We consider repeated two party elections in which voters have monotone preferences over a two dimensional policy space and each party is a high demander of a different issue. In each period the government faces a feasibility constraint which is not observed by the public. This monitoring problem makes it impossible for voters to determine whether the government is selecting an appropriate policy given the constraint or shirking in its policy choice. Pure strategy equilibria with the maximal possible amount of control exhibit the following features: (i) parties that are favored (not favored) by the current constraint are reelected (not reelected), (ii) the identity of the government varies with no party remaining in office indefinitely, (iii) the distributions of policies enacted by each party are distinct and biased in the direction of their preferences, (iv) reelection sometimes follows the enactment of policies that have a very high quantity of a party’s desired coordinate and (v) voters treat the parties differently (i.e., the set of polices that will result in reelection differ across parties). We find that voters prefer these equilibria to ones that select the best ex-ante party. In mixed strategies full control is possible.

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1 Introduction

In February 2002, newly elected New York City Mayor Michael Bloomberg faced a public challenge. Financial strain on city resources and a faltering economy exposed the city to a dramatic budgetary problem. Aside from the policy challenge of managing a city under tight conditions Bloomberg faced a difficult but subtle political problem in choosing a budget. Voters cannot fully comprehend the complex trade-offs associated with running a city under changing economic conditions. Accordingly, a moderate or centrist voter has a hard time discerning if Bloomberg’s policy represents a desirable compromise given the complex trade-offs, or a policy that inappropriately favors Bloomberg’s agenda. The former would merit reelection while the latter would not. In response Bloomberg offered what the New York Times labeled “a budget that hurts everyone” (Cooper 2002). The typical voter may not be able to discern if they were hurt more than necessary as a result of ideological shirking or whether the mayor acted in the voters’ interest under difficult constraints.

This description of executive/party politics and the associated informational asymmetry is distant if not disjoint from the theoretical literature on elections. We present a theory of repeated two party elections involving this type of monitoring problem. The structure of the model is distinct from extant theories in two important ways. First, agents have monotone preferences over two dimensions of services or goods that the government can provide (a classic example is defense spending-guns and social spending-butter, a more timely example is privacy and security)—voters want more of everything and each party cares most about a different issue. Second, the government faces a feasibility constraint. The party in office knows the constraint, but the public and out party do not. In contrast to existing formal studies of elections, which often start with induced preferences (often Euclidean) over policy or treat the government as choosing a level of costly effort, preference divergence is over the willingness to substitute between issues that are viewed as goods. Here, induced preferences over policies and information asymmetries are the product of an explicitly modeled feasibility constraint. This approach represents an alternative starting point for models of politics. Elections serve as a process for aggregating primitive preferences over goods (like security and privacy) into policy decisions that satisfy certain feasibility requirements. The analysis leads to five conclusions.

- Using non random voting rules, the public can at best partially control the government.
- With mixed strategies, in the form of a probabilistic reelection function, perfect control is possible as the government is made to internalize the public’s preferences.
- Voters will treat the parties differently—the set of policies that result in reelection for one party is not the same as the corresponding set for the other party,
- Adoption of policies that are very desirable to the in party does not necessarily result in removal, as some such polices can be justified as optimal for the voters given some constraint.
• With a monitoring problem of this form, the best pure strategy equilibria do not involve always electing and retaining the ex-ante best party.

The first two points are descriptions of the equilibrium analysis. The pure strategy equilibria speak to the types of speeches that may credibly be made by incumbents when justifying their actions to the constituency (Fenno 1978). From this perspective trust in the government is a phenomena partially dependent on uncontrollable features of the political landscape (specifically stochastic constraints) and partially dependent on the choices made by the party in office. The sharp difference between mixed and pure strategy results demonstrates that control becomes easier when the set of actions available to the public is enlarged. The third and fourth points describe the pure strategy equilibria and speak to the types of voting behavior we might observe (in say U.S. presidential or gubernatorial contests). The model offers an explanation for why voters treat the parties differently. Policies (or more generally outcomes) that are considered acceptable if implemented by one party may be viewed as unacceptable when the other party is in power.

The last point speaks directly to a broad question about the relative importance of accountability and representation: Do elections serve as a means to control governmental actions or select the best available governments? Fearon (1999) presents a clear review of the principal agent based formal literature on accountability and forwards a general conclusion. He argues that viewing elections as a means to select high quality candidates is better than viewing elections as a means to induce accountability by sanctioning poor performance. Fearon defends this claim by describing conditions under which the selection explanation is more plausible than the control explanation,

(i) repeated elections do not work well as a mechanism of accountability, because [voters] believe that their ability to observe what politicians do and to interpret whether it is in the public interest is so negligible; and (ii) there actually is relevant variation in the types of candidates for political office, and these can be distinguished to some extent,.... (p. 68)

While this general conjecture is intuitively reasonable and consistent with existing theories, it needs further investigation. In potential contrast, Besley and Case (1995a, 1995b) find evidence consistent with the sanctioning hypothesis: (1) voters reward governors that outperform those of neighboring states and (2) term limited governors tend to shirk in fiscal policy making relative to non limited governors. The tension between Fearon’s conclusion and these empirical findings as well as the fact that existing theories of elections do not adequately capture the two conditions that Fearon highlights motivate analysis of a different type of model. We capture the monitoring problem through hidden knowledge – the in government party observes the feasibility constraint prior to policy selection and the public and out party never observe this information. Variation in candidates is captured by two identifiable infinitely lived parties that each desire to maximize a different policy issue. Contrary to the selection explanation we find that in the public’s most
preferred equilibrium parties of quite different types (with one ex-ante preferred to the other) are controlled
in a manner that makes voters unconcerned with the selection problem.\textsuperscript{2}

The non-standard view of policy preferences and constraints is motivated by work in political behavior. With
overwhelming regularity scholars of public opinion report that voters “want to have their cake and eat it too” (Zaller 1998). Voters want lower taxes and more social spending without increasing the deficit; voters in the 1960’s and 70’s wanted the U.S. to stand strong against Communist expansion in Vietnam but they did not want to see American casualties; voters in the wake of the World Trade Center and Pentagon attacks wanted increased airline security but they did not want increased airline prices or excessive delays. Second, trade-offs are clearly required as policy makers face feasibility constraints.\textsuperscript{3} Increased spending requires funding from somewhere; a strong military stand often means the loss of American lives; increased airline security requires costlier security measures and the resulting inconvenience. A third feature of the political landscape that we include is uncertainty on the part of voters about what policies are actually possible in any period. These features suggest that a standard assumption of spatial models, that voters have interior bliss points in the relevant region of the policy space, may be inappropriate—missing the interaction between required tradeoffs and informational asymmetries about the nature of these tradeoffs. The model involves a two dimensional policy space. In each of an infinite number of periods the government receives a constraint set or policy technology which is characterized by a relative price and a resource level. The government then selects a feasible policy. Voters observe the policy but not the constraint and decide whether to keep the incumbent party in office or elect the challenging party.

We consider both pure and mixed strategy stationary perfect Bayesian equilibria because it is interesting
to note when control by voters requires randomization in voting decisions. The monitoring problem makes perfect control impossible in pure strategies. In the case where there is no uncertainty about the resource level of the constraint, at best the public can adopt a strategy of retrospective voting, reelecting the government only if it is possible to discern that the chosen policy was optimal for the public.\textsuperscript{4} It is

\textsuperscript{2}Standard models in agency theory emphasize the concepts of moral hazard and/or adverse selection. Strictly speaking the current model does not involve adverse selection or moral hazard. Instead the model involves hidden knowledge which limits the set of control schemes available to the public. The extent to which one party may have preferences that are more aligned with the voters’ than the other suggests that one agent might be ex-ante more desirable and thus the phrase ”solving the selection problem” has a natural meaning. Similarly, while the choice variable of the parties is not simply effort, the extent to which incentives might induce the party to act as if she were trying to maximize the public’s utility renders the term ”control” meaningful. Accordingly, we will use the terms control and selection as adjectives describing equilibria. These concepts have similarities with the terms moral hazard and adverse selection which tend to describe models.

\textsuperscript{3}Even if voters are unaware or unconcerned with feasibility, the government faces limitations on the policy bundles it can offer. The model assumes that voters are aware that constraints exist, but that they are poorly informed about the exact nature of these constraints at any given time.

