A Formal Model of Distributive Politics with an Executive and a Budget Cap*

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Abstract

In this paper, I present a distributive politics model based on Baron(1993). The model adds two real-world political institutions to a distributive politics framework: (i) a veto-wielding executive who is subject to legislative override; and (ii) a legislature that can impose a budget cap on itself. I show that the legislature improves its welfare when it can impose a budget cap before it determines allocations in legislative districts. Its welfare also improves when a conservative executive is in office. In addition, several counterintuitive propositions emerge. For instance, suppose the legislature can limit its own spending; then it actually spends more when the executive is conservative (i.e., prefers less distributive spending) as opposed to liberal. In addition, regardless of the executive’s preferences, a budget cap has the effect of lowering spending, relative to not having a budget cap in effect. These theoretical results generate several testable predictions about spending patterns, which can be analyzed by taking advantage of the rich institutional variation in the U.S. states.
1 Introduction

“[T]here is [a] tendency that is common among people. When they can, they wish to live and prosper at the expense of others.” – Frederic Bastiat (The Law)

Bastiat, a 19th century student of government, did not study distributive politics per se, but he expressed the essence of the problem in his writing: politicians love to bring projects to their districts when they can impose costs on the general tax system. Every year, Congress provides funding for distributive projects, which benefit a concentrated geographic region but are funded out of general revenues.¹ Scholars and political professionals have long been interested in the study of these allocations, which sometimes are derisively referred to as “pork-barrel” spending.² Political scientists and economists have focused their attention on who gets what,³ whether projects

¹A distributive good has the property that a change to the good in a given locality does not affect the presence of the good in other localities. It is not the opposite of a public good; it may or may not be a “local public good,” in that the benefits of the good are nonrivalrous and nonexcludable (e.g., clean air in a legislative district).

²For instance, the nonpartisan Citizens Against Government Waste publishes a book called the Congressional Pig Book, which details the recipients of federal largesse. And from 1975 to 1988, Senator William Proxmire awarded a monthly Golden Fleece Award for “wasteful, ridiculous or ironic use of the taxpayers’ money” (Proxmire 1988).

³Riker (1962) and Buchanan and Tullock (1962) ushered in the modern era by predicting that minimum winning coalitions should arise in bargaining over a fixed pie. The authors reasoned that the addition of other members to a minimum winning coalition is unnecessary, as it decreases coalition members’ utility and does not affect the outcome of a vote. Invariably such coalitions tend to benefit the majority at the expense of the minority, and the composition of the coalition varies from vote to vote. Weingast (1979) and others countered that legislators respond to this uncertainty by forming universal coalitions in which legislators are masters of their domain and are allowed to select their preferred projects. (See Baron (1989) for a discussion of this approach’s limitations.)
are efficient,\textsuperscript{4} and how legislative structures affect outputs.\textsuperscript{5}

In this paper, I construct a distributive politics model, based on Baron’s (1993) sequential game, to analyze how institutions affect distributive politics bargaining, spending levels, and allocations. The main contribution of this paper is to add important institutional arrangements that previous work fails to consider. Typically in models of distributive politics, the legislature operates without spending caps or veto-wielding executives standing in its way. This stylized view ignores the constraints imposed by an executive or budget rules. Many real-world legislatures, especially at the state level, have rules in place that limit their spending. State governments cannot mint money, are beholden to bondholders, and operate under balanced-budget rules.\textsuperscript{6} In addition, the public views executives as budgetary stewards. At both the state and the federal level, executives propose a budget on an annual or biannual basis, though often these budgets are not binding. Symbolically, this links the executive to the final product.

I consider two institutions that address these issues: an executive with veto authority that is subject to legislative override, and a binding budget cap that the legislature enacts in advance of bargaining over allocations. I show that the effect of an executive’s spending preferences depend on the

\textsuperscript{4}For instance, in the universalism literature, legislators are akin to diners who decide to split the check on dinner, with no oversight regarding what meals will be ordered. The universalism “norm” eliminates costly uncertainty over the composition of the winning coalition, but it has a downside. Legislators choose inefficient programs in a universalistic world, because they fail to internalize the full costs of the project. This creates a legislative version of the tragedy of the commons(Hardin 1968). This tendency is sometimes referred to as the “Law of 1/n,” since inefficiency increases as the number of legislators increases.

\textsuperscript{5}Baron and Ferejohn (1989) and Baron (1989) developed the first non-cooperative sequential games of distributive politics and coalition building, which study the role of the agenda setter and the amendment structure on coalition sizes and the distribution and amount of spending. Coalitions in these models are never unanimous, the agenda setter gains at the expense of other legislators, and cooperation and coalitions emerge endogenously. In related work, Baron (1993) solves an infinite-horizon sequential bargaining model with a randomly-selected agenda setter, who is to propose projects in legislative districts.

\textsuperscript{6}Enforcement of these balanced-budget rules varies, as I discuss later in this paper.
budget cap. When a legislature can impose a budget cap on itself, it spends more when the executive is conservative than when he is liberal; when a legislature cannot cap its spending, it spends more when the executive is liberal. Also, I show that budget caps always reduce spending, relative to a model without caps, regardless of the executive’s spending preferences. I demonstrate that total net benefits for the legislature increase as the size of the override requirement gets larger, and that the effect of the override requirement on spending depends on whether a budget cap is in effect. I also establish that the legislature most prefers to limit its own spending and have a conservative executive in office. In addition, this paper sounds a cautionary note to policymakers who propose budgetary reforms.

The paper proceeds as follows. In section two, I discuss the literature. In section three, I describe the model and present the baseline Baron equilibrium as well as an extension to the baseline model. I add a budget cap to the model in section four, an executive in section five, and both a budget cap and an executive in section six. In section seven, I compare the results, discuss the implications of my findings, and suggest ways to test this theory.

2 Foundations

2.1 Distributive Politics

In the distributive politics literature, one of the long-standing puzzles has been that theory predicts the existence of minimum winning coalitions in legislative politics (e.g., Riker 1962), while in practice supermajority coalitions are the norm. After Riker (1962) and Buchanan and Tullock (1962)

\footnote{In this paper, I define minimum winning coalitions as coalitions that consist of the bare number of legislators necessary for legislation to pass. Technically, if a supermajority of $s$ legislators is required to pass a piece of legislation, then $s$ is the minimum winning coalition. Ambiguity arises when a piece of legislation requires a majority to pass, but a supermajority is required to override a veto of that legislation. In this case, an agenda setter may build a supermajority coalition to forestall a veto attempt. This coalition...}
posited the first theories of coalition size, empirics and theory clashed.\textsuperscript{8} Ferejohn (1974), Mayhew (1974) and later Wilson (1986), among others, found that oversized (though mostly non-unanimous) coalitions were the norm in a distributive politics setting.\textsuperscript{9} These results cast doubt on theories of minimum winning coalitions.\textsuperscript{10}

Weingast (1979) and others addressed the problem with a theory of universalism, a legislative norm in which every member of an institution receives a benefit if they desire it.\textsuperscript{11} This overcomes every legislator’s fear that she will be left out of the minimum winning coalition. In one variant on this approach, Weingast, Shepsle and Johnsen (1981) allow each legislator to select the size of a particularistic project in her district. A project of a given size $x$ has known benefits and costs, which are reflected in benefit and cost functions $B(x)$ and $C(x)$. Costs are divided equally across districts. This implies that a legislator will set $x^*$, the project size in her district, such that $B'(x^*) = \frac{1}{n}C'(x^*)$, which is inefficient, since the optimal project solves $B'(x) = C'(x)$.\textsuperscript{12} Because no legislator wishes to unilaterally reduce the size of his project, and since the norm ensures passage of the projects, a Pareto-dominated outcome results: a set of inefficient projects passes, even though there is a set of projects that makes everybody better-off. WSJ refer to this result as the “Law of $1/n$.” Their model provides valuable insights into the

\textsuperscript{8}For a cogent review of the distributive politics literature, see Collie (1988a) and Baron (1989).

