Democratic Policy Making with Reconsideration*

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Abstract

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1 Introduction

In the spring of 2001, President George W. Bush signed into law landmark legislation touted as a $1.35 trillion tax cut, the largest in a generation. While the legislation included relatively small short term tax reductions, the most significant provisions were scheduled to phase in over a ten year horizon. Senate Democratic leader Thomas A. Daschle immediately denounced the deferred provisions, promising that "(w)e will revisit these issues. We will try to find ways to make corrections."\(^1\) Likewise, according to reports in the popular press, "(t)ax analysts warned... that some provisions phase in very slowly, and – if history is a guide – a number of them may never materialize."\(^2\) For example, in evaluating the provisions pertaining to the estate tax, one practitioner noted: "The changes have been stretched out for so many years, you can’t think that [lawmakers are] not going to come back and revisit this."\(^3\)

The preceding example illustrates two principles: first, that policy makers can and do engage in lawmaking well in advance of implementation; second, in such instances, policies are ordinarily subject to reconsideration after adoption (but prior to implementation). It is important to clarify up front the formal sense in which we use the term reconsideration. Imagine that policy makers convene in time period \(s\) to consider the policy that will be in effect in some period \(t > s\). If this group chose to disband in period \(s\) and not meet again prior to period \(t\), there would still be some public policy in place for period \(t\). We refer to this as the status quo policy for period \(t\) as of period \(s\). If policy making during period \(s\) can alter the status quo policy for period \(t\) as of the next period \((s+1)\), and if the deliberating body has further opportunities to modify the status quo policy for period \(t\) between periods \(s + 1\) and \(t − 1\), we say that the policy is subject to reconsideration.

Passing a bill with phased-in provisions, and subsequently repealing or altering those provisions before they take effect, is an obvious example of reconsideration. However, the phenomenon is considerably more general. Consider, for example, the common case where, in some period \(s\), policy makers adopt a law with no

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specific expiration date, and later reform the law in period $t > s$. The initial law is a bundle of time dated polices (one for each $s' \geq s$), and the reform amounts to reconsideration of the law as applied to all periods $t' \geq t$. In contrast, in a legislative context, approving an amendment to a bill, and then subsequently removing or altering that amendment before the bill comes to the floor for a final vote, does not constitute reconsideration (if the legislature disbands before taking further action, the bill fails to pass, and the outcome is identical, regardless of whether an amendment is approved).

Policy reconsideration is potentially important for at least two reasons. First, in environments with uncertainty and limitations on the ability to adopt sufficiently complex state-contingent policies, imperfectly predictable changes in the environment may alter the desirability and/or feasibility of a given policy. Second, if policy makers anticipate reconsideration (as our discussion of the 2001 tax reform suggests), a variety of interesting strategic possibilities come into play. In particular, a law-maker may support a given initiative not because she desires its implementation, but rather because she believes its adoption will alter the status quo policy in a way that ultimately leads, through reconsideration, to a more desirable outcome.

The purpose of this paper is to examine the strategic implications of reconsideration. To avoid confounding strategic and non-strategic motives for reconsideration, we examine environments with no uncertainty (alternatively, one could also assume that arbitrarily complex state-contingent policies are feasible). Our first objective is to understand the implications of reconsideration within “static” collective choice problems, wherein the selection of the policy that is to prevail as of some single future period $t$ is considered in isolation. We argue that our analysis extends to “dynamic” problems (wherein the group simultaneously contemplates the policies that will prevail in all future periods) in the following sense: under certain conditions, the dynamic problem decomposes into a collection of static problems (one for each future period). Note that, in any given static collective choice problem, $t$ is fixed. Consequently, these problems inherently involve finite horizons, and hence finite rounds of consideration and reconsideration prior to implementation. Henceforth, we will use the word “period” to refer to units of time for policies, and we will use the word “round” to refer to units of time for proposals. For a static collective choice problem, a sequence of proposal rounds is followed by a single policy period.

To identify and understand underlying forces and tendencies, we study the sim-
plest democratic process in which one can address the strategic implications of reconsideration. There is a group of individuals (possibly citizens, members of a legislature, or even professors belonging to some academic department) who meet periodically to make a collective choice concerning a policy to be implemented in some specific future period. There is also some initial status quo policy, possibly one inherited from previous rounds of deliberation. Individuals are recognized sequentially in some predetermined order. Once recognized, an individual makes a proposal, which is immediately put to a vote. Individuals are permitted to condition both their proposals and their votes on all preceding events, including other proposals and votes. If a proposal passes, it becomes the new status quo and supersedes all previously passed proposals. Passage of a proposal requires a simple majority. The policy that emerges from this process is implemented.

In general, the aforementioned process selects Pareto efficient outcomes. Consequently, the focus of our analysis is on distributional politics. Under fairly weak conditions, when the policy space contains a Condorcet winner, this alternative emerges as the ultimate outcome. Thus, for environments with single-dimensional policy sets and single-peaked preferences, the desires of the median voter prevail (as in Downs’ [1957] model of electoral competition). However, as is well-known, Condorcet winners typically do not exist for multidimensional policy spaces, especially when the government has access to a reasonably flexible set of distributional instruments.

For some familiar classes of policy spaces that give rise to rich distributional politics, we obtain a surprising result: provided that a sufficient number of individuals have opportunities to make proposals, the collective choice process is effectively equivalent to one in which the last proposer is a dictator. Under conditions identified in the text, the group selects a policy that makes every single member worse off (relative to inaction), save for the last proposer. This occurs despite the fact that a majority is required to pass any particular proposal. In this setting, apparently democratic reforms can have the unintended effect of concentrating political power. Ironically, the last proposer need not have dictatorial powers unless a sufficient number of individuals can make proposals. Thus, with reconsideration, guaranteeing a “right to be heard” may have decidedly undemocratic consequences.

The process described above is one of the simplest settings in which one can address the strategic implications of reconsideration. It is not intended as a reali-
tic depiction of some specific policy making institution. In other work (Bernheim, Rangel, and Rayo [2001]), we extend our analysis to collective choice processes operating under a variety of alternative rules and procedures, many of which are motivated by the characteristics of actual institutions. One can view this work as an attempt to understand the ways in which observed institutions and rules of procedure have evolved to combat the disturbing tendency towards high concentration of political power that arises in the presence of reconsideration and distributional politics. As it turns out, it is surprisingly difficult to avoid dictatorial or near-dictatorial outcomes.

The current paper is most closely related to previous work on agenda control, wherein one party (or possibly a collection of parties) establishes an agenda consisting of a finite sequence of proposals. Voters consider the proposals in successive pair-wise comparisons, with the victor of each comparison facing the next proposal in the sequence. This process permits reconsideration of proposals after passage, in the sense that a passed proposal is not implemented if it is subsequently displaced by something else. McKelvey [1976] analyzed such process under the assumptions that voters are completely myopic (in the sense that they vote in favor of their preferred policy in each round irrespective of the implications for the final outcome), and that the sequence of proposals is fixed in advance of the first round. Shepsle and Weingast [1984] allowed for strategically sophisticated voting, but maintained McKelvey's second assumption. In contrast, in our model, the dynamics of proposing and voting are interwoven, so that an individual's proposal can vary with the prevailing status quo. We view this as an essential aspect of reconsideration in practice. It is also critical for our central results.

This paper is also related to a sizeable literature on the theory of legislative institutions. For the most part, this literature examines static collective choice problems without allowing for reconsideration in the sense that we have defined these terms above: in these models, the outcome would be the same irrespective of prior proposals and votes if the legislature disbanded at any point prior to concluding deliberations. Some notable examples include Ferejohn, Fiorina, and McKelvey [1987] as well as Baron and Ferejohn [1989] and the subsequent literature on stochastic bargaining (e.g. Merlo and Wilson [1995], Banks and Duggan [1998], and Eraslan [1998]). Some of these papers allow legislators to amend proposals prior to voting on passage, and/or to alter amendments prior to voting on their approval, but
they do not allow for reconsideration of adopted policies.

A small number of papers, including Baron [1996], Epple and Riordan [1987], and Ingberman [1985] examine the resolution of dynamic collective choice problems (the selection of a sequence of policies). Although one can construe these models as allowing for reconsideration, they differ from our framework in several important respects, which we discuss in section 3.6. Most importantly, all assume that the evolution of the status quo follows what Ingberman terms "dynamic reversion." This implies that the status quo for period $t$ as of some period $s$ is equal to the period $s$ policy. In other words, once adopted, a static (single period) policy stays in place until it is overturned. In our view, this imposes an artificial and implausible restriction on proposals. As our opening example makes clear, a lawmaker is free in practice to propose a dynamic policy (in effect, a vector) specifying a potentially distinct single-period policy for every future period. Even when a dynamic policy stays in place until overturned, the applicable status quo may change from period to period. The notion of dynamic reversion artificially links strategic manipulation of future outcomes (through anticipation of reconsideration) to the current outcome, thereby confounding disparate sources of incentives, and potentially obscuring the role of reconsideration.

The remainder of this paper is organized as follows. Section 2 lays out the basic model. Section 3 presents some general results, including the selection of Pareto efficient outcomes and Condorcet winners (where they exist). It also establishes the irrelevance of early legislative activity. We use the latter result to identify special circumstances in which one can decompose a dynamic collective choice problem into a sequence of static collective choice problems. Section 4 specializes to a familiar policy space with rich distributional politics, and proves our dictatorship results. Section 5 discusses other policy spaces. We conclude in section 6 with a summary of our findings, and a discussion of results from Bernheim, Rayo, and Rangel [2001] concerning alternative institutions, rules, and procedures. To familiarize the reader with our analytic techniques, we include the proofs of several key propositions in the text. Other proofs are contained in the appendix.

2 The Model

Consider a decision-making body ("the group") consisting of $N$ individuals, labelled $l = 1,\ldots,N$, where $N \geq 5$. To avoid complications arising from tie votes, we assume
for convenience that $N$ is odd. Let $M \equiv \frac{N+1}{2}$ denote the size of the smallest majority coalition.

2.1 Policies and Payoffs

The group must select a policy $p \in P$, where $P$ denotes the set of feasible policies. Let $v_l(p)$ denote the payoff to individual $l$ if policy $p$ is implemented. Note that one can think of a policy $p$ as a point $\pi = v(p) \equiv (v_1(p), \ldots, v_N(p))$ in some feasible payoff set $\Pi$, where $\Pi$ is the image of $P$ under $v$. Except where indicated, we impose the following two assumptions throughout:

**Assumption A1:** The policy space $P$ is finite.

**Assumption A2:** Individuals have strict preferences over policies: $p \neq p' \Rightarrow v_l(p) \neq v_l(p')$.

Assumptions A1 and A2 are relatively innocuous. Indeed, given A1, any failure of A2 is non-generic. We nevertheless acknowledge that these assumptions rule out some interesting and important cases, including the familiar “divide-the-dollar” problem. We examine this problem separately in section 5. Since one can exploit indifference to contrive elaborate history-dependent strategies, the analytics of the divide-the-dollar problem are considerably more complicated. However, as we will see, our central conclusions emerge largely intact.