\textsuperscript{4}Our usage of the phrase retrospective voting, differs from that of Fiorina (1981). By retrospective voting we mean behavior in which voting decisions over tomorrow’s government are based only on the policy choice of today’s government. In contrast to typical applications of the concept, we do not require that retrospective voting involve a utility cut-off rule or aspiration
convenient to describe constraints as relatively desirable and undesirable to a party based on the relative price of the issue that the party values. In equilibrium a government that attains a relatively desirable constraint will be successfully controlled. It selects the public’s optimal policy. This is true because given a desirable constraint any policy which the government prefers to the public’s constrained optimum is either infeasible or not optimal for the public under any constraint. Conversely, when the constraint is not desirable to the government, it is impossible for the government to select a policy that will result in reelection. This is true because given an undesirable constraint any policy choice that is optimal for the public can be replaced by a feasible policy which is preferred by the government and optimal for the public under some other constraint. Thus, following a desirable (undesirable) constraint the government selects a policy that results in reelection (loss of office). As a function of the quantity of a party’s preferred issue enacted, reelection occurs for very high levels and moderate levels that also involve a substantial quantity of the other issue.

If there is uncertainty about both the relative price of each policy issue and the overall resource level then there are no pure strategy equilibria in which the government’s actions are controlled with positive probability. This finding is striking but quite intuitive. In pure strategies the only control open to the public is a threat of not reelecting a government that is known to have shirked. With uncertainty about the resource level and relative price, for almost every constraint, it is the case that the public’s optimal policy is less desirable to the government than some feasible policy that is optimal for the public under some other constraint. Accordingly, the public can almost never discern if the government has shirked. In this case, following even a policy that is optimal for the public given the constraint, the incumbent would be incapable of convincing the public that she did not shirk. There is too much uncertainty for the public to trust the government.

In contrast, as long as the value of office is not too low, mixed stationary perfect Bayesian equilibria involving perfect control exist. In these equilibria the public selects policy contingent reelection probabilities that cause the government to internalize the public’s preferences. In an equilibrium of this form the probability of reelection is decreasing in the magnitude of the policy coordinate of the government’s favored issue. Mixed strategy equilibria of this form represent first best control for the voters as the government selects the voters constrained optimal policy in every period. The finding, that perfect control is possible in mixed strategies but not in pure strategies illustrates an important general property of agency problems. The richer is the set of actions available to the principal the more likely it is that control is possible. Allowing for mixed strategies dramatically enlarges the space of possible mechanisms available to the public. Thus, when the importance of office is high enough to make the government care about small changes in the probability of reelection, control is possible. Contrasting the pure and mixed strategy results provides an additional insight about the model. In pure strategies perfect control is infeasible because the voter cannot level, just that voting strategies are dependent only on the last observed policy.
discern when to punish (use the stick) and when to reward (use the carrot). The problem is not related to the sizes of the carrot and stick. In mixed strategies it is possible to create the right incentives without discerning if the government has shirked as long as the stick and carrot are big enough.

Existing principal agent models of elections focus on the problem of creating incentives for the government to undertake the optimal amount of costly but publicly desirable effort. Barro (1973), Austen-Smith and Banks (1989), and Ferejohn (1986) consider the moral hazard aspects of this problem, and Banks and Sundaram (1993, 1998) and Ashworth (2001) consider the problem of both moral hazard and adverse selection. While focused on the control of shirking the current paper is conceptually quite distinct. Whereas the above moral hazard models deal with the creation of incentives for the government to not shirk in its effort choice (a variable upon which the principal and agent have diametrically opposed preferences), we deal with the creation of incentives for the government to not shirk in its policy selection (a variable upon which preferences may be aligned but not identical). Aside from this critical departure, the current model is similar to the two party model that Ferejohn considers. Both involve informational asymmetries, an infinite horizon, and a pool of two long lived parties. In both models voting serves to constrain governmental action and the voter must be indifferent between having either party in office. The equilibrium results and intuition differ in most other respects.

Banks and Duggan (2001) analyze a repeated election citizen candidate model in which preelection commitment is not possible. Their model involves a large population of ex ante indistinguishable candidates, and uncertainty only about the preferences of candidates. Banks and Duggan more directly bridge the gap between social choice theory and the restrictive Downsian/Hotelling world while the current model more directly addresses representation and accountability in two party elections with complex or changing political environments. While Banks and Duggan predict convergence to a particular government and policy for reasonable parameterizations we predict stochastic oscillation between the two parties and non-convergence of policy. Caines-Wrone, Heron and Shotts (2001) present a two period model which explains shirking, in the sense of policy choice that differs from the public optimum, even when preferences are perfectly aligned. The explanation hinges on the executive’s incentive to convince the voter that it is competent and should thus be retained. Given the preference alignment the question is not why do candidates act on the behalf of the voter. Rather the puzzle is why do they sometimes shirk. In contrast, the presence of non-shirking behavior needs an explanation in the current paper as the incumbent has an ideological/preference motivation to not select policies desirable to the electorate. Rogoff and Sibert (1988) analyze a dynamic model of macroeconomic policy in which the government has temporarily private information about its fitness and find that this short-term informational asymmetry can generate political business cycles. While there are similarities in terms of the number of agents and the sequence of play, in Rogoff and Sibert the

\[^{5}\text{See also Duggan's (2000) unidimensional model.}\]

\[^{6}\text{We do not necessarily predict that each party is in office with equal probability.}\]
uncertainty pertains to the competency of the government, and policy is unidimensional—the provision of a good. These differences result in a starkly different equilibrium intuition.

In section 2 we begin by formulating a model involving just uncertainty about the relative trade-offs between each policy coordinate. After describing the model and equilibrium concepts we show that perfect monitoring is impossible in pure strategies. We then characterize the pure strategy equilibria that exhibit the maximal amount of control possible. In section 3 we consider the case where there is uncertainty about both the relative price of each issue and the total amount of resources available. We show that no degree of control is possible in pure strategy equilibria. We then establish the existence of mixed strategy equilibria exhibiting perfect control if the value to office is sufficiently high for the parties. Section 4 raises a few natural extensions to the basic model and discusses the robustness of the findings to these variations. In section 5 we conclude with a discussion.

2 The basic model

We consider a model that is quite distinct from existing theories of representation and accountability. To make clear the incentives we focus on two parties and a representative voter, each infinitely lived and concerned with discounted streams of per period payoffs. The policy space is 2-dimensional, with parties each seeking to maximize one of the two issues. Voters have well behaved preferences in which each issue is a "good". Feasibility constraints are represented by linear constraints, and information asymmetry about the constraint is captured by assuming that initially the slope (and then eventually the resource level also) are observed only by the in-government party. Since the analysis can rely on spatial intuition as opposed to algebraic manipulation we avoid specifying specific functional forms for utility functions and stochastic distributions. Instead a few key assumptions impose the relevant structure on the problem. Since the structure of the model is distinct from existing agency and election theories in several places we have foregone generalizations which do not alter the qualitative properties of the results but do complicate the exposition/notation. (For example there are ways to: allow parties to care about both issues; relax the assumption that the constraint surface is always linear; consider an arbitrary (odd) population of voters. These (and other) points are taken up in section 4.)

2.1 Players and preferences

We consider a sequence of elections involving three players. A set of two parties $P = \{l, r\}$ compete for office in each period, and the representative voter, $m$, selects between the parties. Parties are sometimes denoted as $p$ and $-p$. We consider only the case of a single voter to emphasize the difficulties of controlling parties. The addition of more voters/principals in a multidimensional model would only introduce additional
complexity by muddling the concept of the public’s preference. The policy space is \( X = \mathbb{R}_+^2 \), the positive orthant of two dimensional Euclidean space. We use bold letters to denote a policy which is a vector, and non bold typeface with subscripts 1 or 2 to denote the coordinates of a policy. Thus \( \mathbf{x} = (x_1, x_2) \). We assume that each party cares about the policy enacted and values holding office. If policy \( \mathbf{x}^t \) is chosen in period \( t \) and party \( p \) is in office during period \( t \) the period \( t \) payoff is

\[
u_p(\mathbf{x}^t) + \eta_p.
\]

The party specific term \( \eta_p \geq 0 \) measures the non-policy rents associated with holding office. In contrast if party \( p \) is not in office but policy \( \mathbf{x}^t \) is chosen the \( t \) period payoff is \( \nu_p(\mathbf{x}^t) \). We assume that the policy specific utility function \( \nu_p(\mathbf{x}) : X \rightarrow \mathbb{R} \) is twice differentiable.

Voter \( m \) cares only about policy and has a twice differentiable utility function \( \nu_m(\mathbf{x}) \). Players \( l, r, m \) are assumed to have globally non-satiated preferences. We are interested in the case where party \( l \) is a high demander of dimension 2, party \( r \) is a high demander of dimension 1 and \( m \) likes more of each dimension. This approach represents a departure from standard models of politics which usually assume that agents have ideal points. These ”bliss-point” or spatial preferences are generally motivated as stemming from required trade-offs and traditional economic (non-satiated) preferences over goods (like consumption, safety, privacy, ...). In standard models both of these features are unmodeled and the analysis starts with spatial preferences over policy. Here, the feasibility constraint is explicitly modeled, so the right starting point is preferences over primitive issues (like privacy and safety). Endogenous to the model is how policy should and will balance competing desires.

