different from 0.5, opportunists would cease to join one party, it would become dominated by militants, and fade from the scene. So a long-run evolutionary approach to

does not. We can conjecture, more generally, that to the extent that militants are powerful, democratic dynamics will tend to bring about equality, by which I mean that if we are fairly close to the lower boundary of Figure 2, then equality will be generated in the long run -- but I have no theorem that makes this claim precise.

These convergence results are independent of the sequence of actual victories of the parties. Recall that a PUNEEP only specifies the two parties' policies and their probability of victory. Thus, at each date in a sequence of equilibria, a random event at each date determines which party wins. As long as these probabilities do not approach zero or one, so that *each party wins an infinite number of times*, the theorem holds. (To be more precise, we only need require that the Left win an infinite number of times for equilibria in sequence A.) In the fuller model, this is assured by fixing the dynasty to which the pivotal type belongs, over time.

##### 5. Private educational investment: Topping off

Thus far, we have assumed that all educational finance is public. One might argue, however, that this assumption already stacks the deck in favor of equality, because the advent of publicly financed education was an important political victory of democracy. Therefore, it would be more compelling to *not* assume *a priori* that education will be publicly financed. We now modify the model in this way.

To be precise, we now propose that education can be funded by any mix of public and private contributions. Each family can augment the public contribution,  $r(h)$ , that it receives for its child with any amount of private investment, taken from its own after-tax

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parties will predict that probabilities of victory are near 0.5.

income. In the lingo, mothers can top off public investment with private investment in schooling. If a family spends  $r^{Pr}(h)$  privately on the education of its child, then the total investment in the child is simply  $r(h) + r^{Pr}(h)$ , and that is the investment that determines the child's future wage. Thus, private and public funds are perfect substitutes.

We are able to derive the following result. The equilibria in the revised model are isomorphic to the equilibria in the original model, in the following sense. Take any equilibrium in the revised model, let  $(\psi(h), r(h))$  be the victorious electoral policy, let  $r^{Pr}(h)$  be the function that describes how much each mother tops off public investment with private investment (from her after-tax income), and let  $R(h)$  be the total investment in education of an  $h$  family. Then there is an equilibrium in the *original* model in which the public investment function proposed by one party is  $R(h)$ .

It follows that our theorem continues to hold even if topping off is permitted.

Intuitively, what happens in the model where investment is not constrained to be entirely public is that parties are forced, by competition, to augment the after-tax income of families if they do not propose to invest in education as much as the families would like. For every family, the *total* resources invested in education are the same in the equilibria of the two models.

Our model, in summary, says that whether or not education is publicly funded is of no consequence to any citizen, for if it is not, then competitive forces will augment after-tax income to allow citizens to top off privately. In this aspect, the model fails to capture an important aspect of reality, that public funding of education makes a real difference in the amount of education (some) children will receive.

I believe that, in reality, publicly financed education often delivers more education to poor families than they would finance privately, were they to be given the cash equivalent of that educational investment. This is, formally, a critique of the utility function that I have proposed for adults. Suppose that, instead of (2.1), utility were defined as:

$$u^*(x, h') = \text{Log}(x - C) + \gamma \text{Log} h', \quad (5.1)$$

where  $C$  is a positive number. With these preferences, households with income less than or equal to  $C$  would devote *all* their income to consumption, and none to education. This is indeed the situation in poor countries, where many poor families send their children to work at an early age – that is, the subsistence constraint is all determining, and the family's entire labor supply must be devoted to earning consumption.

In contrast to (5.1), equation (2.1) defines preferences in which the 'subsistence level' is zero, and so, as long as income is positive, the family will devote some it to education. It is therefore appropriate to interpret our model as one applying only to countries at fairly high levels of development.

It is furthermore important to note that there may be *other* differences between public and private education that are important for the learning of children, and that we have not modeled here. One thinks, for example, of 'neighborhood effects:' it may well improve the educational outcomes of children from poor families to integrate them with children from better-off families in school; this will happen with public, but often not with private, schooling. This effect is ignored in our specification of the educational technology. And there is a public-good aspect to education that is ignored in our model, that my child may benefit from the education of other children. These positive

externalities to education will work in the direction, I think, of increasing the probability of convergence to equality in the distribution of human capital.