In section 4, we provide structure to distributional politics by assuming that the policy space has the following structure. For each individual, there is an associated “elementary policy.” Let $E \equiv \{1, \ldots, N\}$ denote the set of all elementary policies. Each $l \in E$ produces highly concentrated benefits and diffuse costs. In particular, policy $l$ generates a net benefit $b_l > 0$ for individual $l$, and a cost $c_l > 0$ for every individual (including $l$). A policy $p$ is a collection of elementary policies. The set of feasible policies $P$ is the power set of $E$; that is, the set of all possible combinations of elementary policies. $P$ includes the empty set $\emptyset$, which represents inaction (nothing is implemented). Payoffs are additively separable:

$$v_l(p) = -\sum_{j \in p} c_j + \begin{cases} b_l & \text{if } a \in p \\ 0 & \text{otherwise} \end{cases}.$$  

When $P$ is generated from elementary policies in the manner described above, we say that it is a CBDC policy set (for concentrated benefits, diffuse costs). Models
with similar payoff structures appear elsewhere in the theoretical literature concerning legislative policy making (see e.g., Ferejohn, Fiorina, and McKelvey [1987], and Gabel and Hager [2000]). Except where indicated, we impose two additional assumptions on CBDC policy sets:

**Assumption A3:** Total costs are increasing in the number of elementary policies. Specifically, \(|p| < |p'| \Rightarrow \sum_{j \in p} c_j < \sum_{j \in p'} c_j\).

**Assumption A4:** A mutually beneficial policy exists for all coalitions consisting of \(M\) or fewer individuals. In particular, for every policy \(p\) with \(|p| \leq M\), \(b_l > \sum_{j \in p} c_j\) for all \(l \in p\).

When all elementary policies are equally costly, Assumption A3 is trivially satisfied. Consequently, this assumption effectively restricts the degree to which costs can vary across elementary policies. For certain results, it is possible to relax this assumption considerably.

Assumption A4 guarantees the existence of policies that are preferred to inaction by a majority of voters. It also guarantees that there exists such a policy for any bare-majority coalition. If there does not exist a policy that is mutually beneficial for all members of some majority coalition, then, for the institutions considered below, the group selects \(p = \emptyset\) (proof omitted). Ironically, the ability to assemble majoritarian coalitions is therefore essential for the emergence of the dictatorial outcomes derived below. Note finally that, under assumption A4, the universalistic policy \(p = E\) need not maximize social surplus. Consider, for example, the case of \(N = 5\) with \(b_1 = \ldots = b_5 = 8, c_1 = c_2 = c_3 = 2,\) and \(c_4 = c_5 = 1\). Assumption A4 is clearly satisfied, but the surplus maximizing policy is \(\{4, 5\}\).

### 2.2 Procedures for Collective Choice

The collective choice process consists of a sequence of \(T\) “proposal rounds.” Activity prior to each round \(t\) establishes some “status quo” policy, \(p_{t-1}\). Round \(t\) begins when individual \(i(t)\) is recognized. For now, both the initial status quo policy, \(p_0\), and the order of recognition, \(i : \{1, \ldots, T\} \rightarrow \{1, \ldots, N\}\), are predetermined and known to all individuals as of round 1. Recognition provides an individual with the opportunity to make a proposal, \(p_t^m\), which can be any element of \(P\). The proposal is then put to an immediate vote. If it receive majority approval (“passes”), it displaces \(p_{t-1}\) as the status quo policy \((p_t = p_t^m\)). If it does not pass, the status
quo policy remains the same \( p_t = p_{t-1} \). One can think of any given proposal as adding to, deleting, or replacing portions of the prevailing status quo policy.

It may at first seem odd to assume that a new proposal, once passed, displaces all policies previously passed. However, this assumption involves essentially no loss of generality. It is important to keep in mind that a policy (and therefore a proposal), as we have defined it, involves a complete description of all government actions, and not merely the component actions pertaining to some particular subset of issues.

To illustrate, consider the following example. Imagine that the government faces two choices: whether to build bombers, and whether to save the whales. In each instance, there are two possibilities: build the bombers \( B \) or not \( NB \), and save the whales \( S \) or not \( NS \). There are four possible policies: \( B, S \), \( NB, S \), \( B, NS \), and \( NB, NS \). Imagine also that the initial status quo policy \( p_0 \) involves no action \( NB, NS \). If the first recognized individual wishes to propose to build bombers, he will propose \( B, NS \). If this passes, and if the second individual wishes to save the whales, she proposes \( B, S \). Though the second proposal, if passed, technically displaces the first, it is clear that individuals are actually voting on the incremental component policy \( S \). If the second individual wished to repeal the bomber legislation before implementation and save the whales, she would instead propose \( NB, S \). Alternatively, if the initial bomber proposal does not pass, and if the second individual still wishes to save the whales, she proposes \( NB, S \). From this perspective, it is perhaps more natural to think of the policy proposed in round \( t \) as consisting of the differences between \( p_{t-1} \) and \( p^m_t \), rather than simply as \( p^m_t \).

The ultimate fate of the proposal that emerges from the policy development stage, \( p_T \), is determined in some final stage of the collective choice process. We focus here on the simplest possibility: \( p_T \) is simply enacted into law. Nevertheless, for reasons that will become clear in the next section, it is analytically useful to allow for greater generality at the outset. For our current purposes, we will abstract from institutional details and simply assume that it is possible to derive some reduced form representation of the final stage, \( \Omega : P \to P \). In other words, when the policy \( p_T \) emerges from the final stage, the ultimate outcome is \( \Omega(p_T) \). Obviously, this framework includes the special case of a degenerate final stage, wherein \( p^F \) becomes law without further modification (\( \Omega(p) = p \)).

To model other collective choice processes, one can vary the rules and procedures
of the proposal rounds, append an initial process that generates the initial status quo, $p_0$, and/or adopt some alternative mapping $\Omega$ to represent a more complex final stage. Consider the following example: a committee generates a proposal, legislators can amend the proposal through a sequence of motions and votes, the final amended proposal is put to an up-or-down vote against existing law, and the bill, if passed, is enacted into law without reconsideration. To model this institution, we would append some initial game between committee members to generate $p_0$, as well as a final stage consisting of an up or down vote between $p_T$ and some default policy $p^D$.

2.3 Behavioral assumptions

Throughout our analysis, we assume that (1) individuals are strategically sophisticated, and (2) they always vote as if they are pivotal. We make the second assumption to deal with the familiar problem of indifference among non-pivotal voters, which otherwise gives rise to a vast multiplicity of equilibria. The equilibria that we rule out through the second assumption are unreasonable because agents cast votes that are contrary to their true preferences. Together, our two assumptions imply that individuals compare the continuation equilibrium if a proposal passes with the continuation equilibrium if it is defeated, and cast their vote for the option that yields the preferred continuation path. We also confine attention to pure strategy subgame perfect equilibria. Henceforth, the term “equilibrium” should therefore be construed as indicating a pure strategy subgame perfect equilibrium with the preceding characteristics.

3 Some general results

We begin our analysis with some general results. For the purpose of this section, we assume only that $P$ is finite and generic (assumptions A1 and A2).

3.1 An equivalence result for final stages

Let $\Omega(P)$ denote the image of all points in $P$ under the mapping $\Omega$. Plainly, the final policy outcome must belong to the set $\Omega(P)$. Let $J \equiv \{ j \mid j = i(t) \text{ for some } t = 1, \ldots, T \}$; this is the set of individuals who are recognized at least once. Similarly, let $J(t, t')$ denote the set of individuals recognized at least once in periods $t$, $t + 1$, $\ldots$, $t'$. 
Our first result establishes an extremely simple yet important equivalence principle:

**Lemma 1:** Consider a policy set $P$ satisfying $A1$ and $A2$. An institution with policy space $P$, initial status quo $p_0$, and final stage $\Omega$ yields the same policy outcome as an otherwise identical institution with policy space $\Omega(P)$, initial status quo $\Omega(p_0)$, and a degenerate final stage.

The proof of lemma 1 is completely straightforward, and involves relabeling of branches and nodes in the extensive form of the game, as well as deletion of redundant branches. The theorem is important because it implies that we can understand all institutions in this class by studying institutions with degenerate final stages. In particular, if one wishes to know the outcome generated by an institution with a non-degenerate final stage, one need only derive a reduced form mapping for the final stage ($\Omega$), and then consider an equivalent institution with a smaller policy space ($\Omega(P)$) and a degenerate final stage.

### 3.2 The recursive structure of equilibria

Lemma 1 is also important because it allows us to provide a useful recursive characterization of the equilibria for these models. This requires some additional notation.

For any $P' \subseteq P$ and $p' \in P'$ define

$$Z(p', P') \equiv \{ q \in P' \mid \exists S \text{ with } |S| \geq M \text{ and } v_l(q) \geq v_l(p') \text{ for all } l \in S \}. $$

This is the set of policies in $P'$ that (weakly) defeat $p'$ by majority rule. The use of weak inequalities here implies that $p' \in Z(p', P')$. However, in light of our genericity assumption, strict inequalities hold for all other $p \in Z(p', P')$. Next, define

$$\varphi_l(p', P') \equiv \arg \max_{q \in Z(p', P')} v_l(q).$$

This represents individual $l$'s most preferred element of the set $Z(p', P')$. Under assumptions $A1$ and $A2$, this function is well defined. Finally, define

$$\Phi_l(P') \equiv \{ q \in P' \mid q = \varphi_l(p', P') \text{ for some } p' \in P' \}.$$ 

This is simply the image of the set $P'$ under the mapping $\varphi_l(\cdot, P')$.

Now we exhibit the recursion. Consider first the following institution:
**Institution #1:** $T$ proposal rounds, a recognition order $i(t)$ (for $t = 1, \ldots, T$), a policy space $P$, an initial status quo $p_0$, and a degenerate final stage.

Observe that, without altering the game in any substantive way, one can think of the final proposal round as part of the final stage. The policy that emerges from round $T - 1$, $p_{T-1}$, then serves as the input for the final stage. For any particular $p_{T-1}$, solving this final stage is straightforward: $i(T)$ proposes the policy in $P$ she most prefers among those that (weakly) defeat $p_{T-1}$. In other words, $\Omega(p_{T-1}) = \varphi_i(T)(p_{T-1}, P)$. Lemma 1 tells us that this is in turn equivalent to the following institution:

**Institution #2:** $T - 1$ proposal rounds, a recognition order $i(t)$ (for $t = 1, \ldots, T - 1$), a policy space $\Phi_i(T)(P)$, an initial status quo $\varphi_i(T)(p_0, P)$, and a degenerate final stage.

The preceding argument demonstrates that a basic institution with $T$ proposal rounds and a degenerate final stage is equivalent to another basic institution with $T - 1$ proposal rounds and a degenerate final stage, where the policy space has been appropriately reduced, and where the initial status quo has been appropriately transformed. The same argument implies that these institutions are in turn equivalent to another basic institution with $T - 2$ proposal rounds and a degenerate final stage, where the policy space has been further reduced (to $\Phi_i(T-1) \circ \Phi_i(T)(P)$), and where the initial status quo has been further transformed.

Where does this argument ultimately lead? Recursive application of the same equivalence principle implies that the original institution is equivalent to a basic institution with zero proposal rounds and a degenerate final stage, where the policy space is

$$\Phi_i(1) \circ \cdots \circ \Phi_i(T-1) \circ \Phi_i(T)(P),$$

and where the initial status quo has been appropriately transformed. Since this institution is completely degenerate, the initial status quo is simply enacted into law.