Formally, we define the marginal rate of substitution at \( \mathbf{x} \) for player \( i \) as

\[
MRS_i(\mathbf{x}) = \frac{\partial u_i(\mathbf{x})}{\partial x_1} / \frac{\partial u_i(\mathbf{x})}{\partial x_2}
\]

and assume that for any \( \mathbf{x} \in X \) \( MRS_l(\mathbf{x}) < MRS_m(\mathbf{x}) < MRS_r(\mathbf{x}) \). For simplicity we take the extreme case where \( MRS_l(\mathbf{x}) = 0 \) and \( MRS_r(\mathbf{x}) = \infty \) for every \( \mathbf{x} \in X \). This holds when \( u_l(\mathbf{x}) = h_l(x_2) \) and \( u_r(\mathbf{x}) = h_r(x_1) \) with \( h_l(\cdot) \) and \( h_r(\cdot) \) strictly increasing functions. We assume that the voter, \( m \), has strictly convex preferences. Figure 1 depicts a generic representation of policy specific indifference curves for the three agents.

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7See section 4 for a discussion of how the model can be generalized to multiple voters. With an odd number of voters that are ordered by their marginal rates of substitution a representative voter will exist.

8In section 4 we discuss how this assumption can be relaxed.
We consider an infinite sequence of elections. In period \( t \) nature, a non strategic player, randomly selects a constraint set \( B^t \subset X \) and the government, \( g^t \in P \) after observing \( B^t \), selects a policy point \( x^t \in B^t \). The voter knowing \( x^t \) but not \( B^t \) then casts a ballot \( v^t \in \{0,1\} \) where a vote of 1 is a vote to keep the incumbent and a vote of 0 is a vote to replace the incumbent with party \( P \cup g^t \). In period \( t+1 \) a new constraint \( B^{t+1} \) is realized, a new policy \( x^{t+1} \in B^{t+1} \) is selected by \( g^{t+1} \) and a new election occurs. Without loss of generality we assume that period 1 involves selection of \( x^1 \in B^1 \) by government \( g^1 = l \). This game form necessitates that we extend the policy utility functions \( u_B \) is isomorphic to \( B \) constraint on the parameters we sometimes denote a constraint by \( \mu \). Similarly, the voter's utility over such a sequence is

\[
U_p(\{x^t, g^t\}) = (1 - \delta)^{\sum_{t=1}^{\infty} \delta^{t-1} [u_p(x^t)] + \eta_p 1_p(g^t)}.
\]

where \( \delta \in (0,1) \) is a common discount rate\(^9\) and \( 1_p(g^t) \) is an indicator taking the value 1 if \( g^t = p \) and 0 otherwise. Similarly, the voter's utility over such a sequence is

\[
U_m(\{x^t, g^t\}) = (1 - \delta)^{\sum_{t=1}^{\infty} \delta^{t-1} u_m(x^t)}.
\]

We consider an infinite sequence of elections. In period \( t \) nature, a non strategic player, randomly selects a constraint set \( B^t \subset X \) and the government, \( g^t \in P \) after observing \( B^t \), selects a policy point \( x^t \in B^t \). The voter knowing \( x^t \) but not \( B^t \) then casts a ballot \( v^t \in \{0,1\} \) where a vote of 1 is a vote to keep the incumbent and a vote of 0 is a vote to replace the incumbent with party \( P \cup g^t \). In period \( t+1 \) a new constraint \( B^{t+1} \) is realized, a new policy \( x^{t+1} \in B^{t+1} \) is selected by \( g^{t+1} \) and a new election occurs. Without loss of generality we assume that period 1 involves selection of \( x^1 \in B^1 \) by government \( g^1 = l \). This game form necessitates that we extend the policy utility functions \( u_B \) is isomorphic to \( B \) constraint on the parameters we sometimes denote a constraint by \( \mu \). Similarly, the voter's utility over such a sequence is

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\[
U_m(\{x^t, g^t\}) = (1 - \delta)^{\sum_{t=1}^{\infty} \delta^{t-1} u_m(x^t)}.
\]

[Figure 2 about here]

We assume that any feasible constraint is of the form \( B^t = \{ x \in X : b^t x_1 + (1 - b^t) x_2 \leq 1 \} \) where \( b^t \in [\gamma, 1 - \gamma] \). In other words a constraint is given by a relative price \( b^t \). The constant \( 0 < \gamma < \frac{1}{2} \) is a measure of the spread of the support on the random relative price. To make clear the dependence of the constraint on the parameters we sometimes denote a constraint by \( B(b) \). The set of possible constraints is isomorphic to \( \mathbb{B} = [\gamma, 1 - \gamma] \). Figure 2 depicts the set of policies which are feasible for some constraint, \( \beta = \cup_{b \in [\gamma, 1 - \gamma]} B(b) \). By \( \hat{\beta} \) we denote the efficient boundary of \( \beta \). This is the set of points that satisfy \( B(b) \) with equality for some \( b \in [\gamma, 1 - \gamma] \). In section 3 we introduce uncertainty not just about the relative price but also about the total size of the pie. In this case the constraint is \( \{ x \in X : b^t x_1 + (1 - b^t) x_2 \leq c^t \} \) and both \( b^t \) and \( c^t \) are treated as random variables.\(^10\)

We assume that the common belief is that for every \( t \) the parameter \( b^t \) is given by an independent draw from the continuous and strictly increasing distribution function \( F_b(\cdot) \) on support \( [\gamma, 1 - \gamma] \).\(^11\) The parties \( l \) and \( r \) only observe the value \( b^t \) if they are in office at period \( t \). The parameter \( b^t \) is never revealed to players other than \( g^t \). The game, thus, involves hidden knowledge about the period state variable \( b^t \).

\(^9\)The assumption of common discount rates is made purely to simplify the notation.

\(^10\)In section 4 we discuss convex constraints with non linear boundaries.

\(^11\)The assumption that \( b \) is generated by iid draws is unnecessary. In section 4 we discuss the extension to non independent \( b^t \)’s.
We introduce a convenient partial ordering on $X$. We say $x \lessdot y$ if $x_2 > y_2$ and $x_1 < y_1$. Intuitively $x \lessdot y$ means that $x$ is to the northwest of $y$. For any constraint $B(b)$ compactness and convexity of the constraint and continuity and strict convexity of the preferences insure that the set of induced ideal policies for agent $i$

$$x_i^*(b) = \arg \max_{x \in B(b)} u_i(x)$$

exist and are singletons. Moreover, by the theorem of the maximum (Berge 1963) this function is continuous. We impose two additional assumptions on the preferences of $m$.

**Assumption 1:** For some $b^* \in (\gamma, 1 - \gamma)$

$$x_m^*(b^*) = (1, 1).$$

This condition states that for some feasible $b^*$, $m$’s optimal policy subject to the constraint $B(b^*)$ corresponds to the point $(1, 1)$. Note that this is the unique point that lies on the boundary of every feasible constraint. This assumption is satisfied if

$$\frac{\gamma}{1 - \gamma} < MRS_m((1, 1)) < \frac{1 - \gamma}{\gamma}. \quad (7)$$

**Assumption 2:** If $b < b'$ then $x_m^*(b') \lessdot x_m^*(b)$.

This assumption requires that $m$ respond to increases in the relative price of issue 1 by selecting less of issue 1 and more of issue 2. Jointly assumptions 1 and 2 ensure that $m$ has well behaved preferences.

### 2.2 Interpretations

One stylized interpretation of the model is to think about issue 1 as defense spending and issue 2 as welfare or redistributive policies. In this interpretation $c$ represents the available revenue (from taxing and deficit spending). Party $l$ is then the Democrat party and $r$ is the Republican party. An alternative interpretation is closer in spirit to the public finance literature. Define $x_1 = 1 - \tau$ were $\tau$ is the tax rate, and let $x_2$ denote the amount of government redistribution. The constraint is $b(1 - \tau) + (1 - b)x_2 \leq c$, and the production function on redistribution is $x_2 = \frac{c - b(1 - \tau)}{(1 - b)}$ which is stochastic with random parameters $b$ and $c$. Party $l$ seeks the maximization of welfare spending, $x_2$, and party $r$ seeks the minimization of taxation. The representative voter seeks to balance the marginal cost and benefit of redistribution when she has strictly convex preferences over $(1 - \tau)$ and $x_2$. The model may also be applicable to areas of regulatory politics in which the executive or congress can select from a set of feasible agencies in defining discretion. Additionally, it is possible that within agency decision making, may also be described in this manner as a principal chooses between different departments each staffed with agents that have certain policy biases.