\textsuperscript{9}For a point-counterpoint on this issue, see Bickers and Stein (1994) and Weingast (1994). For a discussion of the legislative coalitions over time, see Collie (1988b).

\textsuperscript{10}See Baron (1991) and Ferejohn, Fiorina and McKelvey (1987) for later research that predicts minimum winning coalitions. Recent work (e.g., Baron 1989; Groseclose and Snyder 1996; Krehbiel 1998; Carrubba and Volden 2000) provides theoretical explanations for the presence of oversized (or larger than majority-sized) coalitions in legislative politics.

\textsuperscript{11}Papers in this tradition include Fiorina (1981), Shepsle and Weingast (1981), Weingast, Shepsle and Johnsen (1981), and Niou and Ordeshook (1985).

\textsuperscript{12}This holds only for $B(x)$ and $C(x)$ such that $B(x) - C(x)$ is strictly concave. For example, if benefits and costs are both linear, then it is possible that there is no maximum.
problem of cost-sharing and has implications for fiscal federalism (e.g., Del-Rossi and Inman 1999), but this approach raises concerns. Most importantly, the norm of universalism is never established an equilibrium phenomenon, but rather is assumed.\footnote{To be sure, Shepsle and Weingast (1981) present a more general form of this model which generates the conditions under which universalism is preferred to majoritarianism. However, it fails to address how the norm is sustained over time. Further, it begs the question: if the legislature could sustain universalism, why could it not sustain the variant of universalism that maximized the legislature’s welfare? See Schwartz (1994) for a defense of universalism.}

Game theoretic approaches have improved the literature significantly. Baron (1989) presents a theory of coalition formation which predicts supermajority coalitions under certain circumstances, and Baron and Ferejohn (1989) study the role of different institutional arrangements on bargaining outcomes in a divide-the-dollar framework. Baron (1991) offers a theory of distributive politics with predictions of minimum winning coalitions and an explanation for the presence of inefficient projects. Baron (1993) extends his 1991 model by allowing for project sizes to be chosen independently district-by-district, rather than from a fixed pie. Gilligan and Krehbiel (1994) use a divide-the-dollar framework to discuss the “gains from exchange” hypothesis in legislative studies. Carrubba and Volden (2000) show that coalition formation ranges from minimum-winning to universal and depends on the size of the legislature and time preferences.

Most game theoretic analyses do not include an executive with veto authority, though some exceptions can be found. McCarty (2000) extends Baron (1991) and shows that an executive with preferences over the division of the distributive pie can affect both the size of the coalition and which legislators will be in the coalition. In addition, Inman and Fitts (1990) and Fitts and Inman (1992) argue that Presidents can utilize resources at their disposal to overcome the universalistic (and inefficient) tendencies of the legislature; commitment to a particular position is central to this ability.\footnote{Ingberman and Yao (1991) present a model of presidential commitment in a non-}
limitation of Inman and Fitts’ work is that it does not overcome the fundamental problem with the research on universalism: universalistic models never specify a mechanism by which the phenomenon will emerge and be sustained. Finally, Lohmann and O’Halloran (1994) study the effect of divided government on trade policy, where trade policy outputs are distributive goods.

2.2 Can Budget Rules “Stick”?

Also absent from distributive politics theories is a discussion of total spending on distributive goods. In most models, the pie is either fixed (i.e., a divide-the-dollar game) or the size of spending is a byproduct of project selection. However, legislatures may be able to set spending limits in advance of the allocation of projects. To the best of my knowledge, this is the first formal model that has explored the effect of this institution on distributive spending.\footnote{Ferejohn and Krehbiel (1987) use a social choice model to examine the effect of institutional arrangements on budget size; they show that rules are not neutral with respect to budgetary outcomes. Gabel and Hager (2000) use a numerical example to show that a balanced budget rule that allows deficits when a supermajority votes for them can lead to higher spending. While Gabel and Hager’s intuition is exactly right—reforms often carry with them unintended consequences—I show elsewhere Primo (n.d.) that their specific result is highly contingent on the parameter values chosen.}

To be sure, legislators are adept at circumventing the rules they place on themselves. For instance, in 1990 Congress adopted the Budget Enforcement Act. Under the Act, discretionary spending is capped every year. Not surprisingly, Congress included a giant loophole in the law. Legislators may enact emergency appropriations if they so choose. Conveniently, emergency appropriations are not defined, and in a remarkable coincidence, there have been many emergencies since 1990. Since sovereign governments can literally mint money in the near-term, it may not make sense to model budget caps as externally enforceable. To that end, in Appendix B, I sketch a model in
which caps are self-enforcing.

Despite this, there are examples of governments which are required to abide by spending limitations to some degree, notably the American states. As McKinnon and Nechyba (1997) discuss, American states face a hard constraint because they cannot mint money, cannot expect to be bailed out by the federal government, and will face an exodus if they mismanage the budget. State bondholders will hold the state accountable if fiscal mismanagement occurs. Poterba and Rueben (1999) demonstrate that states with stricter budget rules have lower costs of borrowing, and Bayoumi, Goldstein and Woglom (1995) find that “credit markets do appear to provide incentives for sovereign borrowers to restrain borrowing.” Further, many states have strict balanced budget requirements, such that any end-of-year deficits must be eliminated immediately, and cannot be carried over to the next fiscal year. (Of course, states can engage in accounting tricks as much as any other entity.) In an important paper, Bohn and Inman (1996) demonstrate that states which have externally enforced balanced-budget requirements tend not to run deficits in the general fund, and in fact often run surpluses. Similarly, Eichengreen (1994) and Eichengreen and Bayoumi (1994) demonstrate that states with more stringent budget rules tend to have lower deficits (higher surpluses), though they find no relationship between these rules and the level of spending. This suggests that the effect of these rules is on taxation rather than spending. Alesina and Bayoumi (1996) find that stricter fiscal rules tend to reduce deficits but do not affect state outputs.16

Can the U.S. government constrain itself in a similar way? If a Constitutional amendment set budget rules, and if these rules were enforced by an independent body (say, the Supreme Court), then perhaps. This may be why supporters of balanced budget requirements at the national level seek a Constitutional amendment.

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16For a different viewpoint, see von Hagen (1991) on the weak effect of fiscal restraints on state debt.
In this paper, one can think of the budget cap model as a world with externally-enforced caps, and the baseline Baron model as a world where rules may exist, but are not enforceable. From a game theoretic perspective, non-enforceable rules amount to cheap talk that have no effect on the model.