According to the preceding argument, for any initial status quo $p_0 \in P$, the initial institution must generate an outcome in the set $\Phi_i(1) \circ \cdots \circ \Phi_i(T-1) \circ \Phi_i(T)(P)$. Notice that we can solve for this set through mechanical application of the $\Phi_i$ mappings. This allows us to completely characterize all possible outcomes of the legislative process, allowing for any conceivable initial status quo.
For some of the arguments appearing later in this paper, it is also convenient to define a function $Q_t(p_{t-1})$ that maps the status quo $p_{t-1}$ in round $t$ to the eventual final outcome. The map is defined recursively as follows:

$$Q_T(p_{T-1}) = \varphi_i(T)(p_{T-1}, P)$$

and, for $t < T$,

$$Q_t(p_{t-1}) = \varphi_i(t)(Q_{t+1}(p_{t-1}), Q_{t+1}(P)).$$

This construction is intuitive. Consider the problem of individual $i(t)$ in round $t$ when the status quo is $p_t$. If proposal $p'$ passes in round $t$, the status quo for round $t + 1$ is $p'$, and the eventual outcome is $Q_{t+1}(p')$. If no new proposal passes in round $t$, the status quo for round $t + 1$ is $p_t = p_{t-1}$, and the eventual outcome is $Q_{t+1}(p_{t-1})$. Thus, $i(t)$’s problem is to choose the best policy in the set of continuation outcomes $Q_{t+1}(P)$ that can (weakly) defeat the continuation status quo $Q_{t+1}(p_{t-1})$ by majority rule. The solution is $\varphi_i(t)(Q_{t+1}(p_{t-1}), Q_{t+1}(P))$.

Note that $Q_t(P) = \Phi_{i(t)} \circ \ldots \circ \Phi_{i(T-1)} \circ \Phi_{i(T)}(P)$. Thus, $Q_t(P)$ denotes the set of policies that can emerge as final outcomes if one places no restrictions on the status quo for round $t$, $p_t$. Since $\Phi_i(Q) \subseteq Q$, every application of a $\Phi_i$ mapping shrinks the set of possible final outcomes. It follows that the sets $\{Q_t(P)\}_{t=1}^T$ are nested: $Q_1(P) \subseteq Q_2(P) \subseteq \ldots \subseteq Q_T(P)$.

### 3.3 Pareto efficiency

One can evaluate institutions with respect to the efficiency and distributional characteristics of the outcomes they generate. With respect to efficiency, we have the following simple result.

**Theorem 1:** Consider an institution with a degenerate final stage and a policy set $P$ satisfying $A1$ and $A2$. Then the outcome, $p_T$, is Pareto efficient in $P$.

**Proof:** We know that $p_T \in \Phi_{i(T)}(P)$. Consequently, we need only demonstrate that all points in $\Phi_{i(T)}(P)$ are Pareto efficient in $P$. Consider some $p \in \Phi_{i(T)}(P)$, and suppose contrary to the theorem that it is not Pareto efficient in $P$. In light of assumption $A2$, there is some $p^* \in P$ such that every individual strictly prefers $p^*$ to $p$. We know that there is some $p'$ such that $p \in \varphi_{i(T)}(p', P) = \arg \max_{q \in Z(p', P)} v_{i(T)}(q)$. But $p^* \in Z(p', P)$, and $v_{i(T)}(p^*) > v_{i(T)}(p)$, which is a contradiction. Q.E.D.
Theorem 1 assures us that the collective outcome will always lie on the Pareto frontier. Consequently, the remainder of our analysis focuses on distributional politics. In much of this paper, we therefore specialize to policy sets that bring out distributional issues.

3.4 Selection of Condorcet winners

Bearing in mind the equivalence result of section 3.1, we will continue to focus on processes with degenerate final stages. In general, there is no reason to believe that the policy set $P$ will contain a Condorcet winner (defined as a policy that is majority preferred to all other policies). However, it is natural to wonder whether the collective choice process will select a Condorcet winner if one exists. As it turns out, this question is central to a number of the results proven in later sections.

Plainly, there are institutions of the form considered here that do not select Condorcet winners. As an example, consider an institution with a single proposal round. For any given initial status quo $p_0$, there is no particular reason to believe that the Condorcet winner, $p^c$, is the recognized individual’s preferred outcome in $Z(p_1, P)$. Indeed, it is entirely possible that this individual prefers $p_0$ to $p^c$.

Despite the preceding observation, the group will select a Condorcet winner, assuming that one exists, provided that a sufficiently diversified set of individuals have opportunities to make proposals.

**Theorem 2:** Consider an institution with a degenerate final stage, and a policy set satisfying $A1$ and $A2$. Suppose that there is a Condorcet winner $p^c$ in $P$. Then $p^c$ is the final outcome regardless of the initial status quo (i.e. $Q_1(P) = p^c$) whenever:

1. $|J| \geq M$, or
2. $p^c$ is the preferred policy in $P$ for some individual $l \in J$.

**Proof:** Note that, for any $Q$ with $p^c \in Q$, $Z(p^c, Q) = \{p^c\}$, from which it follows that $p^c = \varphi_i(p^c, Q) \in \Phi_i(Q)$ for all $i$. This in turn implies that $p^c \in Q_t(P)$ for all $t \in \{1, \ldots, T\}$.

Next, consider any $Q \subseteq P$ with $p^c \in Q$. Suppose that $\nu_i(p^c) > \nu_i(p')$ for some $p' \in Q$. We claim that $p' \notin \Phi_i(Q)$. Suppose $p' \in \Phi_i(Q)$. Then $\exists q' \in Q$ such that $p'$ solves $\max_{q'' \in Z(q', Q)} \nu_i(q'')$. But $p^c \in Z(q', Q)$ (since $p^c$ is a Condorcet
winner in $P$, and hence in $Q$, and $v_i(p') > v_i(p')$; this contradiction establishes the claim.

Finally, consider any $p \in P$ other than $p^c$. In case (1), we know there is a set of players $S_p$ with $|S_p| \geq M$ such that $v_i(p^c) > v_i(p)$ for $i \in S_p$. Note that $J \cap S_p \neq \emptyset$ (since both sets are at least of size $M$). Thus, for some $t' \geq 1$, $i(t') \in S_p$. But then, by our previous claim, $p \notin \Phi_i(Q_{t'+1}(P)) = Q_{t'}(P)$.

Since the sets $\{Q_t(P)\}_{t=0}^T$ are nested, $p \notin Q_{t-s}(P) \forall s \geq 0$. The same argument applies in case (2) for $t'$ such that $i(t') = l$. Q.E.D.

For environments with single-dimensional policy sets and single-peaked preferences, theorem 2 implies that the desires of the median voter prevail (just as in Downs' [1957] model of electoral competition). From lemma 1, we know that an analog of theorem 2 holds for basic institutions with non-degenerate final stages whenever there exists a Condorcet winner in $\Omega(P)$. This latter observation will prove useful in section 4.

### 3.5 An irrelevance result for early activity

We know that the outcome of the collective choice process must lie in $Q_1(P)$. If $|Q_1(P)| = 1$ then this outcome is necessarily independent of the initial status quo, $p_0$. Moreover, if for some $t > 1$, $|Q_t(P)| = 1$, the actions in proposal rounds 1 through $t-1$ have no effect on the final policy. In contrast, if $|Q_1(P)| > 1$, both the initial status quo and the actions taken in early rounds are potentially important.

Theorem 2 identifies a set of conditions under which $|Q_t(P)| = 1$ for some $t \geq 1$. When those conditions are satisfied, the outcome is necessarily the Condorcet winner; the initial status quo and actions taken in sufficiently early proposal rounds are irrelevant. Our next result demonstrates that the irrelevance of early stages is completely general, and does not depend on the existence of a Condorcet winner in $P$ (or in $\Omega(P)$ for institutions with non-degenerate final stages).

**Theorem 3:** Consider an institution with a degenerate final stage and a policy set satisfying A1 and A2. Suppose that at least $M$ individuals make proposals in the first $T - |P| + 2$ rounds. Then the outcome is independent of the initial status quo $p_0$.

**Proof:** Suppose $\Phi_i(Q) = Q$ for some $i$. We claim that there exists a Condorcet winner in $Q$. Let $q^i$ solve $\min_{q \in Q} v_i(q)$. Since $\Phi_i(Q) = Q$, $\exists q^i \in Q$ such that
\( \hat{q} \) solves \( \max_{q'' \in Z(\hat{q}', Q)} v_i(q'') \). This can only be the case if \( Z(\hat{q}', Q) = \{ q' \} \). Since it is always the case that \( q' \in Z(\hat{q}', Q) \), we know that \( q' = \hat{q}' \). Thus, \( Z(\hat{q}', Q) = \{ q' \} \). Take any other \( q \in Q \). Since \( q \notin Z(\hat{q}', Q) \), \( \#S \) with \( |S| \geq M \) such that \( v_j(q) \geq v_j(q') \) \( \forall j \in S \). But then \( v_j(q') > v_j(q) \) for some set \( S' \) with \( |S'| \geq M \). Since this is true \( \forall q \in Q \setminus \{ q' \} \), \( \hat{q}' \) is a Condorcet winner in \( Q \).

We now claim that there exists a Condorcet winner in some \( Q_t(P) \) with \( t \geq T - |P| + 3 \). Suppose not. Then, by the preceding argument, \( |Q_t(P)| \leq |Q_{t+1}(P)| - 1 \) (with \( |Q_{T+1}(P)| \equiv |P| \)). But then \( |Q_{T-|P|+3}(P)| \leq 2 \) which contradicts the non-existence of a Condorcet winner in \( Q_{T-|P|+3}(P) \). Combining this claim with theorem 2 (and invoking the equivalence property from theorem 1) establishes the theorem. Q.E.D.

Though we have stated this result for institutions with degenerate final stages, theorem 1 implies that it also holds for institutions with non-degenerate final stages. Thus, the irrelevance of the initial status quo and actions taken in early proposal rounds is quite general (provided that the number of rounds is sufficiently large, and that the set of recognized individuals is sufficiently diversified). This observation has a number of important implications. To illustrate, consider the alternative institution mentioned at the end of section 2.2: a committee generates a proposal, legislators can amend the proposal through a sequence of motions and votes, the final amended proposal is put to an up-or-down vote against existing law, and the bill, if passed, is enacted into law without reconsideration. Suppose that we model this institution by appending to the basic institution some initial game between committee members to generate \( p_0 \), as well as a final stage consisting of an up or down vote between \( p_T \) and some default policy \( p^D \). Theorem 3 tells us that, as long as the amendment process is sufficiently long and inclusive, the committee’s deliberations are irrelevant.

### 3.6 An extension to dynamic collective choice

Throughout this paper, we confine attention to static collective choice problems (the selection of the policy that will prevail at a given point in time). In this section, describe an extension to dynamic collective choice problems (the selection of policies for all future periods) that follows directly from theorem 3.

Suppose that the group must select the policy that will prevail in each period \( k = \)
1, 2, ..., $K$, where $K$ is potentially large but finite. Let $p^k$ denote the policy selected from period $k$. A dynamic policy as of period $k$, $d_k$, is a vector of policies for $k$ and all subsequent periods: $d_k = (p^k, p^{k+1}, ..., p^K)$. The group convenes in a “session” before the first period, and between each successive pair of periods. Each session consists of multiple proposal rounds. Activity prior to round $t$ of session $k$ establishes a dynamic status quo policy for period $k$ onwards, $d_{kt} = (p^{k}_{kt}, p^{k+1}_{kt}, ..., p^{K}_{kt})$. The initial dynamic status quo, $d_{10}$, is predetermined. Let $d_{kt}^m$ denote the dynamic proposal made in the $t$-th round of the the $k$-th session (which convenes between periods $k − 1$ and $k$). This proposal consists of a policy for the upcoming period and every subsequent period: $d_{kt}^m = (p^{k,m}_{kt}, p^{k+1,m}_{kt}, ..., p^{K,m}_{kt})$, where $p^{j,m}_{kt}$ denotes the policy proposed in round $t$ of session $k$ for period $j$. If the proposal passes, it displaces the previous status quo; if it does not pass, the previous status quo is unchanged. At the end of session $k$, the final status quo policy for period $k$, $p^{k}_{kT}$, is implemented. The final session $k$ status quo policy for any period $j > k$, $p^{j}_{kT}$, carries over as the initial status quo for the next legislative session ($p^{j}_{k+1,0} = p^{j}_{kT}$).