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12 In this case, one might interpret $\frac{b}{1 - \tau}$ as the relative price of missile defense systems in terms of subsidized health insurance.
2.3 Strategies and equilibria

We focus on stationary perfect Bayesian equilibria (SPBE). A SPBE consists of a government policy function \( \psi_p(b) : [\gamma, 1 - \gamma] \rightarrow B(b) \) for each \( p \in P \), a ballot function \( v(x, g) : \beta \times P \rightarrow \{0, 1\} \) for the voter, \( m \), and a voter belief \( \pi(b | x, g) \) about the constraint faced by the government conditional on the policy chosen and the government identity. This belief mapping is a distribution function on \( B \) conditional on a policy \( x \in \beta \) and identity \( p \in P \). The policy function of \( p \) needs to be optimal given the ballot function and the policy function of \(-p\), the ballot function needs to be sequentially rational relative to the belief mapping and the policy functions, and the belief mapping needs to satisfy Bayes’ rule when it is defined. The assumption of stationarity is satisfied by this description as we have required strategies to hinge only on the state variable \((x^t, g^t)\).

The assumption of stationarity is tenable as voting or policy selection strategies that hinge on a long history of past elections seem peculiar when these past elections provide no payoff relevant information. Additionally, we will see that relaxing the stationarity assumption does not increase the amount of control that is possible, although it does enlarge the set of parameterizations for which partial control is possible in pure strategies. Since this is a model of incomplete information we focus on perfect Bayesian equilibria to ensure that the equilibria do not hinge on unreasonable voter beliefs. In this game the only relevant uncertainty is faced by the voter, \( m \), when she must decide whether to retain or remove the incumbent. Accordingly, SPBE require that beliefs about \( b^t \) be consistent with the observation \( x^t \) and the strategy \( \psi_g(\cdot) \). Unlike many models of imperfect information the beliefs are not very important to the analysis. Under a fixed profile of stationary strategies \( \psi_l(b), \psi_r(b) \) the voter’s preference for retaining or removing \( g \) is not dependent on \( b^t \). Accordingly, the extent to which a ballot strategy \( v(x, g) \) is sequentially rational does not depend on the beliefs. Intuitively, sequential rationality would constrain prospective behavior (in this case voting), but the information observable to \( m \) is of no value in predicting future play under a given pair of stationary policy functions. Given this observation we suppress the beliefs from subsequent statements of and arguments about equilibria. We characterize strategy profiles as supportable as SPBE when there exists a belief mapping for which the strategy profile and belief would constitute a SPBE. The relevant constraint that sequential rationality imposes on the ballot function \( v(x, g) \) is simple.

**Condition 1** Given \( \psi_l(b), \psi_r(b) \), the mapping \( v(x, g) \) is sequentially rational iff

\[
\int u_m(\psi_g(b))dF_b(b) > (\) \int u_m(\psi_{-g}(b))dF_b(b)
\]

implies \( v(x, g) = 1(0) \).

Given this condition in any SPBE in which one party is not always in office we must have
\[
\int u_m(\psi(b))dF_b(b) = \int u_m(\psi_r(b))dF_b(b).
\] 

(8)

This means that in any non-trivial equilibrium, the ballot function will not be prospective in nature. In fact given the desire of \( m \) to create incentives for \( \psi_g(b) \) to be close to \( x^*_m(b') \) it is natural to think about the game as a mechanism design problem, where \( m \) selects a ballot function satisfying condition 1 to maximize the left and right hand side of (8) and the policy mappings \( \psi_p(\cdot) \) are mutual best responses to the ballot function. Our analysis will serve to characterizes the relevant incentive compatibility constraints on ballot functions.

**Definition 1** We say a SPBE exhibits **perfect control** if for every \( p \in P \), \( \psi_p(b) = x^*_m(b) \) for almost every \( b \). We say a SPBE exhibits **partial control** if for every \( p \in P \), \( \psi_p(b) = x^*_m(b) \) for \( b \in D \) with \( D \) a subset of \([\gamma, 1 - \gamma]\) having positive Lebesgue measure.

Intuitively, under SPBE with perfect control the in government party (almost) always adopts the voter’s most preferred feasible policy. In an SPBE exhibiting partial control, the in government party sometimes adopts the voter’s most preferred feasible policy and sometimes it shirks adopting its most preferred feasible policy. Given a pure strategy ballot function the set of polices which will result in an incumbent victory is given by the acceptance sets

\[
A_l = \{ x : v(x, l) = 1 \} \\
A_r = \{ x : v(x, r) = 1 \}.
\]

(9)

The compliments of these sets result in loss of office. By choosing the acceptance sets \( m \) influences the policies that governments will enact. If she makes \( A_p \) to small or restrictive then when party \( p \) is in office it will select a policy to maximize its current utility over policy and not retain office next period. Conversely, if \( A_p \) is too large or unrestrictive when party \( p \) is in office it will be able to retain office while selecting a policy that is far from the voter’s optimum (subject to \( B(b') \)). Agent \( m \) would like to select \( A_p \) so that when party \( p \) is in office she selects \( x^*_m(b') \), the voter’s most preferred policy in \( B(b') \). Perfect control by the voter is difficult because she never learns \( b' \) and thus may not be able to discern if a chosen (and observed) policy was indeed in her best interest.

### 2.4 The impossibility of perfect control in pure strategies

We now show that in pure strategies there are no SPBE in which the voter perfectly controls the candidates. This finding is of more than just technical interest. The finding indicates that the monitoring problem
faced by the public is severe and insolvable with simple voting strategies that either retain or remove the in
government party deterministically. Here, the institutional controls open to the public (a binary action) are
insufficient to create the right incentives for the government. The problem that the voter faces is that when
the price $b$ is low (high) party $l$ ($r$) can choose an inefficient policy $(x_1b + (1 - b)x_2 < 1)$ which is optimal
for $m$ under some other higher (lower) price $b' > (<)b$. This inefficient policy will be more desirable to the
party than $m$’s most preferred policy given $b$. If $m$ conjectures that parties are always choosing $x_m^*(b)$ she
will not be able to discern a deviation of the form just described since she does not know $b$.

Given any constraint $B(b)$ the incumbent party $p \in P$ must decide whether to select a policy that results
in reelection if such a policy is feasible (i.e., $A_p \cap B(b) \neq \emptyset$). On purely policy grounds $p$’s most preferred feasible policy is given by

$$x_p^*(b) = \begin{cases} 
(0, \frac{1}{b}) & \text{if } b < b' \\
(\frac{1}{1 - b}, 0) & \text{otherwise}
\end{cases}$$

(10)

Alternatively, given $A_p$ and the constraint $B(b)$ the optimal policy that will result in reelection is given by

$$x_w^p(b) = \arg \max_{x \in B(b) \cap A_p} u_p(x).$$

(11)

Throughout, the ballot functions we consider will yield $B(b, c) \cap A_p$ sets that are singletons or empty.
Accordingly, the set $x_w^p(b, c)$ is either a singleton or empty.

Figure 3 exhibits the intuition. In any SPBE with perfect control in which party $p$ is elected with positive
probability, for almost every $b$ it must be the case that $x_w^p(b) = x_m^*(b)$. By assumption 1 there exists a $b^*$ s.t.
$x_m^*(b^*) = (1, 1)$. We will repeatedly refer to this special slope $b^*$. Now consider $b^* < b < b'$. Prices $b$ and
$b'$ both induce a constraint with a boundary that is steeper than the boundary of $B(b^*)$. By assumption 2,
$x_m^*(b') \setminus x_m^*(b)$. This and $b < b'$ imply that $x_m^*(b') \in B(b)$ and $u_l(x_m^*(b')) > u_l(x_m^*(b))$. Accordingly party $l$
facing a constraint $b$ will strictly prefer to enact policy $x_m^*(b')$ which is feasible under constraint $B(b)$. The
following proposition states the conclusion we have just demonstrated.

**Proposition 1** In pure strategies there are no SPBE that exhibit perfect control.

The problem that prevents us from constructing SPBE with perfect control is slightly peculiar. When
$m$ observes $l$, a high demander of $x_2$, choose a low value of $x_2$ she cannot tell if the government shirked. In
contrast when $l$ chooses high values of $x_2$ the voter can be certain that no shirking occurred. This is true
because when \( b < b^* \) a deviation from \( x_m^*(b) \) that increases \( x_2 \) cannot be in the constraint set \( B(b) \). Thus, if \( m \) tried to monitor the parties she would fail, not when the enacted policies involved high quantities of the government’s preferred coordinate, but the monitoring would fail when the enacted policies involved low quantities of the government’s preferred coordinate and very low quantities of the other coordinate. The intuition being, when the constraint favors party \( r \), party \( l \) has an incentive to select an inefficient policy that makes it look like the constraint favors party \( l \) by a little less. There are always regions of the set \( B \) where party \( l \) can get away with this. It should be noted that even in non stationary strategies perfect control is not possible. The barrier to control is not the size of the stick and carrot, but rather the inability of the voter to determine when to use the stick and when to use the carrot.