Additionally, in this paper I treat the override requirements and the institution of a budget cap as exogenous. However, at some point these were objects of choice. Since these rules tend to be difficult to change, it is reasonable to treat them as exogenous. This is especially true if they have been in place long enough that the connection between the institutional choices and the spending choices is weak.\footnote{See Rueben (1995) for an analysis of tax and expenditure limitations (TELs) in which TELs are treated as endogenous. In empirical work, she finds that this changes her estimates of the effect of the TELs on budgetary outcomes.}

3 Structure of the Game

3.1 Actors and Preferences

$N$ identical legislators, where $n$ is odd for convenience, comprise a legislature $L$, who have preferences over a vector of projects $X = \{x_1, x_2, x_3, \ldots, x_n\}$, $x_i \in [0, \infty)$. The legislator in district $i$ evaluates the benefits and costs of projects with the function

$$bx_i - \frac{1}{2}cx_i^2.$$ 

This implies that the efficient project scale is $\frac{b}{c}$. Net benefits for legislator $i$ are

$$NB_i(X, b, c) = bx_i - \frac{1}{2}c \sum_{j=1}^{n} x_j^2.$$ 

The benefits of the projects do not spillover into other districts (i.e., no positive externalities are present), and districts share the total cost of projects
equally.\footnote{This is the function selected by Baron (1993) as an example that generates a closed-form solution. General results can be proved with the assumption that the net benefit function is strictly concave and hence guarantees a unique maximum for each legislator.} Given this function, legislator $i$’s ideal vector of programs consists of $x_i = \frac{nb}{c}$ and $x_j = 0 \ \forall j \neq i$. Later I will introduce an executive, $E$, who has preferences over the overall cost of the projects, not the allocation.\footnote{Some research suggests that Presidents may be concerned about allocation, due to the presence of the Electoral College. For example, in seminal work, Wright (1974) demonstrates that electorally important states tended to benefit more from New Deal largesse. Grier, McDonald and Tollison (1995) show that Presidents weight more heavily the votes of Senators from electorally important states in the decision to veto legislation. Mebane and Wawro (1993) use county-level data to demonstrate that Presidents target “local federal expenditures” with an eye toward Electoral College constituencies. A similar argument does not apply to state governors, since they are elected in a system where all votes count equally. In future research, I would like to examine how my model is affected when the executive has preferences over allocations as well as overall spending levels.}

### 3.2 Legislative Organization

This model is based on Baron’s 1993 distributive politics model; in the rest of the paper, I refer to it as the baseline model. A legislature $L$ must select a vector of projects, $X = \{x_1, x_2, x_3, \ldots, x_n\}$. The game is an infinite-horizon bargaining model with the following structure. At the beginning of every period, nature selects a legislator at random to serve as an agenda setter. The agenda setter makes a proposal, which is a project vector. Let $x$ represent the size of the proposer’s project, and $y$ the size of projects for those members who receive an offer from the proposer. (By symmetry, in equilibrium $y$ will be the same for all members who receive a project.) The legislature, operating under a closed rule (i.e., no amendments allowed), then votes on the proposal by majority rule. If the proposal is accepted, the game ends. If the proposal is rejected, nature selects a new agenda setter at random, and the new agenda setter offers a new proposal. The game continues indefinitely until an agreement is reached. For simplicity, I set the discount factor for
the legislators to one.\footnote{Baron (1993) discusses the effect of changes in the discount factor on equilibrium outcomes. They are not central to the analysis here, which is focused on how institutional arrangements change predictions of coalition size and budget size.} The equilibrium concept is subgame perfect Nash restricted to the consideration of stationary strategies, in which players must take the same actions at every node in which the game is structurally identical.\footnote{See Baron and Kalai (1993) for a discussion of the “focal quality” of the stationary equilibrium. They show that the stationary equilibrium is the “simplest” equilibrium of a Baron-Ferejohn (1989) game with infinitely-many subgame perfect Nash equilibria.} The structure of the game guarantees that, in equilibrium, \( x^* \) and \( y^* \) will be the same in all rounds of bargaining.\footnote{Also, note that the structure of this game implies that the \textit{ex ante} expected net benefit in the game will be equal to the net benefit realized by a supporter of the coalition. Therefore, unless otherwise noted, when I refer to net benefits, this can be taken to mean either the expected value of the game \textit{ex ante} for all legislators or the actual benefits received by a supporter of the winning coalition.}

I add two features to the model. First, I allow the legislature to select a budget cap before the agenda setter is known. The agenda setter must propose a set of projects that does not exceed the budget cap. Then, I include an executive with preferences over total spending. After an allocation passes, the executive can sign the bill into law, or he can veto the legislation, which requires a supermajority of \( s \in \left( \frac{n+1}{2}, n \right] \) legislators for an override. If he successfully vetoes the legislation, the legislature selects a new budget and the game begins again. Figure 1 depicts the sequence of play in a model with both an executive and a budget cap. I now turn to the equilibrium results of the baseline model.

3.3 Baseline Equilibrium with Majority Rule

\textbf{Proposition 1 (Baron 1993)} The subgame perfect Nash equilibrium in stationary strategies is as follows. In every period, the agenda setter proposes \( y^* = \frac{b}{c} \) to \( \left( \frac{n-1}{2} \right) \) legislators and \( x^* = \frac{(n+1)b}{2} \). In every period, those members who receive an offer of at least \( y^* \) vote for it, and all other legislators vote against it. The agenda setter accepts offers of at least \( y^* \), and since \( x^* > y^* \), the agenda setter votes for the proposal. The supporters’ project scales are...
efficient, and ex post net benefits for all legislators except the agenda setter are negative.

In this equilibrium, every legislator in the coalition receives an efficient project, except for the proposer, who gains at the expense of those without a project. A summary of the equilibrium values can be found in Table 1. I define the budget $I$ as the cost of the projects, or $\frac{c}{2}(x^2 + \frac{n-1}{2}y^2)$. In this case, $I = \frac{b^2(n^2+4n-1)}{8c}$.

Surprisingly, the ex post net benefits for all supporters in this equilibrium are negative, as are the expected net benefits at the beginning of the game. If all legislators except the agenda setter are worse off in this game than if no spending took place, then how can this be an equilibrium? Why wouldn’t the legislators simply vote against such a proposal indefinitely? The answer lies in their expectations about what happens if a proposal fails. If the proposal fails and bargaining continues, then the legislator offered $y^*$ in round $t$ has a $\frac{n-1}{2n}$ chance of being left out of the coalition altogether in round $t+1$, which is an even worse outcome. Thus, defection is not profitable. If this were a one-shot game, then a minimum winning coalition of legislators would have to receive non-negative benefits, or else they would vote down the proposal and take zero spending. In this infinite-horizon model, continuation values (the expected value of the game for a legislator who rejects an offer) are negative, which drives the result.

Later I will show that a budget cap that limits spending affects the outcome in a similar way. Because legislators always have the option of setting spending to zero if they expect negative outcomes in expectation for all positive budgets, non-negative continuation values obtain in equilibrium.

3.4 Baseline Equilibrium with Supermajority Rule

The next equilibrium extends the model to consider legislatures that require a supermajority of $s$ legislators to enact legislation. I use this result later in the paper. Proofs appear in the appendix.
Proposition 2 (Baseline Model With a Supermajority Requirement)

The subgame perfect Nash equilibrium in stationary strategies is as follows. In every period, the agenda setter proposes $y^* = \frac{b}{c}$ to $s - 1$ legislators and $x^* = (n - s + 1)\frac{b}{c}$. In every period, those members who receive an offer of at least $y^*$ vote for it, and all other legislators vote against it. The agenda setter accepts offers of at least $y^*$, and since $x^* > y^*$, the agenda setter votes for the proposal. The supporters’ project scales are efficient. Net benefits are positive for the supporters for sufficiently high $s$.

The supermajority requirement makes all legislators except for the agenda setter better off ex post than in a majority-rule setting; and expected net benefits are higher than in a majority-rule setting. The supermajority requirement increases the probability that the legislator will get a project in the next period. This implies that the agenda setter needs to make the proposal, $y^*$, more attractive to the legislators. The optimal way to do this, because of the quadratic costs, is to reduce his project to fund new projects, and to give all supporters an efficient project. Note that if $s = n$, then all legislators receive an efficient project. Further, the total cost of the projects, noted in Table 1, decreases in $s$. Thus, the supermajority requirement in this model increases the number of efficient projects that are provided and lowers the size of spending.