Imagine that the number of proposal rounds in each session is sufficiently large, and that the recognition orders are sufficiently inclusive, so that theorem 3 applies. In that case, it is easy to solve the dynamic collective choice problem by backward recursion. By theorem 3 we know that the initial status quo for session $K$ is irrelevant. The period $K$ outcome is simply the policy selected by our static model of collective choice. In light of this observation, proposals in session $K − 1$ cannot affect session $K$ outcomes. Consequently, the period $K − 1$ outcome is also the policy selected by our static model of collective choice. Continuing the recursion, we see that the dynamic problem decomposes into a sequence of static problems, each of which can be treated separately. Note that this conclusion also holds when the recognition order is determined randomly and revealed at the outset of each session.

It is useful to contrast this framework with the dynamic collective choice models of Baron [1996], Barron and Herron [1998], Eppele and Riordan [1987], and Ingher-
man [1985]. Four differences merit emphasis. First, each of these papers assumes that the status quo for future periods is determined by dynamic reversion. This

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4It is worth mentioning one additional difference between our model and Ingher-
man’s: Ingher-
man assumes that some third party establishes the voting agenda (a similar comment applies to related papers by McElvkey [1976] and Kramer [1972]). Such models may be appropriate when considering public referenda, but are probably ill-suited for describing legislative processes wherein members share responsibility for the generation of proposals.
amounts to imposing the restriction that \( p_{k+1}^{k,m} = p_{k+1}^{k+1,m} = \ldots = p_{K}^{K,m} \). In other words, once adopted, a static (single period) policy stays in place until it is overturned. As mentioned in section 1, this restriction is not imposed in practice, and it artificially links strategic manipulation of future outcomes (through anticipation of reconsideration) to the current outcome, thereby confounding disparate sources of incentives, and potentially obscuring the role of reconsideration.

Second, all of these papers assume that it is possible to make only one proposal per period. In extending our analysis to dynamic choice problems, we have implicitly taken the view that policy deliberations are more rapid than policy implementation, so that individuals have the ability to make and consider several rounds of proposals within each policy period (think, for example, of a legislative session establishing policy for the subsequent year). When the number of proposal rounds per session is small, dynamic collective choice problems do not necessarily decompose into sequences of static collective choice problems.

Third, in contrast to these papers, we focus on a finite horizon problem. Epple and Riordan are primarily concerned with identifying sustainable outcomes in an infinite horizon setting where it is possible to contrive subtle subgame perfect punishments. In contrast, Baron is troubled by the resulting indeterminacy. He imposes a stationarity refinement, which isolates the same set of equilibria that one would obtain by considering a sequence of finite horizon models with the length of the horizon converging to infinity. By focusing on finite-horizon problems, we avoid the implications of the folk-theorem entirely. One can also construct an equilibrium for the infinite-horizon model by decomposing it into an infinite sequence of static collective choice problems, but (as in Baron’s analysis) there are other equilibria.

Finally, Baron and Ingberman both restrict attention to one-dimensional policy spaces with single-peaked preferences, while Epple and Riordan limit their analysis to situations involving pure redistribution among three voters (Baron and Herron examine multidimensional choice problems). Our analysis considers more general policy spaces, and subsumes these cases. One can, of course, specialize our model to particular dynamic collective choice problems. Focusing on one-dimensional policy spaces with single-peaked preferences, we obtain the median voter outcome in every period (this is a consequence of theorem 2). This contrasts with Baron’s central result, which establishes convergence to the median voter outcome. Problems of pure redistribution are considered in section 5; our results concerning the concentration
of political power contrast with the indeterminacy noted by Epple and Riordan.

4 Dictatorship results

To characterize the possible outcomes of the collective choice process with greater precision, one must place some restrictions on feasible policies. In this section, we restrict attention to CBDC policy sets. We extend our analysis to some other types of policy sets in section 5.

Analysis of the simple collective choice process described in section 2 yields an important insight: seemingly democratic institutions can yield highly undemocratic outcomes. Since a proposal must receive majority support to pass, it is natural to conjecture that final outcomes must benefit a majority of the individuals. This is not the case. Indeed, under surprisingly weak conditions, the democratic process considered here produces dictatorial outcomes. Moreover, seemingly democratic reforms, such as increasing the number of individuals who are given opportunities to make proposals, can accentuate the concentration of political power.

We divide this discussion into three subsections. The first considers environments in which many individuals have opportunities to make proposals ("inclusive recognition orders"). We demonstrate that, as long a sufficient number of individuals are recognized at some point during deliberations, a dictatorial outcome emerges for every recognition order and every initial status quo. The second considers environments in which relatively few individuals have opportunities to make proposals ("exclusive recognition orders"). Our analysis of these environments provides a sense for the frequency with which the dictatorial policy emerges when this outcome is not guaranteed. It also demonstrates that our central result is robust with respect to some important variations in institutional procedures. In particular, the third subsection considers some implications for environments in which recognition is determined randomly during the course of deliberations ("random recognition orders").

4.1 Inclusive recognition orders

Some collective choice processes plainly yield majoritarian outcomes. Consider, for example, an institution with a degenerate final stage and one proposal round \((T = 1)\). This institution permits no reconsideration. Imagine that the initial status quo is inaction \((\pi_0 = \emptyset)\). Then the outcome necessarily consists of \(M\) elementary
policies. Specifically, the policy includes the elementary policy $i(1)$ and the $M - 1$ least costly elementary policies other than $i(1)$.

Compare the institution discussed in the previous paragraph to one in which there are extensive opportunities for reconsideration. In particular, imagine that a large fraction of the individuals – perhaps all of them – have opportunities to make proposals (an inclusive recognition order). The latter institution certainly seems more democratic. It better reflects the egalitarian principle that every interested party has a right to be heard. Surprisingly, it produces a much less democratic outcome. Indeed, our next result suggests that the right to be heard can concentrate all political power in the hands of a single individual.

**Theorem 4:** Consider an institution with a degenerate final stage, a CBDC policy set satisfying A1-A4, and $N \geq 5$ individuals. Provided that $|J| > M$, the unique outcome is the policy $p = \{i(T)\}$.

The simple intuition for this result is as follows. The last proposer only needs to secure the approval of a minimally decisive coalition. Consequently, she never leaves anything on the table for some group of $M - 1$ individuals (their elementary policies are excluded). The individuals who anticipate exclusion have a common interest in making sure that as few elementary policies as possible are ultimately implemented. Anyone expecting to be one of the remaining beneficiaries joins in this objective. This includes the last proposer, since she can always make sure that her elementary policy is adopted. Adding the last proposer to those who are excluded produces a majority in favor of dropping all other elementary policies.

Since this result is somewhat counterintuitive, it is important to go through the argument carefully with the object of elaborating on the preceding intuition. This is best accomplished by integrating the formal proof with a less formal discussion. After presenting and discussing the proof, we provide some further comments concerning the theorem.

### 4.1.1 Proof of the theorem

We start by demonstrating that $\Phi_{i(T)}(P)$ has the following three properties:

**Property 1:** $\{i(T)\} \in \Phi_{i(T)}(P)$.

**Property 2:** $i(T) \in p$ for all $p \in \Phi_{i(T)}(P)$.
Property 3: \( p \in \Phi_{i(T)}(P) \Rightarrow |p| \leq M. \)

Property 1 is straightforward: if the status quo in the last round is \( p_{T-1} = \{i(T)\} \), \( i(T) \) proposes \( \{i(T)\} \), thereby assuring that \( \{i(T)\} \) is implemented.

Now we show that regardless of the status quo at the beginning of the last round, \( i(T) \) proposes a policy that includes the elementary policy \( i(T) \), and contains at most \( M \) elementary policies; moreover, this proposal passes. For the purposes of the proof, it is useful to distinguish between the following five cases. In each case, we identify \( i(T) \)'s best proposal.

Case 1: \( i(T) \in p_{T-1} \). For this case, we claim that the best choice for \( i(T) \) is to propose the policy \( p' \) obtained by dropping the min \( \{|p_{T-1}| - 1, M - 1\} \) highest cost elementary policies in \( p_{T-1} \) other than \( i(T) \). The proposed policy strictly improves the payoff to any individual associated with an elementary policy that is not dropped. Since this group forms a majority, the proposal passes. If \( p' = \{i(T)\} \), then it is clearly the best choice for \( i(T) \). Now suppose that \( p' \) also contains other elementary policies. In light of A3 and A4, for any policy \( p'' \) preferred by \( i(T) \) to \( p' \), \( |p''| \leq |p'| \). But since \( p'' \neq p' \), this means that more than \( M - 1 \) elementary policies have been dropped from \( p_T \) in constructing \( p'' \). All of the associated individuals, of whom there are at least \( M \), strictly prefer \( p_{T-1} \) to \( p'' \). Consequently, any such \( p'' \) does not pass.

Case 2: \( i(T) \notin p_{T-1} \) and \( |p_{T-1}| \geq 2 \). For this case, we claim that the best choice for \( i(T) \) is to propose the policy \( p' \) obtained by dropping the min \( \{|p_{T-1}|, M\} \) highest cost elementary policies in \( p_{T-1} \), and adding \( i(T) \). The proof is essentially the same as for case 1 (except one invokes A4 to establish that the proposed policy strictly improves the payoff to any individual associated with an elementary policy that is not dropped).

Case 3: \( i(T) \notin p_{T-1} \) and \( p_{T-1} = \{j\} \) for some \( j \neq i \) with \( c_j > c_{i(T)} \). Then \( i(T) \)'s best choice is obviously to propose \( p' = \{i(T)\} \), which passes almost unanimously.

Case 4: \( i(T) \notin p_{T-1} \) and \( p_{T-1} = \{j\} \) for some \( j \neq i \) with \( c_j < c_{i(T)} \). We claim that \( i(T) \)'s best choice is to propose a policy \( p' \) consisting of \( i(T) \) and the \( M - 1 \) lowest-cost elementary policies other than \( i(T) \) and \( j \). The proposed policy strictly improves the payoff to any individual associated with an elementary policy that is included in \( p' \). Since there are \( M \) such individuals, the policy passes. Consider any other policy \( p'' \) that receives majority support (including \( p_{T-1} \)). Either (1) \( p'' = \emptyset \), (2) \( p'' \) contains a single elementary policy \( k \) with \( c_k \leq c_j \), (3) \( p'' \) contains \( M \)
elementary policies and \( j \notin p' \), or (4) \( p'' \) contains more than \( M \) elementary policies. Note that \( i(T) \) prefers \( p' \) to any such \( p'' \).

Case 5: \( p_{T-1} = \emptyset \). We claim that \( i(T) \)'s best choice is to propose a policy \( p' \) consisting of \( i(T) \) and the \( M-1 \) lowest-cost elementary policies other than \( i(T) \). The proposed policy strictly improves the payoff to any individual associated with an elementary policy that is included in \( p' \). Since there are \( M \) such individuals, the policy passes. Consider any other policy \( p'' \) that receives majority support (including \( p_T \)). Either (1) \( p'' = \emptyset \), or (2) \( p'' \) contains at least \( M \) elementary policies. Note that \( i(T) \) prefers \( p' \) to any such \( p'' \).

In each of the five cases mentioned above, it is easy to check that \( i(T) \in p' \) and \(|p'| \leq M \) as required to establish properties 2 and 3.