Why does proposition 1 require the condition \textit{in pure strategies}? If following \( x_t \), player \( m \) retains \( l \) with probability \( \lambda(x, l) \) and this function is decreasing in the first coordinate of \( x \) it might be possible for \( l \) to prefer selection of \( x_m^*(b) \). In a subsequent section we explore this possibility and derive a mixed strategy SPBE in which the voter’s mixed ballot function creates the right incentives for the in government party. The existence of such an equilibrium hinges on the slopes of the party utility functions (over policy) not being too steep relative to the term \( \delta \eta_p \).

### 2.5 Imperfect control in pure strategies

While perfect control cannot occur in a pure strategy SPBE, sometimes there are pure strategy SPBE in which the voter can exert partial control on the parties. To develop the basic intuition we first consider cases where there is a fair amount of symmetry making it easy to satisfy condition 1.

\textbf{Definition 2} We say \textit{symmetry} is satisfied if the voter is indifferent between the following two lotteries

\[
\psi_l(b) = \begin{cases} 
 x_m^*(b) & \text{if } b \geq b^* \\
 x_l^* \text{ otherwise} 
\end{cases} \\
\psi_r(b) = \begin{cases} 
 x_m^*(b) & \text{if } b \leq b^* \\
 x_r^* \text{ otherwise} 
\end{cases}
\]

Recall that since \( b \) is a random variable in each period these policy mappings induce lotteries over policy. An example satisfying symmetry involves \( F_b(\cdot) \) uniform and \( u_m(x) = x_1x_2 \). This is not the only parameterization that satisfies the condition. Symmetry is a joint restriction on the distribution \( F_b(\cdot) \) and the voter’s preferences \( u_m(\cdot) \). When symmetry is satisfied we can characterize strategy profiles that are supportable as SPBE in which each party selects the voter’s constrained optimum when \( b \) is on its desirable side of \( b^* \) and it selects the party constrained optimum when \( b \) is on its undesirable side of \( b^* \). The construction uses the fact that when \((1,1) \setminus x^t \) and \( x^t \in \hat{\beta} \) the public can trust that \( r \) has not shirked and when \( x^t \setminus (1,1) \) and and \( x^t \in \hat{\beta} \) the public can trust that \( l \) has not shirked. In the converse cases (with
$x^t \in \tilde{\beta}$ it is not possible to infer that the parties are not shirking. By $x_m^{-1}(x)$ we denote the inverse of $x_m(b)$. Thus $x_m^{-1}(x) = \{b : x_m(b) = x\}$. In addition to symmetry two additional conditions are needed for pure strategy SPBE with partial control.

**Proposition 2** If symmetry is satisfied the following profile is supportable as a SPBE with partial control

$$
A_l = \{x \in \tilde{\beta} : x \wedge (1, 1)\} \\
A_r = \{x \in \tilde{\beta} : (1, 1) \wedge x\}
$$

$$
\psi_l(b) = \begin{cases} 
  x_m^*(b) & \text{if } b \geq b^* \\
  x_l^*(b) & \text{otherwise}
\end{cases}
$$

$$
\psi_r(b) = \begin{cases} 
  x_m^*(b) & \text{if } b \leq b^* \\
  x_r^*(b) & \text{otherwise}
\end{cases}
$$

if the following conditions are satisfied

$$
\frac{\max_{b \in [b^*, 1-\gamma]}[u_l(x^*_m(b)) - u_l(x_m^*(b))]}{\eta_l + \int [u_l(\psi_l(b')) - u_l(\psi_r(b'))] dF_b(b')} \leq \delta \tag{C1}
$$

$$
\frac{\max_{b \in [\gamma, b^*]}[u_r(x^*_m(b)) - u_r(x_m^*(b))]}{\eta_r + \int [u_r(\psi_r(b')) - u_r(\psi_l(b'))] dF_b(b')} \leq \delta. \tag{C2}
$$

**Proof:** Given symmetry is satisfied each party’s policy function induces a lottery over policy with the same expected utility for $m$ and thus condition 1 is satisfied, so the ballot function is sequentially rational. It remains only to verify that the policy mappings are mutual best responses.

-Consider party $l$: Assume that $\psi_r(b)$ and $A_l, A_r$ are given by the proposition. It is sufficient to show that no unilateral single-period deviation from $\psi_l(b)$ is desirable. If $b > b^*$ then selection of $x_m^*(b)$ involves re-election and selection of any other feasible policy involves either loss of office or less of $x_2$. We let $v_l^i(b)$ denote the continuation value to $l$ from being in office with constraint parameter $b$ and $v_r^i(b)$ be the continuation value to $l$ for having $r$ in office with constraint parameter $b$. We define

$$
Ev_l = [F_b(1 - \gamma) - F_b(b^*)] \int v_l^i(b') dF_b(b') + [F_b(b^*) - F_b(\gamma)] \int v_r^i(b') dF_b(b').
$$

The continuation value to $l$ from selecting $x_m^*(b)$ (with $b > b^*$) and staying in office is
\[ v_l'(b) = u_l(x^*_m(b)) + (1 + \delta)\eta_l + \delta \int \psi_l(b')dF_b(b') + \delta^2 E v_l. \]  

(12)

The continuation value to \( l \) from selecting \( x^*_l(b) \) and losing office is

\[ v_r'(b) = u_l(x^*_r(b)) + \eta_l + \delta \int \psi_r(b')dF_b(b') + \delta^2 E v_l. \]  

(13)

Subtracting and rearranging demonstrates the deviation is not desirable for any \( b > b^* \) if (C1) is satisfied. Now if \( b < b^* \) the strategy profile \( \psi_l(b) \) is clearly optimal as no policy that would attain reelection is in \( B(b) \) and thus selection of \( x^*_l(b) \) is a best response. Interchanging \( l \) and \( r \) and the appropriate ranges of \( b \) in the argument yields the result for party \( r \). ■

This SPBE involves successful control over governments that receive constraints which they find relatively desirable, and no control over governments that receive constraints that are not desirable. In the latter case the government shirks, giving itself as desirable a policy as possible and then leaves office. In the event of a desirable constraint the government forgoes the opportunity to shirk because it values the prospect of retaining office. The value to office consists of the exogenous term \( \eta_p \) and the endogenous term

\[ \int [u_p(\psi_p(b')) - u_p(\psi_{-p}(b'))] dF_b(b'). \]  

(14)

Note that because the equilibrium is stationary the punishment to shirking derives only from 1 period of play. It is important to note that \( \delta \) and \( \eta_p \) effect whether this SPBE exists but they do not effect observable behavior in such a SPBE. In otherwords if the SPBE exists under the triple \( (\delta, \eta_l, \eta_r) \) then a triple \( (\delta', \eta'_l, \eta'_r) \) which is bigger in each coordinate induces an observationally equivalent partial control SPBE. It should also be noted that this equilibrium involves purely retrospective voting in the sense that voters base voting decisions on what the incumbent has done for them lately. The voter’s action serves to control shirking by parties, since the voter will punish any shirking that is observable. This feature of voting is in contrast to the Banks and Duggan model. The key distinction is that the stochastic element of the current model is not correlated with the actions of parties accordingly there is no room for the voter to learn about the future from past play. Thus, voting here is best characterized by the heuristic “I will vote to keep you in office only if I trust that you did not shirk in your last term.” Finally, note that with the assumption of symmetry there is no selection problem. Figure 4 exhibits the shape of a generic acceptance set \( A_l \).

\[ \text{[Figure 4 about here]} \]

\(^{13}\)Recall, that our notion of retrospective voting differs from Fiorina’s, in that the voter does not use a cutpoint rule.
Given the argument preceding proposition 1 we see that it is impossible (in a pure strategy SPBE) to control \( l \) (\( r \)) for \( b < (>)b^* \). This means that this SPBE (if it exists) involves the maximal amount of control. Exactly half of the possible \((b, g)\) pairs can be controlled.\(^{14}\)

**Corollary 1** If the SPBE in proposition 2 exists no other pure strategy SPBE exhibits control on any more pairs \((b, g)\).

When symmetry is satisfied but C1 or C2 fail, it may be possible to attain SPBE with control for a smaller subset of the possible \((b, g)\) pairs. Alternatively, partial control may be possible in non-stationary strategies when C1 or C2 fail. We do not consider these extensions as no additional intuition is gained, and C1 and C2 are satisfied as long as \((\eta_l, \eta_r, \delta)\) are big enough. Symmetry on the other hand involves a knife edged condition and we want to understand what happens when the condition does not hold. If symmetry is violated and the parties use the policy functions defined in proposition 2, punishment of one of the parties is no longer a best response for the voter as condition 1 would require that the one party is always reelected and the other party is never reelected. The voter strictly prefers having one party in office and that party will not find the threat of punishment credible following a single period deviation. In this case the selection problem seems to make credible solution of the control problem impossible. However, we can modify this SPBE to accommodate failures of symmetry. The modification involves reducing the cases where the advantaged or favored party is controlled.