The legislators in this model are not bound by a budget constraint. I now add an endogenously-chosen budget constraint to the model and examine the implications of this choice.

4 The Budget Cap Model

In this section I extend the baseline model to include a pre-bargaining period in which the legislature sets the level of spending. Because all legislators are identical, in the pre-bargaining period the action of a representative (dictatorial) legislator can be used to predict the budget constraint that is selected.23

23The reason there is no dissension over the selection of $I$ is that ex ante legislators are equally likely to be selected as the agenda setter or as a member of the winning coalition.
After the legislature sets the budget cap, the agenda setter makes a proposal that satisfies the cap, and the legislature votes on it. If the proposal fails, the legislature selects a new cap, and a new round of bargaining begins.

The model builds-in within-period enforcement of the budget constraint. Once an agenda setter is chosen, he may have the incentive to propose a bill that both rescinds the budget constraint and requests an allocation that does not satisfy the budget constraint. Under many circumstances, the other legislators would support the legislation, if they believed that agenda setters in subsequent rounds would behave similarly. To avoid this, I assume that the budget constraint cannot be changed except at the beginning of any bargaining round. Perhaps it is constitutionally mandated, or changes require high costs such that the agenda setter does not wish to challenge the cap.\textsuperscript{24}

As discussed earlier, this is a reasonable assumption in many U.S. states. I now state the equilibrium when an enforceable budget cap is in effect.

**Proposition 3 (Majority Rule Budget Cap Model)** The subgame perfect Nash equilibrium in stationary strategies is as follows. In the budget stage, the legislature unanimously selects 
\[ I^* = \frac{2b^2n^2}{c(n^2+4n-1)} \]
Then, in every period of bargaining, the agenda setter offers 
\[ y^* = \frac{4n}{n^2+4n-1} \frac{b}{c} \]
to 
\[ n^* = \frac{n+1}{2} \frac{4n}{n^2+4n-1} \frac{b}{c} \]
and everybody who receives an offer of at least \( y^* \) votes for it. The agenda setter votes for a proposal that gives him at least \( y^* \), and since \( x^* > y^* \), the agenda setter votes for the proposal. Net benefits are positive for all legislators who receive a project.

Now consider the supermajority rule budget cap model, in which a supermajority of legislators must approve allocations. The majority-rule result is just a special case of the supermajority result where \( s = \frac{n+1}{2} \). I utilize

\footnote{Therefore, most any collective choice mechanism suffices to generate this result.}

\footnote{It is possible to make the budget cap “stick” endogenously with a slight modification of the model. Suppose that the agenda setter can either propose to wave the budget cap in subsequent periods or can propose some allocation. It is easy to show that he would never use his proposal to undo the budget constraint, since he has no guarantee of being the agenda setter in the following period. Also, see Appendix B for a possible modification of the model, which allows the constraint to be challenged in a reconciliation process.}
the general supermajority case later in the paper. However, when I compare
different institutional arrangements, I always assume majority rule in the
legislature.

**Proposition 4 (Supermajority Rule Budget Cap Model)** The subgame
perfect Nash equilibrium in stationary strategies is as follows. In the budget
stage, the legislature unanimously selects \( I^* = \frac{b^2 n^2}{2c[(n-s+1)^2+(s-1)]} \). Then, in ev-
ery period of bargaining, the agenda setter offers \( y^* = \frac{nb}{c[(n-s+1)^2+(s-1)]} \) to \( s-1 \)
legislators and \( x^* = \frac{nb(n-s+1)}{c[(n-s+1)^2+(s-1)]} \), and everybody who receives an offer of
at least \( y^* \) votes for it. The agenda setter votes for a proposal that gives him
at least \( y^* \), and since \( x^* > y^* \), the agenda setter votes for the proposal. Net
benefits for all legislators who receive a project are positive.

The next two remarks establish a relationship between the baseline model
and the budget cap model.

**Remark 1** In both the baseline and budget cap models, net benefits increase as
the supermajority requirement to pass legislation increases. In the baseline
model, spending decreases as the supermajority requirement increases. In
the budget cap model, spending increases as the supermajority requirement
decreases.

This result relates to the interaction between executives and budget caps
later in the model. The interaction effect occurs because the maximization
problem changes when the legislature selects spending levels, and this induces
the agenda setter to make different choices. This result plays a role in the
budget cap executive model. As I show later in the paper, a low-spending
executive forces the agenda setter to build an override coalition, but this is
observationally equivalent to an increase in the legislators necessary to pass
legislation. This is what drives the results we observe in the budget cap
executive model.

**Remark 2** The expected net benefits in the budget cap model are positive
and greater than those realized in the baseline model, and spending is lower
than in the baseline model.
This is a main result of the paper. A budget cap imposed by the legislature simultaneously lowers spending and makes the legislature better off. (This result holds when an executive is in place as well.) In the baseline model, net benefits are negative even for legislators offered a project by the agenda setter. In the budget cap model net benefits are positive for those legislators. The equilibrium level of spending drops when the legislature selects a budget cap because the equilibrium scale of all projects drops. The supporters’ project scales are now smaller than the efficient size, while the agenda setter’s project remains larger than optimal.

The budget cap adds a constraint to the agenda setter’s problem: he must select \( x \) and \( y \) such that he stays within the spending level set by the legislature. Why must the legislature always be better off in this case? First, the legislature can never do worse than in the baseline model, since it can always select the spending level from that model as its budget constraint. Next, the baseline model produces negative benefits whenever \( n \geq 5 \). This implies that the legislature can never do worse than a net benefit of 0 whenever \( n \geq 5 \), since it can always select zero spending. This provides the rationale for why a sufficiently large legislature always makes itself better off with a budget cap when \( n \) is reasonably large.\(^{25}\)

Further, the agenda setter, at the proposal stage, and the legislature, at the budget cap stage, have different incentives. The agenda setter maximizes his utility, subject to the constraint that a winning coalition votes for his proposal. The legislature selects \( I \) such that the average benefits of a legislator will be maximized.\(^{26}\) This prompts the legislature to select a smaller \( I \).

\(^{25}\)Since legislatures are rarely smaller than 5 members, I focus on cases where \( n \) is sufficiently large, but the proofs are calculated for all \( n \).

\(^{26}\)The average benefits of a legislator must be equal to the net benefits of a member of a winning coalition, by construction of the continuation value. A legislator accepts an offer of \( y \) in period \( t \) if \( NB(X) \geq v \), where \( v \) is the continuation value. This will hold with equality given the functional forms I have chosen. One interpretation of \( v \) is that it is the average benefit of the game, which implies that net benefits for a legislator who receives a project must be the same as the average benefit of the game.
How does the legislature manage to lower spending and increasing its net benefits, just by selecting \( I \) properly? First, it is easy to show that the ratio between \( x \) and \( y \) remains unchanged when the legislature sets a budget cap in the first period. But \( I \) now appears in the relations for \( x \) and \( y \). I show in the appendix that

\[
y^* = \sqrt{\frac{2I}{c[n^2 + s(n - s + 1) + s(n - 1)]}} \quad \text{and} \quad \text{and} \\
x^* = (n - s + 1) \sqrt{\frac{2I}{c[n^2 + s(n - s + 1) + s(n - 1)]}}.
\]

Notice that \( x \), the agenda setter’s project scale, declines in \( I \) \( (n-s+1) \) times faster than \( y \) does. Because the interests of the agenda setter and the legislature conflict, the legislature will clearly want to move to a lower \( I \) than the one preferred by the agenda setter. When it selects \( I \), the legislature takes into account the expected benefits of the game for a given \( I \). The agenda setter, once chosen, maximizes his utility. Put differently, a legislator who does not yet know who the agenda setter or coalitions members will be gets a smaller “bang for the buck” than the agenda setter, who knows that he gets a large share of every additional dollar of total spending for his district.