Having established that \( \Phi_{i(T)}(P) \) does indeed satisfy properties 1 through 3, we argue next that \( \{i(T)\} \) is a Condorcet winner in \( \Phi_{i(T)}(P) \). By property 1, we know that \( \{i(T)\} \) is contained in \( \Phi_{i(T)}(P) \). Consider any other policy \( p' \in \Phi_{i(T)}(P) \). By properties 2 and 3, there are at least \( M-1 \) individuals whose associated elementary policies are excluded from both \( \{i(T)\} \) and \( p' \). By property 2, all of these excluded individuals prefer \( \{i(T)\} \) to \( p' \). Obviously, individual \( i(T) \) also prefers \( \{i(T)\} \) to \( p' \). Thus, a majority prefers \( \{i(T)\} \) to \( p' \). In general, the identity of the winning majority coalition depends on the choice of \( p' \) (see, however, the discussion of lemma 2, below).

The desired conclusion now follows almost immediately from lemma 1 and theorem 3. By lemma 1, the basic institution under consideration is equivalent to one in which there are \( T-1 \) proposal rounds in the policy development stage, and for which the policy space is \( \Phi_{i(T)}(P) \) (one must also transform the initial status quo appropriately, but this is inconsequential). The preceding arguments establish that \( \{i(T)\} \) is a Condorcet winner in \( \Phi_{i(T)}(P) \). By theorem 3 part (1), the institution therefore selects \( \{i(T)\} \) as long as there are at least \( M \) distinct individuals are recognized in proposal rounds 1 through \( T-1 \). If \( J > M \), this condition is plainly satisfied. This establishes theorem 4. Q.E.D.

The recursive structure of the proof shows that the power of the last mover resides in the properties of \( \Phi_{i(T)}(P) \). The final proposer can always contribute to implement a policy that includes \( i(T) \), and always averts the implementation of policies with more than \( M \) elementary components. As a result, any round-\( T \) continuation path producing \( \{i(T)\} \) is preferred to any other feasible round-\( T \) continuation path.
by a majority of the individuals; that is, it is a Condorcet winner in the set of feasible continuation paths. All individuals whose associated elementary policies are excluded from the continuation outcome, as well as \( i(T) \), find it in their interests to make and support proposals that ultimately lead to the effectively dictatorial outcome \( \{i(T)\} \).

### 4.1.2 Some remarks on the theorem

Theorem 4 identifies conditions under which the last proposer, \( i(T) \), is a dictator in the following sense: she obtains her most preferred outcome, \( \{i(T)\} \), irrespective of the initial status quo, the order of recognition, or the costs and benefits associated with any particular elementary policy (provided that A1 through A4 are satisfied). It is important to emphasize the perversity of this outcome. When, for example, the initial status quo is the null policy \( \emptyset \), all individuals other than \( i(T) \) strictly prefer it to the final outcome. If the group simply failed to meet, everyone would be better off except \( i(T) \). The group produces a result that is contrary to the interests of almost every member, even though no proposal can pass without majority support.

Theorem 4 also demonstrates that apparently democratic reforms have decidedly undemocratic effects. For example, a majoritarian outcome results when the group entertains only a single proposal, but dictatorship emerges when every individual is allowed to make a proposal.

A few further remarks concerning theorem 4 are in order. First, we have assumed that individuals are sufficiently flexible to make different proposals when they inherit different status quos. As mentioned previously, \( i(T) \)'s power depends upon her ability to implement a policy that includes the elementary policy \( i(T) \), and to avert the implementation of policies with more than \( M \) elementary components, irrespective of the round \( T \) status quo, \( p_{T-1} \). No single proposal accomplishes these objectives for all possible \( p_{T-1} \). Thus, \( i(T) \) must have the flexibility to select an appropriate proposal for each round-\( T \) status quo. Institutions that deprive \( i(T) \) of this flexibility do not, in general, give rise to dictatorial outcomes. As an example, imagine that each individual in the set \( J \) must commit herself to round-specific proposals prior to round 1 (any individual who is recognized more than once makes several round-specific commitments). This is, in effect, the assumption employed by Ferejohn, Fiorina, and Mc Kelvey [1987] (these authors also add a non-degenerate final stage consisting of an up-or-down vote versus prevailing law), as well as in
portions of Shepsle and Weingast [1984]. One does not generally obtain dictatorial outcomes in such settings. Indeed, with a degenerate final stage, though the precommitment assumption simplifies the strategy spaces, it makes the model extremely difficult to solve, and in some numerical examples is inconsistent with the existence of pure strategy equilibria.

Second, aside from the requirement that $|J| > M$, we have placed no restrictions on the order of recognition. Some individuals may be recognized once or more than once, while others never have opportunities to make proposals. There is no need to cycle through those who are recognized in any particular order. Indeed, a single individual may be recognized in several consecutive rounds. It is natural to conjecture that consecutive proposals are redundant, but this is not the case. Somewhat surprisingly, an individual may be able to accomplish some objective with two consecutive proposals, but not with a single proposal. For example, with $T = 1$, the institution produces a policy with $M$ elementary components including $i(T)$. However, with $T > 1$ and $i(T - 1) = i(T)$, the outcome is $\{i(T)\}$ (this follows because $\{i(T)\}$ is a Condorcet winner on $\Phi_{i(T)}(P)$ and by part (2) of theorem 3).

Third, theorem 4 also holds for more general policy spaces. For CBDC policy spaces, one can substantially relax A3. In particular, the same result holds as long as $c_i(T) < \sum_{j \in L_{M-1}} c_j$, where $L_K$ is defined as the set of the $K$ least costly elementary policies in $E \setminus i(T)$ (to understand why, note that properties 1 through 3 still hold under this alternative assumption). Likewise, one can allow for some variation across individuals in the rankings of elementary policies by cost. Using an alternative argument, one can also prove the same result for environments in which different elementary policies have the same costs (this violates assumption A2).\(^5\)

In section 5, we also demonstrate that our result holds as an approximation when the policy space involves “splitting a dollar” among a large number of parties, even though this case violates both assumptions A1 and A2.

Finally, the theorem does not hold for institutions with three individuals ($N = 3$).\(^6\) The proof breaks down when one tries to establish property 2. To illustrate, suppose that $T = 3$, $i(t) = t$, and $c_1 < c_2 < c_3$. Then the set of continuation

\(^5\)An alternative argument is required because $\phi(p', P')$ may be set-valued.

\(^6\)To our embarrassment, we initially discovered theorem 4 by “proving” it for the case of $N = 3$, generalizing the arguments to cases with $N > 3$, and then discovering that our initial proof was incorrect.
outcomes for any status quo \( p_T \) is given by

\[
\begin{array}{c|c}
p_{T-1} & Q_T(p_{T-1}) \\
\hline
\emptyset & \{1,3\} \\
\{1\} & \{2,3\} \\
\{2\} & \{1,3\} \\
\{3\} & \{3\} \\
\{1,2\} & \{1\} \\
\{1,3\} & \{3\} \\
\{2,3\} & \{3\} \\
\{1,2,3\} & \{1,3\} \\
\end{array}
\]

Note that if \( p_{T-1} = \{1,2\} \) the eventual outcome is \( \{i(1)\} \). Since 1 and 2 prefer \( \{i(1)\} \) to \( \{i(3)\} \), the latter is no longer a Condorcet winner in \( Q_T(P) \). This undermines the dynamics that generate dictatorial outcomes. In this case, depending on the initial status quo, the outcome is either \( \{i(1)\} \) or \( \{i(2), i(3)\} \). Since most collective choice problems involve more than three decision makers in practice, we regard this as a technical curiosity.

4.2 Exclusive recognition orders

It is natural to question the general applicability of theorem 4. Several objections immediately come to mind. First, the result requires individuals to know the recognition order as of round 1. In practice, there may be considerable uncertainty concerning who will be recognized two or three rounds, let alone twenty or thirty rounds, in the future. A second related concern is that individuals must know the number of proposal rounds as of round 1. Though it is plausible to assume that there is a finite upper bound on the number of proposal rounds that can precede any time-dated policy, deliberations on any given proposal may vary randomly in length, creating variation in the realized number of rounds. A third concern is that the result appears to require highly sophisticated strategic reasoning. The familiar centipede game admits a single subgame perfect equilibrium, but this solution presupposes an ability to think through many layers of strategy. In practice, play of the centipede game fails to unravel as predicted by theory. Conceivably, our result may be vulnerable to the same criticism. Finally, the requirement that \( |J| > M \) is particularly demanding for groups with large numbers of members.

In this section, we describe one potential avenue for addressing all of these criticisms simultaneously. Each of the concerns mentioned above relates in some way
to the number of proposal rounds. When relatively few individuals have opportunities to make proposals (formally, \(|J| < M\), one can show that there are always recognition orders and initial status quo for which \(\{i(T)\}\) is not the outcome. In this sense, one cannot “improve” upon the requirement that \(|J| < M\). However, it turns out that non-dictatorial outcomes are unusual: a high fraction of possible recognition orders generate \(\{i(T)\}\) for all initial status quos even when \(|J|\) is small and \(M\) is large. Consequently, we obtain a dictatorial or near-dictatorial outcome “most of the time” (in a sense made precise below), regardless of the group’s size, as long as individuals can think ahead strategically only a small number of steps, and as long they properly anticipate the number of remaining rounds and the order of recognition once the end of deliberations draws near. We present some basic results in this section, and elaborate on their implications for random recognition processes (including random order and random closure) in the next subsection.

We begin our analysis of exclusive recognition orders by deriving several conditions under which the dictatorial outcome emerges even for small \(|J|\). The statement of this theorem requires the following definitions: \(H_K\) denotes the set of individuals associated with the \(K\) most costly policies in \(E \setminus i(T)\), and \(i^*_K\) is the individual associated with the \(K\)-th least costly policy in \(E \setminus i(T)\).

**Theorem 5:** Consider an institution with a degenerate final stage, a CBDC policy set satisfying A1-A4, and \(N \geq 5\) individuals.

(a) If \(T \geq 2\) and \(i(T-1) \neq i^*_{M-1}\), then the outcome is either \(\{i(T)\}\) or \(\{i(T-1), i(T)\}\).

(b) Under either of the following conditions, the unique outcome is the policy \(p = \{i(T)\}\):

(b1) some member of \(H_{M-2} \cup \{i(T)\}\) has the opportunity to make at least one proposal prior to round \(T\)

(b2) \(i(t) \neq i(T-1) \neq i^*_{M-1}\) for some \(t < T-1\).

Part (a) follows from computing \(\Phi_{i(T-1)} \circ \Phi_{i(T)}(P)\) under the assumption that \(i(T-1) \neq i^*_{M-1}\). To establish part (1) of (b), one supplements the proof of theorem 4 with a few additional arguments. First one shows that if \(j \in H_{M-2}\), then \(j \notin p\) for any \(p \in \Phi_{i(T)}(P)\). This follows from an inspection of \(i(T)\)'s optimal proposal, \(p'\), for each of the five cases mentioned in the previous proof. In combination with
property 2, this implies that all members of \( H_{M-2} \cup \{i(T)\} \) prefer \( \{i(T)\} \) to all other elements \( \Phi_{i(T)}(P) \). Thus, if some member of \( H_{M-2} \cup \{i(T)\} \) has the opportunity to make a proposal prior to round \( T \), condition (2) of theorem 3 is satisfied (where \( \{i(T)\} \) is the Condorcet winner in \( \Phi_{i(T)}(P) \)). This in turn implies that the process yields \( \{i(T)\} \), as claimed. To establish part (2) of part (b), one invokes part (a). All individuals other than \( i(T-1) \) prefer \( \{i(T)\} \) to \( \{i(T-1),i(T)\} \). Consequently, if any \( j \neq i(T-1) \) is recognized in any previous period, she will make a proposal for which the continuation path leads to \( \{i(T)\} \) rather than to \( \{i(T-1),i(T)\} \), and the proposal will pass.