**Definition 3** We say that party \( l \) is advantaged if given the two policy mappings

\[
\psi_l(b) = \begin{cases} 
  x_m^*(b) & \text{if } b \geq b^* \\
  x_l^*(b) & \text{otherwise}
\end{cases}
\]

\[
\psi_r(b) = \begin{cases} 
  x_m^*(b) & \text{if } b \leq b^* \\
  x_r^*(b) & \text{otherwise}
\end{cases}
\]

we have \( \int u_m(\psi_l(b))dF(b) > \int u_m(\psi_r(b))dF(b) \).

Specifically, if \( l \) is the advantaged party then for some \( b^\# > b^* \) whenever \( b > b^\# \), \( l \) will select \( x_l^*(b) \) and \( m \) will not punish \( l \). This makes the value of having \( l \) in office decrease. Accordingly, in constructing a SPBE with partial control when symmetry fails we will use a \( b^\# \) which is chosen to equate the expected utility to \( m \) of having each party in office. The advantaged party will then shirk for some values of \( b \) on the desirable side of \( b^* \) (namely the extreme ones with \( b > b^\# \)). It is obvious that analysis with \( r \) advantaged would be completely analogous. We now formalize this extension.

\(^{14}\)Of course control of a particular party may happen more or less than half the time in the SPBE, as the distribution \( F_b(b) \) may assign any probability to the set \([\gamma, b^*] \).
We first define the critical point $b^\#$ by the equation

$$\int_{b}^{b^\#} u_m(x_m^\#(b))dF(b) + \int_{b^\#}^{b^\ast} u_m(x_m^*(b))dF(b) + \int_{b}^{1-\gamma} u_m(x_m^*(b))dF(b) = \int u_m(\psi_r(b))dF(b)$$

(15)

where the mapping $\psi_r(b)$ is identical to that in definition 2. In the appendix we establish the existence and uniqueness of the point $b^\#$ when $l$ is advantaged.

**Lemma 1** If $l$ is advantaged exactly one point $b^\# \in (b^\ast,1-\gamma)$ exists that solves (15).

The analogue to proposition 2 when $l$ is advantaged can now be stated and proven.

**Proposition 3** If $l$ is advantaged the following profile is supportable as a SPBE with partial control

$$A_l = \{ x \in \bar{\beta} : x \setminus (1,1) \text{ and } x^*_l(b^\#) \setminus x \} \cup \{ x \in \beta : x_2 \geq \frac{1}{b^\#} \}$$

$$A_r = \{ x \in \bar{\beta} : (1,1) \setminus x \}$$

$$\psi^l_r(b) = \begin{cases} x^*_m(b) & \text{if } b \in [b^\ast,b^\#) \\ x^*_l(b) & \text{otherwise} \end{cases}$$

$$\psi^r_r(b) = \begin{cases} x^*_m(b) & \text{if } b \leq b^\ast \\ x^*_l(b) & \text{otherwise} \end{cases}$$

if the following conditions are satisfied

$$\frac{\max_{b \in [b^\ast,b^\#]} [u_l(x^*_l(b)) - u_l(x^*_m(b))]}{\eta_l + \int [u_l(\psi^l_r(b')) - u_l(\psi^l_r(b))] dF(b')} \leq \delta$$

(C1’)

$$\frac{\max_{b \in [\gamma, b^\#]} [u_r(x^*_r(b)) - u_r(x^*_m(b))]}{\eta_r + \int [u_r(\psi^r_r(b')) - u_r(\psi^r_r(b))] dF(b')} \leq \delta.$$  

(C2’)

**Proof:** By construction $b^\#$ is chosen so that condition 1 is satisfied by $\psi^l_r(b)$ and $\psi^r_r(b)$. It remains only to show that the policy functions are mutual best responses.

-Consider party $l$: Assume that $\psi^l_r(b)$ and $A_l, A_r$ are given by the proposition. It is sufficient to show that no unilateral single-period deviation from $\psi^l_r(b)$ is desirable. If $b > b^\#$ then selection of $x^*_l(b)$ is clearly optimal as it results in reelection and is the optimal feasible policy for $l$. Thus no deviation from $\psi^l_r(b)$ is desirable in this case. If $b \in [b^\ast, b^\#)$, no policy in $\{ x \in \beta : x_2 \geq \frac{1}{b^\#} \}$ is feasible and thus $l$ faces exactly the choice she did under the ballot function in proposition 2. Thus the proof of the optimallity of $\psi^l_r(b)$ for $b \in [b^\ast, b^\#)$ is the same as that for the optimallity of $\psi_r(b)$ for $b \in [b^\ast, 1-\gamma]$ in the proof of proposition 2 and the associated condition C1’ attains. Similarly the optimallity of $\psi^l_r(b)$ follows from a similar argument.
It is important to note that when $l$ is advantaged it is not possible to support SPBE in which for values of $b$ that are moderately higher than $b^*$ $l$ chooses $x^*_l(b)$ and for values of $b$ that are substantially higher than $b^*$ $l$ chooses $x^*_m(b)$. A ballot function like this is not incentive compatible for values of $b > b^*$. If $l$ is supposed to select $x^*_m(b)$ for a moderately high $b$ but not a very high $b$, following a very high $b$ she could always select a policy $x$ which is feasible and coincides with $x^*_m(b')$ for a moderately high $b'$ but which $l$ prefers to $x^*_m(b)$. The voter would not be able to determine if shirking had occurred. That is, as $b$ goes from $b^*$ to $1 - \gamma$ incentive compatibility requires that we only transition from requirements of $x^*_m(b)$ to $x^*_l(b)$.

Given the fact that the ballot function in the SPBE of propositions 2 and 3 make $m$ indifferent between voting for or against the incumbent it might seem possible to support SPBE in which $v(x, l) = 1$ when $x = x^*_l(b)$ and $b < b^*$ and $v(x, r) = 1$ when $x = x^*_r(b)$ and $b > b^*$. While $m$ is indifferent, this change creates an incentive compatibility problem. If $v(x, l) = 1$ when $x = x^*_l(b)$ and $b < b^*$ then there are values of $b > b^*$ in which $l$ would prefer to deviate from $x^*_m(b)$ and select $x^*_l(b)$. The voter would not be able to determine when this behavior occurs. Thus, creation of incentives for partial control requires that the voter not reelect parties when they receive an undesirable constraint. Finally, note that even though party $l$ is advantaged, the selection problem is of secondary importance to the control problem. In equilibrium the parties adopt strategies which make the voter indifferent between having either party in office.

Since $b^#$ is unique, if the SPBE in proposition 3 exists it maximizes the amount of control (when pure strategies are required).

**Corollary 2** If the SPBE in proposition 3 exists no other pure strategy SPBE exhibits control on any more pairs $(b, g)$.

Corollaries 1 and 2 have an alternative interpretation. Instead of considering the set of pairs $(b, g)$ for which control occurs, we can consider the probability that the government enacts the voter's constrained optimum $x^*_m(b)$. The nature of the equilibrium in proposition 3 and the argument proceeding corollary 3 imply the following result.

**Corollary 3** In the equilibrium of proposition 3, (1) a government of the right party selects the voter’s constrained optimum with probability $F_b(b^*)$, (2) a government of the left party selects the voter’s constrained optimum with probability $F_b(b^#) - F_b(b^*)$, and (3) no pure strategy equilibrium involves control with higher party dependent probabilities.

In principle it may be possible for $m$ to prefer always having party $l$ in office uncontrolled. The single period expected utility to $m$ of this arrangement is

$$E u^l_m = \int u_m(0, \frac{1}{b}) dF_b(b). \tag{16}$$
Since \( m \) is indifferent between having either party in office in the equilibrium of proposition 3, the single period expected utility to \( m \) from the equilibrium in proposition 3 is

\[
\int_{b^*}^{b} u_m(0, \frac{1}{b}) dF(b) + \int_{b^*}^{b} u_m(x_m^*(b)) dF(b) + \int_{b^*}^{1-\gamma} u_m(0, \frac{1}{b}) dF(b).
\]

(17)

By inspection, we see that the latter is clearly higher than the former as \( u_m(x_m^*(b)) \geq u_m(0, \frac{1}{b}) \) for every \( b \).