## 6. Discussion of the Theorem

Inequalities (3.2) and (3.3) are not motivated, at least within the terms of the model, by politics, and hence I called them social norms. Inequality (3.2) effectively constrains the Left party, which might otherwise like to propose that each rich family receive fewer total resources than each poor family, and inequality (3.3) constrains the Right party, for it says that total resources cannot increase too rapidly – so that if the Right proposes very little for the members of the Left party, it cannot jump up rapidly and propose a lot for its own members.

Rather than trying to argue that these constraints are realistic – although I think they are, at least in advanced democracies – let me propose another motivation for them. Our benchmark is *laissez-faire*, and the total resource solution in *laissez-faire* obeys both constraints. (Indeed, as I noted, in *laissez-faire*, the rate of increase of total resources with respect to  $h$  is one.) Thus, our set of feasible policies contains the *laissez-faire* policy, and so our study has asked: If we restrict ourselves to a set of feasible policies that contains the *laissez-faire* policy, how will democracy differ from *laissez-faire* in what it delivers?

How demanding is the upper bound (3.3)? Substantially, I think. For many poor countries, it may be that  $\psi' + r' > 1$  for part of the income distribution, for income is often taxed at very low rates and only the children of the relatively wealthy attend publicly financed schools, at least at the secondary and tertiary levels. Thus, it may well

be, in such countries, that inequality (3.3) fails for some regions of the pre-tax income distribution -- that is, the total resources going to families increase faster than their pre-tax incomes. These democracies would redistribute from the relatively poor to the relatively rich.

Our central result says that, if democratic politics have an ideological character -- where militants dominate in intra-party bargaining -- then convergence to equality of human capital is achieved. On the other hand, if political entrepreneurs who desire only to hold office dominate, then equality is not achieved in the long-run. Note the sharp contrast between this result and the Downsian result. With unidimensional policy spaces, Downsian intuitions predict that, if politicians are opportunist, and if median human capital is less than mean human capital, then complete leveling will occur! (Of course, the Downsian model we outlined in section 1 was insufficiently rich to permit both a redistributive consumption policy and an educational investment policy, because of its unidimensional nature.) We suggest that this shows the pitfalls of the restrictive Downsian model: in other words, modeling political competition on large-dimensional policy spaces is not just a mathematical nicety.

What happens if we relax the constraint  $b+c=1$ , and say that  $b+c < 1$ , which may be empirically more realistic? In that case, *both laissez-faire and democracy unambiguously generate equality in the long-run*. Therefore, to evaluate the effect of democracy in this case, we would have to get into the business of comparing *rates of convergence* of the two kinds of regime, a more delicate mathematical enterprise. As I said earlier, the assumption  $b+c=1$  is motivated by its property of generating a clean benchmark -- namely, a constant degree of inequality (in the sense of coefficient of

variation of wages) over time. We may conjecture that if  $b+c < 1$ , then we would have an analogue to our theorem of the following sort: If militants are powerful, then convergence to equality will occur much faster with democracy than with laissez-faire, and if opportunists are powerful, the rates of convergence of the two regimes will be more similar.

### 7. Why don't we observe more convergence in reality?

Our model is highly simplified and abstract. The model, as it stands, suggests only one reason that we do not observe convergence, and that is, that the militants are not particularly powerful in intra-party bargaining. But perhaps the more important reasons are ones that we have failed to model. Let me list several of the most important.

(1) Our model has assumed that there is no variation in talent or in effort among children from the same socio-economic background. This is empirically false. To model this kind of variation, we can add a stochastic term to the educational production function, a random variable  $\varepsilon$ , and write, in lieu of (2.2):

$$h' = \varepsilon \alpha h^b r^c \quad (7.1)$$

where  $\varepsilon$  has a mean of unity. If we substitute (7.1) for (2.2), then our theorem becomes modified as follows: for equilibria of type *A*, it is the case, after many periods, that the human capital of individuals is independent of the human capital of their ancient ancestors, while in equilibria of type *B*, the effect of the level of human capital of the original ancestor may be persistent forever. Thus, in equilibria of type *A*, we *do* reach a

situation of ‘protracted equality of opportunity,’ in the sense that a person’s family background has, at least in the long-run, no impact on her economic fortune.