Theorem 5 would seem to imply that basic institutions with short recognition orders can produce non-dictatorial outcomes only in relatively unlikely circumstances. We formalize this observation by deriving a lower bound on the fraction of recognition orders that generate the dictatorial outcome \( \{i(T)\} \) for all initial status quo.

**Theorem 6:** Consider an institution with \( T > 1 \) proposal rounds, a degenerate final stage, a CBDC policy set satisfying A1-A4, and \( N \geq 5 \) individuals. The fraction of recognition orders that generate the outcome \( \{i(T)\} \) for all \( p_0 \in P \) is not less than

\[
B(N,T) = 1 - \frac{1}{2N}[(\frac{1}{N})^{T-3} + (\frac{1}{2})^{T-3}].
\]

If one imagines that a recognition order is selected at random in the initial stage, and that this selection process is governed by a uniform distribution over the set of all feasible recognition orders, then \( B(N,T) \) provides a lower bound on the probability that the collective choice process yields \( \{i(T)\} \). Figure 4 illustrates the manner in which this bound changes with the numbers of rounds and individuals. Notice that, regardless of whether \( N \) is large or small, \( B(N,T) \) approaches unity for relatively small values of \( T \). Also notice that the bound is more sensitive to the number of proposal rounds than to the number of individuals. To understand why this is the case, consult part (b1) of theorem 5. If any member of \( H_{M-2} \cup \{i(T)\} \) is recognized prior to round \( T \), the outcome is \( \{i(T)\} \). The probability of not recognizing a member of this group in any particular round is approximately 1/2 for all \( N \). This probability compounds rapidly with the number of rounds, thereby generating the observed convergence with \( T \). Finally, notice that, for \( T > 2 \), the bound is actually increasing in the number of individuals. This suggests that,
contrary to the apparent implications of theorem 4, dictatorial outcomes are even more likely in large groups than in small ones. However, the bound is not tight. For example, if $N = 5$ and $T = 4$, theorem 4 implies that every ordering generates a dictatorial outcome, even though $B(5, 4) = 0.95$.

The function $B(N, T)$

The preceding result concerns the fraction of possible recognition orders that produce $\{i(T)\}$ for all initial status quos when $|J| < M$. We now consider the conditions under which a particular initial status quo produces $\{i(T)\}$ regardless of the recognition ordering, again assuming $|J| < M$.

**Theorem 7:** Consider an institution with a degenerate final stage, a CBDC policy set satisfying A1-A4, and $N \geq 5$ individuals. An initial status quo $p_0 \in P$ leads to the outcome $\{i(T)\}$ provided that at least one of the following conditions is satisfied:

(i) $\sum_{j \in p_0} c_j > c_i(T)$ and either $|p_0| \leq M$ or $|J| > 2$

(ii) $p_0 = \emptyset$ and $|J| > 2$

(iii) $|J| > |Q_T(p_0)|$
Part (i) tells us that the legislative process tends to generate \( \{i(T)\} \) when the initial status quo is more costly than \( \{i(T)\} \). This requires one of two conditions: either the initial status quo consists of no more than \( M \) elementary policies, or at least three individuals are recognized. Part (ii) tells us that the collective choice process also generates \( \{i(T)\} \) when the initial status quo is inaction, provided again that at least three individuals are recognized. Together, parts (i) and (ii) imply that, with \(|J| > 2\) (a very weak condition indeed), \( \{i(T)\} \) can be avoided only if the initial status quo consists of a single elementary policy that is less costly than \( \{i(T)\} \).

This is a small fraction of all feasible initial status quos; moreover, this fraction goes to zero as the number of individuals, \( N \), becomes large. Consequently, if a status quo is selected at random in the initial stage, and if at least three individuals are recognized, a large group is almost certain to produce the dictatorial outcome \( \{i(T)\} \).

Part (iii) tells us that any initial status quo \( p_0 \) leads to the outcome \( \{i(T)\} \) provided that the number of recognized individuals exceeds the number of elementary policies that would be implemented were \( i(T) \) to inherit \( p_0 \) as the round \( T \) status quo. Note that \(|Q_T(p_0)| < M\) for any initial status quo other than \( p_0 = \emptyset, p_0 = E \), and \( p_0 = \{j\} \) for \( j \) with \( c_j < c_i(T) \).

### 4.3 Random recognition orders

Obviously, our basic results subsume cases in which the recognition order and number of rounds are determined randomly and revealed prior to the initial proposal round. In this section, we tackle a more difficult question: what happens if the recognition sequence and/or the number of rounds are realized randomly (or at least revealed) during the course of deliberations?

Theorem 5 has important implications for institutions with random recognition (both with respect to the sequence of proposers and the number of rounds). Consider in particular the combined implications of parts (a) and (b1) of the theorem. Suppose that the recognition order and number of rounds are both determined randomly. Assume, however, that, as of the second-to-last round, the proposer knows who will propose next, and is aware that only one round remains. Henceforth, we will refer to this assumption as **penultimate certainty**. (It holds, for example, when all individuals are guaranteed a fixed number of opportunities to make proposals.)

Parts (a) and (b1) tell us that, with penultimate certainty, the outcome is dictatorial (\( \{i(T)\} \)) if \( i(T - 1) \in H_{M-2} \cup \{i(T)\} \), and either dictatorial or nearly dictatorial
(\{i(T)\} or \{i(T-1),i(T)\}) if i(T-1) \in L_{M-1}. Thus, if all individuals stand an equal chance of being selected as i(T-1), the probability of achieving a dictatorial or near dictatorial outcome is at least \(\frac{2(M-1)}{N} = \frac{N-1}{N}\) (which converges to unity for large groups), while the probability of achieving a dictatorial outcome is at least \(\frac{M-1}{N} = \frac{N-1}{2N}\) (which converges to \(\frac{1}{2}\) for large groups).

To obtain stronger results for random recognition orders, one must make further assumptions. As an example, consider the following class of recognition processes. The number of rounds, \(T\), may be stochastic, and the (potentially stochastic) algorithm for recognizing individuals prior to round \(T-K+1\) for some \(K \leq N\) is not restricted. However, between rounds \(T-K\) and \(T-K+1\), individuals are informed that each member of some set \(\Theta\) with \(|\Theta| = K\) (e.g., elders, faction leaders, or possibly even the entire group) will be recognized one final time for “closing proposals.” In round \(T-K+1\), an individual is selected at random (with equal probabilities) to make a closing proposal. In subsequent rounds, another individual is selected at random (with equal probabilities) from among those who have not yet made closing proposals. We will say that such recognition processes are characterized by final proposal phase. Note that a random recognition process with a final proposal phase satisfies the assumption of penultimate certainty.

Now imagine that period \(T-2\) has arrived, that a proposer for \(T-2\) has been selected. Let \(i^A\) and \(i^B\) denote the two individuals in \(\Theta\) who have not yet been recognized in phase 2. Suppose in addition that \(i^*_{M-1} \notin \{i^A,i^B\}\) (which occurs with probability \(\frac{N-2}{N}\)). According to part (a) of theorem 5, all individuals know that the final outcome will definitely include the elementary policy for whichever individual is chosen to propose last, and may also include the elementary policy for whichever individual is chosen to propose second to last. By adopting any one of any number of policies in \(T-2\) (e.g., any policy consisting of two elementary policies), the group can guarantee that the outcome will include only the elementary policy for whichever individual is chosen to propose last. For all individuals other than \(i^A\) and \(i^B\), this is strictly preferable to all other possible continuation outcomes. Accordingly, the individual recognized in round \(T-2\) will make a proposal that guarantees this outcome, and a majority of individuals (all but \(i^A\) and \(i^B\)) will vote in favor (unless the status quo for \(T-2\) would generate the same outcome). Thus, we have:

**Theorem 8:** Consider an institution with a degenerate final stage, a CBDC policy
set satisfying A1-A4, \( N \geq 5 \) individuals, and a random recognition process with a final proposal phase. Then, with probability not less than \( \frac{N-2}{N} \), the group adopts the elementary policy corresponding to whichever individual is randomly chosen to propose last, and nothing else.

Observe that the probability that the last proposer is effectively a dictator converges to unity as the size of the group becomes large.

5 An Alternative Policy Space

In section 4, our analysis focused on CBDC policy spaces. It is natural to wonder whether our central conclusions also hold for other types of policy spaces. In this section, we explore the generality of the basic dictatorship result (theorem 4) by considering a specific, natural alternative: the problem of dividing a dollar. The policy space for this model violates assumption A1 and, more importantly, A2 (the generic no-indifference condition). This undermines the uniqueness of continuation equilibria, and thereby complicates the analysis considerably. Nevertheless, provided that one adopts a reasonable and consistent rule for resolving this indiffrence, the outcome is approximately dictatorial.

For the purposes of the following discussion, we will say that \( q \in Q \) is a weak Condorcet winner within \( Q \) iff, for all \( q' \in Q \), a majority of individuals weakly prefer \( q \) to \( q' \). Notice that, in general, nothing assures the uniqueness of a weak Condorcet winner.

Suppose we use any method of resolving indifference that is consistent with the existence of \( \varphi_{i(T)}(p, P) \) for all \( p \in P \). We claim that \( (\varphi_{i(T)})_i(p, P) = 0 \) for at least \( M-1 \) individuals. We establish the claim as follows. Let \( S \) be a set of \( M-1 \) individuals other than \( i(T) \) who vote in favor of \( \varphi_{i(T)}(p, P) \) (since this proposal defeats \( p \), we know that such a set exists). Suppose contrary to the claim that, for some \( p \), \( (\varphi_{i(T)})_j(p, P) = 0 \) for fewer than \( M-1 \) individuals. Then there exists some \( j \notin S \) for whom \( (\varphi_{i(T)})_j(p, P) > 0 \). Consider the policy \( p' \) formed from \( p \) by extracting all surplus from \( j \) and distributing it equally among \( i(T) \) and members of \( S \). Individual \( i(T) \) and all members of \( S \) strictly prefer this outcome to \( \varphi_{i(T)}(p, P) \) and therefore to \( p \). Consequently, \( p' \) would definitely pass if \( i(T) \) proposed it. Since \( i(T) \) prefers \( p' \) to \( \varphi_{i(T)}(p, P) \), this contradicts the premise that \( \varphi_{i(T)}(p, P) \) maximizes \( i(T) \)'s payoff within the set \( Z(p, P) \).
It follows immediately that the dictatorial outcome \((p_{i(T)} = 1 \text{ and } p_l = 0 \text{ for } l \neq i(T))\) is a weak Condorcet winner in the set \(\Omega_{i(T)}(P)\). It is also evident that one of the individuals, \(i(T)\), strictly prefers this outcome to all others. Somewhat surprisingly, it is also possible to show that this outcome is the unique weak Condorcet winner in \(\Omega_{i(T)}(P)\). For any other element of this set, \(p'\), there is some other element, \(p''\), such that a majority of individuals strictly prefers \(p''\) to \(p'\).

If it were possible to prove an analog of theorem 2 for unique weak Condorcet winners, then, based on the preceding observation, the implications of theorem 4 would generalize immediately to the problem of dividing a dollar. When the policy set includes a unique weak Condorcet winner \(p^{we}\), and when the recognition order is sufficiently inclusive, there does indeed exist an equilibrium that selects \(p^{we}\). However, by appropriately contriving the resolution of indifference at various stages of the game, one can in many instances achieve other outcomes.