Figure 5 depicts (for a generic example) the set of policies that can be enacted in the equilibrium of proposition 3 and those that can be enacted under the selection equilibrium of always reelecting party \( l \). Inspection of the two sets of feasible pictures demonstrates the algebraic argument. The conclusion is a strong contradiction of Fearon’s conjecture, “Introduce any variation in politician’s attributes or propensities relevant to their performance in office, and it makes sense for the electorate to focus completely on choosing the best type when it comes to vote” (p. 77). Instead, we have just established the following result.

**Proposition 4** The public, \( m \), would rather solve the control problem by using the SPBE in proposition 3, then the selection problem of always deferring to the advantaged party, \( l \).

### 3 Uncertainty about the price and level of the constraint

We now consider the case where there is more uncertainty about the form of the feasibility constraint. Let \( B(b,c) = \{ x \in X : bx_1 + (1-b)x_2 \leq c \} \). We assume that in each period \( b’ \) and \( c’ \) are generated by independent draws from the continuous and strictly monotone joint distribution function \( F_{bc}(b,c) \). The support is assumed to be \([\gamma, 1-\gamma] \times [\varsigma, 1+\varsigma] \) with \( 0 < \varsigma < 1 \). We redefine \( B \) in the natural manner. We define \( x_m^*(b,c) \) and \( x_{p}^*(b,c) \) and \( \psi_p(b,c) \) in the natural manner. Extension of the concepts partial and full control is natural.

**Definition 4** We say a SPBE exhibits perfect control if for every \( p \in P \) \( \psi_p(b,c) = x_m^*(b,c) \) for almost every \( b,c \). We say a SPBE exhibits partial control if for every \( p \in P \), \( \psi_p(b,c) = x_m^*(b,c) \) for \( b,c \in D \) with \( D \) a subset of \([\gamma, 1-\gamma] \times [\varsigma, 1+\varsigma] \) having positive Lebesgue measure.

Uncertainty about the resource level \( c \) has dramatic implications for the possibility of monitoring and control. If \( m \) expects \( l \) to select \( x_m^*(b,c) \) for any pair \((b,c)\) with \( c < 1 + \varsigma \) then there is always a deviation to a feasible policy \( x’ = x_m^*(b’,c’) \in B(b,c) \) with \( c’ < c \) and \( b’ > b \) that is desirable for \( l \). Similarly, \( r \) has an incentive to deviate from \( x_m^*(b,c) \) to a policy \( x'' = x_m^*(b'',c'') \in B(b,c) \) with \( c'' < c \) and \( b'' < b \).
Accordingly, now it is not possible to attain control on any subset of the space $B$ in pure strategies. Figure 6 illustrates this point by plotting two constraints $B(b, c)$ and $B(b', c')$ and two possible points $x^*_m(b, c)$ and $x^*_m(b', c')$. Upon observing $x^*_m(b', c')$ the voter cannot determine if $b, c$ have attained and $l$ has shirked or if $b', c'$ have attained and $l$ has not shirked. With the addition of uncertainty about $c$, equilibria of the type in proposition 3 cannot be constructed. Here the monitoring problem is almost always severe. This leads us to the following conclusion.

**Proposition 5** With uncertainty about $b$ and $c$ there are no pure strategy SPBE in which partial control occurs.

However, if the slope of $h_p(\cdot)$ is not too steep there are mixed strategy SPBE in which perfect control occurs. The construction hinges on creating mixed ballot functions that induce each party to choose $x^*_m(b, c)$. We let $\lambda(x, p)$ denote the probability that $p$ is retained if she selects policy $x$. Using the single deviation principal we can derive the incentive compatibility condition that a mixed ballot function must satisfy. Suppose both parties will select $x^*_m(b, c)$ whenever they are in office (except possibly for party $l$ this period). Given this, a ballot function $\lambda(x, l)$ and a constraint $B(b, c)$, party $l$ must solve the problem

$$\arg \max_{x \in B(b, c)} h_l(x_2) + \lambda(x, l)\delta \eta_l + k$$

where $k$ is a constant with respect to the choice variable $x$. By definition we have

$$x^*_m(b, c) = \arg \max_{x \in B(b, c)} u_m(x).$$

Since (18) and (19) involve the same constraint, if $\lambda(x, l)$ is chosen so that the first order conditions from problem (18) are equated with the first order conditions from (19), party $l$ will have an incentive to choose $x^*_m(b, c)$. This requires

$$\frac{\partial h_l(x_2)}{\partial x_2} + \delta \eta_l \frac{\partial \lambda(x, l)}{\partial x_2} = \frac{\partial u_m(x)}{\partial x_2}$$

$$\delta \eta_l \frac{\partial \lambda(x, l)}{\partial x_1} = \frac{\partial u_m(x)}{\partial x_1}$$

Rearranging yields

$$\frac{\partial \lambda(x, l)}{\partial x_2} = \frac{1}{\delta \eta_l} \left( \frac{\partial u_m(x)}{\partial x_2} - \frac{\partial h_l(x_2)}{\partial x_2} \right)$$

$$\frac{\partial \lambda(x, l)}{\partial x_1} = \frac{1}{\delta \eta_l} \left( \frac{\partial u_m(x)}{\partial x_1} \right).$$
A function that satisfies this condition is

$$\lambda(x_1, x_2, l) = \frac{1}{\delta \eta_l} (u_m(x) - h_l(x_2)) + q$$

where $q$ is a scaler. It remains only to verify that it is possible to construct a mapping $\lambda(x, l)$ with image $[0, 1]$ that satisfies these conditions. This requires that

$$\max_{x \in \beta} \lambda((x_1, x_2), l) - \min_{x \in \beta} \lambda((x_1, x_2), l) < 1. \quad (23)$$

This difference is bounded by

$$\frac{1}{\delta \eta_l} \left( h_l(\frac{1}{\gamma} - h_l(0) \right). \quad (24)$$

Thus, if

$$h_l(\frac{1}{\gamma} - h_l(0) \leq \delta \eta_l \quad (C7)$$

an incentive compatible mixed ballot function can be constructed. Similar logic yields the conditions

$$\frac{\partial \lambda(x, r)}{\partial x_1} = \frac{1}{\delta \eta_r} \left( \frac{\partial u_m(x)}{\partial x_1} - \frac{\partial h_l(x_1)}{\partial x_1} \right) \quad (25)$$

$$\frac{\partial \lambda(x, r)}{\partial x_2} = \frac{1}{\delta \eta_r} \left( \frac{\partial u_m(x)}{\partial x_2} \right)$$

$$h_r(\frac{1}{\gamma} - h_r(0) \leq \delta \eta_r. \quad (C8)$$

We are then left with the following conclusion.\textsuperscript{15}

**Proposition 6** With uncertainty about $b$ and $c$, if conditions C7 and C8 are satisfied there exist scalers $q_l$ and $q_r$ such that full control is supportable in a mixed strategy SPBE with the following ballot functions:

$$\lambda(x, l) = \frac{1}{\delta \eta_l} (u_m(x) - h_l(x_2)) + q_l$$

$$\lambda(x, r) = \frac{1}{\delta \eta_r} (u_m(x) - h_r(x_1)) + q_r.$$

The mixed strategy equilibria characterized above are first-best for the public. Since the stationarity assumption only limits the size of the carrot and stick, When C7 and C8 are satisfied (and thus the carrot and stick are big enough), the restriction to stationary strategies does not limit the public's ability to control the government in mixed strategies, as there are no equilibria that do better than these stationary mixed

\textsuperscript{15}The mixed strategy equilibria can also be constructed in the simpler model where there is no uncertainty about $c$. 


strategy equilibria. When conditions C7 and C8 are not satisfied allowing punishment to last for multiple periods can make full control in mixed strategies possible.

More generally, relaxing the restriction of stationary strategies has no effect on the extent of control when \( \delta \eta \) are high enough for the equilibria in propositions 2, 3 and 6 to exist. However, when the value of retaining office is too small multi-period punishments and rewards may enlarge the set of parameterizations for which partial (and in the case of mixed strategies full) control is possible.

4 Extensions

In this section we discuss how the results are affected by a few extensions. With respect to the number of voters the model is more general than it seems. If we assume that there is an odd number of voters and that the voters are ordered by their marginal rate of substitution (MRS) then a representative voter will exist. Accordingly, the generalization to \( n \) (odd) voters is immediate if \( MRS_i(x) \leq MRS_{i+1}(x) \) for every \( x \) for \( i = \{1, 2, ..., n-1\} \). In the space of preference profiles that are monotone and twice differentiable this condition holds on subsets with non-empty interior. Accordingly, the existence of a representative voter is not a knife-edged assumption. Meirowitz (2001) shows that if voters have strictly convex and monotone preferences on \( \mathbb{R}^2 \) then choice restricted to any bounded convex set will have a non-empty majority rule core. This result is not sufficient for generalizing the model to agents that are not ordered by their MRS. An analogue to Assumption 2, requiring that the majority rule core respond nicely to changes in the slope would also be needed.