To see this formally, I consider the net benefit function for any legislator \( i \) at the budget constraint stage (denoted \( NB_i \)), as a function of \( I \):

\[
NB_i = b \sqrt{\frac{2I}{c[n^2 + s(n - s + 1) + s(n - 1)]}} - \frac{I}{n}.
\]

To show that the legislature wants to lower spending, note that the marginal benefit of an additional dollar of spending for legislator \( i \) is

\[
\frac{b}{2\sqrt{I}} \sqrt{\frac{2}{c[n^2 + s(n - s + 1) + s(n - 1)]}},
\]

while the marginal cost of an additional dollar of spending for legislator \( i \) is \( 1/n \).
Recall that the equilibrium level of spending in the supermajority baseline model is

\[ I = \frac{b^2}{2c} \left[ (n - s + 1)^2 + (s - 1) \right]. \]

At this level of spending, \( MB_i - MC_i \) is equal to

\[ \frac{1}{(n - s + 1)^2 + (s - 1)} - \frac{1}{n}, \]

which simplifies to

\[ \frac{[(n - s + 1) - (n - s + 1)^2]}{n[(n - s + 1)^2 + (s - 1)]}. \]

This quantity is negative, as is the second derivative. Because the function is concave, this indicates that the maximum net benefit is achieved by lowering \( I \).

5 The Executive Model

In this section, I consider a baseline model in which an executive has veto authority. If he vetoes the agenda setter’s bill, the legislature can override the veto with a supermajority of \( s \) legislators, \( s \in (\frac{n+1}{2}, n] \). I assume that the executive has preferences only over the size of the budget, NOT the allocation of the budget, and these preferences are common knowledge. Further, I assume that his discount rate is strictly less than one. If the executive vetoes a bill and the veto is not overridden, spending is set at zero for that period, the legislature chooses a new spending level, and the game resumes. Let the executive’s ideal point for spending be denoted by \( E \), where \( E = \{L, H\} \). If \( E = L \), then the executive is said to be a low spender who prefers no distributive spending. If \( E = H \), where \( H \) is large enough such that the executive always prefers more spending than the legislature, then the executive is said to be a high spender. I also use the terms conservative and liberal, respectively, to refer to these two types of executives. Let \( U_P(I) \) denote the executive’s utility as a function of the level of spending, and let \( Z \)
be the spending level such that the high-spending executive vetoes all budgets greater than this amount, and signs all budgets lower than this amount. Assume that $Z > \frac{20n^2}{c(n^2+4n-1)}$, the budget selected by the agenda setter in the baseline model.\footnote{I do not consider a continuum of executive ideal points for spending in this model or the next, because this complicates the analysis and does not add any insights to the model. Consider 3 intervals of ideal points: call them low, medium, and high. For low ideal points, the agenda setter always builds override coalitions. For high ideal points, the agenda setter always constructs majority coalitions. The equilibrium in the low and high cases are invariant to the ideal points within the interval. Think of the equilibrium levels of spending as a step function in the low and high intervals of executive ideal points. For medium ideal points, the agenda setter and the legislature must consider whether to set spending to a level that allows them to build a majority coalition, or whether to set spending to a level that requires an override coalition. The equilibrium I present considers only low and high ideal points, because the medium case depends to a large degree on the values of the $b$ and $c$ terms, which are not central to the model. Therefore, this region of ideal points adds little to the analysis.}

The equilibrium to this model combines two results previously proven, so it is not stated formally. If the executive is a low spender, then the agenda setter must build an override coalition to secure a positive budget. Therefore, this game is equivalent to the one described in Proposition 2, the baseline model with supermajority rule, because the override requirement acts like a supermajority requirement. If the executive is a high spender, then the constraint he imposes does not bind, and the equilibrium policy outcome will be the one described in Proposition 1, the baseline model. Note that in this model, the low-spending executive induces lower spending. Recall that expected net benefits increase as $s$ increases. Therefore, the low-spending executive improves the legislature’s welfare because he forces the agenda setter to build a larger coalition. This is summarized in the following remark, presented without proof:

**Remark 3** Given $s \in (\frac{n+1}{2}, n]$, and when the executive is a low spender, the net benefits in the executive model are positive and greater than those attained in a baseline model with majority rule, and spending is lower than in the baseline model. If the executive is a high spender, the equilibrium is unchanged from a majority-rule baseline model.
6 The Budget Cap Executive Model

This section presents the main model which includes both a budget cap and an executive, where the executive is either a low spender or a high spender. The path of play is as follows. The legislature selects a spending level. The agenda setter proposes an allocation, which is voted upon by majority rule. If rejected, the game begins again. If accepted, the executive either signs or vetoes the bill containing the allocation. If he signs the legislation, the allocation is enacted, and the game ends. If he vetoes, the legislation, the legislature then either overrides the veto with \( s \) legislators, in which case the allocation is enacted, or it fails to override the veto, in which case spending is set to zero and a new round of bargaining begins with a new budget set by the legislature. Refer to Figure 1 for the path of play.

**Proposition 5 (Budget Cap Executive Model)** The subgame perfect Nash equilibrium in stationary strategies is as follows. If \( E = L \), then in the budget stage, the legislature unanimously selects \( I^* = \frac{b n^2}{2c[(n-s+1)^2+(s-1)]} \). Then, in every period of bargaining, the agenda setter offers \( y^* = \frac{n}{(n-s+1)^2+(s-1)} b \) to \( (s-1) \) legislators and \( x^* = \frac{n(n-s+1)}{(n-s+1)^2+(s-1)} c \), and legislators who receive an offer of at least \( y^* \) vote for it. The agenda setter votes for a proposal that gives him at least \( y^* \), and since \( x^* > y^* \), the agenda setter votes for the proposal. The executive vetoes all legislation in which \( I > 0 \). If \( E = H \), then in the budget stage, the legislature unanimously selects \( I^* = \frac{2b^2 n^2}{c(n^2+4n-1)} \). Then, in every period of bargaining, the agenda setter offers \( y^* = \frac{4n}{n^2+4n-1} c \) to \( \frac{n-1}{2} \) legislators and \( x^* = \frac{n+1}{2} \frac{4n}{n^2+4n-1} c \), and everybody who receives an offer of at least \( y^* \) votes for it. The agenda setter votes for a proposal that gives him at least \( y^* \), and since \( x^* > y^* \), the agenda setter votes for the proposal. The executive never vetoes legislation on the equilibrium path.

The main finding is that there is an interaction effect between the executive and the budget cap. When an executive is a low spender and the legislature can set a budget cap, spending increases, relative to if no budget cap were in place. When an executive is a high spender and the legislature caps spending, spending decreases, relative to if no budget cap were in
place. Even more interesting, moving from a liberal to a conservative executive affects spending differently depending on whether a budget cap is in force. When a budget cap is in place, a conservative executive produces higher spending than a liberal executive. When no budget cap is in effect, the liberal executive produces greater spending than the conservative executive.