From our perspective, the most reasonable equilibria in such circumstances are the ones that selects \(p^{we}\). To sustain other outcomes, one must assume that individuals who will receive zero payoffs in all continuation paths, and who therefore have absolutely nothing at stake, resolve their indifference when casting their votes either in favor of or in opposition to a proposal by selecting the course that inflicts the most damage on individual \(i(T)\). It is difficult to sustain such outcomes once one rules out such malevolence by imposing a consistent rule for resolving indifference.

In relaxing assumptions A1 and A2 simultaneously, we introduce some technical problems related to continuity and openness (through A1), as well as the aforementioned issues related to the resolution of indifference (through A2). To focus exclusively on the latter concerns, we suppose that the policy space is a discretized version of the unit simplex in \(\mathbb{R}^N\). Specifically, select some positive integer \(m\), and let \(\varepsilon = \frac{1}{m}\). Define

\[
P^\varepsilon \equiv \left\{ p \in \mathbb{R}^N \mid p \geq 0, \sum_{l=1}^{N} p_l = 1, \text{ and } p_l = n\varepsilon \text{ for some } n \in \{0, 1, \ldots, m\} \right\}
\]

For our next result, we assume that individuals vote in favor of a proposal only if they expect to be strictly better off should the proposal pass. We also rule out complex history dependent punishments by focusing on Markov-perfect equilibria, which can be described by \((Q_t)_{t=1}^T: P \rightarrow P\). We demonstrate that, with these restrictions, the final proposer receives virtually all of the surplus. This holds for every possible initial status quo, including equal division. In such cases,
an approximately dictatorial outcome emerges even though every other individual would be strictly better off if the group took no action, and even though every proposal requires the approval of a majority to pass. The outcome is approximately dictatorial in the following sense: as ε approaches zero, i(T)'s equilibrium payoff converges to unity.

**Theorem 9:** Consider an institution with a degenerate final stage and a policy set $P^*$ with $\epsilon < N^{-2}$. Suppose $N \geq 3$ and $|J| > M$. Consider any Markov-perfect equilibrium $(Q^*)_{t=1}^T$ under which each individual $l$ votes in favor of $p^m_l$ in round $t$ if and only if $Q_{t+1}(p^m_l) > Q_{t+1}(p_{l-1})$. Then, $p^F_{i(T)} \geq 1 - N \epsilon$.

A natural alternative assumption is that individuals resolve their indifference in favor of the current proposal, rather than against it. This case is considerably more complex. However, one can demonstrate that the outcome satisfies the following two properties: (i) all surplus is divided between $i(T)$, $i(T - 1)$, and $i(T - 2)$, and (2) if $\epsilon < \frac{\delta}{N}$ for some sufficiently small $\delta$, then as $N$ goes to infinity, the surplus received by $i(T - 1)$ goes to zero at the rate $\frac{1}{N}$, and the surplus received by $i(T - 2)$ goes to zero at the rate $\frac{1}{N^2}$. Thus, in large groups, the last proposer again receives essentially all of the surplus. One can extend these results to the non-discretized simplex by invoking suitable equilibrium refinements.

6 Summary and Conclusions

In this paper, we have explored the effect of reconsideration on democratic policy making. Our analysis reveals a surprisingly robust tendency for a natural and simple class of democratic institutions to produce high concentrations of political power. In particular, under surprisingly weak conditions, the individuals with the last opportunity to make a proposal is effectively a dictator. Moreover, this outcome is more likely to arise when more individuals have opportunities to make proposals. Thus, seemingly democratic (inclusive) reforms can have the perverse effect of further concentrating political power. We have also demonstrated that institutions belonging to the class considered here yield Pareto efficient outcomes and select a Condorcet winner when one exists. Provided that the deliberation process is sufficiently long, actions taken in the early stages of deliberation never affect final outcomes. The latter finding suggests that, in the absence of restrictions
on amendments, certain activities undertaken by legislative committees, such as drafting initial proposals, are irrelevant.

In Bernheim, Rangel, and Rayo [2001], we examine the sensitivity of our central conclusions to variations in institutional rules. Some apparently minor procedural details matter a great deal, while seemingly important rules are actually of little consequence. Supermajority requirements do little to overcome the dictatorial power of the final proposer. Endogenizing the order of recognition has no effect on the high concentration of political power when a chair chooses the order in advance of deliberations, or when the chair makes these decisions round by round but is aligned with a single member of the group. In the latter case, even a chair with universalistic objectives may find it impossible in some instances to manipulate the order of recognition so as to enact a policy that benefits more than two individuals. When the rules of the institution permit members to preclude reconsideration through collective action, the power of the last proposer may evaporate. However, the particular outcome depends on the details of the termination rule. For the least restrictive rule (one that allows individuals to bundle policy proposals with motions to preclude reconsideration), political power is simply transferred from the final proposer to the first proposer (and perhaps to one other individual) in a significant fraction of environments. In contrast, when individuals are not permitted to bundle policy proposals with motions to preclude reconsideration, one can obtain almost anything from inaction to a universalistic outcome, depending on the initial status quo. When members of the group are allowed to amend each others’ proposals, the power of the last proposer evaporates, but in some instances the group nevertheless selects policies that benefit small minorities (even a single individual) at the expense of large majorities. A no-repeal rule leads to outcomes that benefit groups no larger than minimal majorities. Ironically, when a non-repeal rule is combined with a supermajority requirement, the final outcome benefits a minority of members at the expense of a majority, and the number of individuals benefiting from the final outcome shrinks with the size of the required supermajority. Limitations on the introduction of new business near the conclusion of deliberations promotes inaction.
Appendix

For the proof of lemma 1, we use the notation introduced in section 3.2, except that we write $Q_t(p; P, \Omega)$ rather than simply $Q_t(p)$ to make explicit the dependence of this function (suppressed in the notation of the text) on both the policy space $P$ and the final stage $\Omega$. Also, we use $\mathbb{I}$ to denote the identity mapping.

**Proof of Lemma 1:** Fix $P, \Omega$, and $p \in P$. We proceed by induction. Using the definition of $Q_T$ we obtain:

$$Q_T(p; P, \Omega) = \varphi_{i(T)}(\Omega(p); \Omega(P)) = \varphi_{i(T)}(\mathbb{I} \circ \Omega(p); \mathbb{I} \circ \Omega(P)) = Q_T(\Omega(p); \Omega(P), \mathbb{I}),$$

which proves the claim for $t = T$.

Now suppose the claim is true for $t + 1$, which together with the definition of $Q_t$ implies:

$$Q_t(p; P, \Omega) = \varphi_{i(t)}(Q_{t+1}(p; P, \Omega); Q_{t+1}(P; P, \Omega)) = \varphi_{i(t)}(Q_{t+1}(\Omega(p); \Omega(P), \mathbb{I}); Q_{t+1}(\Omega(P); \Omega(P), \mathbb{I})) = Q_d(\Omega(p); \Omega(P), \mathbb{I}),$$

establishing the result. Q.E.D.

**Proof of Theorem 6:** First some notation. Let $\overline{A}$ denote the set of $M - 2$ players with the highest costs other than $i(T)$; $A$ the set of $M - 1$ players with the lowest costs other than $i(T)$; and $A^*$ the player with the $M$'th lowest cost other than $i(T)$.

**Step 1.** We claim that:

$$\Phi^{(T-1)}(P) = \begin{cases} \{i(T)\} & \text{for } i(T-1) \in i(T) \cup \overline{A}, \\ \{i(T)\}, \{i(T), i(T-1)\} & \text{for } i(T-1) \in A, \\ \{i(T)\}, \{i(T), S\} & \text{for } i(T-1) = A^*, \text{ if } i(T) \text{ is not the lowest cost elementary policy}, \\ \{i(T)\} & \text{for } i(T-1) = A^*, \text{ if } i(T) \text{ is the lowest cost elementary policy}. \end{cases}$$

Where $S$ is of size $M - 1$, and consists of $i(T - 1)$ and $M - 2$ players in $A$.

The player in $A$ that is not in $S$, is the highest cost player in $A$ with a lower cost than $i(T)$. 

We consider the four possible cases used above.

Case (i): $i(T - 1) \in i(T) \cup A$. If $i(T - 1) = i(T)$, that $\Phi^{i(T - 1)}(P) = \{i(T)\}$ follows from theorem 1.ii, since $\{i(T)\}$ is a Condorcet winner in $\Phi^{i(T)}(P)$, and $i(T - 1)$’s preferred policy. On the other hand, whenever $i(T - 1) \in \overline{A}$, $i(T - 1)$ will be excluded from every policy in $\Phi^{i(T)}(P)$. Therefore, the result will again follow from theorem 1.ii, because $\{i(T)\}$ is $i(T - 1)$’s preferred policy in $\Phi^{i(T)}(P)$.

Case (ii): $i(T - 1) \in A$. Since $\{i(T)\}$ is a Condorcet winner in $\Phi^{i(T)}(P)$, we must have $Q_{T-1}(\{i(T)\}) = \{i(T)\}$, and therefore $\{i(T)\} \in \Phi^{i(T - 1)}(P)$. Moreover, since $\{i(T), i(T - 1)\} \in \Phi^{i(T)}(P)$ (e.g. $Q_{T}(\{i(T), i(T - 1), A, A^{*}\}) = \{i(T), i(T - 1)\}$), and it is the preferred policy in $\Phi^{i(T)}(P)$ for $i(T - 1)$, we must have $\{i(T), i(T - 1)\} \in \Phi^{i(T - 1)}(P)$. Now suppose there is a policy $x \in \Phi^{i(T - 1)}(P)$ such that $x \neq \{i(T)\}$, $\{i(T), i(T - 1)\}$. Notice that we must have $i(T) \in x$. Let $y \in P$ be such that $Q_{T-1}(y) = x$. We have two cases to consider.

Case (ii.1): $i(T - 1) \in x$. Let $z \in P$ be such that $Q_{T}(z) = \{i(T), i(T - 1)\}$. Notice that we must have $M \geq |Q_{T}(z)| \geq |x| \geq 3$, because if else $i(T - 1)$’s proposal would not have passed. Now suppose that at the beginning of stage $T - 1$, upon receiving the status quo $y$, $i(T - 1)$ deviates to the proposal $z$. From the above inequalities such a proposal would pass and give a strictly higher payoff to $i(T - 1)$ than does $x$, a contradiction.

Case (ii.2): $i(T - 1) \notin x$. In which case, upon receiving status quo $y$, $i(T - 1)$ could deviate to the proposal $\{i(T)\}$, which would pass and result in the final policy $\{i(T)\}$, that is strictly preferred for $i(T - 1)$ over $x$, a contradiction.

Case (iii): $i(T - 1) = A^{*}$, and $i(T)$ is not the lowest cost elementary policy. That $\{i(T), S\} \in \Phi^{i(T - 1)}(P)$, follows from the fact that $Q_{T}(x) = \{i(T), S\}$ whenever $x$ is the highest cost player in $\overline{A}$ with a lower cost than $i(T)$. On the other hand, to see that there is no policy in $\Phi^{i(T - 1)}(P)$, other than $\{i(T)\}$ and $\{i(T), S\}$, we must consider two cases. Let $x$ be the status quo that $i(T - 1)$ receives.