Slightly relaxing the assumption that parties care only about one issue has no effect on the results. This assumption dramatically simplifies the notation without altering the general incentives of the problem. As long as \( MRS_l(x) \) is sufficiently low and \( MRS_r(x) \) is sufficiently high for every \( x \) the analysis presented here holds. In this sense the results of the current model are not knife-edged with respect to this assumption. More extreme departures, that still satisfy the condition \( MRS_l(x) < MRS_m(x) < MRS_h(x) \) are not problematic, as the analysis can be applied to the portion of the feasibility constraint on which party and voter preferences are not aligned. If this ordering is not satisfied then the incentives may be quite different, and the results will not generally hold.

Relaxing the assumption that the feasibility constraint has a linear boundary is quite simple. A natural extension is to assume that every constraint is compact and convex and that the intersection of all possible constraints is a singleton \( x^* \). This point then corresponds to the point \((1,1)\) in the model in which \( c = 1 \). If the family of constraint boundaries represent rotations about this common intersection point results similar to propositions 1-4 can be attained in a model of this form.\(^{16}\) More precisely stated, if \( \{B_\rho\}_\rho \) is the collection of feasible constraint boundaries then propositions 1-4 would extend if \( \{\cap_\rho B_\rho\} := x^* \) is a singleton and for

\(^{16}\)See Meirowitz (2001) for analysis of social choice on convex constraint sets.
every $\rho \neq \rho'$, $B_\rho \cap B_{\rho'} = x^*$. The logic behind these results breaks down if there are pairs of constraints that have some intersections that are to the northwest and others that are to the southeast of the point $x^*$. More generally, as long as the public constrained optima and the parties constrained optima are well defined the construction in proposition 6 can be used to establish the existence of mixed strategy equilibria with full control when agents value office enough.

A simplifying assumption of the model is that the constraint parameters $b^t$ or $(b^t, c^t)$ are independently and identically distributed over time. If only the identical part of the assumption is relaxed the pure strategy analysis extends (without the stationarity restriction) as long as time dependent versions of condition 1 and C1’ and C2’ are satisfied. In mixed strategies the equilibrium characterized in proposition 4 will exist unchanged since the voter is indifferent between having either party in office when both parties select $x^*_m(b)$. If it is assumed that $b^t$ or $(b^t, c^t)$ are not independent over time then again the mixed strategy equilibrium with full control still exists unchanged for precisely the same reason, the voter is indifferent between either party enacting $x^*_m(b)$ regardless of the distribution on $b^t$ and $c^t$. The implications for the pure strategy analysis are more serious and further study is needed. If $b^t$ and $b^{t-1}$ are correlated a more severe selection problem surfaces. The voter will want to select $l$ ($r$) if $b^{t+1}$ is likely to be high (low). Formally, non independence requires that a history dependent symmetry condition hold in order for $m$ to be willing to remove a particular party. Additionally, the possibility that $x^t$ can effect $m$’s beliefs about $b^{t+1}$introduces potential incentives for $p$ to manipulate $m$’s beliefs. We could also allow $b^t, c^t$ to be correlated with $x^{t-1}$. When the policy area involves control of economic factors this extension might be quite defensible. These extensions are beyond the scope of the current paper.

5 Discussion

In comparison to other models of representation the current predictions are novel. We predict policy differences over time and find that while governments may sometimes be retained they will eventually be thrown out. Both of these findings are in contrast to the repeated election models and obviously can’t be compared with the static election models in the Downsian tradition. In the pure strategy equilibria with partial control we find a non-monotonic relationship between reelection and the quantity of a party’s preferred issue even though the voter’s preferences are static. Most striking, is the prediction that governments that enact policies that have a very high quality of the issue they care about will be retained. This last fact is a pattern that the analyst would observe from election data, but it is not a statement about the causality. When policies that contain a high degree of the party’s preferred issue are feasible, the monitoring problem is solvable and thus these polices are also those that the public would enact under the constraint.

In the pure strategy partial control equilibria voters treat incumbents from different parties differently. This prediction does not surface in either the quality based moral hazard and adverse selection models
(Barro, Ferejohn, Austen-Smith and Banks, Banks and Sundaram, Ashworth) or the spatial representation models (Duggan, Banks and Duggan). While many scholars have focused on the role of party in elections, there is a paucity of empirical specifications that test the interaction of incumbent party and how voters assess the incumbent’s performance on different policy issues. Estimating acceptance sets for Democrat and Republican incumbents (say at the Gubernatorial and Presidential level) might yield a direct empirical test of one of the model’s predictions.

The model offers a perspective on the types of rhetoric that candidates might use when justifying their in office actions. Fenno defends the importance of this feature of legislator-constituent relationships, and it is reasonable to imagine that executives also must justify their record to the voters. While the model does not involve explicit communication between voters and parties, the equilibrium intuition is easy to interpret in this manner. Following a constraint and policy choice which is supposed to result in reelection the incumbent can make speeches espousing her policy selection as in the public’s interest given the constraint. There is no compelling retort that the challenger can give. In contrast, following either an out of equilibrium policy or any policy that is feasible given a constraint that does not favor the incumbent, the challenger will be able to persuade voters that the incumbent (might have) shirked. The analysis formalizes how the public, not knowing the constraint can adjudicate this type of debate.

In terms of the principal agent perspective the theory is primarily one of control and not selection. While the model offers the potential for a non-trivial selection problem as one party might be ex-ante preferred, we find that in the desirable equilibrium the public is indifferent between the pool of available agents/parties and thus the selection problem is resolved. The control problem, is persistent and we find only partially solvable in pure strategies. In contrast, with mixed strategies the problem is solvable. This demonstrates that a finer control device (choice of mixtures as opposed to a binary decision) allows the principal to exert increased control.

In Fearon’s comparison of selection and control explanations he notes that ”if politicians vary in policy preferences, even a little, then voters are no longer generically indifferent between the incumbent and possible replacements.” (p.75). This conclusion does not hold in the current model. Indifference over the ideal policies of the parties is not necessary for indifference between an incumbent or challenger in equilibrium. The relevant condition is more endogenous. In equilibrium the voter must be indifferent between the lotteries induced by having either party use its equilibrium strategy in office. The intuition hinges on the analysis when symmetry is not satisfied. In this case, one party (l) is more desirable than the other in a clear policy sense. Even in this case the voter is indifferent between having either party in office. Moreover, Fearon’s conclusion that the gain to voters from solving the selection problem is higher than the gain from solving the control problem is not valid in the current model. With only uncertainty about \( b^* \), proposition 4 indicates that the public would rather partially solve the control problem than fully solve the selection problem. In
the model with uncertainty about $b^t$ and $c^t$ the mixed strategy SPBE are first best for the public as they induce the policy $x^*_m(b^t, c^t)$ in every period. Our conclusion that control is a relevant feature of the voter-government relationship is consistent with the empirical findings of Besley and Case (and others). From an institutional choice perspective the possibility of control and its reliance on the future horizon and rents to holding office has important implications for debates about term limits and office compensation.
6 Appendix.

Proof of Lemma 1: Let the left hand side of (15) be denoted by the function \( \phi(b^\#) : (b^*, 1 - \gamma) \rightarrow \mathbb{R}^1 \). Note that by continuity

\[
\lim_{b^\# \rightarrow b^*} \phi(b^\#) = \int_{\gamma}^{b^*} u_m(x^*_l(b))dF(b) + \int_{b^*}^{1-\gamma} u_m(x^*_m(b))dF(b). \tag{26}
\]

Since \( b < b^* \) implies \( x^*_l(b) \rightarrow (1, 1) \backslash x^*_m(b) \) the right hand side of the above is less than \( \int u_m(\psi_r(b))dF_b(b) \), thus

\[
\lim_{b^\# \rightarrow b^*} \phi(b^\#) < \int u_m(\psi_r(b))dF_b(b). \tag{27}
\]

On the other hand

\[
\lim_{b^\# \rightarrow 1-\gamma} \phi(b^\#) = \int_{\gamma}^{b^*} u_m(x^*_l(b))dF(b) + \int_{b^*}^{1-\gamma} u_m(x^*_m(b))dF(b). \tag{28}
\]

The right hand side of this is greater than \( \int u_m(\psi_r(b))dF_b(b) \) because \( l \) is advantaged. Since \( \phi(\cdot) \) is continuous there is a value \( b^\# \) solving (15) by the intermediate value theorem. Since \( \phi(\cdot) \) is strictly monotone this value is unique.■
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Figure 1 Indifference curves
Figure 2  The set
Figure 3 Monitoring problem for uncertain $b$
Figure 4 The set $A_1$
Figure 5 Control vs. selection

Control eqm. policies

Selection eqm. policies
Figure 6 Monitoring problem for uncertain \( b,c \)