(2) Our model allows only a very pleasant kind of technical change, namely, a change in the parameter  $\alpha$  over time. This kind of technical development is neutral in the sense that it leaves unchanged the ratios of wages in any two dynasties. In particular, it rules out the kind of inequality-increasing phenomenon that the United States and Great Britain have experienced in the past twenty years, in which the ratio of skilled to unskilled wages has increased. Whether the cause of this increased inequality in recent years is due to the information technology revolution, or to increased international competition, it would be modeled, in our set-up, as an exogenous shock which changes the educational technology in a non-neutral way. If non-neutral technical changes of this kind are historically important, then our model misses an important contributor to inequality.

(3) Our model assumes a *perfectly representative democracy*, in the sense that:

- (a) every citizen belongs to one party, and
- (b) each party aggregates the preferences of its members by giving equal weight to each member.

In reality, neither of these assumptions is true. In particular, in democracies with private campaign financing, (b) is very far from true.

(4) We have assumed that the only issues on the political agenda are redistribution of income and educational finance. In reality, many other issues exist. In particular, there are issues of *values* and *race/culture* that alter, in important ways, the politics of redistribution. For instance, in the United States, there is a substantial set of voters who, although poor, vote for the Republican party, which is anti-redistributive, because it is the party with the conservative view on race, which these voters share. Thus racism can fundamentally alter the redistributive agenda – a fact long obvious in the United States, and one becoming more salient in Europe, with the advent of significant immigration from poor countries. (See Roemer [1998] and Lee and Roemer [2002].)

(5) We have assumed that individuals have a perfectly inelastic labor supply. Indeed, this generates the flat sections of the total-resource functions in Figure 1, where, for intervals of income, the marginal tax rate is 100%. In reality, labor supply would be (very) elastic with total confiscation of income above a given level. This fact can only be expected to reduce the degree of redistribution compared to what our model predicts.

On the other hand, there are, as well, aspects of reality, not depicted in the model, that work in the direction of convergence to equality. Education is to some degree a public good, so that the well-off may be interested in educating the children of the less well-off, even absent altruism.

With these caveats, the reader is entitled to ask: What is the use of such an abstract analysis? The answer is that it is worthwhile to know whether democracy *under the best of conditions* -- conditions, that is, described by the model of our politico-economic environment – will, by itself, be significantly more effective in engendering

equality of opportunity, in the sense of eliminating the effect of family disadvantage on the economic fortunes of children, than a laissez-faire economy (one with a minimal state that enforces property rights but does not interfere with market incomes). Clearly, our analysis is only the beginning of a thorough investigation.

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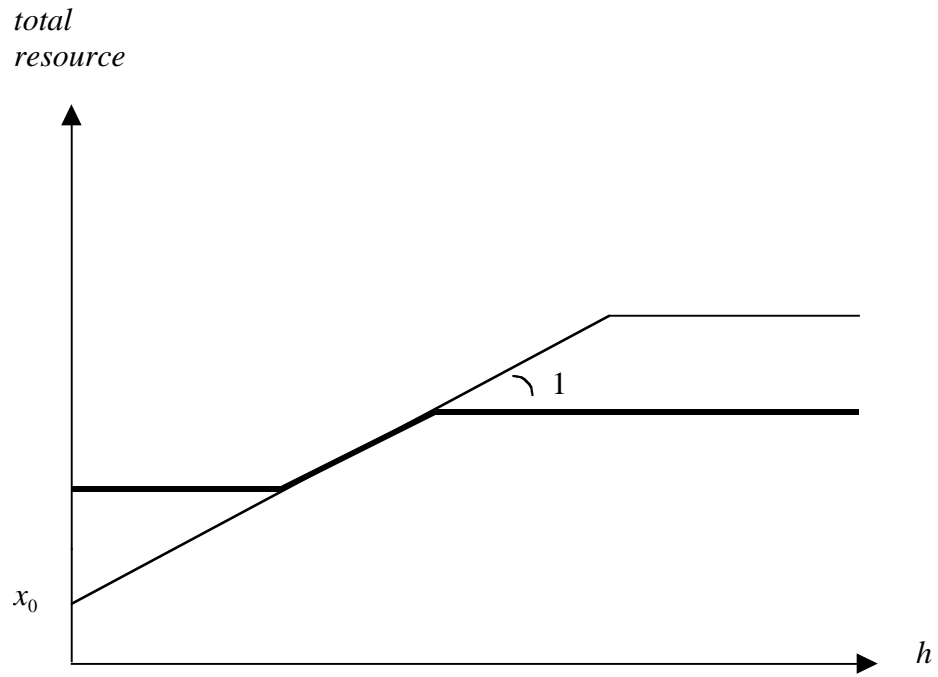
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Figure 1 Left (bold) and Right educational investments in a quasi-PUNE or PUNEEP