This paradox suggests that a low-spending executive who cannot commit to signing legislation will cause more spending than if he were liberal or did not have veto authority.\(^\text{28}\) This occurs because building an override coalition requires more spending when a budget cap is present. The net benefits of the model show that the presence of an executive can never hurt the legislature, and a frugal executive improves the legislature’s welfare. In addition, when the executive is a low spender, he ends up worse off than he would be if he had no veto authority.

In large measure, the conservative executive affects the legislature by forcing an override coalition to form. As noted in Remark 1, the effect of coalition sizes on spending depends on whether a cap is in effect. When the legislature can set a budget \textit{ex ante}, its net benefits are increasing in the size of the coalition needed to pass legislation. It would prefer the opportunity to select \(I\) such that a supermajority coalition will form, and one way to do that is to select an \(I\) which would be vetoed. This forces the agenda setter to build a large override coalition. When no budget cap is in place, the legislature cannot select spending levels. Rather, spending levels are a function of the decisions made by the agenda setter. It turns out that net benefits here are increasing in the size of coalitions, while spending is declining in the size of coalitions. This is what causes the results to reverse. See Figures 2 and 3 for a graphical illustration of how coalition sizes affect spending and net benefits.

The main results are summarized in the following remark.

\(^{28}\)If this result is not supported empirically, it may suggest the importance of executive commitment with regard to vetoes (see Ingberman and Yao 1991).
Remark 4 Among all the institutions considered, the legislature’s expected net benefits are highest when (i) the executive has low spending preferences and (ii) the legislature can impose a budget cap on itself. When a budget cap is in place, spending is higher under a conservative executive than a liberal one. When a budget cap is not in effect, spending is higher under a liberal executive than a conservative one.

7 Discussion

Several important points emerge in this paper. First, I demonstrate the importance of the interaction between institutions. For instance, a higher override requirement increases the size of the budget when a budget cap is present, but lowers it when a budget cap is not present. Similarly, spending is greater under a conservative executive than a liberal executive when a budget cap is in place, but the opposite relationship exists when a budget cap is not in effect.

Second, I show that a conservative executive can help discipline a legislature that cannot buckle down and pass optimally-sized programs. The low-spending executive with veto authority induces supermajority coalitions to form, and the budget cap constrains spending and makes legislators better off. In fact, the legislature prefers to have a budget cap and a conservative executive in place, compared with all other institutional arrangements considered.29

Third, I offer several testable propositions regarding the effect of override requirements, executive spending preferences, and budget caps on the size of distributive spending. This theory is therefore falsifiable. Some testable propositions include the following:

29Earlier it was noted that in this paper the override requirements and the institution of a budget cap are exogenous, but that at some point, these were objects of choice. A puzzle for political scientists and economists is why we observe less than unanimous requirements for distributive politics legislation, since unanimity rule (which is different than a universalism norm) appears to produce optimally-sized programs. Further, one might consider why every state does not have an enforceable budget cap in place.
1. When a budget cap is in effect, budgets are smaller under high-spending than low-spending executives.

2. When a budget cap is not in effect, budgets are larger under high-spending than low-spending executives.

3. When a budget cap is in effect and a low-spending executive is in office, spending is increasing in the override requirement.

4. When a budget cap is not in effect and a low-spending executive is in office, spending is decreasing in the override requirement.

5. When a high-spending executive is in office, spending does not depend on the override requirement.

6. Coalition sizes are larger under low-spending than high-spending executives.

Some of these predictions differ from previous work. For instance, McCarty’s distributive politics model features a project of fixed size that is to be allocated among the districts. In McCarty’s model, stronger veto authority tends to produce lower spending.\textsuperscript{30} In my model, the effect of stronger veto authority (i.e., a higher override requirement) depends on whether a budget cap is in place.

\textsuperscript{30}Technically, spending in McCarty’s model is either zero or an exogenously given fixed quantity. McCarty determines the minimum efficiency requirements that generate positive spending.
The first five implications can be tested with a regression:

\[
\text{Spending} = \beta_1 \text{Republican Executive} \\
+ \beta_2 \text{Spending Limitation} \\
+ \beta_3 \text{Override Requirement} \\
+ \beta_4 \text{Republican Executive} \times \text{Spending Limitation} \\
+ \beta_5 \text{Republican Executive} \times \text{Spending Limitation} \times \text{Override Requirement} \\
+ \beta_6 \text{Republican Executive} \times \text{No Spending Limitation} \times \text{Override Requirement} \\
+ \alpha + \text{controls} + \epsilon
\]

While several models make predictions regarding the signs of \(\beta_1\), \(\beta_2\), and \(\beta_3\), I am aware of none before mine that make predictions regarding the interaction terms. Namely, I predict that \(\beta_4 > 0\), \(\beta_5 > 0\), and \(\beta_6 < 0\).

The American states provide an excellent opportunity for testing these hypotheses, since there is variation on all the key institutions. In ongoing research, I am testing the theory using state budget data. If my theory is correct, the effect of liberal and conservative governors will depend on the budgetary restrictions and override requirements.

Next, a natural extension is a model in which the executive has preferences both over the distribution of spending and the size of spending. This would combine the work of Baron (1993), McCarty (2000), and my work presented here. The effect of other budgetary reforms could also be considered in a similar framework. Also, as I suggest in Appendix B, future research should study the effect of reconciliation procedures on self-enforcing budget caps. In addition, a fruitful avenue might be to consider the effect of incomplete or private information regarding the costs and benefits of projects.

Finally, my model offers some advice for reformers, especially advocates of a balanced budget rule. Specifically, the effect of a balanced budget rule is not as simple as it first appears. For instance, if the executive is conservative, such a rule can actually cause spending to increase. Further, the effect of supermajority passage or override requirements will vary depending on other
budget rules that are in effect. Budget rules that work for one locale may not work for another. Failure to understand the institutional environment in which legislators operate may lead to outcomes that are the opposite of what reformers intended.
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<tbody>
<tr>
<td>$y^*$</td>
<td>$\frac{b}{c}$</td>
<td>$\frac{n+1}{2}$</td>
<td>$s$</td>
<td>$\frac{n+1}{2}$</td>
<td>$s$</td>
<td>$\frac{n+1}{2}$</td>
</tr>
<tr>
<td>$x^*$</td>
<td>$\frac{n+1}{2}$</td>
<td>$\frac{b}{c}$</td>
<td>$\frac{4nb}{c(n^2+4n-1)}$</td>
<td>$\frac{b(n-s+1)}{c}$</td>
<td>$\frac{n+1}{2}$</td>
<td>$\frac{4nb}{c(n^2+4n-1)}$</td>
</tr>
<tr>
<td>Spending</td>
<td>$\frac{b^2(n^2+4n-1)}{8c}$</td>
<td>$\frac{b^2n^2}{c(n^2+4n-1)}$</td>
<td>$\frac{b^2[(n-s+1)^2+(s-1)]}{2c}$</td>
<td>$\frac{b^2(n^2+4n-1)}{8c}$</td>
<td>$\frac{b^2[(n-s+1)^2+(s-1)]}{2c}$</td>
<td></td>
</tr>
<tr>
<td>Expected</td>
<td>$\frac{b^2(-n^2+4n+1)}{8cn}$</td>
<td>$\frac{b^2n}{c(n^2+4n-1)}$</td>
<td>$\frac{b^2[-(n-s+1)^2-(s-1)+2n]}{2nc}$</td>
<td>$\frac{b^2(-n^2+4n+1)}{8cn}$</td>
<td>$\frac{b^2n}{c(n^2+4n-1)}$</td>
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Table 1: Equilibrium values of coalition size, $y$, $x$, spending, and expected net benefits for the models.
<table>
<thead>
<tr>
<th></th>
<th>No Executive</th>
<th>High Executive</th>
<th>Low Executive</th>
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<tbody>
<tr>
<td><strong>Budget Cap</strong></td>
<td>$\frac{2b^2n}{c(n^2+4n-1)}$</td>
<td>$\frac{2b^2n}{c(n^2+4n-1)}$</td>
<td>$\frac{b^2n}{2c[(n-s+1)^2+(s-1)]}$</td>
</tr>
<tr>
<td><strong>No Budget Cap</strong></td>
<td>$\frac{b^2(-n^2+4n+1)}{8cn}$</td>
<td>$\frac{b^2(-n^2+4n+1)}{8cn}$</td>
<td>$\frac{b^2[-(n-s+1)^2-(s-1)+2n]}{2nc}$</td>
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Table 2: Equilibrium expected net benefits depending on the institutions in place and the preferences of the executive.
<table>
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<tr>
<th></th>
<th>No Executive</th>
<th>High Executive</th>
<th>Low Executive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Budget Cap</td>
<td>( \frac{2b^2n^2}{c(n^2+4n-1)} )</td>
<td>( \frac{2b^2n^2}{c(n^2+4n-1)} )</td>
<td>( \frac{b^2n^2}{2c(n-s+1)^2+(s-1)} )</td>
</tr>
<tr>
<td>No Budget Cap</td>
<td>( \frac{b^2(n^2+4n-1)}{8c} )</td>
<td>( \frac{b^2(n^2+4n-1)}{8c} )</td>
<td>( \frac{b^2(n-s+1)^2+(s-1)}{2c} )</td>
</tr>
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</table>