Case (iii.1): $x$ is not an elementary policy in $\overline{A}$ with a lower cost than $i(T)$. In which case, $i(T - 1)$ will never be included in $Q_{T-1}(x)$. To see this, notice that the total cost of $Q_{T}(x)$ will be lower than the cost of any policy in $\Phi^{i(T)}(P)$ that includes $i(T)$ (as a consequence of $i(T)$’s optimal behavior). Therefore,
no \( T - 1 \) stage proposal, that would result in a final policy including \( i(T - 1) \), would pass. So that, upon receiving \( x \), \( i(T - 1) \)'s best response is to propose a policy that results in \( \{i(T)\} \), which is \( i(T - 1) \)'s preferred attainable policy.

**Case (iii.2):** \( x \) is an elementary policy in \( \mathbb{A} \) with a lower cost than \( i(T) \). From \( i(T) \)'s optimal behavior, \( \{i(T), S\} \) is the preferred policy for \( i(T - 1) \) in \( \Phi^{(T)}(P) \). Moreover, it has a total cost that is no larger than the total cost of \( Q_T(x) \). Therefore, if \( i(T - 1) \) proposes a policy \( y \) such that \( Q_T(y) = \{i(T), S\} \), such a proposal will pass. As a consequence, we must have \( Q_{T-1}(x) = \{i(T), S\} \).

**Case (iv):** \( i(T - 1) = A^* \), and \( i(T) \) is the lowest cost elementary policy. In which case, from \( i(T) \)'s optimal behavior, we will have that \( i(T - 1) \) is excluded from every policy in \( \Phi^{(T)}(P) \). Therefore, the result follows from theorem 1.ii, because \( \{i(T)\} \) is a Condorcet winner in \( \Phi^{(T)}(P) \), as well as \( i(T - 1) \)'s preferred policy in that set.

**Step 2.** We now proceed to prove the theorem. Two cases are considered, according to the identity of \( i(T) \).

**Case (i):** \( i(T) \) is not the lowest cost elementary policy. In which case, according to step 1, the only way to have \( \Phi^{(1)}(P) \neq \{i(T)\} \) is if either:

(i) \( i(T) = i(T - 1) \in \mathbb{A} \) for all \( t < T - 1 \), which occurs in the fraction of recognition orders:

\[
\frac{M - 1}{N} \left(\frac{1}{N}\right)^{T-2},
\]

or (ii) \( i(T - 1) = A^* \), and \( i(T) \in S \) for all \( t < S \), which occurs in the fraction of recognition orders:

\[
\frac{1}{N} \left(\frac{M - 1}{N}\right)^{T-2}.
\]

Therefore, the fraction of orders that produce outcome \( \{i(T)\} \) is at least (for some values of \( \rho_0 \) the fraction will be higher):

\[
1 - \frac{M - 1}{N} \left(\frac{1}{N}\right)^{T-2} - \frac{1}{N} \left(\frac{M - 1}{N}\right)^{T-2}.
\]

**Case (ii):** \( i(T) \) is the lowest cost elementary policy. In which case, according to step 1, the only way to have \( \Phi^{(1)}(P) \neq \{i(T)\} \) is for \( i(T) = i(T - 1) \in \mathbb{A} \).
for all \( t < T - 1 \), which occurs in the fraction of orders:

\[
\frac{M - 1}{N} \left( \frac{1}{N} \right)^{T-2}.
\]

Therefore, the fraction of orders that produce \( \{i(T)\} \) is at least:

\[
1 - \frac{M - 1}{N} \left( \frac{1}{N} \right)^{T-2}.
\]

The proposition follows from combining both cases, and replacing \( M - 1 \) with \( \frac{N-1}{2} \). Q.E.D.

**Proof of Theorem 7: Step 1.** We start with part (i) for the case in which \( |p_0| \leq M \).

We show that \( Q_t(p_0^s) = \{i(T)\} \) for all \( t \), by induction. That the claim is true for \( t = T \) follows from the fact that \( \{i(T)\} \) majority defeats \( p_0^s \). Now suppose the claim is true for \( t + 1 \) but not for \( t \). This implies that, in stage \( t \), when facing status quo \( p_0^s \), player \( i(T) \) proposed a policy \( z \) that passed and such that \( \{i(T)\} \neq Q_{t+1}(z) := Q_t(p_0^s) \supseteq Q_{t+1}(p_0^s) \), because \( i(T) \in Q_{t+1}(z), Q_{t+1}(p_0^s) \). But since \( |Q_{t+1}(z)| \leq M \), we must have that \( i(T) \) together with every player not included in \( Q_{t+1}(z) \) (there are at least \( M - 1 \) such players) voted against the proposal \( z \), a contradiction to the fact that it passed.

**Step 2.** We claim that the total cost of \( Q_t(p_0^s) \) cannot be greater than the total cost of \( Q_T(p_0^s) \).

To show this, we proceed by induction. The claim is trivially true for \( t = T \). Now suppose the claim is true for \( t + 1 \) but not for \( t \). This implies that, in stage \( t \), when facing status quo \( p_0^s \), player \( i(T) \) proposed a policy \( z \) that passed and such that the total cost of \( Q_{t+1}(z) \) was larger than the total cost of \( Q_T(p_0^s) \). But since \( |Q_{t+1}(z)| \leq M \), and \( i(T) \in Q_{t+1}(z), Q_T(p_0^s) \), we must have that \( i(T) \) together with every player not included in \( Q_{t+1}(z) \) (there are at least \( M - 1 \) such players) voted against the proposal \( z \), a contradiction to the fact that it passed.

**Step 3.** We proceed with part (i) for the case in which \( |J| > 2 \).

Step 3.1: \( Q_1(p_0^s) \subset \{i(T - 1), i(T)\} \). Suppose not. From step 1 in the proof of theorem 6, it must be the case that \( i(T - 1) = A^* \). Furthermore, notice that
Lemma 9.1: We begin with two lemmas.

Step 3.2: The claim follows from step 3.1 and theorem 1.ii (that the hypothesis of this theorem is met, follows from the assumption $|J| > 2$).

Step 4. We now prove part (ii).

Step 5. Finally, we prove part (iii).

Proof of Theorem 9: We begin with two lemmas.

Lemma 9.1: For any $p^s_{T-1} \in P^c$ such that $(p^s_{T-1})_{i(T)} \leq 1 - N\epsilon$, there exists a set $S(p^s_{T-1})$ composed of $M - 1$ players with the highest values of $(p^s_{T-1})_i$, other than $i(T)$, such that:

(i) $Q_T(p^s_{T-1})_i = 0$ for all $i \in S(p^s_{T-1})$.
(ii) $Q_T(p^s_{T-1})_i = p^s_{T-1} + \epsilon$ for all $i \notin S(p^s_{T-1})$, $i \neq i(T)$.
(iii) $Q_T(p^s_{T-1})_{i(T)} = (p^s_{T-1})_{i(T)} + \sum_{i \in S(p^s_{T-1})}(p^s_{T-1})_i - (M - 1)\epsilon$. 

$Q_1(p^s_0) \subseteq \{i(T - 1), i(T)\}$ implies that $i(T - 1) \in Q_1(p^s_0)$, and $|Q_1(p^s_0)| = M$. Which in turn implies that the total cost of $Q_1(p^s_0)$ is greater than the total cost of $Q_T(p^s_0)$, a contradiction to step 2.

$\Phi_{k(1)}(P) \neq \{i(T)\}$ is for $i(T) = i(T - 1) \in A$ for all $t < T - 1$, which cannot occur when $|J| > 2$.

Step 5.1: $|Q_t(p^s_0)| \leq |Q_T(p^s_0)|$ for all $t$. We proceed by induction. The claim is trivially true for $t = T$. Now suppose the claim is true for $t + 1$ but not for $t$. This implies that, in stage $t$, when facing status quo $p^s_0$, player $i(T)$ proposed a policy $z$ that passed and such that $|Q_{t+1}(z)| > |Q_T(p^s_0)|$. But since $|Q_{t+1}(z)| \leq M$, and $i(T) \in Q_{t+1}(z)$, $Q_T(p^s_0)$, we must have that $i(T)$ together with every player not included in $Q_{t+1}(z)$ (there are at least $M - 1$ such players) voted against the proposal $z$, a contradiction to the fact that it passed.

Step 5.2: $Q_1(p^s_0) = \{i(T)\}$. If not, from step 5.1 and the fact that $|J| > |Q_T(p^s_0)|$, there is a least one player in $J$, excluded from $Q_1(p^s_0)$, who would find it profitable to deviate to the proposal $\{i(T)\}$. Q.E.D.
Proof of lemma 9.1: Given \((p^s_{T-1})_{i(T)}(p^s_{T-1})_{i(T)} \leq 1 - N\epsilon\), it is easy to see that the optimal strategy for \(i(T)\) is to select a set \(Z\) of \(M - 1\) players with the highest values of \((p^s_{T-1})_{i(T)}\), other than \(i(T)\), and propose \(x\) such that: \(x_i = 0\) for all \(i \in Z\), and \(x_i = (p^s_{T-1})_i + \epsilon\) for all \(i \notin Z\) other than \(i(T)\). Since \(\sum_{i \in Z} x_i \geq M\epsilon\), such a proposal will pass: every player outside \(Z\) -including \(i(T)\)- will vote in favor.

Lemma 9.2: If \(x\) is such that \(x_i = \epsilon\) for \(M - 1\) players other than \(i(T)\), and 
\[x_i(T) = 1 - (M - 1)\epsilon\] 
Then \(Q_t(x) = x\) for all \(t\).

Proof of lemma 9.2: Step 1: The claim is true for \(t = T\). The only way for \(Q_T(x) \neq x\) is to have \(M\) players such that \(Q_T(x)_i > x_i\), but this would imply \(Q_T(x)_{i(T)} < x_{i(T)}\), a contradiction.

Step 2: If claim is true for \(t + 1\), then it is true for \(t\). Suppose not, then only way for \(Q_t(x) \neq x\) is to have \(M\) players such that \(Q_t(x)_i > x_i\), but this would imply \(Q_t(x)_{i(T)} < x_{i(T)}\), \(Q_t(x)_j \geq 2\epsilon\) for a player \(j\) such that \(x_j = \epsilon\), and that \(Q_t(x)_j \geq \epsilon\) for at least \(i\) players other than \(j\) and other than \(i(T)\). But this in turn would imply that \(i(T)\) does not follow an optimal behavior in stage \(T\).

We now prove the theorem, i.e., \(Q_1(p^s_0)_{i(T)} > 1 - N\epsilon\).

Suppose on the contrary that \(Q_1(p^s_0)_{i(T)} < 1 - N\epsilon\). Then -form lemma 16.1(i)-there exists a set \(Z\) of \(M - 1\) players such that \(Q_1(p^s_0)_i < \epsilon\) for all \(i \in Z\), and therefore there exists a \(t\) such that \(i(T) \in Z\).

Case (i): \(Q_{t+1}(p^s_{t-1})_{i(T)} < 1 - N\epsilon\). Suppose \(i(T)\) deviates to the proposal \(x\) such that: (a) \(x_i = \epsilon\) for \(M - 1\) players that would have received less than \(\epsilon\) under \(Q_{t+1}(p^s_{t-1})\) (i.e., if \(i(T)\)’s proposal does not pass), including \(i(T)\), and (b) \(x_{i(T)} = 1 - (M - 1)\epsilon\). If such a proposal passes, it will become the final outcome (from lemma 16.2), hence it will pass and constitute a profitable deviation for \(i(T)\), a contradiction.

Case (ii): \(Q_{t+1}(p^s_{t-1}) > 1 - N\epsilon\). This case cannot arise because, if else, \(i(T)\)’s equilibrium proposal would not have passed. To see this note that \(i(T)\) and every player in \(Z\) would have voted against such equilibrium proposal. Q.E.D.
References