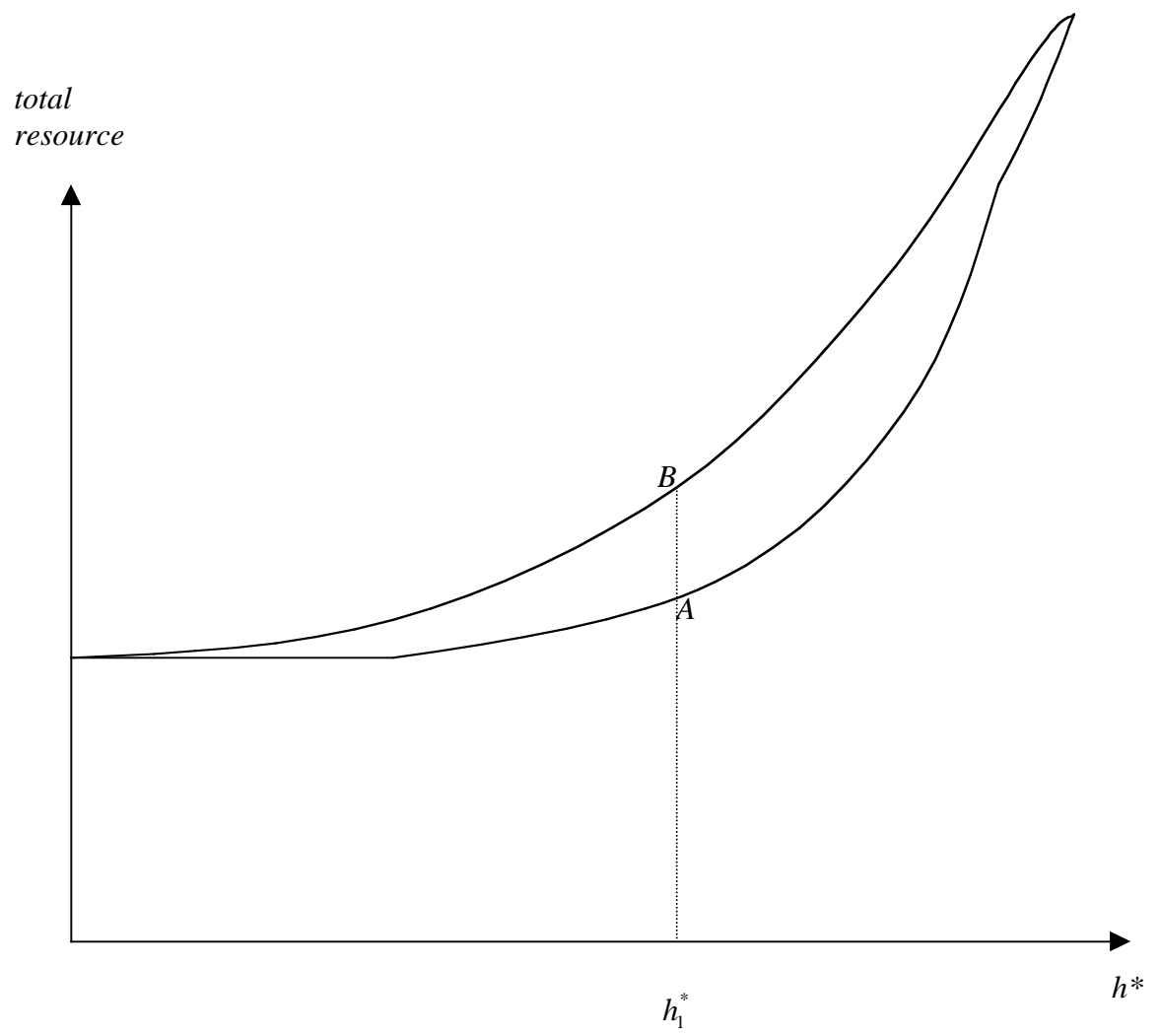


Figure 2 The manifold of political equilibria