Table 3: Equilibrium spending depending on the institutions in place and the preferences of the executive.
Legislature selects budget.

Baron initiates proposal from Baron game sent to executive for veto or signature.

If signed, policy enacted.

If vetoed, sent back to legislature for override by legislators.

If overridden, spending continues.

If override fails, game continues.

Figure 1: Sequence of Play in Budget Cap Executive Model
Effect of Override/Supermajority Requirement on Expected Net Benefits, With No Budget Cap

![Graph 1: Net Benefits](image1)

Effect of Override/Supermajority Requirement on Expected Net Benefits, With A Budget Cap

![Graph 2: Net Benefits](image2)

Figure 2: Net Benefits
Effect of Override/Supermajority Requirement on Spending, With No Budget Cap

![Graph 1: Effect of Override/Supermajority Requirement on Spending, With No Budget Cap](image1)

Effect of Override/Supermajority Requirement on Spending, With A Budget Cap

![Graph 2: Effect of Override/Supermajority Requirement on Spending, With A Budget Cap](image2)

Figure 3: Spending
A Proofs

Proof of Proposition 1: Baseline Model for Majority Rule
Note that majority rule is just a special case where \( s = \frac{n+1}{2} \), and so the proof for Proposition 1 is simply the proof of Proposition 4, replacing \( s \) with \( \frac{n+1}{2} \).

Proof of Proposition 2: Baseline Model for supermajority rule
The agenda setter choose \( x \) and \( y \) to maximize

\[
\begin{align*}
& bx - \frac{c}{2n}(x^2 + (s-1)y^2) \\
\text{s.t. } & by - \frac{c}{2n}(x^2 + (s-1)y^2) - v \geq 0,
\end{align*}
\]

where \( v \) is the equilibrium continuation value of a legislator receiving an offer of \( y \), where

\[
v = \frac{bx^*}{n} + \frac{(s-1)by^*}{n} - \frac{c(x^{*2} + (s-1)y^{*2})}{2n}.
\]

The Lagrangian is

\[
\mathcal{L} = bx - \frac{c}{2n}(x^2 + (s-1)y^2) + \lambda(by - \frac{c}{2n}(x^2 + (s-1)y^2) - v),
\]

which gives three first-order conditions:

\[
\frac{\partial \mathcal{L}}{\partial x} = b - (1 + \lambda)\frac{cx^*}{n} = 0
\]

\[
\frac{\partial \mathcal{L}}{\partial y} = -(1 + \lambda)\frac{(s-1)cy^*}{n} + \lambda b = 0
\]

\[
\frac{\partial \mathcal{L}}{\partial \lambda} = by - \frac{c}{2n}(x^{*2} + (s-1)y^{*2}) - v = 0.
\]

Solving for \( x^* \) and \( y^* \) in terms of \( \lambda \) gives
\[ x^* = \frac{bn}{(1+\lambda)c} \quad \text{and} \]

\[ y^* = \frac{\lambda bn}{(1+\lambda)(s-1)c}. \]

Substituting these values, as well as the value of \( v \), into the 3rd first-order condition and solving for \( \lambda \) gives

\[ \lambda = \frac{(s-1)}{(n-s+1)}. \]

Substituting back into \( x^* \) and \( y^* \) gives

\[ x^* = (n-s+1) \frac{b}{c} \quad \text{and} \]

\[ y^* = \frac{b}{c}. \]

Spending \( I \) is

\[ \frac{c}{2}(x^*^2 + (s-1)y^*^2), \]

which reduces to

\[ I = \frac{b^2}{2c} \left[ (n-s+1)^2 + (s-1) \right]. \]

Net benefits for supporters, equivalent to \( v \), are

\[ NB = v = \frac{b^2}{2nc} \left[ -(n-s+1)^2 - (s-1) + 2n \right]. \]

By construction, any legislator offered \( x^* \) will accept, since he can do no better by rejecting. Further, the agenda setter always accepts his own offer, since \( y^* > x^* \). That a supporter never wants to deviate from the equilibrium strategy is obvious.
To show that the agenda setter would never want to offer something different than $x^*$ in any round, note that to offer $\tilde{y} < y^*$ or $\tilde{x} > x^*$ would make him worse off. If he offered $\tilde{y} > y^*$ or $\tilde{x} < x^*$, then the supporters would reject the offer, which makes him worse-off. This shows that no defection with regard to project scales is rational.

To show that the agenda setter never builds an oversized coalition (larger than $s$), note that if the agenda setter adds a member to the coalition, it is as if he is building a coalition of size $s + 1$. The agenda setter’s net benefit as a function of $s$ is

$$NB_{as} = \frac{b^2}{2cn} [sn + s - n].$$

The first-derivative of this function is $\frac{b^2}{2cn} [-2s + 1 + n]$, which is negative for all $s > \frac{n+1}{2}$. Therefore, adding to the coalition is not optimal for the agenda setter. This proves that this is an equilibrium. Uniqueness is easily established by contradiction. ■

**Proof of Proposition 3: Budget Cap Model for majority rule**

Note that majority rule is just a special case where $s = \frac{n+1}{2}$, and so the proof for Proposition 3 is simply the proof of Proposition 4, replacing $s$ with $\frac{n+1}{2}$.

**Proof of Proposition 4: Budget Cap Model for supermajority rule**

The agenda setter chooses $x$ and $y$ to maximize

$$b x - \frac{c}{2n} (x^2 + (s - 1)y^2)$$

s.t. $by - \frac{c}{2n} (x^2 + (s - 1)y^2) - v \geq 0$ and

$$I - \frac{c}{2} (x^2 + (s - 1)y^2) \geq 0$$

where $v$ is the equilibrium continuation value of a legislator receiving an offer of $y$, and $I$ is the budget selected by the legislature in the first period. Recall that
The Lagrangian is

\[ L = bx - \frac{c}{2n}(x^2 + (s-1)y^2) + \lambda \left( by - \frac{c}{2n}(x^2 + (s-1)y^2) - v \right) + \gamma \left( I - \frac{c}{2}(x^2 + (s-1)y^2) \right) \]

which gives four first-order conditions:

\[
\frac{\partial L}{\partial x} = b - (1 + \lambda )\frac{cx^*}{n} - \gamma cx^* = 0
\]

\[
\frac{\partial L}{\partial y} = -(1 + \lambda )\frac{(s-1)cy^*}{n} + \lambda b - \gamma c(s - 1)y^* = 0
\]

\[
\frac{\partial L}{\partial \lambda} = by^* - \frac{c}{2n}(x^*^2 + (s-1)y^*^2) - v = 0
\]

\[
\frac{\partial L}{\partial \gamma} = I - \frac{c}{2}(x^*^2 + (s-1)y^*^2) = 0
\]

Solving for \( x^* \) and \( y^* \) in terms of \( \lambda \) and \( \gamma \) gives

\[ x^* = \frac{bn}{(1 + \lambda + \gamma n)c} \quad \text{and} \]

\[ y^* = \frac{\lambda bn}{(1 + \lambda + \gamma n)(s - 1)c}. \]

This implies that \( y^* = \frac{\lambda}{s-1}x^* \).

The third first-order condition and the definition of \( v \) requires that \( x^* = (n - s + 1)y^* \).

Combining these two relations implies that \( \lambda = \frac{s-1}{n-s+1} \).
Next, to calculate $\gamma$, substitute the relations for $x^*$ and $y^*$ into the fourth first-order condition. Algebraic simplification implies that

$$\gamma = \frac{1}{n-s+1} \left[ b \sqrt{\frac{(n-s+1)^2 + (s-1)}{2cI} - 1} \right].$$

Substituting this back into the relation for $x^*$ and $y^*$ gives the equilibrium values

$$y^* = \sqrt{\frac{2I}{c[(n-s+1)^2 + (s-1)]}}$$

and

$$x^* = (n-s+1) \sqrt{\frac{2I}{c[(n-s+1)^2 + (s-1)]}}.$$

In the first period, the legislature selects the value of $I$ that makes it best off, given expectations about the agenda setter’s behavior in the distributive game. Since all legislators are equally likely to be either an agenda setter or a member of the coalition receiving projects, I consider the decision of a generic legislator. All legislators will vote identically.

Formally, his problem is to choose the $I$ that maximizes

$$b \frac{(n-s+1)}{n} \sqrt{\frac{2I}{c[(n-s+1)^2 + (s-1)]}} + b \frac{(s-1)}{n} \sqrt{\frac{2I}{c[(n-s+1)^2 + (s-1)]}}$$

$$-\frac{c}{2n^2} \left[ (n-s+1)^2 \frac{2I}{c[(n-s+1)^2 + (s-1)]} + (s-1) \frac{2I}{c[(n-s+1)^2 + (s-1)]} \right].$$

Simplifying, taking the first-order conditions, and verifying the second-order conditions for a maximum implies that

$$I^* = \frac{b^2 n^2}{2c[(n-s+1)^2 + (s-1)]}.$$  

Substituting $I^*$ back into $x$ and $y$ gives

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\[ y^* = \frac{bn}{c[(n-s+1)^2 + (s-1)]} \text{ and} \]

\[ x^* = (n-s+1)\frac{bn}{c[(n-s+1)^2 + (s-1)]}. \]

This implies that net benefits for supporters are

\[ \text{NB} = v = \frac{b^2n}{2c[(n-s+1)^2 + (s-1)]}, \]

a positive number. This implies that setting spending to zero is not optimal.

Verifying that defection will never occur proceeds identically as in the proof of Proposition 2.

**Proof of Proposition 5: Budget Cap Executive Model**

The proof simply combines the results of previous models. Consider first a low-spending executive who vetoes any budget bill containing positive spending. (If his discount rate is less than 1, then this is a credible threat.) In this case, the agenda setter must build a supermajority coalition if he wishes to pass a distributive bill. Also, he must adhere to the budget cap set by the legislature in the first period. Notice that this is the same constraint faced by the agenda setter in a budget cap supermajority model, and is also the same as that faced by the entire legislature in selecting \( I \). Hence, the equilibrium specified there holds here as well.

Next, consider a high-spending executive whose indifference budget, \( Z \), is greater than the \( \frac{b^2(n^2+4n-1)}{8c} \), the equilibrium spending level in a Baron budget model. Consider the legislature’s decision: it can propose spending greater than \( Z \) and provoke a veto, thereby forcing the agenda setter to build a supermajority coalition. Or it can propose the optimal \( I \) such that a majority coalition forms. If it chooses to induce a majority coalition, clearly
it will pick the $I$ described in the majority rule budget cap model. Suppose that it chooses to build a supermajority coalition. First, note that it would never choose a budget larger than $Z + \epsilon$, since the net benefit function is declining in $I$ in this interval. (This is easily verified.) Therefore, let the $I$ selected be $Z + \epsilon$. Again, because net benefits are declining in $I$ in this interval, I can consider just the case where $Z + \epsilon = \frac{b^2(n^2 + 4n - 1)}{8c} + \epsilon$. (For simplicity, the equations below will omit the $\epsilon$.)

Recall that the expected net benefit of the majority-rule coalition is

$$NB = \frac{2b^2}{c(n^2 + 4n - 1)}.$$ 

The net benefit of a supermajority coalition member is

$$NB = \frac{b^2}{2c} \sqrt{\frac{(n^2 + 4n - 1)}{(n - s + 1)^2 + (s - 1)^2}} - \frac{b^2(n^2 + 4n - 1)}{8nc}.$$ 

The supermajority net benefits can be rewritten as

$$NB = 8n\sqrt{n^2 + 4n - 1} \left[ \frac{1}{n\sqrt{(n - s + 1)^2 + (s - 1)^2}} - \sqrt{n^2 + 4n - 1} \right].$$ 

It is straightforward to show that the supermajority net benefits are negative and that the majority net benefits are positive, implying that the majority equilibrium will form. ■
B Alternative Budget Cap Rule

In future work, I may consider the following self-enforcing mechanism for the budget cap, which eliminates the need for external enforcement. Consider the following modification to the game, which resembles the budget procedure in the U.S. Congress. Think of the budget constraint as a set of “instructions” to the agenda setter regarding spending levels. (I call this a budget resolution.) To prevent the agenda setter from violating the instructions, I add the following “reconciliation” procedure to the legislative bargaining game. After an allocation is voted upon favorably, a new agenda setter is chosen for the reconciliation procedure if spending breaks the budget. He can choose to do nothing, which leaves the allocation alone, or he can offer an amendment, consisting of a new allocation that satisfies the budget cap. If this proposal is rejected, the initial allocation stays in effect. This changes the strategy of the game.

The equilibrium strategies of the legislature and the agenda setter in the budget cap and bargaining stages must take into account the effect of the reconciliation stage. This complicates the model significantly. (I continue to work on this model as part of my research agenda, and plan to write another paper, or extend the next version of this paper, to discuss this modification.) I offer one result with a sketch of a proof: expected net benefits of this game are positive. To see this, suppose not. Then this implies that the agenda setter successfully enacted a budget (different than the amount set by the legislature) which resulted in negative net benefits for all players except the agenda setter. But during reconciliation, any agenda setter would propose a modified allocation that satisfies the budget constraint. Since the budget constraint must always provide positive expected benefits, this guarantees positive net benefits in equilibrium.

I also wish to determine how often the budget resolution will be ignored in this model versus how often it will be self-enforcing. I conjecture that it is always self-enforcing, but I have yet to prove it.
References


